SLAC – PUB – 3991
June 1986
(T/E)

SPECTROSCOPY WITHOUT QUARKS: A SKYRME-MODEL SAMPLER*

Marek Karliner and Michael P. Mattis
Stanford Linear Accelerator Center
stanford University, Stanford, California 94305

ABSTRACT

A potpourri of Skyrme-model results for the meson-baryon system is surveyed.

This talk is devoted to a study of meson-nucleon scattering in skyrmion models of the nucleon.[1,2] We shall focus on the characteristic energy-range of the baryon resonances, typically 1.5-2.5 GeV. This is well beyond the point where the chiral Lagrangians are conventionally applied; nor is it known at present how to apply QCD directly in this domain. Thus it is especially interesting to see what insights emerge in this regime from skyrmion physics. The results presented here will only be valid to leading order in $1/N_c$, where N_c is the number of colors of the underlying gauge theory.[3]

The object of our investigations will be effective Lagrangians (Skyrme's included) of the form

$$\mathcal{L}_\pi = \frac{f_\pi^2}{16}\,\mathrm{Tr}\,(\partial_\mu U \partial^\mu U^\dagger) + \cdots. \tag{1}$$

The leading term is the usual 2-flavor or 3-flavor nonlinear sigma model, depending on whether $U \in SU(2)$ or $U \in SU(3)$. The dots stand for higher-derivative terms, which are not usually exploited in traditional soft-pion physics. Nevertheless, they are needed to stabilize a soliton, or "skyrmion," whose topological charge (following Skyrme) is interpreted as baryon number. The standard identification of the pion field in (1) in the baryon-number-0 sector of the 2-flavor theory is via:

$$U(x) = \exp(\frac{2i}{f_\pi}\vec{\pi}(x)\cdot\vec{\sigma}). \tag{2}$$

Thus the pions can be thought of as "small fluctuations" about the trivial vacuum $U(x) \equiv 1$.

It is a straightforward procedure to introduce additional fields into (1) in such a way as to preserve chiral invariance.[4] In particular, the traditional approach to studying the coupling of pions to the nucleon isodoublet N is to set

$$\mathcal{L}_{\pi N} = \frac{f_\pi^2}{16}\,\mathrm{Tr}\,(\partial_\mu U \partial^\mu U^\dagger) + \bar{N}(i\gamma^\mu \mathcal{D}_\mu - m)N + g_A \mathcal{D}_\mu \vec{\pi}\cdot \bar{N}\vec{\tau}\gamma^\mu\gamma^5 N. \tag{3}$$

Here $\mathcal{D}$ is the covariant derivative appropriate to the manifold G/H, where $G = SU(2)_L \times SU(2)_R$ and $H = SU(2)_{\mathrm{isospin}}$. From this Lagrangian, all soft-pion theorems pertaining to the πN interaction, such as Weinberg's calculation of the S-wave scattering lengths,[5] can be derived.

---

* Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.

Invited talk presented at the 2nd Conference on the Interactions Between Particle and Nuclear Physics, Lake Louise, Canada, May 23-31, 1986

It is the moral of this talk that the purely mesonic Lagrangian (1) contains *at least* as much information as does (3)! Not only does (1) properly encompass soft-pion physics, as Schnitzer has shown,[6] but in addition—well beyond the soft-pion regime—it yields surprisingly accurate predictions concerning the spectrum of nucleon and Δ resonances and the qualitative behavior of the large majority of πN and $\overline{K}N$ partial-wave amplitudes.

The study of meson-nucleon scattering in skyrmion models involves splitting the Goldstone field $\vec{\pi}$ into two pieces: a spatially-varying c-number piece, *i.e.*, the skyrmion, and a fluctuating piece, which we identify with physical mesons.[7] The skyrmion will be assumed to be of the hedgehog form:

$$U_\circ(\vec{x}) = \exp(iF(r)\hat{\mathbf{r}}\cdot\vec{\sigma}). \tag{4}$$

Calculating the T-matrix then reduces to a problem of potential scattering, from which partial-wave phase-shifts can be extracted in the usual manner. In addition, it is necessary to fold in a little group theory, as we now describe.

For simplicity, let us focus on the non-strange processes $\pi N \to \pi N$, $\pi N \to \pi\Delta$ and $\pi\Delta \to \pi\Delta$. The quantum numbers needed to describe such processes are: the initial and final pion angular momenta L and L'; the initial and final spin (or isospin) representation of the baryon s and s', which equal $\frac{1}{2}$ for nucleons and $\frac{3}{2}$ for Δ's; and the total pion-baryon isospin and angular momentum $\mathbf{I}$ and $\mathbf{J}$. The T-matrix describing such processes can then be shown to be:[9,10]

$$\begin{aligned}\mathbf{T}(\{Ls\mathbf{I}\mathbf{J}\} \to \{L's'\mathbf{I}'\mathbf{J}'\}) &= \delta_{II'}\delta_{I_zI'_z}\delta_{JJ'}\delta_{J_zJ'_z}\,(-1)^{s'-s} \\ &\times \sqrt{(2s+1)(2s'+1)}\sum_{K=L-1}^{L+1}(2K+1)\begin{Bmatrix} KIJ \\ s'L'1\end{Bmatrix}\begin{Bmatrix} KIJ \\ sL1\end{Bmatrix}\mathcal{T}_{KL'L}\,.\end{aligned} \tag{5}$$

The expressions in curly brackets are $6j$-symbols. The quantities $\mathcal{T}_{KL'L}$, which are functions of pion energy, are the "reduced amplitudes" of the model, obtainable numerically from a phase-shift analysis about the skyrmion. The Kronecker δ's express the reassuring fact that $\mathbf{I}$ and $\mathbf{J}$ are conserved in these models, as they ought to be.

Although, in Eq. (5), K plays the part of a dummy index, it actually has an interesting physical interpretation. Specifically, K can be viewed as the vector sum of the pion's angular momentum and isospin in the unphysical frame in which the pion scatters, not from a nucleon, but rather from an unrotated hedgehog soliton. This frame is "unphysical" in that a nucleon properly corresponds to a *rotating* hedgehog soliton in the skyrmion approach.[2]

Pleasingly, all the model-dependence in (5) arising from the details of the terms indicated by dots in the Lagrangian (1) is subsumed in the reduced amplitudes $\mathcal{T}_{KL'L}$; the $6j$-symbols, in contrast, follow purely from the assumed hedgehog symmetry of the skyrmion. Equation (5) is thus analogous to the Wigner-Eckart theorem in that a large number of physical matrix elements (the $\mathbf{T}$'s) are expressed in terms of a substantially smaller set of reduced matrix elements (the $\mathcal{T}_K$'s) weighted by appropriate group-theoretical coefficients.

One can carry the analogy further by finding those special linear combinations (analogous to the Gell-Mann-Okubo formula) for which the model-dependent right-hand side of (5) cancels out; the net result will be a set of energy-independent linear relations between physical scattering amplitudes that serve as a test of the applicability of skyrmion physics to the real world. For example, for $\pi N \to \pi N$, one can for each value of $L \geq 1$ solve for the two independent isospin-$\frac{3}{2}$ amplitudes (*i.e.*, with $J = L \pm \frac{1}{2}$) as linear combinations of the two isospin-$\frac{1}{2}$ amplitudes![9,10] How well do these relations work as applied to the experimental scattering data? Consider the case of the F waves (*i.e.*, $L = 3$). Figures 1*a* and 1*b* depict the real-world F_{35} and F_{37} amplitudes,[11] respectively (indicated by solid lines), compared with the linear combinations of the F_{15} and F_{17} amplitudes (dotted lines) to which they should correspond, if Eq. (5) holds true.[9] The degree of agreement is impressive.

Similar relations hold[9] for $\pi N \to \pi \Delta$. Figure 1*c* displays the FP_{15} amplitude compared with the appropriate multiple of the FP_{35} amplitude predicted by Eq. (5). One can even find relations between $\pi N \to \pi N$ and $\pi N \to \pi \Delta$ amplitudes; a typical such prediction is illustrated in Fig. 1*d*. In both cases, although the sizes of the two comparison curves are not in especially close accord, the disagreement is on the order of the typical error-bars of the data. We should point out that the agreement in the *signs* of the amplitudes is in itself a completely nontrivial result.

It is straightforward to generalize Eq. (5) to the case when the initial and/or final meson has arbitrary spin and isospin; one need only replace the $6j$ symbols by $9j$ symbols.[12] This allows us to treat in the skyrmion framework such experimentally accessible processes as $\pi N \to \rho N$ and $\pi N \to \omega N$. A typical relation for $\pi N \to \rho N$ is illustrated in Fig. 1*e*. Alternatively, one can generalize Eq. (5) to incorporate *strangeness*, which entails interpreting U in Eq. (1) as an $SU(3)$ matrix.[13,14] This enables us to study KN and $\overline{K}N$ scattering as well. In this context, one can obtain predictions relating $\pi N \to \pi N$ to $\overline{K}N \to \overline{K}N$,[15] as exemplified in Fig. 1*f*.

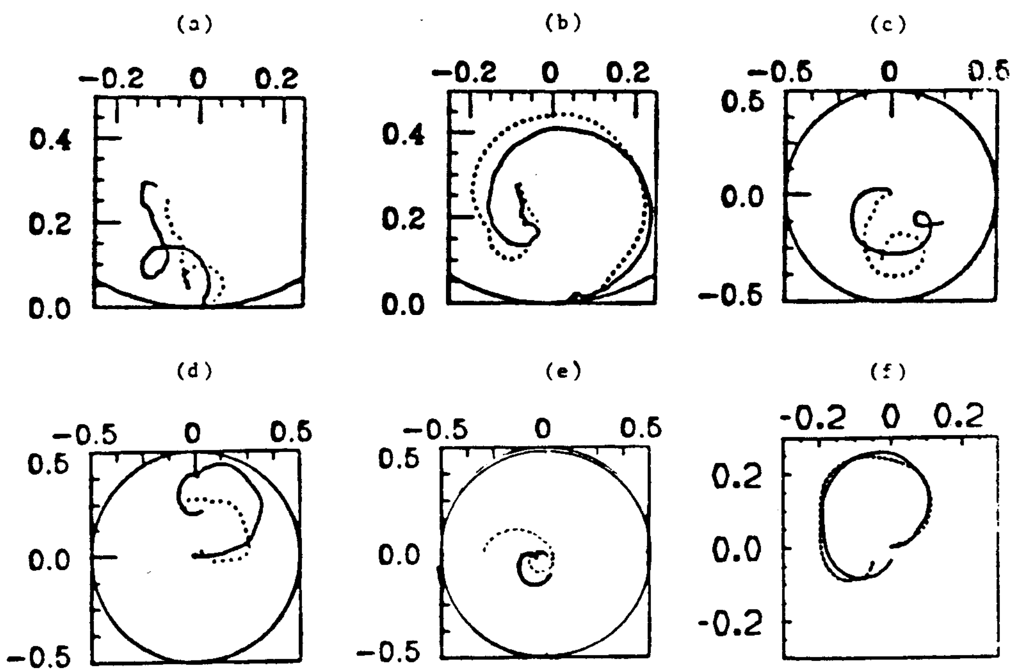


Fig. 1. Model-independent linear relations between experimental amplitudes. (a)[9] $\pi N \to \pi N$: F_{35} *vs.* $\frac{1}{7}F_{15} + \frac{6}{7}F_{17}$. (b)[9] $\pi N \to \pi N$: F_{37} *vs.* $\frac{9}{14}F_{15} + \frac{5}{14}F_{17}$. (c)[9] $\pi N \to \pi\Delta$: FP_{15} *vs.* $\sqrt{10}FP_{35}$. (d)[9] $\pi N \to \pi N$ *vs.* $\pi N \to \pi \Delta$: $\frac{1}{\sqrt{2}}(P_{11} - P_{13})$ *vs.* $\frac{5}{4}PP_{33} - \frac{1}{8}PP_{11}$. (e)[12] $\pi N \to \rho N$: $-\frac{1}{2}SD_{31}$ *vs.* SD_{31}. (f)[15] $\pi N \to \pi N$ *vs.* $\overline{K}N \to \overline{K}N$: $\frac{1861}{2395}F_{15} - \frac{10661}{14370}F_{37}$ *vs.* $F_{05} - \frac{1952}{1437}F_{17}$. In all cases, the first-named amplitude is depicted by a solid line, the second by a dotted line. As usual, the graphs depict the T-matrices in the complex plane.

We should reiterate that all the curves in Fig. 1 are drawn from experiment. In other words, they are examples of *model-independent* predictions of skyrmion physics. (As far as we know, no such relations follow from $SU(6)$, or from the quark model.) In our eyes, Fig. 1 is compelling evidence for the validity of the hedgehog-soliton picture of the nucleon.

Of course, one can also adopt a *model-dependent* approach to Eq. (5). This involves specifying an effective meson Lagrangian, calculating the $\mathcal{T}_{KL'L}$'s numerically, reconstructing the complete partial-wave T-matrix using (5), and then comparing with experiment.[8,10] Let us focus on the venerable Skyrme model, in which the first term of Eq. (1) is supplemented by the soliton-stabilizing term $\frac{1}{32e^2}\,\mathrm{Tr}[(\partial_\mu U)U^\dagger,(\partial_\nu U)U^\dagger]^2$.

Figure 2 presents the partial-wave T-matrix of the Skyrme model, juxtaposed with experiment. The overall degree of accord is quite striking. The exceptions are in the lower partial waves, the S_{31}, P_{11}, P_{33} and D_{35} channels especially. We have argued at length elsewhere[8,14] that the poor agreement in these channels can be attributed to our failure to factor out the rotational and translational zero-modes of the skyrmion in our formalism. As such, they represent failures, not of the Skyrme model *per se*, but rather of our treatment of it. We are confident that a higher-order $1/N_c$ analysis would dramatically improve the agreement in these channels. Fortunately, in higher partial waves, which do not mix with the zero-modes, there is no such problem.

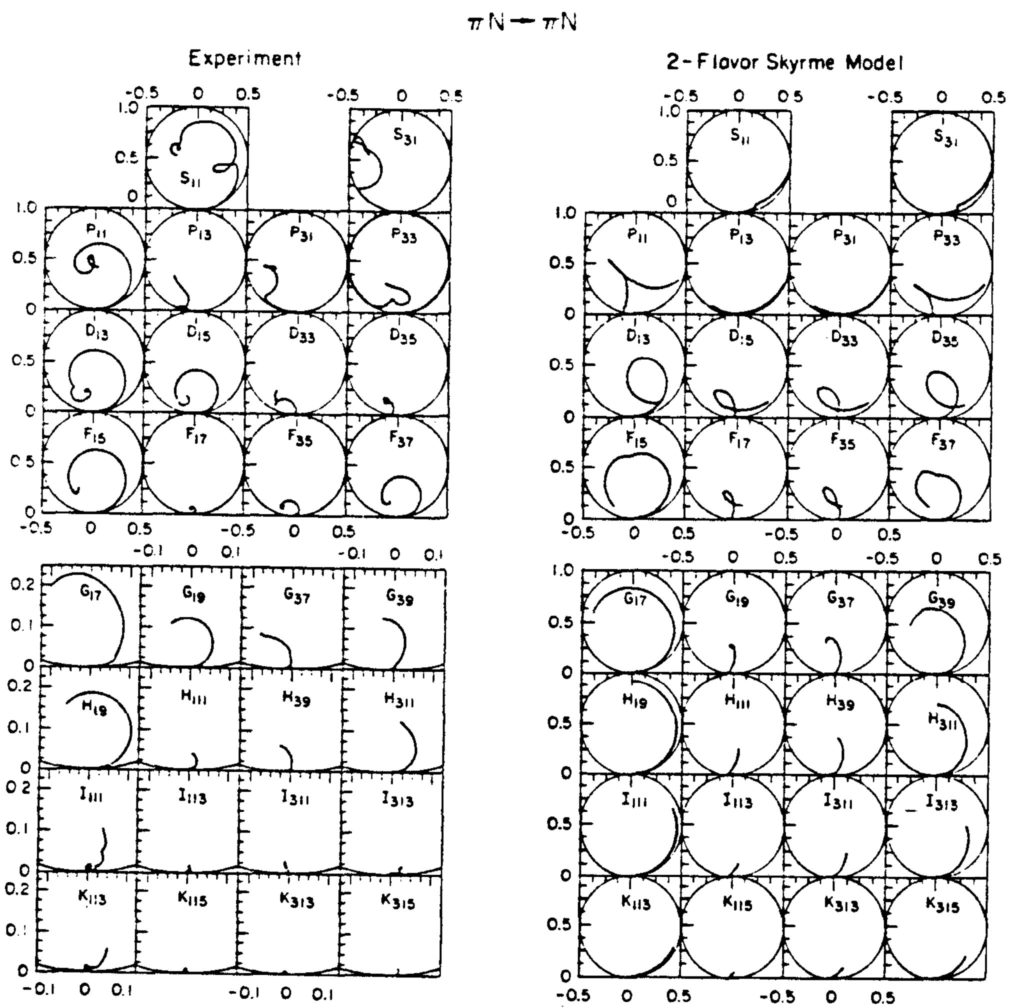


Fig. 2. $\pi N \to \pi N$: comparison between 2-flavor Skyrme model and experiment (from Ref. 14).

The most intriguing feature of Fig. 2 is how the Skyrme model reproduces the pattern of size alternation evident in the experimental curves for each value of $L \geq 1$. For example, the F_{15} and F_{37} curves are much bigger than the F_{17} and F_{35} amplitudes. This "big-small-small-big" pattern finds a natural group-theoretic explanation in the context of skyrmion physics.[9] According to Eq. (5), the physical scattering amplitudes can be expressed as linear combinations of reduced amplitudes. In the F-wave sector, one finds, for example:

$$\mathbf{T}(F_{15}) = \frac{5}{9} \cdot \mathcal{T}_{233} + \frac{4}{9} \cdot \mathcal{T}_{333}, \tag{6a}$$

$$\mathbf{T}(F_{17}) = \frac{1}{4} \cdot \mathcal{T}_{333} + \frac{3}{4} \cdot \mathcal{T}_{433}, \tag{6b}$$

$$\mathbf{T}(F_{35}) = \frac{5}{63} \cdot \mathcal{T}_{233} + \frac{5}{18} \cdot \mathcal{T}_{333} + \frac{9}{14} \cdot \mathcal{T}_{433}, \tag{6c}$$

$$\mathbf{T}(F_{37}) = \frac{5}{14} \cdot \mathcal{T}_{233} + \frac{3}{8} \cdot \mathcal{T}_{333} + \frac{15}{56} \cdot \mathcal{T}_{433}. \tag{6d}$$

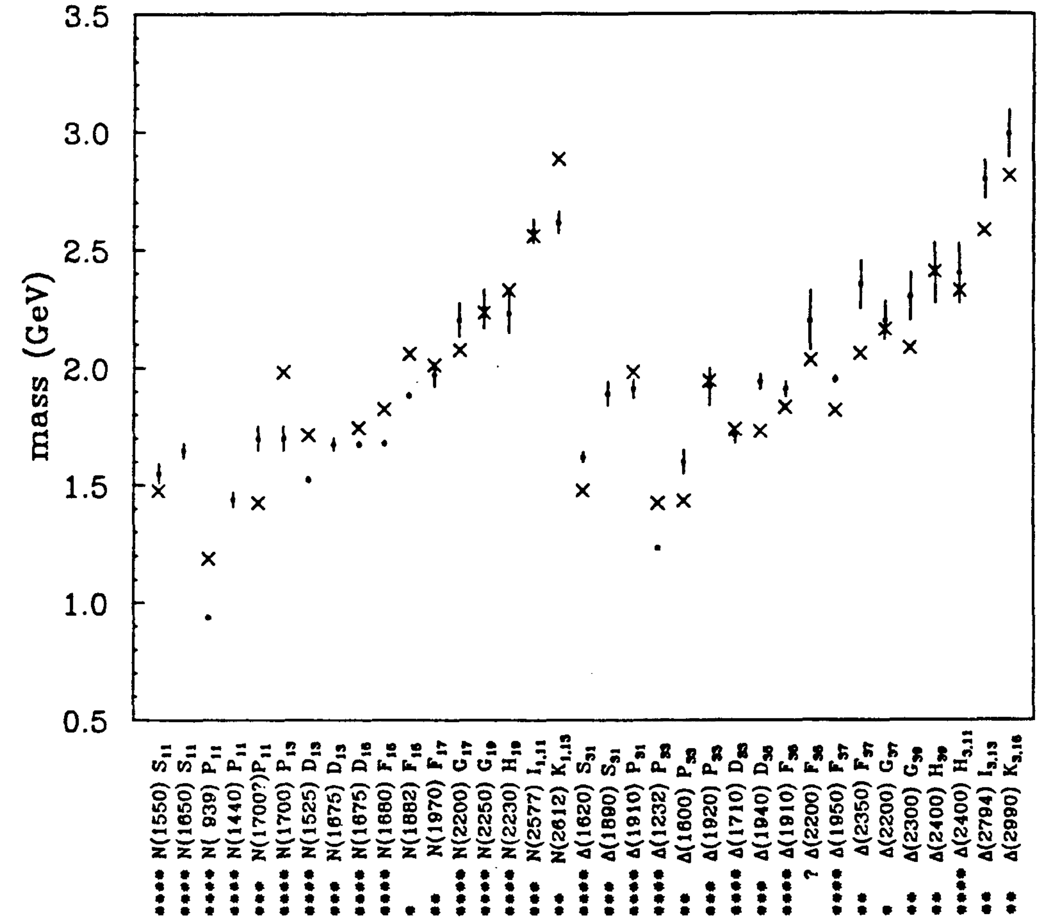


Fig. 3. Spectrum of N and Δ resonances: Skyrme model *vs.* experiment (from Ref. 14). The experimental masses (with uncertainties) are indicated by dots, the Skyrme-model masses by crosses. The Skyrme-model values for m_N and m_Δ are obtained from Eq. (9) of Ref. 2, using our "best fit" parameters $\{e = 4.79,\ f_\pi = 150\,\text{MeV}\}$.

Note that, whereas the F_{17} and F_{35} amplitudes receive the bulk of their contributions from $\mathcal{T}_{433}$, the F_{15} and F_{37} amplitudes are composed primarily of $\mathcal{T}_{233}$ and $\mathcal{T}_{333}$. *The big-small-small-big pattern can thus be simply explained by the assumption that $\mathcal{T}_{433}$ is small compared with $\mathcal{T}_{233}$ and $\mathcal{T}_{333}$, etc.* Indeed, this turns out to be true numerically in the Skyrme model; presumably it is true as regards the optimal effective Lagrangian of Nature, as well.

Figure 3 presents the spectrum of nucleon and Δ resonances of the Skyrme model compared with experiment. The Skyrme-model masses are the results of a least-squares fit to the data, with all resonances weighted equally. (Recall that the model has two "free parameters," f_π and e.) The agreement with Nature is surprisingly good—better than 7% on average up to 3 GeV. Interestingly, two resonances, the $N(1882)$ F_{15} and the $\Delta(2350)$ F_{37}, are not present in the 2-flavor Skyrme model but only in the 3-flavor model; other than this, the two versions yield identical spectra.[16]

Figure 4 displays both the 2-flavor and 3-flavor Skyrme-model amplitudes for the process $\pi N \to \pi\Delta$, indicated by dotted and solid curves, respectively. In general, the agreement with experiment is excellent. It is especially pleasing that the model correctly predicts a negative DD_{13} amplitude, in stark contrast to all the other PP, DD and FF channels. Likewise, in the model as in Nature, the FF_{15} amplitude curves around much more than its three F-wave counterparts. We should also note that—unlike $SU(6)$—the Skyrme model correctly predicts the signs of the $\pi N \to \pi\Delta$ amplitudes in channels in which the pion's angular momentum jumps by 2 units (not depicted).

Finally, Figs. 5-7 present a sampler of results corresponding to the processes $\overline{K}N \to \overline{K}N$, $\pi N \to K\Lambda$, and $\overline{K}N \to \pi\Lambda$, in which the 3-flavor Skyrme model is compared with experiment. In general, the agreement is quite satisfactory. (As one would expect, the accord is much less good in the S- and P-wave sectors, which we have not depicted, due to mixing with the zero-modes of the skyrmion.) Although the Skyrme-model graphs are often much too large, as in Fig. 6, the *relative* sizes and signs between amplitudes are well rendered by the model. In particular, for $\overline{K}N \to \overline{K}N$, note that the D_{03}, F_{05} and G_{07} amplitudes dominate their counterparts in both the model and experiment; we have dubbed this the "big-small-small-small" pattern, in analogy with the big-small-small-big pattern discussed earlier. Likewise the model successfully predicts a "down-up" pattern of sign alternation for $\pi N \to K\Lambda$ and $\overline{K}N \to \pi\Lambda$. Not surprisingly, these patterns, too, find a natural group-theoretic explanation in the skyrmion framework, just as in the case of the big-small-small-big pattern.

In closing, we would like to express our wonderment that so much detailed structure of the meson-nucleon T-matrix—much of it in reasonable accord with Nature—can emerge from a simple meson Lagrangian with no explicit quark or nucleon fields.

Fig. 4. $\pi N \rightarrow \pi\Delta$

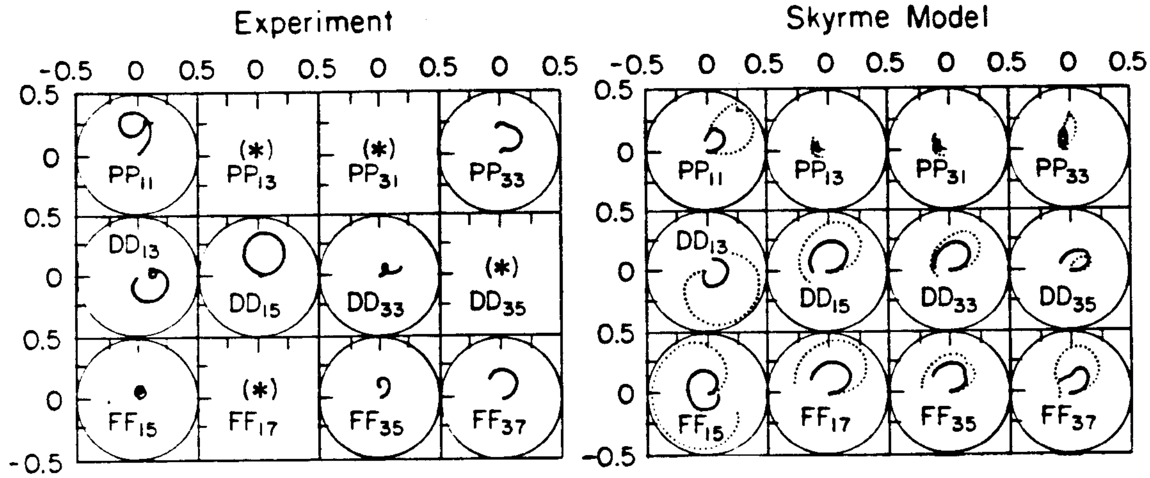


Fig. 5. $\bar{K}N \rightarrow \bar{K}N$

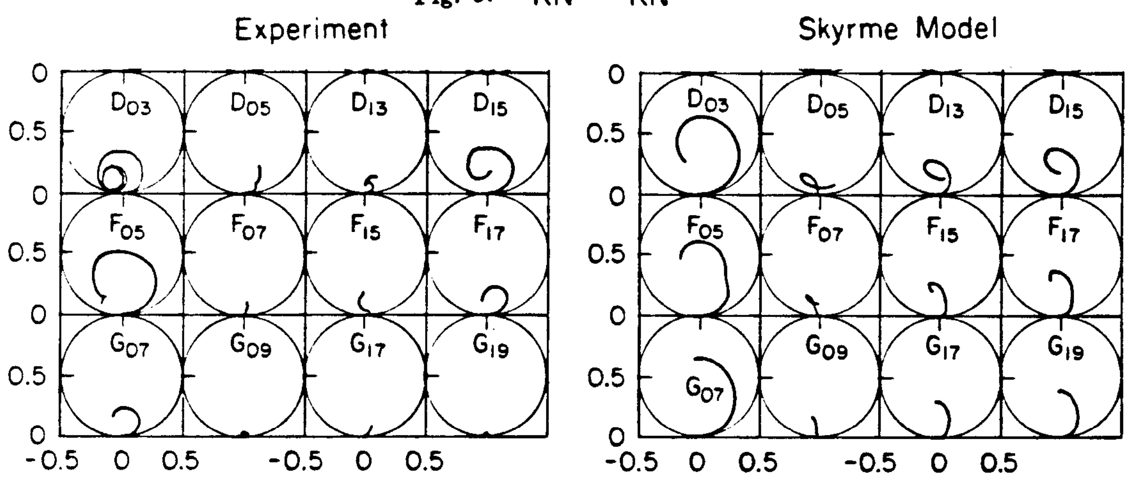


Fig. 6. $\pi N \rightarrow K\Lambda$

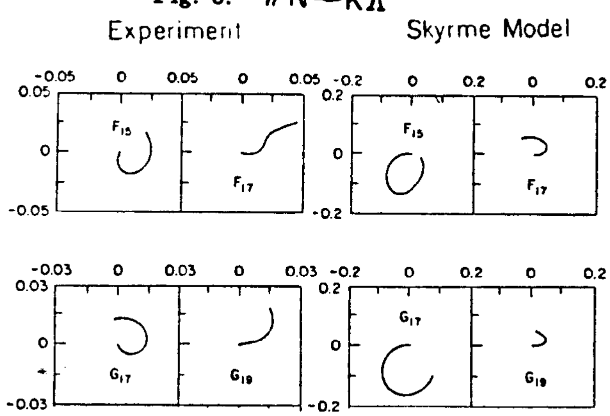
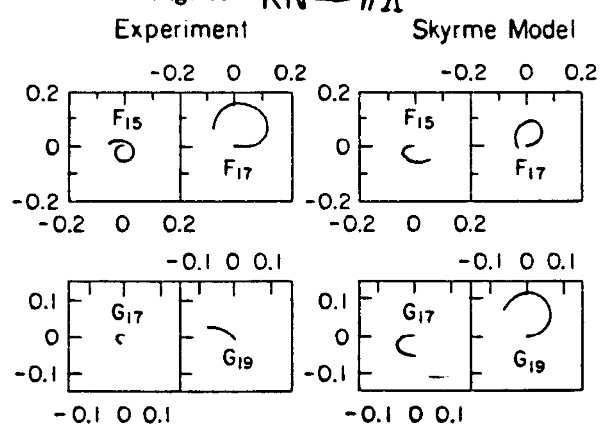


Fig. 7. $\bar{K}N \rightarrow \pi\Lambda$



Figures 4–7 are taken taken from Ref. 14

REFERENCES

1. T. H. R. Skyrme *Proc. Roy. Soc.* **A260** (1961) 127; *Nucl. Phys.* **31** (1962) 556.
2. G. Adkins, C. Nappi, and E. Witten, *Nucl. Phys.* **B228,** 552 (1983).
3. The parameters of the effective Lagrangians have well defined scaling properties as functions of N_c. (For a detailed exposition of this issue see Ref. 2 and references therein). Consequently in the large-N_c limit certain terms in the meson equations of motion are formally supressed and these equations can therefore be linearized in a consistent fashion.[8,9]
4. S. Coleman, J. Wess, B. Zumino, *Phys. Rev.* **177** (1969) 2239; C. Callan, S. Coleman, J. Wess, B. Zumino, *ibid.* 2247.
5. S. Weinberg, *Phys. Rev. Lett.* **17,** 168 (1966).
6. H. Schnitzer, *Phys. Lett.* **139B,** 217 (1984); *Nucl. Phys.* **B261** (1985) 546.
7. In truth, not all fluctuations correspond to *bona fide* mesonic excitations: some serve only to rotate or translate the skyrmion, and should therefore be viewed as *baryonic* degrees of freedom. These *zero-modes* mix only with S-, P- and D-wave pions, and tend to spoil the results in these channels.
8. M. P. Mattis and M. Karliner, *Phys. Rev.* **D31** (1985) 2833.
9. M. P. Mattis and M. Peskin, *Phys. Rev.* **D32** (1985) 58.
10. A. Hayashi, G. Eckart, G. Holzwarth, and H. Walliser, *Phys. Lett.* **147B,** 5 (1984).
11. We shall follow the customary notation for partial-wave amplitudes: $\pi N \to \pi N$ and $\pi N \to K\Lambda$ amplitudes are indicated by $L_{2I,2J}$, $\overline{K}N \to \overline{K}N$ and $\overline{K}N \to \pi\Lambda$ amplitudes by $L_{I,2J}$, and $\pi N \to \pi\Delta$ and $\pi N \to \rho N$ amplitudes by $LL'_{2I,2J}$, where L and L' are the incoming and exiting pion orbital angular momenta, and I and J are the total meson-baryon isospin and angular momentum.
12. M. P. Mattis, *Phys. Rev. Lett.* **56** (1986) 1103.
13. M. P. Mattis, *Aspects of Meson-Skyrmion Scattering*, SLAC-PUB. 3795, in S. Brodsky and E. Moniz, eds., Proceedings of the 1985 ITP Workshop on Nuclear Chromodynamics, World Scientific Press.
14. M. Karliner and M. P. Mattis, *πN, KN and $\overline{K}N$ Scattering: Skyrme Model vs. Experiment,* SLAC-PUB-3901, 1986.
15. M. Karliner, *How chiral solitons relate $\overline{K}N$ and πN scattering,* SLAC-PUB 3958, 1986.
16. M. Karliner and M. P. Mattis, *Phys. Rev. Lett.* **56** (1986) 428.