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How chiral solitons relate $\overline{K}N$ and πN scattering*

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ABSTRACT

Large-N arguments suggest that baryons correspond to soliton solutions of the optimal low-energy Lagrangian of QCD. Such solitons are characterized by a hedgehog symmetry which mixes isospin and space rotations. We show that this symmetry implies linear relations between experimental $\overline{K}N$ and πN elastic partial wave scattering amplitudes. At least in one case these linear relations are satisfied with an extremely high accuracy.

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Several years ago it was suggested by Witten[1] that the low-energy effective Lagrangian of QCD is some kind of a generalized non-linear sigma model in which baryons correspond to soliton configurations of the chiral meson field. That picture strongly relies on the intuition obtained by formally considering QCD in the limit where the number of colors becomes large. The Skyrme model[2–4] is the simplest example of such a non-linear sigma model admitting soliton solutions. Recently there has been a great deal of activity in calculating the properties of baryons in the Skyrme model and several similar models.

In this paper we pursue a somewhat different direction. We will examine the consequences of the assumption that the optimal low energy effective Lagrangian of Nature is characterized by the same symmetry as the Skyrme model. That refers in particular to the assumption that the soliton which corresponds to the nucleon has a "hedgehog" shape[2] and is only invariant under simultaneous isospin and space rotations. While the detailed structure of the optimal Lagrangian is unknown, one can rely on the symmetry alone in order to derive model-independent predictions for relations between experimental scattering amplitudes. This approach has been previously applied to elastic and inelastic pion-nucleon scattering with rather satisfactory results.[5–8] Here we shall extend that framework to include processes involving strange particles. This extension involves some additional (mild) technical assumptions[8] and leads to linear relations between strange and non-strange partial-wave amplitudes which are rather well satisfied in Nature.

Consider a meson-baryon scattering process in which both the initial and the final states consist of a pseudoscalar meson octet and a $\frac{1}{2}^+$ octet or a $\frac{3}{2}^+$

decuplet baryon. The initial and the final state are described by the set of quantum numbers $\{LsRR_{\rm tot}\gamma I_{\rm tot}I_{z\rm tot}Y_{\rm tot}\mathbf{J}\}$, where L denotes the orbital angular momentum of the meson; s and R are the spin and the flavor representation of the baryon [*i.e.*, $(s,R) = (\frac{1}{2},\mathbf{8})$ or $(\frac{3}{2},\mathbf{10})$]; $\mathbf{J}$ is the total angular momentum of the meson-baryon system; $R_{\rm tot}$, $Y_{\rm tot}$, $I_{\rm tot}$ and I_{ztot} are the total $SU(3)_{\rm flavor}$ quantum numbers: the $SU(3)$ representation, the hypercharge, the isospin and its third component, respectively; and finally γ is a discrete index which serves to distinguish between degenerate representations that can occur in the product of two $SU(3)$ representations, as for example the $\mathbf{8}_{\rm sym}$ and $\mathbf{8}_{\rm antisym}$ in $\mathbf{8}\times\mathbf{8}$.

If the effective Lagrangian describing that process shares the symmetry of the Skyrme model, then one can show[5–9] that any experimental meson-baryon scattering amplitude $\mathbf{T}^{physical}$ can be expressed as a linear combination of a number of "*reduced*" amplitudes $\mathbf{T}^{reduced}$ labelled by a quantum number $\mathbf{K}$, which corresponds to vector sum $\mathbf{I}+\mathbf{L}$, of isospin $\mathbf{I}$ and orbital angular momentum $\mathbf{L}$. ($\mathbf{K}$ is the generator of the symmetry under which a chiral soliton remains invariant.)

This relation between the experimental and the reduced amplitudes can be schematically written as

$$\mathbf{T}^{physical} = \sum_n C_n \mathbf{T}_n^{reduced} \tag{1}$$

Here C_n are group theoretical factors, depending only on the hedgehog symmetry. They are tabulated in Ref. 8. The reduced amplitudes in the real world are not known, since they depend on the detailed structure of the optimal effective Lagrangian.

On a more explicit level, eq. (1) corresponds, in the three flavor case, to

$$
\begin{aligned}
&\mathbf{T}(\{LsRR_{\text{tot}}\gamma I_{\text{tot}}I_{z\text{tot}}Y_{\text{tot}}\mathbf{J}\} \to \{L's'R'R_{\text{tot}}\gamma' I_{\text{tot}}I_{z\text{tot}}Y_{\text{tot}}\mathbf{J}\}) \quad = \\
&(-1)^{s'-s}\frac{\sqrt{\dim R\cdot\dim R'}}{\dim R_{\text{tot}}}\sum_{\{IY\}}\sum_{i}\sum_{K}(2i+1)(2K+1)\begin{Bmatrix} KiJ \\ s'L'I \end{Bmatrix}\begin{Bmatrix} KiJ \\ sLI \end{Bmatrix} \\
&\times \quad \left(\begin{array}{c|cc} R_{\text{tot}}\gamma' & R' & 8 \\ i,1+Y & s'1 & IY \end{array}\right)\left(\begin{array}{cc|c} R & 8 & R_{\text{tot}}\gamma \\ s1 & IY & i,1+Y \end{array}\right)\mathcal{T}^{\{IY\}}_{KL'L}\,.
\end{aligned} \tag{2}
$$

where $\mathcal{T}^{\{IY\}}_{KL'L}$ are the reduced matrix elements; the quantities in parentheses are $SU(3)$ isoscalar factors[10] ; the quantities in braces are 6j-symbols; the pair $\{IY\}$ is summed over $\{1,0\},\{0,0\}$, and $\{\frac{1}{2},\pm1\}$ and the index K assumes integral values when $\{IY\} = \{1,0\}$ or $\{0,0\}$ and odd-half-integral values when $\{IY\} = \{\frac{1}{2},\pm1\}$, while the index i assumes odd-half-integral and integral values, respectively, in these cases. The derivation of (2) is carried out in detail in Refs. 8 and 9.

The role of the $\mathbf{K}$ symmetry in (2) can be understood by invoking an analogy with the role of isospin in $\pi^{\pm}N$ scattering: experimentally one measures four distinct elastic amplitudes involving a charged pion and a nucleon – π^+p, π^+n, π^-p and π^-n. However, since isospin is a good symmetry of Nature, these four physical amplitudes can be expressed as linear combinations of only two reduced $I = 1/2$ and $I = 3/2$ amplitudes:

$$
\mathbf{T}_{\pi^+n} = C_{1/2}\mathbf{T}_{I=1/2} + C_{3/2}\mathbf{T}_{I=3/2} \qquad \text{etc.} \tag{3}
$$

where $C_{1/2}$ and $C_{3/2}$ are Clebsch-Gordan coefficients. As a consequence, there is one linear relation connecting any three elastic $\pi^{\pm}N$ amplitudes. In a similar fashion, one can use eq. (2) to derive more extensive linear relations relating various partial wave meson-baryon scattering amplitudes. Such relations for non-strange processes are in general quite successful.[5,8] Here we shall extend this framework to relate strange and non-strange processes.

Meson-baryon scattering processes with one meson and one baryon in the initial and the final state can be divided into two categories. In the first, the angular momentum of the outgoing meson is equal to that of the incoming meson. In the second, the angular momentum of the meson L jumps by two units. ($|\Delta L| = 1$ is forbidden by parity conservation).

In what follows we shall concentrate on the processes in which L remains unchanged. In that case for, a given value of L, any physical meson-baryon scattering amplitude allowed by $SU(3)_{flavor}$ can be expressed as a linear combination of the eight reduced amplitudes: $\mathcal{T}^{\{1,0\}}_{L\pm1,LL}$, $\mathcal{T}^{\{1,0\}}_{LLL}$, $\mathcal{T}^{\{0,0\}}_{LLL}$, $\mathcal{T}^{\{\frac{1}{2},1\}}_{L\pm\frac{1}{2},LL}$, and $\mathcal{T}^{\{\frac{1}{2},-1\}}_{L\pm\frac{1}{2},LL}$. In the Skyrme model, where the reduced amplitudes can be explicitly calculated, it turns out that, for each $L \geq 3$, the three reduced amplitudes,

$$\mathcal{T}^{\{1,0\}}_{L-1,LL},\ \mathcal{T}^{\{1,0\}}_{LLL},\ \text{and}\ \mathcal{T}^{+\}}_{L-\frac{1}{2},LL} \equiv \tfrac{1}{2}\left(\mathcal{T}^{\{\frac{1}{2},1\}}_{L-\frac{1}{2},LL} + \mathcal{T}^{\{\frac{1}{2},-1\}}_{L-\frac{1}{2},LL}\right)$$

are significantly larger than the other five.[8] This two-tiered hierarchy results from three different sources: (a) smallness of the Wess-Zumino term[1,11] contribution to reduced amplitudes; (b) repulsive contribution to reduced amplitudes with $K > L$; (c) vanishing of $\mathcal{T}^{\{0,0\}}_{LLL}$. The hierarchy among reduced amplitudes suggested by the Skyrme model is most probably also present in Nature, since it gives the simplest explanation of the relative sizes of the experimental πN and $\overline{K}N$ elastic partial wave amplitudes and it is also responsible for the success of the linear relations between non-strange experimental amplitudes.[5,8]

With this in mind, we can neglect five of the reduced amplitudes and make an assumption that, for a given L, any meson- baryon scattering process in Nature can be approximately expressed, via Eq. (2), as a linear combination of three reduced amplitudes. This implies one linear relation between any four physical amplitudes with the same L. In order to decide which physical amplitudes we should look at, it is useful to remind oneself that experimentally elastic πN scattering is characterized by an interesting pattern relating the magnitudes of the four independent amplitudes with a given L:

$$\mathbf{T}^{\pi N}_{I=1/2,J=L-1/2}, \quad \mathbf{T}^{\pi N}_{I=1/2,J=L+1/2}, \mathbf{T}^{\pi N}_{I=3/2,J=L-1/2} \quad \text{and} \quad \mathbf{T}^{\pi N}_{I=3/2,J=L+1/2}.$$

Typically $\mathbf{T}^{\pi N}_{I=1/2,J=L-1/2}$ and $\mathbf{T}^{\pi N}_{I=3/2,J=L+1/2}$ are much larger than $\mathbf{T}^{\pi N}_{I=1/2,J=L+1/2}$ and $\mathbf{T}^{\pi N}_{I=3/2,J=L-1/2}$.

An analogous albeit less sharp pattern holds for $\overline{K}N$ scattering with $I = 0$ and $I = 1$ replacing $I = 1/2$ and $I = 3/2$.

Thus for a given L, there are eight independent elastic πN and $\overline{K}N$ amplitudes, out of which typically four are larger than the others. That pattern emerges naturally in the context of the soliton picture of the baryon:[5,8] since the physical amplitudes are linear superpositions of reduced amplitudes, the question which physical amplitudes will be large is determined by the relative size of the group theoretical coefficients multiplying the reduced amplitudes. The physical amplitudes which are relatively large typically receive relatively big contributions from the three large reduced amplitudes. In other words, the approximation in which five small reduced amplitudes are neglected is expected to be more accurate for the physical amplitudes which are dominated by the large reduced amplitudes. It is therefore natural to write down a linear relation connecting these four amplitudes.

Taking the group-theoretical factors from eq. (2), we have the following prediction:

$$\begin{aligned}\frac{410L+631}{479(L+2)}\mathbf{T}^{\pi N}_{I=1/2,J=L-1/2} &- \frac{702L^2+1279L+506}{958L(L+2)}\mathbf{T}^{\pi N}_{I=3/2,J=L+1/2} = \\ \mathbf{T}^{\overline{K}N}_{I=0,J=L-1/2} &- \frac{559L+275}{479L}\mathbf{T}^{\overline{K}N}_{I=1,J=L+1/2}\end{aligned} \tag{4}$$

In the Skyrme model the agreement between theory and experiment is usually best for $L=3$, *i.e.* F-waves.[8,12–14] . We expect this to be true not only in the Skyrme model but rather in the whole class of models to which eq. (2) applies. This expectation is reinforced by the results of Ref. 5 which examines model-independent relations between non-strange experimental amplitudes.

The reason for this pattern may be understood with the help of the following observation. In a model in which the nucleon is treated as a soliton of a chiral Lagrangian, the physical pseudoscalar mesons correspond to small fluctuations of the chiral field around the soliton solution. Some fluctuations of the chiral field however *do not* correspond to mesons, but rather reflect the motion of the soliton as a whole, either under translation or under rotation. A proper treatment of such *zero-modes* should disentangle them from the physical meson excitations. Eq. (2) is derived in the leading order in the $1/N_c$ expansion, and therefore it neglects nucleon recoil. As a result, the unphysical zero-modes mix with physical meson excitations in the low partial waves – S, P and D.[5,13] On the other hand the experimental results are less accurate for higher L, because as L grows extraction of partial waves from the experimental cross-sections becomes more difficult. Consequently, we expect eq. (4) to work best for the lowest L which is free from the zero-mode problem – $L = 3$, *i.e.* the partial waves F_{15} and F_{37} in $\pi N \to \pi N$ on the one hand, and F_{05} and F_{17} in $\overline{K}N \to \overline{K}N$ on the other hand. (πN partial waves are labelled by $L_{2I,2J}$ while $\overline{K}N$ partial waves are labelled by $L_{I,2J}$.)

The relation (4) is derived in the idealized case of exact $SU(3)_{flavor}$, *i.e.* $m_s = m_u = m_d$. In Nature, the u and d quarks are very light, while the strange quark mass is about 150 MeV and in order to compensate for that we need to shift the elastic $\overline{K}N$ amplitudes by 150 MeV. Other than this, the relevant amplitudes are taken directly from the experimental partial wave analyses.[15,16]

The two linear combinations are plotted on Fig. 1. They agree to a remarkable degree of accuracy. It should be stressed that two curves result directly from superposition of experimental data and the only outside input is the strange quark mass. The lower limit of the center of mass energy range shown is determined by the $\overline{K}N$ threshold (minus 150 MeV) while the upper limit is taken somewhat above the energy range covered by the existing $\overline{K}N$ experimental partial wave-analysis[16]. The purpose is to provide a prediction which will be tested when the $\overline{K}N$ partial-wave analysis is extended to higher energies.

Although the case $L = 3$ is expected, on theoretical grounds, to work best, it is instructive to examine higher angular momenta as well. The relevant experimental $\overline{K}N$ partial-wave solutions published so far go only up to $L = 4$, *i.e.* G-waves, and this is the case we examine in Figure 2. The shapes of the two curves in the $Im(T)$ *vs.* $Re(T)$ representation are rather similar but the magnitudes and the energy dependence differ substantially. This can be traced back to the smallness of the experimental G_{19} elastic $\overline{K}N$ amplitude in the partial wave solution.[16] The fact that G_{19} is small violates the earlier mentioned pattern relating the magnitude of the four independent amplitudes with a given L. It would be very interesting to re-analyze the experimental data regarding that particular issue.

At this point it should be emphasized that there is no known way of deriving these relations using more conventional symmetries like $SU(3)_{flavor}$ or $SU(6)$. While $SU(3)_{flavor}$ is part of the symmetry used to derive Eq. (2), it is clear that $SU(3)_{flavor}$ alone cannot produce such relations, since they mix amplitudes with different total angular momentum. As for $SU(6)$, the relevant candidate symmetry is the so-called $SU(6)_W$. As far as I know, no linear relations analogous to the ones presented here have been published. In addition one needs to remember that $SU(6)_W$ can only relate various strange and non-strange partial wave amplitudes under two rather stringent assumptions: a) the amplitudes have to be purely resonant; b) the mass differences of the relevant resonances can be neglected compared to their width. The second of these assumptions is certainly not satisfied by the F-wave data.

If conventional symmetries cannot explain the success of the F-wave linear relations, than these relations should be viewed as evidence in favor of the view that a nucleon is indeed a soliton in a field of mesons, as suggested by large-N arguments. We hope that in future the connection between this viewpoint and QCD will be put on a more quantitative basis.

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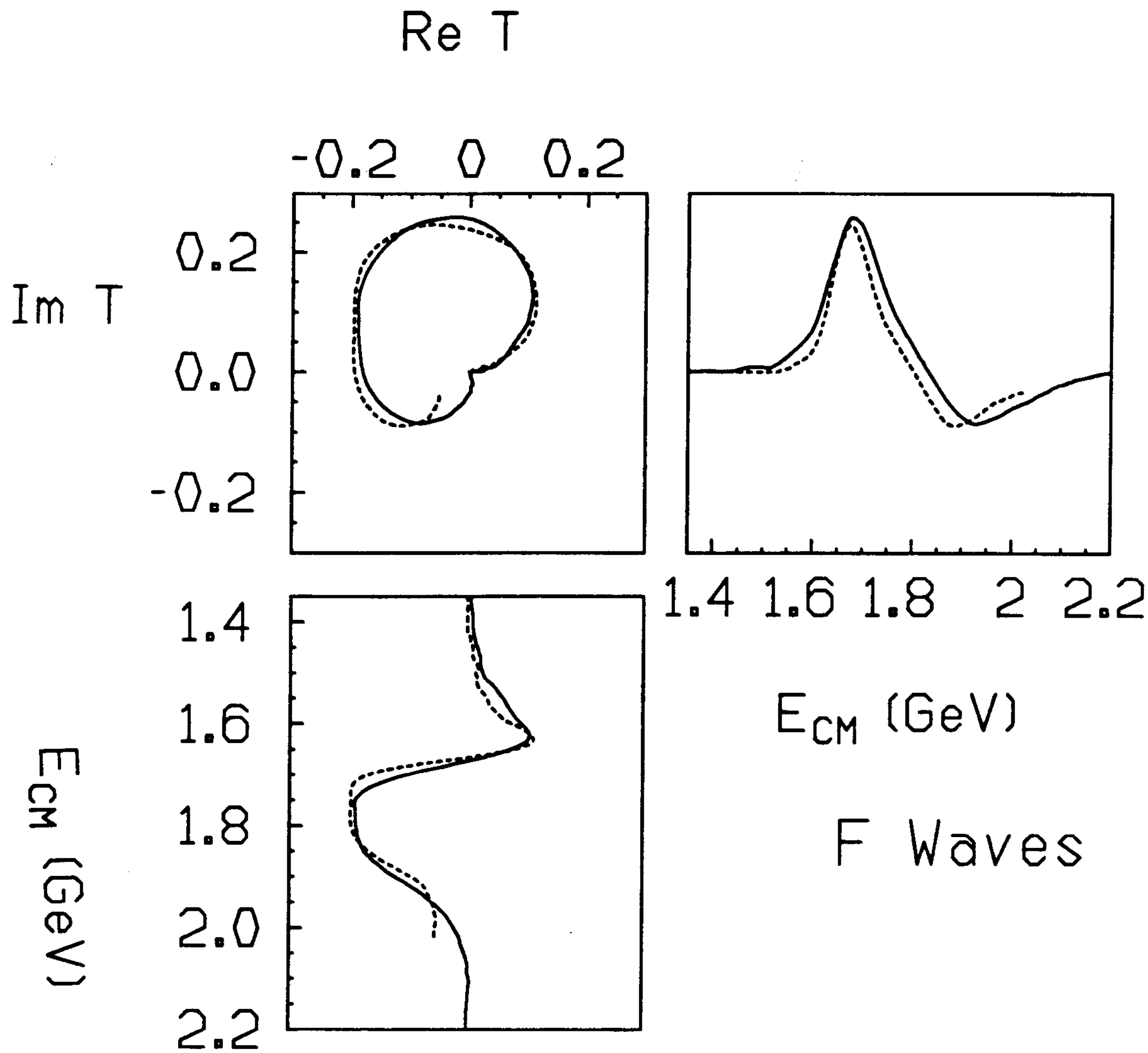

Fig. 1. Test of the linear relation (3) for F-waves. The scattering matrix T is plotted both as function of energy and in $Im(T)$ *vs.* $Re(T)$ representation. Continuous lines show the linear combination of F_{15} and F_{37} experimental πN amplitudes while dotted lines show the linear combination of F_{05} and F_{17} experimental $\overline{K}N$ amplitudes. $\overline{K}N$ amplitudes here and in Figure 2 are shifted by $m_s \approx 150$ MeV.

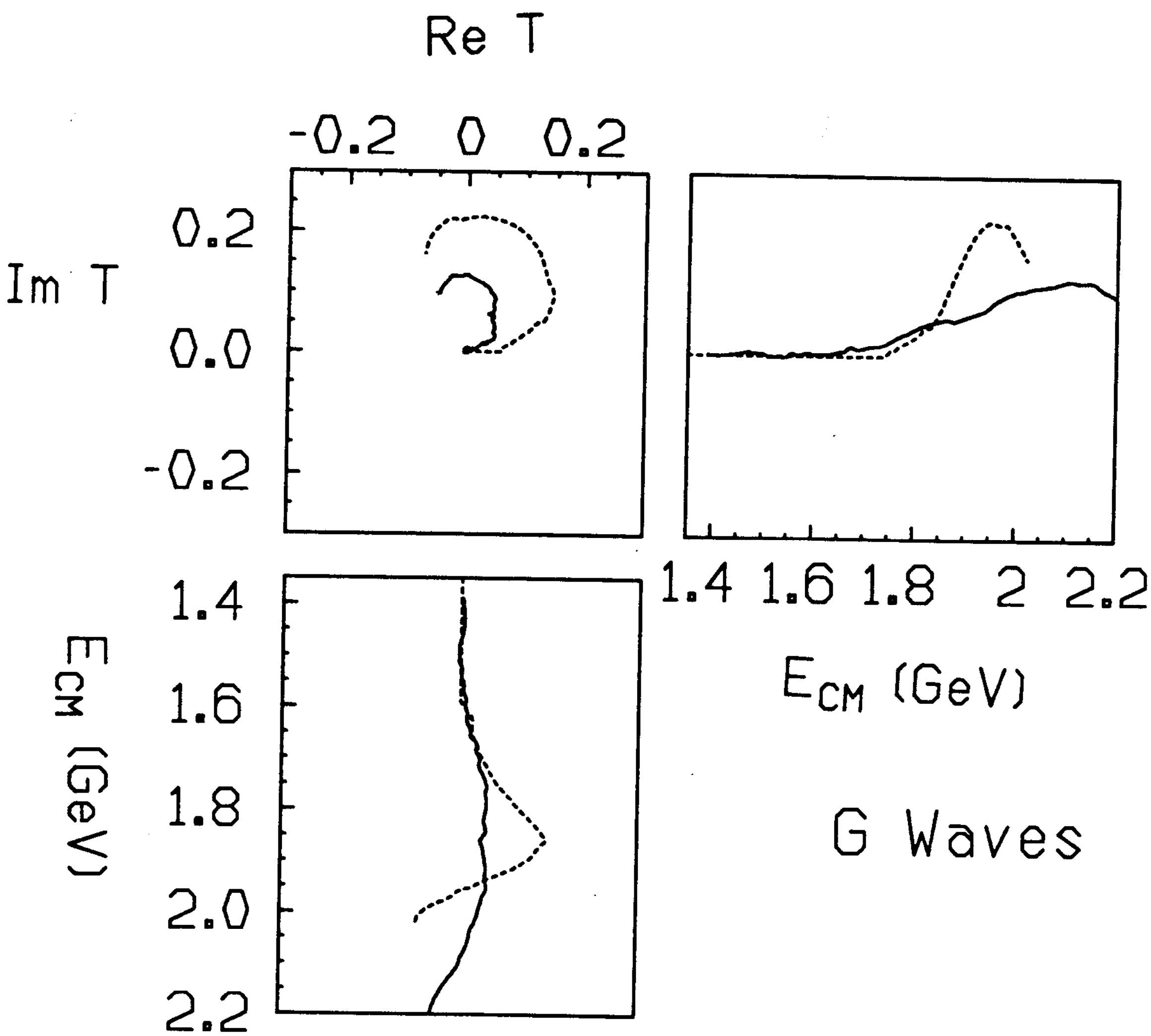

Fig. 2. Test of the linear relation (3) for G-waves. Continuous lines show the linear combination of G_{17} and G_{39} experimental πN amplitudes while dotted lines show the linear combination of G_{07} and G_{19} experimental $\overline{K}N$ amplitudes.