# AXIORECOMBINATION: A NEW MECHANISM FOR STELLAR AXION PRODUCTION* 

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#### Abstract

We consider axiorecombination, $e^{-}+Z \rightarrow\left(e^{-}, Z\right)+a$ as a source of light pseudoscalar axions in stars. This process dominates the energy loss in axions for low mass main sequence stars, $M \lesssim 0.2 M_{\odot}$. For the sun, axiorecombination accounts for $\sim 1 / 4$ of total axion flux at energies above $\sim 2 \mathrm{keV}$.


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## INTRODUCTION

The axion, a byproduct of a natural solution to the strong CP problem, ${ }^{1}$ has proven singularly elusive, raising fears that it may be one of the endangered species of hypothetical particles. Its habitat, the range of possible scales $F$ of Peccei-Quinn symmetry breaking, has been restricted on astrophysical and cosmological grounds to $10^{7} \mathrm{GeV} \lesssim F \lesssim 10^{12} \mathrm{GeV}$. The upper bound on $F$, from limits on the cosmological mass density $\left(\Omega_{\text {axion }} \leqslant 2\right),{ }^{2}$ may be strengthened to $F \lesssim 10^{10} \mathrm{GeV}$ due to axion emission from cosmic strings. ${ }^{3}$ The lower limit on $F$ is obtained by requiring that the axion luminosity of the sun not exceed the photon luminosity. ${ }^{4,5}$ This bound is improved by a factor $\approx \pi$ by including the effects of axions on the sun self-consistently; ${ }^{6}$ even stronger lower bounds, $F \gtrsim 10^{9} \mathrm{GeV}$, may result from consideration of $H e$ ignition in red giants and from the cooling of white dwarfs and neutron stars. ${ }^{4,7,8}$

The lower bounds on $F$ arise by considering stellar axion emission through bremsstrahlung, Primakoff, and Compton processes; ${ }^{4,5}$ in these interactions, the initial and final state electrons are unbound (free-free transitions). In this letter, we point out that free-bound transitions, in which a free electron is captured by a heavy ion into an atomic K -shell and emits an axion, can also be an important source of stellar energy loss if $F$ is near the lower bound. ${ }^{9}$ This 'axiorecombination' process, depicted in Fig. 1, is the inverse of the 'axioelectric effect' ${ }^{10}$ and displays the same cross-section enhancement (over that of hydrogen) for large-Z atoms at the $\mathcal{O}(\mathrm{keV})$ energies typical of stellar interiors.

The free-bound axion luminosity per unit mass scales more weakly with temperature, $\mathcal{E}_{f b} \sim T^{3 / 2}$ than for any other process (e.g., for bremsstrahlung, $\mathcal{E}_{b r} \sim T^{5 / 2}$ and for Compton scattering $\mathcal{E}_{c} \sim T^{6}$ ). Thus, the free-bound process
will dominate the energy loss at low central temperatures, i.e., for low mass stars. In addition, since the nuclear energy generation rate drops sharply, $\mathcal{E}_{n u c} \sim T^{6}$ (for the $p-p$ chain), at low temperature, ${ }^{9}$ the effects of axions on the structure of low-mass stars may be dramatic. ${ }^{6}$ For the sun, we will see that the free-bound process contributes substantially to the axion flux above about 2 keV , with a signature which could be seen in underground detectors. ${ }^{11}$

## FREE-BOUND AXION PR.ODUCTION

It is straightforward to work out the axiorecombination cross-section and consequent stellar axion and energy fluxes. Recall that the photoelectric crosssection for hydrogen-like atoms is ${ }^{12}$

$$
\begin{equation*}
\sigma_{p e}(\omega)=\frac{7.9 \times 10^{-18}}{Z_{*}^{2}} g_{1 f}\left(\frac{\omega_{1}}{\omega}\right)^{3} \mathrm{~cm}^{2} \quad, \omega>\omega_{1} \tag{1}
\end{equation*}
$$

where the K-shell ionization energy is $\omega_{1}=Z_{*}^{2}(13.6 \mathrm{eV}), Z_{*}$ is the effective nuclear charge seen by K -shell electrons due to plasma and atomic screening, and the gaunt factor

$$
\begin{equation*}
g_{1 f}=8 \pi \sqrt{3} \frac{\omega_{1}}{\omega} \frac{e^{-4 n^{\prime} \cot ^{-1} n^{\prime}}}{1-e^{2 \pi n^{\prime}}}, n^{\prime}=\left(\frac{\omega_{1}}{\omega-\omega_{1}}\right)^{1 / 2} \tag{2}
\end{equation*}
$$

is within $20 \%$ of unity for the frequencies of interest (near threshold). For stellar interiors, except for the heaviest elements, states more weakly bound than the K -shell are screened into the continuum by plasma effects. To good approximation, the ionization of K-shell electrons for complex atoms can be treated using hydrogenic wavefunctions by modifying the nuclear charge ( $Z_{\text {eff }} \simeq Z-0.3$ ) to include the effects of atomic screening. ${ }^{13}$

From detailed balance, the photorecombination (free-bound) cross-section (for the empty K-shell) is

$$
\begin{equation*}
\sigma_{p r}=\frac{2 \omega^{2}}{m_{e}^{2} v_{e}^{2}} \sigma_{p e} \tag{3}
\end{equation*}
$$

where $\frac{1}{2} m_{e} v_{e}^{2}=\omega-\omega_{1}$.
From the axion coupling to electrons, $\mathcal{L}=\frac{2 X_{c}^{\prime} m_{e}}{F} a \bar{e} i \gamma_{5} e$, in the nonrelativistic limit one finds the 'axioelectric' cross-section ${ }^{10}$

$$
\begin{equation*}
\sigma_{a e}=\left(\frac{2 X_{e}^{\prime}}{F}\right)^{2} \frac{\omega^{2}}{16 \pi \alpha_{e m}} \sigma_{p e} \tag{4}
\end{equation*}
$$

where now $\omega$ is the axion energy and $X_{e}^{\prime}$ is a constant of order unity. ${ }^{14}$ From Eqs. (1-4), the axiorecombination (free-bound) cross-section is

$$
\begin{align*}
\sigma_{f b} & =\frac{1}{2}\left(\frac{\sigma_{a e}}{\sigma_{p e}}\right) \sigma_{p r} \\
& =5.3 \times 10^{-52} \frac{Z_{*}^{4}}{F_{7}^{2}}\left(\frac{\omega}{\omega-\omega_{1}}\right) g_{1 f} \mathrm{~cm}^{2} \tag{5}
\end{align*}
$$

where $F / 2 X_{e}^{\prime}=10^{7} F_{7} \mathrm{GeV}$, and the factor $1 / 2$ arises because the axion has only 1 helicity state, compared to 2 for the photon.

The axion production rate $\left(\mathrm{gm}^{-1} \mathrm{sec}^{-1}\right)$ in a star is given by

$$
\begin{equation*}
G=\sum_{Z} \frac{n_{z} n_{e}}{\rho}\left\langle\sigma_{f b}^{(Z)} v_{e}\right\rangle \tag{6}
\end{equation*}
$$

where $n_{z}$ is the number density of $Z$-atoms with singly or doubly ionized K -shell, $n_{e}$ is the number density of free electrons, and the bracket indicates averaging over a Maxwell-Boltzmann distribution of free electrons at the stellar temperature. It
is convenient to rewrite Eq. (6) in the form

$$
\begin{equation*}
G=\sum_{Z} \frac{2 \pi \rho\left(1+X_{H}\right)}{A_{Z} m_{u}^{2}} f_{Z} X_{Z}\left(\frac{m_{e} c^{2}}{2 \pi k T}\right)^{3 / 2} \int_{\omega_{1}^{(Z)}}^{\infty} \frac{d \omega 2\left(\omega-\omega_{1}\right)}{m_{e}^{2} c^{3}} \sigma_{f b}^{(Z)} e^{-\left(\omega-\omega_{1}\right) / k T} \tag{7}
\end{equation*}
$$

where we have used the relation $n_{Z} / \rho=X_{Z} f_{Z} / m_{Z}$, with $X_{Z}$ the mass fraction of element $Z$ and $f_{Z}$ the fraction of $Z$-atoms with ionized K -shell, and $A_{Z}$ is the atomic mass in units of $m_{u}=1.66 \times 10^{-24} \mathrm{gm}$; we have also used the approximate relation $n_{e}=\rho\left(1+X_{H}\right) / m_{u}$, where $X_{H}$ is the hydrogen mass fraction. ${ }^{9}$

To find the differential solar axion flux at the earth, we substitute (5) into (7) and integrate $d G / d \omega$ over the sun. To obtain an approximate estimate, we take a uniform sun with temperature $k T=1 \mathrm{keV}$, density $\rho_{c}=160 \mathrm{gm} \mathrm{cm}^{-3}$ and hydrogen mass fraction $X_{H}=0.36$, which gives

$$
\begin{equation*}
\Phi_{f b}=\sum_{Z} \frac{2.8 \times 10^{18}}{A_{Z} F_{7}^{2}} Z_{*}^{4} f_{Z} X_{Z} g_{1 f} \bar{\omega} e^{-\left(\bar{\omega}-\bar{\omega}_{1}\right)} \mathrm{keV}^{-1} \mathrm{~cm}^{-2} \mathrm{day}^{-1} \tag{8}
\end{equation*}
$$

at the surface of the earth; here, $\bar{\omega}$ is the axion energy in keV . The 'exact' freebound solar axion flux, integrated over a standard solar model, ${ }^{9}$ is shown in Fig. 2, where it is compared to the axion bremsstrahlung flux. ${ }^{5,10}$ The axiorecombination peak occurs at the $S i$ threshold at 1.87 keV .

The relevant heavy element parameters for the sun are shown in Table 1. The quantities $X_{Z}, f_{Z}$ and $Z_{*}$ are discussed in the Appendix.

To consider the effects of axions on stars, we need to know the energy loss rate in axions. To compute the free-bound axion luminosity for a star, we simply insert
a factor $\omega$ in the integrand of Eq. (7). The result is (taking $g_{1 f} \simeq$ const. $=0.9$ )

$$
\begin{align*}
\varepsilon_{f b}= & \sum_{Z} 4.7 \times 10^{-2} \rho_{100} T_{7}^{3 / 2} \frac{\left(1+X_{H}\right)}{A_{Z} F_{7}^{2}}  \tag{9}\\
& \times\left(1+1.16 \frac{\bar{\omega}_{1}}{T_{7}}+0.68 \frac{\bar{\omega}_{1}^{2}}{T_{7}^{2}}\right) X_{Z} f_{Z} Z_{*}^{4} \frac{\mathrm{erg}}{\mathrm{gm} \mathrm{sec}}
\end{align*}
$$

where the density $\rho=\rho_{100} 100 \mathrm{gm} \mathrm{cm}^{-3}$ and temperature $T=T_{7} 10^{7} \mathrm{~K}$. For the solar center, this yields $\varepsilon_{\odot}^{f b}=\frac{0.6}{F_{7}^{2}} \mathrm{erg} / \mathrm{gm} \mathrm{sec}$ and $L_{\odot}^{f b} \simeq 3 \times 10^{32} \frac{\mathrm{erg}}{3 e c} \simeq .08 L_{\odot}$, with the main contributions from the elements $O, M g$ and $S i$. For comparison, the differential bremsstrahlung luminosity at the solar core is $\mathcal{E}_{\odot}^{b r}=\frac{14.5}{F_{7}^{2}} \mathrm{erg}$ $\mathrm{gm}^{-1} \mathrm{sec}^{-1}$, , so the axiorecombination luminosity contributes about $4 \%$. A selfconsistent bound on the axion luminosity ${ }^{6}$ is $\left\langle\mathcal{E}_{a}\right\rangle \lesssim 1.2 \mathrm{erg} \mathrm{gm}^{-1} \mathrm{sec}^{-1}$, where the average is over the energy-producing core of the sun. Integrating the combined bremsstrahlung and free-bound axion luminosity over the solar core yields the limit

$$
\begin{equation*}
F / 2 X_{e}^{\prime}>3.2 \times 10^{7} \mathrm{GeV} \tag{10}
\end{equation*}
$$

Note that this is a factor $\pi$ better than the previous solar bound. ${ }^{4,5}$
Since the free-bound luminosity scales as $\sim T^{3 / 2}$, compared to $T^{5 / 2}$ from bremsstrahlung, we expect the axiorecombination process to dominate in cooler stars. In addition, at lower temperatures the bremsstrahlung rate is further relatively reduced by screening. ${ }^{5}$ The two processes are compared in Fig. 3; the axiorecombination luminosity dominates over the bremsstrahlung rate for main sequence stars less massive than $\approx 0.2 M_{\odot}$, or $T_{c} \lesssim 6 \times 10^{6} \mathrm{~K} .{ }^{21}$ This comparison assumes the axion luminosity is small in comparison with the stellar luminoisty. If $F$ is near the lower bound $\simeq 3 \times 10^{7} \mathrm{GeV}$, however, the central temperature
of low mass stars may be substantially increased by axions, and the stellar mass below which the free-bound process dominates will be slightly decreased. ${ }^{6}$ The effects of axions on low mass stars will be discussed in a separate publication. ${ }^{6}$

## CONCLUSIONS

We have identified a new process, axiorecombination, for axion emission in stars. This process is important at low temperature and thus dominates the energy loss in axions for low mass stars. In the sun, axiorecombination accounts for $\approx$ a quarter of the axion flux at high energy. If solar axions are observed, the atomic threshold structure of the free-bound flux should be seen. If the PecceiQuinn scale $F$ is close to its lower bound, axiorecombination axion emission will change the structure and evolution of low mass stars.

## APPENDIX

The heavy element mass-fractions $X_{Z}$, taken from Ref. 15, are the surface abundances; however, since the elements heavier than helium have not undergone significant nuclear processing in the sun, it is assumed that their surface composition has not changed since the sun began its main sequence phase. Further, since the sun is thought to be chemically homogenous at birth, the surface abundances should reflect conditions throughout the star. ${ }^{16}$

To find the screened K -shell binding energy $\omega_{1}$ and ionization fraction $f_{Z}$, we follow the treatment of ionization equilibrium in a screened plasma given by Ulrich. ${ }^{17}$ In a plasma, atomic binding energies are reduced due to screening of the nuclear Coulomb field by free electrons. Screening is important on scales larger than the Debye-Huckel radius $D$ given by

$$
\begin{equation*}
\frac{1}{D^{2}}=\frac{4 \pi \hbar c \alpha_{e m}}{k T}\left(n_{e}+\sum_{Z} Z^{2} n_{Z}\right) \simeq \frac{4 \pi \hbar c \alpha_{e m}}{k T} \frac{\rho}{m_{u}}\left(\frac{3+X_{H}}{2}\right) \tag{11}
\end{equation*}
$$

The reduction in bound state energy is proportional to $a / Z D$, where $a$ is the Bohr radius; more specifically, the $1 s$ state in hydrogen-like atoms disappears when $D_{\text {crit }} / a \equiv 0.84 / Z .{ }^{18}$ If we define the screening parameter $y \equiv 1-D_{c r i t} / D$, then at the solar center, where $D / a=0.41$, we have $y_{\odot}=1-2.05 / Z$. The screened K-shell ionization energy may then be fit by ${ }^{17}$

$$
\begin{equation*}
\omega_{1}=\left(\frac{y^{3}+3 y^{2}}{4}\right) Z^{2}(13.6 \mathrm{eV}) \tag{12}
\end{equation*}
$$

where the factor in parenthesis defines $Z_{*}^{2} / Z^{2}$; Eq. (12) is used to compute the relevant columns of Table $1 .{ }^{19}$

Ionization equilibrium for the heavy elements is determined by the PlanckLarkin modification of the Saha equation ${ }^{17}$

$$
\begin{equation*}
\frac{N_{r+1}}{N_{r}}=f \frac{U_{r+1}}{U_{r}}\left[e^{\omega_{r} / k T}-1-\frac{\omega_{r}}{k T}\right]^{-1} . \tag{13}
\end{equation*}
$$

Here, $N_{r}$ is the relative population of the rth - times ionized atom, $U_{r}$ is the corresponding partition function, $\omega_{r}$ is the screened ionization energy required to go from state $r$ to $r+1$, and $f=0.677 T^{5 / 2} / p_{e}$ (here, $T$ and $p_{e}$ are in cgs units; $p_{e}$ is the electron pressure). For the high temperatures characteristic of stellar interiors, L-shells and higher are generally screened into the continuum, so we consider the statistical equilibrium of the "neutral" and singly ionized K-shell states $N_{1}, N_{2}$ and the completely ionized state $N_{3}$. In this case, the partition functions are just the statistical weights, $U_{1}=U_{3}=1, U_{2}=2$. Using the value $f=5.3$ at the solar center and the screening factor of Eq. (12) to compute the ionization energies $\omega_{r}$ yields the last column of Table 1 for the ionization fraction $f_{Z} \cdot{ }^{20}$ As far as the axion flux is concerned, the effect of screening is to reduce the effective charge $Z_{*}$ and thus the flux (see Eq. (8)); however, some of this loss is made up in the gain in ionization fraction $f_{Z}$ (screened atoms are easier to ionize). We also note that $f_{Z}$ begins to drop for $\omega_{1} \gtrsim k T$, as expected.

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20. To include axiorecombination to both the empty and singly occupied shells, we define $f_{Z}=\left(N_{3}+1 / 4 N_{2}\right) / \sum_{i} N_{i}$ where the coefficient $1 / 4$ in the second factor accounts for the statistical weight suppression of the recombination cross-section to this state.
21. To make this comparison, we have used the zero-age main sequence models of A. Grossman, D. Hays and H. Graboske, Astron. \& Astrophys. 30, 95 (1974), and D. Vandenberg, et al. , Ap. J. 266, 747 (1983) and have assumed the Population I abundances of Table 1. In cooler stars, $f_{Z}$ is somewhat reduced for the heaviest elements, but is essentially unaffected for $C, N$ and $O$. For the bremsstrahlung rate we have included only the lowest order correction due to plasma screening; at low temperature, screening effects become large and inclusion of higher order terms may slightly modify the result shown in Fig. 3. We have also neglected the effect of the onset of partial electron degeneracy at low temperature.

TABLE 1: Solar Heavy Element Parameters. $X_{Z}$ is the inferred mass fraction (with exponent in parenthesis). $Z_{*}$ is the effective nuclear charge due to screening, and $\omega_{1}=Z_{*}^{2}(13.6 \mathrm{eV})$ is the screened $K$-shell binding energy. $f_{Z}$ is the effective ionization fraction. ${ }^{20}$ The quantities $Z_{*}, \omega_{1}$ and $f_{Z}$ are calculated for solar temperature of 1 keV .

| Element | $A_{Z}$ | $Z$ | $Z_{*}$ | $\bar{\omega}_{1}(\mathrm{keV})$ | $X_{Z}$ | $f_{Z}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $C$ | 12 | 6 | 3.7 | 0.19 | $4.0(-3)$ | 1.0 |
| $N$ | 14 | 7 | 4.8 | 0.31 | $9.7(-4)$ | 1.0 |
| $O$ | 16 | 8 | 5.8 | 0.45 | $8.8(-3)$ | 1.0 |
| $N e$ | 20 | 10 | 7.8 | 0.82 | $5.9(-4)$ | 0.92 |
| $M g$ | 24 | 12 | 9.7 | 1.29 | $7.6(-4)$ | 0.78 |
| $S i$ | 28 | 14 | 11.7 | 1.87 | $9.9(-4)$ | 0.55 |
| $S$ | 32 | 16 | 13.7 | 2.56 | $4.0(-4)$ | 0.18 |

## FIGURE CAPTIONS

Fig. 1 The Axiorecombination Process $e^{-}+Z \rightarrow\left(e^{-}, Z\right)+a$.
Fig. 2 Solar axion flux at the earth due to bremsstrahlung and axiorecombination (denoted $f-b$ ).

Fig. 3 Axion luminoisty per unit mass for bremsstrahlung and axiorecombination $(f-b)$ processes, as a function of central stellar temperature and stellar mass.


Fig. 1


Fig. 2


Fig. 3


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