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# UNIFIED FORMULATION FOR LINEAR ACCELERATOR DESIGN*

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## ABSTRACT

Expressions for peak and average powers required to produce a given average gradient in an accelerator section are given. They are valid for both lossy and lossless (superconducting) sections, for both traveling wave and standing wave sections, and for pulsed or continuous wave rf input. The expressions are given in terms of structure parameters that are equally applicable to traveling wave or standing wave. These parameters delineate the effect of wall losses and energy required to build up the field. For both traveling wave and standing wave sections it is possible to make the rf pulse length short enough to make the wall losses negligible at the expense of increased peak power requirement. Therefore the expressions will include the effects of pulse compression.

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## SECTION PARAMETERS

The necessary and sufficient parameters that can characterize traveling wave (TW) or standing wave (SW) accelerator sections are

$$s = \frac{E^2}{w}; \qquad T_o = \frac{2w}{p_d} = \frac{2W}{P_d}; \qquad v_g = \frac{P}{w} \, . \tag{1}$$

For SW section only: $\quad s = \frac{V^2}{WL}; \qquad T_e = \frac{2W}{P_e} \, .$

The names of the symbols in the above definitions with a consistent set of units are:

| | |
|---|---|
| $s$ | elastance per unit length, $M\Omega \; m^{-1} \mu s^{-1}$ |
| $T_o$ | unloaded (internal) time constant, $\mu s$ |
| $v_g$ | group velocity, $m \; \mu s^{-1}$ |
| $E$ | local accelerating gradient, $MV \; m^{-1}$ |
| $w$ | energy stored per unit length, $joule \; m^{-1}$ |
| $V$ | particle voltage, $MV$ |
| $W$ | energy stored in the section, $joule$ |
| $L$ | section length, $m$ |
| $p_d$ | power dissipated per unit length, $MW \; m^{-1}$ |
| $P_d$ | power dissipated in the section, $MW$ |
| $P$ | power transmitted, $MW$ |
| $T_e$ | SW section external time constant, $\mu s$ |
| $P_e$ | power emitted by the SW section, $MW$ |

The elastance is the reciprocal of capacitance. It depends on such factors as the crossectional area of the section, the concentration of the electric field at the axis, and transit time. $T_o$ depends on the ratio of cross section area to surface area. As both stored energy and power dissipated vary linearly with section length, $T_o$ is the same for both SW and TW, if we neglect the power dissipated in the SW section end walls. The group velocity, elastance, internal time constant can be functions of distance along the section.[1] But in this note only uniform group velocity sections are considered.

Both TW and SW sections can be characterized by the same parameters $s$, $T_o$, and fill time $T_f$. For both structures the fill time can be defined as the pulse duration for which the ratio of section energy to pulse energy is maximum. For a TW structure the fill time is the section length divided by the group velocity. For a SW structure, it is a function of $T_e$ and $T_o$ and is independent of $v_g$. $v_g$ depends on coupling between cells and $T_e$ depends on coupling between the section and the generator. For both TW and SW sections, a single bunch beam is injected at one fill time after the beginning of the input RF pulse. For a SW section, however, a long beam pulse is injected when the rate of increase in field due to the RF is balanced by the rate of increase in the field due to the beam so that the field remains constant.[2] For a SW section the time the beam pulse is injected is an alternative definition of fill time.

The group velocity is a local parameter for both a SW and TW section. But its effect on the section fill time differs. For a SW section the external time constant and not the group velocity determines the fill time. Although the group velocity does not determine the fill time and is not germane to peak power requirement, it has to be relatively high so that the assumption that the SW section fills evenly along its length, like air being pumped in a tire, is valid. A TW section fills like a shaft entering a tube.

## PEAK AND AVERAGE POWER

The peak power $P_p$ and average power $P_a$ required to produce a section average gradient $E_a$ are:

$$P_p = \frac{E_a^2 L}{\eta_s s T_f M} \quad , \quad P_a = f_r P_p T_f = f_r \frac{E_a^2 L}{\eta_s s \eta_{pc}} \quad . \tag{2}$$

Here $f_r$ is the pulse repetition frequency and must be less than $1/T_f$. If $f_r = 1/T_f$, we have the continuous wave (CW) case. The peak and average powers are then identical and their expressions coalesce into a single expression.

With no pulse compression the peak and average powers required to produce a given gradient depends only on $s$, $T_o$ and $T_f$. The peak power decreases as the internal time constant increases. Therefore both the elastance and the internal time constant should be as large as practicable. The fill time is determined by the desired trade-off between the peak and average powers.

The effect of pulse compression is included in Eq. (2). Its constants are: the compression factor $C_f$ (the klystron pulse length divided by the compressed pulse length), the power multiplication factor $M$ (the square of the section voltage with compression divided by the section voltage with no pulse compression), and the compression efficiency $\eta_{pc}$ given by $\eta_{pc} = M/C_f$. For a TW section the attenuation in nepers $\tau$, the section efficiency $\eta_s$ and the section length $L$ are

$$\tau = \frac{T_f}{T_o}, \quad \eta_s = \frac{(1-e^{-\tau})^2}{\tau^2}, \quad L = v_g \tau T_o \quad . \tag{3}$$

For a SW section, the efficiency is

$$\eta_s = \frac{2\gamma(1-e^{-\tau_e/\gamma})^2}{\tau_e/\gamma} \tag{4}$$

$$\gamma = \frac{1}{1+T_e/T_o} \quad , \quad \tau_e = \frac{T_p}{T_e} \tag{5}$$

The pulse length $T_p$ when it maximizes $\eta_s$ is the fill time $T_f$ and is given by

$$T_f = 1.257 T_f, \quad T_l = \gamma T_e \quad . \tag{6}$$

$T_l$ is the loaded time constant.

Substituting for $\eta_s$ and $T_f$ from (3) into (2) we obtain for a constant impedance TW section an identical expression for the CW power and pulsed peak power

$$P = \frac{E_a^2 L/sT_o}{(1-e^{-\tau})^2/\tau} \tag{7}$$

For a fixed $s$ and $T_o$, P is minimum when $\tau = 1.257$ that is $T_f = 1.257T_o$. This is similar to the condition that minimizes the average pulsed power for a SW section, except that $T_l$ is replaced by $T_o$. The condition for minimum CW power for a SW section is $T_e = T_o = 2T_l$.

Plots of peak and average powers required to produce 21 MV/m gradient in the SLAC disk-loaded structure operating at 2856 MHz and having a internal time constant of 1.44$\mu$s with no pulse compression, $M = \eta_{pc} = 1$, are shown in Fig. 1. The solid lines are for constant group velocity and hence constant elastance, which is assumed to be 76.4 $M\Omega/m - \mu s$. The dotted lines are for a constant length and variable group velocity and hence elastance. The dashed lines are for a SW section having the same elastance and internal time constant as the constant group velocity TW section. At each fill time the section efficiency was maximized with respect to $T_e$. The value of $T_e$ is given in Ref. 1. In this case, with $E_a$, $s$, $L$, held constant the average power is a constant divided by the section efficiency.

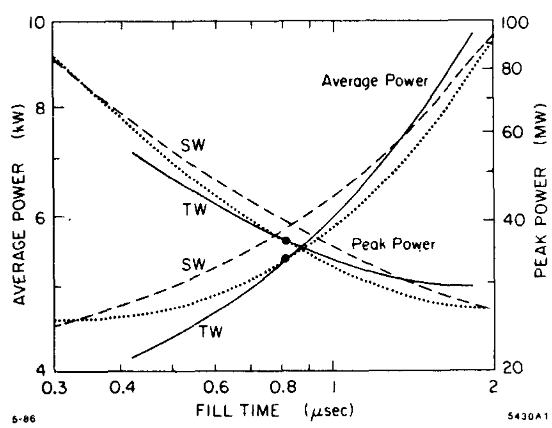


Fig. 1. TW and SW average and peak powers/section.

Define the improvement factor $I_f$ as the section material resistivity divided by the resistivity of copper. For a superconducting section,[8] $\tau << 1$ and the refrigerator ac power in KW is

$$P_r = \frac{2\tau_c R_f}{I_f} P_a \quad . \tag{8}$$

Here $\tau_c$ is the copper section attenuation at room temperature, $R_f$ is the ratio refrigerator power to heat load power. If the section is cooled $\tau$ is much lower than its average power- peak power trade-off value and is determined by refrigerator AC power requirements and by the average power rating of the cooled section.

Peak and average powers vs fill time for a copper section at room temperature and at liquid nitrogen temperature 77°$K$, $I_f = 3$, $R_f = 10$, are shown in Fig. 2. It is apparent from the Fig. 2 that improving the internal time constant can reduce the peak or average power requirements or both. The undesirable effect of cooling the section is the required refrigerator power which is the same order of magnitude as the average power. But for niobium or lead sections at liquid helium temperature 4.2°$K$, $I_f = 4000$, $R_f = 400$, at a few microsecond fill times, the refrigerator power is reduced to a fraction of the average power as shown in Fig. 3.

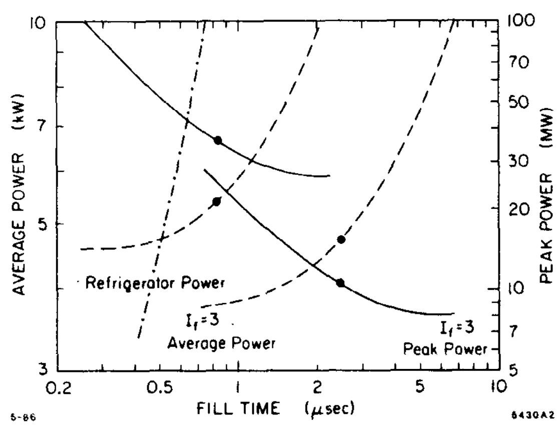


Fig. 2. Peak and average powers for a room temperature and a liquid nitrogen temperature copper TW section.

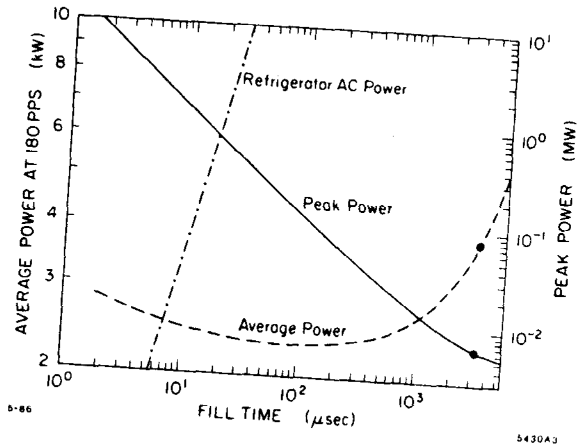


**Fig. 3. Peak and average powers for a liquid helium temperature niobium or lead TW section.**

If $T_f = 1/f_r$ then the rf is CW. At 4.2° operating at fill times approaching CW results in prohibitive refrigerator power and low gradient break down. But one can operate at short fill time. How short depends on the experimentally determined pulse energy and average power rating of the superconducting section. Making a section superconducting reduces the peak and average power by the reciprocal of $\eta_s$. The peak power per unit length can be further reduced by increasing the section length.

High effective gradients can be obtained with reduced peak and average power into the section by using the energy that has already accelerated the beam in one accelerator section to drive a second accelerator section to accelerate the recirculated beam as illustrated in Fig. 4. Or two beams can be accelerated. This can be done because the energy loss in a SC section is essentially zero.

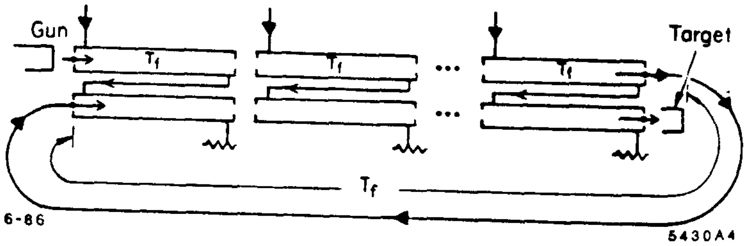


Fig. 4. Double pass recirculation scheme.

## FREQUENCY SCALING

The effect of frequency when keeping either the group velocity, or the section length, or the aperture size constant is now considered. As the linear dimensions vary as the frequency $f$, we infer from their definitions that for the same group velocity, geometry, and mode

$$s \propto f^2 \qquad \text{and} \qquad T_o \propto f^{-3/2} \quad . \tag{9}$$

Dividing $s$ by $f^2$ yields the structure parameter $s_g$ which is invariant with frequency. It depends only on structure geometry and mode. For the SLAC disk-loaded $2\pi/3$ structure $s_g$ and the aperture size $a$ as a function of group velocity are[4]

$$s_g = \frac{13.24}{1 + 0.216\sqrt{v_g}}, \qquad \frac{a}{\lambda} = 0.08 v_g^{1/4} \quad . \tag{10}$$

Here $s_g$ is in $\frac{M\Omega}{m} - ps$ $v_g$ is in $m/\mu s$.

The SLAC section $s_g = 9.37\ \frac{M\Omega}{m} - ps$. The SLAC section fill time and length as a function of frequency and improvement factor are

$$T_f = \tau T_o = \frac{6.95 I_f \tau}{f^{3/2}}, \qquad L = v_g T_f = \frac{6.95 I_f v_g \tau}{f^{3/2}} \tag{11}$$

Here $f$ is in GHz, $T_f$ in $\mu s$ and $L$ in meter. The improvement factor can be due to cooling the section or any other cause. Substituting the SLAC section $\tau = 0.57$, and $I_f = 1$ we obtain

$$T_f = \frac{3.96}{f^{3/2}}, \qquad L = \frac{3.96 v_g}{f^{3/2}} \quad . \tag{12}$$

Substituting the SLC gradient and repetition rate, $E_a = 21 MV/m$, $f_r = 180 pps$, and $\eta_s = 0.581$ into eq. (2) we obtain the peak power per section and the average power per meter

$$P_p\ (MW) = \frac{759.1 v_g}{s_g f^2}, \qquad p_a\ (kW/m) = \frac{136.6}{s_g f^2} \ . \tag{13}$$

The average power per section is

$$P_a\ (kW) = p_a L = \frac{541 v_g}{s_g f^{7/2}} \tag{14}$$

Using the above expressions we obtain the system parameters as a function of frequency listed in Table 1.

Table 1. Peak and average power vs frequency
$E_a = 21 MV/m$, $f_r = 180 pps$, and $\eta_s = 0.581$

| freq GHz | $T_f$ $\mu s$ | $v_g$ $m/\mu s$ | a cm | b cm | L m | $P_p$ MW | $p_a$ kW/m | $P_a$ kW |
|---|---|---|---|---|---|---|---|---|
| 1.0 | 3.96 | 3.66 | 3.31 | 5.95 | 14.5 | 296 | 14.6 | 221 |
| 2.86 | 0.82 | 3.66 | 1.16 | 2.09 | 3.00 | 36.4 | 1.79 | 5.36 |
| 10 | .125 | 3.66 | .331 | .595 | 0.46 | 2.96 | .146 | .0668 |
| 10 | .125 | 60.0 | .858 | 1.50 | 7.50 | 138 | .415 | 3.11 |
| 30 | .024 | 60 | .286 | .500 | 1.44 | 15.3 | .046 | .066 |
| 30 | .024 | 48 | .286 | .610 | 1.16 | 17.8 | .054 | .077 |

The rf to beam energy conversion efficiency is inversely proportional to $p_a$. Hence, it is clear from Table 1 that high frequencies are desirable. But, increasing the frequency decreases the aperture size and length. These can be remedied by increasing the group velocity which has the added advantage of increasing the section bandwidth and improving cooling and vacuum. The decrease in elastance and high peak to accelerating field ratio due to increased group velocity can be remedied by using nose cones and magnetic coupling.

Lines 4 and 5 of Table 1 are parameters for a large aperture disk-loaded structure. Its $s_g$ decreased from 9.37 of the SLAC section, $a/\lambda = 0.111$ to 3.29 for $a/\lambda = 0.286$. The values in the last line in Table 1 are for the zig-zag structure shown in Fig. 5. Changing from a disk-loaded structure to a zig-zag structure further improves the cooling, vacuum, and mechanical construction of the section. At high group velocity the elastance of a zig-zag section is about the same as that of a DLWG: $s_g = 2.82$ for the zig-zag and $s_g = 3.29$ for the DLWG.

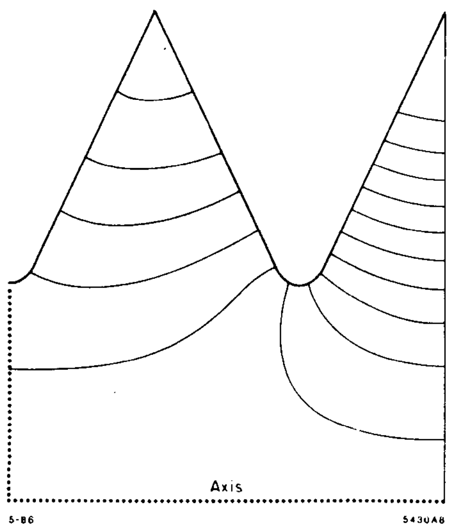


Fig. 5. Zig-Zag structure.

## PULSE COMPRESSION

With pulse compression energy is gathered over a long time interval and released in a short time interval. When used to accelerate a single bunch of charged particles the short time interval is the fill time $T_f$. The process is not 100% efficient. The peak power decreases by the multiplication factor and the average power increases by the reciprocal of the compression efficiency. The greater the compression factor the greater is the increase in average power. This is illustrated in Fig. 6, which shows plots of peak and average powers without pulse compression and with the present SLAC pulse compression system SLED.

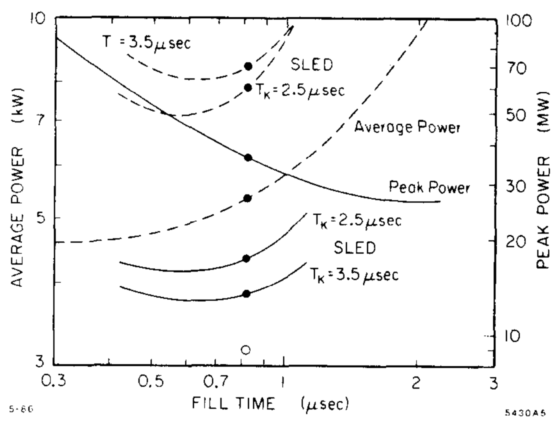


Fig. 6. Peak and average power with and without pulse compression.

The trade-off $T_f$ is lower with than without pulse compression, hence for the same length we have higher group velocity with the advantage of larger aperture size. With 100% efficiency pulse compression the average power is the same as with no compression and the peak power is reduced by the compression factor as shown by the single point in the figure.

## ELASTANCE AND INTERNAL TIME CONSTANT FROM CODES

The SW computer codes, for example URMEL,[5] does not give $s$ and $T_o$ directly, but it does give the maximum possible voltage gained by a charged particle traversing the cavity $V$, the stored energy and power dissipated in the cavity $W$, and $P_d$, and the cavity length $L$. From these we obtain:

$$s = \frac{V^2}{WL}; \qquad T_o = \frac{2W}{P_d}; \tag{15}$$

$s$ can also be computed from the given $R/Q$ or loss parameter $k_1$ and $T_o$ from the given $Q$:

$$s = \frac{\omega R/Q}{L} = \frac{k_1}{4L}; \qquad T_o = \frac{Q}{\pi f} \tag{16}$$

The elastance of a TW section can be obtained from TW computer codes such as TWAP. It can, also, be inferred from a SW cavity solution if the number $m$ of mechanical periods times the phase shift $\phi$ per mechanical period is $m\phi = n\pi$, assuming both ends are neumann boundaries; or if the one of the ends is dirichlet boundary and $m\phi = (2n+1)\pi$. The TW elastance is twice the SW elastance, except for $\pi$ mode when they are equal. This is illustrated in Fig. 7, which shows the SW elastance when $\phi = 2\pi/n$ obtained from URMEL and the TW elastance obtained from the TW code KN7C.

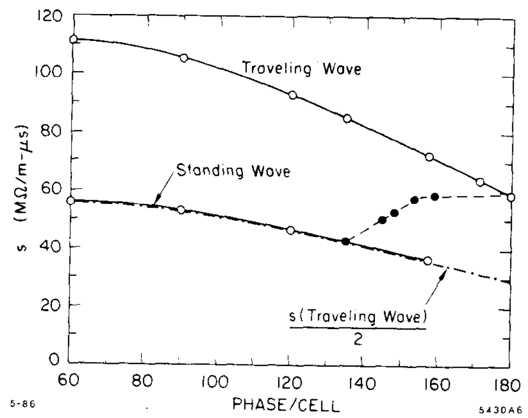


Fig. 7. TW and SW elastance vs phase shift/period.

But if one or more mechanical periods are terminated in a reactance rather than a short,[2] that is a shorted transmission line $\theta$ electrical degrees long, then $m\phi + \theta = n\pi$ and $\phi$ does not have to be a rational fraction of $\pi$ and the TW elastance is no longer double the SW elastance. This is illustrated in Fig. 7 by the line joining the SW and TW plots where the phase shift per cell is between 135 and 180 and is an irrational fraction of 180. It was obtained as follows. Two equal mechanical periods were terminated by a variable length shorted line. As the shorted line was made longer, the period lengths was decreased so as to maintain a 10 cm cavity length. The $b$ dimension was adjusted to resonate the cavity at 3 GHz, that is to make the average phase velocity of the cavity equal to $c$. The phase change/cell was obtained from the arc cosine of the field amplitude ratio between two adjacent cells.

The phase velocity can be obtained from URMEL. It is given by

$$\frac{v_p}{c} = \frac{\lambda}{\lambda_g} = \frac{n\lambda}{2L} \tag{17}$$

Here $n$ is the number of longitudinal phase reversals.

## CONCLUSION

For the same group velocity the elastance and hence both peak and average powers decrease as the square of the frequency therefore higher frequencies are favored. But the internal time constant and hence fill time and section length, for a given section efficiency also decreases. Pulse compression also favors high frequencies because the compression efficiency decreases as the ratio of pulse length to internal time constant increases. But, because here we are not concerned with acceleration, overmoded storage devices can be used and the internal time constant does not have to decrease. This is especially true for the BPM compression system[6] where with the use overmoded round $TE_{01}$ waveguide for energy storing delay lines, the internal time constant decrease as frequency increases.

Expression for peak and average power were given in terms of parameters that are identical for both TW and SW sections. Expressions for beam induced gradient and output power when the section operates in the traveling wave tube mode can also be expressed in terms of $s$, $T_o$, and $v_g$ for TW sections or $s$, $T_o$, and $T_e$ for SW sections. These expressions and expressions for the case of beam loaded sections have been given in a companion paper.[1] After the fill time is determined from peak and average power trade-off (peak and AC power trade-off for superconducting sections) the section parameters and peak and average power requirements of lossy, lossless TW and SW sections operating in the single bunch or CW mode can be obtained from the given expressions.

## REFERENCES

1. Z. D. Farkas, "New Formulation for Linear Accelerator Design," IEEE Transactions of Nuclear Science, NS-32, p. 2738, October 1985.

2. R. D. Miller "SW/TW Structures," this conference.

3. Z. D. Farkas "Superconducting Traveling Wave Accelerators," SLAC/AP-29, April 1984

4. P. B. Wilson "High Energy Electron Linacs," February 1982, SLAC-PUB-2824

5. T. Weiland, "On the computation of resonant modes in cylindrically symetric cavities," Nucl. Instrum. Methods 216 (1983), p. 329-348.

6. Z. D. Farkas "Binary Peak Power Multiplier and its Application to Linear Accelerator Design," SLAC-PUB-3694, May 1985.