SLAC – PUB – 3983
May 1986
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DEFLECTION BY THE IMAGE CURRENT AND CHARGES OF A BEAM SCRAPER*

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Introduction

Scrapers are often used in storage rings and accelerators to "clean-up" the transverse profile of the beam. When the beam is not exactly midway between the jaws of the scraper the transverse electric and magnetic fields produced by the image charges and currents are asymmetric. For a relativistic beam traveling through a longitudinally uniform tube with infinitely conducting walls the transverse force from the electric field is canceled by the transverse force from the magnetic field, to order $1/\gamma^2$, where γ is the particle energy. However, when an off-center particle bunch passes by a longitudinal discontinuity in the beam tube the transverse force from the electric field is no longer canceled by the transverse force from the magnetic field and particles in the bunch will experience a transverse momentum kick which is independent of energy (to order $1/\gamma^2$). While we are particularly interested in the effects of a scraper in a beam channel, the results obtained here apply to any similar discontinuity. The necessary assumption is that the distance between the scraper jaws is small compared to the longitudinal extent of the beam bunch and the scraper. Scrapers also affect the energy of the beam. Since this is normally a negligable effect it will not be discussed in this paper.

We have used the TBCI program[1] to treat the case, shown schematically in Fig. 1, where a circular beam of radius a is off-center by amount ξ in a circular scraper of radius b. The scraper is situated in a beam tube. The length of the scraper is not important as long as it is much greater than its radius.

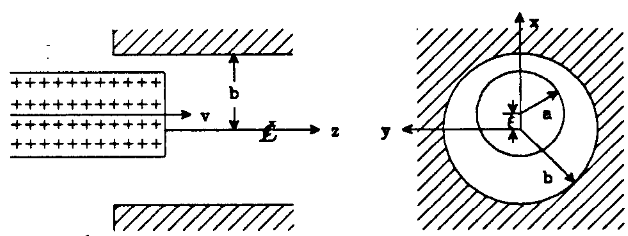

Fig. 1. Off-center beam entering a circular scraper.

Analytic Approximation

Consider a bunch with uniform transverse charge density inside of radius a and displaced an amount $\xi \ll a$. The asymmetric portion of the charge and current density is given by

$$\rho = \frac{e\lambda\xi}{\pi a^2}\delta(r-a)\cos\theta \quad \text{and} \quad j_z = \beta c\rho \quad , \tag{1}$$

with the longitudinal particle density in the bunch, $\lambda = \lambda(\tau)$; the longitudinal position in the bunch, $\tau = (z - \beta ct)$; $\delta(x)$ is the Dirac delta function, the velocity of the bunch, $v = \beta c$; and θ the azimuthal angle. Note that we are using the convention of $\tau > 0$ at the bunch head. For all cases used in the computer calculations the function λ is taken to be given by

$$\lambda(\tau) = \frac{N}{\sqrt{2\pi}\sigma}\exp\left(-\frac{c^2\tau^2}{2\sigma^2}\right) \quad , \tag{2}$$

where N is the number of particles in the bunch. The results obtained below will apply to any other form of λ as long as the variation of λ is small over a longitudinal distance of order b. It is interesting to note that for a line charge (i.e. radius $a \to 0$) displaced by amount ξ the charge distribution is given by

$$\begin{aligned}\rho &= \frac{e\lambda(\tau)}{\xi}\delta(r-\xi)\delta(\theta)\\ &= \frac{e\lambda}{\xi\pi}\delta(r-\xi)\left[\frac{1}{2}+\sum_{m=1}^{\infty}\cos m\theta\right] \quad .\end{aligned} \tag{3}$$

The transverse deflection of a particle at radius r produced by the field of the m^{th} multipole (due to the bunch passing the edge of the scraper) is proportional to the quantity f_m where

$$f_m = \left(\frac{\xi}{b}\right)^m\left(\frac{r}{b}\right)^{m-1} \quad . \tag{4}$$

The above scaling is well known for the transverse wakefields of cavities.[2,3] It is clear that for scrapers it must also hold. We will only consider the leading, dipole term ($m = 1$), so that the transverse deflection is proportional to $f_1 = \xi/b$.

The total electric and magnetic field seen by a particle in the bunch can be divided into two parts. The first part, designated as the self field, would be present in the absence of any image charges or image currents, and has the $1/\gamma^2$ cancellation of the electric and magnetic fields. The second part of the fields is due to the image charges and currents. The electric force of this part is not canceled by the magnetic force from the image current near a discontinuity in the beam pipe.

Normally we are interested in the case where b is small compared to the scraper length, and σ is not small compared to b. In this case the two edges of the scraper can be treated separately. The dipole electric field and magnetic field far away from any discontinuity in a round pipe of radius b, due to the image charges and current, is given by

$$E_x = \frac{2e\lambda\xi}{b^2} \quad \text{and} \quad B_y = \frac{2e\lambda\beta\xi}{b^2} \quad . \tag{5}$$

Near a discontinuity these expressions do not hold.

Consider a long bunch approaching the exit of a scraper. The image charges increase as the bunch approaches the scraper edge, while the image currents decrease. For a test particle within a bunch and with $\xi/b \not\ll 1$ we approximate the fields as shown in Fig. 2b, with the reduction in E_x lagging the reduction in B_y by a distance of b. (The scraper is at $z = 0$.)

* Work supported by the Department of Energy, contract DE–AC03–76SF00515.

Contributed to the 1986 Linac Accelerator Conference,
Stanford, California, June 2-6, 1986

For long bunches the problem becomes more similar to one of statics rather than electrodynamics. Thus positive charges and currents moving past a scraper entrance have an effect similar to negative charges and currents moving past a scraper exit. We therefore expect the fields at the entrance of the scraper to be the mirror images of those at the exit, as shown in Fig. 2a. A consequence of this is that the kick at the entrance is identical to that at the exit of the scraper in both amplitude and sign.

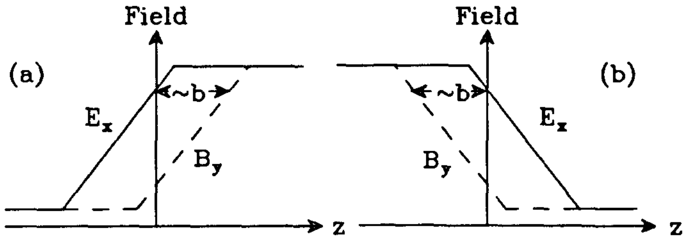

Fig. 2. Fields seen by a test particle when (a) entering and (b) leaving a scraper.

The transverse momentum kick given to a particle as it passes the discontinuity is the area of the parallelogram equal to

$$c\,\Delta p_x(\tau) = e\int_{-\infty}^{\infty} dz\,(E_x - \beta B_y)\Big|_{t=(z/\beta c-\tau)} \approx eb\,\Delta E_x \quad , \quad (6)$$

with ΔE_x the difference between the electric field in the two pipes experienced by a particle far from the discontinuity. Normally the scraper radius is much smaller than the radius of the beam pipe. Therefore, the approximation that applies at each end of the scraper yields

$$c\,\Delta p_x(\tau) \approx \frac{2e^2\lambda(\tau)\xi}{b} \quad . \quad (7)$$

<u>Computer Calculations</u>

In order to check the above approximations we have used the program TBCI to study the transverse fields excited by an off-center bunch of the form given by Eq. 3 truncated at $\pm 4\sigma$. All of the computer calculations have been performed for the ultra relativistic case where $\beta = 1$, and the total charge in the bunch is one pico-coulomb ($N = 6 \times 10^6$). The transverse deflection of a particle in the bunch, due to the bunch passage off-center by the entrance or exit of a small radius pipe, is shown in Fig. 3 as a function of the position of the particle in the bunch for several different bunch lengths. The bunch enters (or exits) off-center by 0.25 mm from a large pipe of radius 4 mm into a small pipe of radius 0.5 mm. The longitudinal density also is shown in the Fig. 3 for comparison. The bunch length σ varies from 0.25 mm to 1.0 mm. Note that, for the longer bunch lengths, the approximation that the transverse kick received by a particle at position τ in the bunch is proportional to the longitudinal particle density at that position is justified.

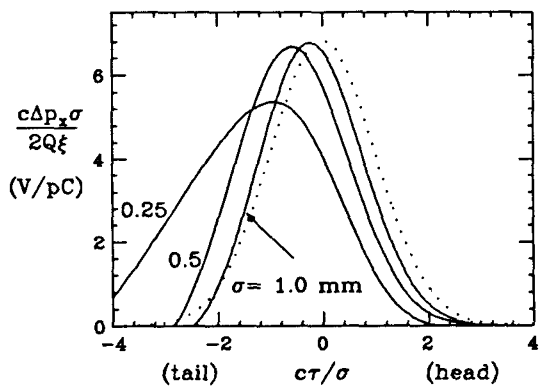

Fig. 3. Kick of one scraper edge for various values of σ, when $b = 0.5$ mm. The dotted line gives the charge distribution.

The electric and magnetic fields experienced by a particle as it travels by the entrance of the small pipe at $\xi/b = 0.5$ are shown in Fig. 4 for a particle in the middle of the bunch ($\tau = 0$). The conditions are the same as those for the $\sigma = 1$ mm bunch in Fig. 3. The test particle feels a total kick of 3.4 volts when passing this scraper edge, if the total charge of the bunch is 1 pico-coulomb. Eq. (7), our simple model, yields a value of 3.6 volts. For a bunch exiting the scraper TBCI gives a result that is the mirror image of Fig. 4.

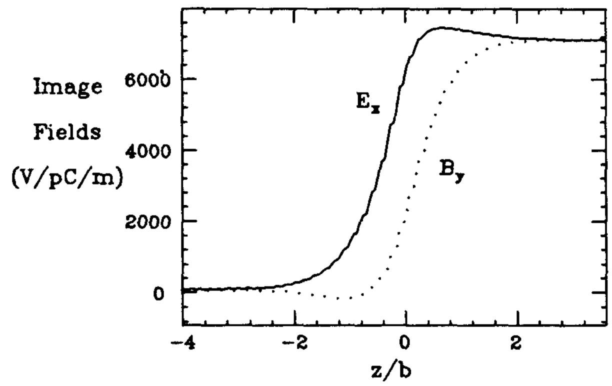

Fig. 4. Computed fields seen by a particle at $\tau = 0$ as it enters a scraper. The scraper edge is at $z = 0$.

The main problem from the scraper is not that the off-center beam receives a transverse deflection, but that the transverse deflection varies, as the bunch density, along the bunch. If the variation of the transverse momentum deflection is comparable to the natural transverse momentum spread the effective beam emittance is increased. For example, a typical SLC bunch of 5×10^{10} particles has a length $\sigma = 1\,\text{mm}$. At a high β point near the end of the machine the beam has an angular divergence of $\sigma_{x'} \sim 2\mu r$. If the bunch passes halfway off-center in a scraper with $b = 0.5$ mm, then a test particle at $\tau = 0$ receives a kick of 0.55 μr due to each scraper edge whereas particles at either end of the bunch receive no kick. The variation due to one full scraper is half as much as the beam's original rms angular spread.

One way of lessening the deflection of the scraper is by reducing the angle of its incoming or outgoing edge. If an angled piece, whose projection in the axial direction is fixed at 2 cm, is cut out of a scraper edge (see Fig. 5 inset) the resultant kick for a particle at $\tau = 0$ is given in Fig. 5. The kick can be reduced by slightly more the 50% if the angle is taken to be 15°. The dashed curve is an extrapolation to an angled scraper edge that is arbitrarily long. Calculation in the dashed region was not possible due to the great number of mesh points needed to model the problem.

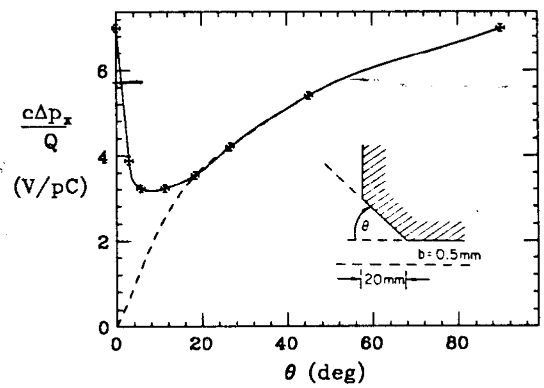

Fig. 5. Peak kick of 1/2 scraper as a function of scraper angle θ, when $\xi/b = 1.0$ mm and $\sigma = 1.0$ mm.

Discussion

We have shown that scrapers that pass close by high peak current beams can significantly degrade the beam emittance. A circular scraper was chosen for this study since, at the moment, it is the only one that we can compute. But it can be expected that these results give a reasonably good approximation for a normal rectangular window scraper, with b representing the half width of the window. Whenever wakefield effects can be important scrapers that only have one side should be avoided since they will give larger kicks than computed here. Furthermore, they have no ideal trajectory along which there is no kick, as the symmetric scrapers have.

References

[1] T. Weiland, DESY **82**-015 (1982) and NIM **212**, 13 (1983).

[2] K. Bane and P. Wilson, Procedings of the 11^{th} Int. Conf. on High-Energy Accelerators, CERN (Birkhäuser Verlag, Basel, 1980), p. 592.

[3] T. Weiland, DESY M83-02 (1983) and NIM **216**, 31 (1983).