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CENTRAL CHARGE OF THE VIRASORO ALGEBRA FOR SUPERSYMMETRIC SIGMA MODELS ON CALABI-YAU MANIFOLDS*

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ABSTRACT

It is shown that the central charge of the Virasoro algebra for a conformally invariant supersymmetric σ-model on a Calabi-Yau manifold remains equal to its free field value to order α'^4 despite the non-Ricci-flatness of the background metric. Various possibilities for higher loop contributions are discussed.

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It has become clear through recent studies that N=2 supersymmetric σ-models on Calabi-Yau manifolds with a Ricci-flat Kahler metric have non-vanishing β-functions at the four loop order[1–4] thus destroying the expectation that such models have vanishing β-functions to all orders in the perturbation theory.[5] It was shown in Ref.[6], however, that it is always possible to choose a Kahler metric on a Calabi-Yau manifold such that the β-function vanishes to all orders in the perturbation theory, thus providing us with a conformally invariant two dimensional field theory. Such theories are very much of current interest since they provide exact solutions of the classical string field equations by the conjectured equivalence between the equations of motion of the massless fields in the string theory and the criteria for the vanishing of the β-functions in two dimensional σ-models.[7–14] In order to satisfy the equations of motion of all the massless fields, however, it is not enough to have all the σ-model β-functions vanish.† It is also necessary that the central charge of the Virasoro algebra in this two dimensional system be identical to its free field value, i.e. the value obtained in the lowest order in the perturbation theory. (The two sets of conditions together imply the conformal invariance of the σ-model in a curved two dimensional background). In this paper we shall show that the central charge of the Virasoro algebra for a conformally invariant supersymmetric σ-model on a Calabi-Yau manifold does not receive any correction to order α'^4, α'^{-1} being the string tension.

An explicit calculation of the σ-model correction to the central charge beyond the lowest non-trivial order is a task of considerable difficulty, although it has been calculated exactly for some manifolds, namely the group manifolds.[15] A different strategy was used by Gross and Witten.[2] They calculated the effective action in the string theory directly by calculating the string scattering amplitudes, and from that derived the equations of motion of various massless fields in the string theory. If these equations are satisfied for a given background, then the

† Throughout this paper the word β-function will refer to the standard σ-model β-functions in flat two dimensional space, and will not include the central charge of the Virasoro algebra.

corresponding σ-model should automatically have vanishing β-functions as well as vanishing correction to the central charge. This is the approach we shall pursue in this paper. It will be shown that all the equations of motion are satisfied to order α'^3, despite the fact that the background metric is not Ricci flat. This implies the vanishing of the central charge to order α'^4.

We start by writing down the most general effective action for type II superstring theory involving the dilaton (ϕ) and the graviton ($G_{\mu\nu}$) field:

$$S = \int d^{10}x e^{\phi} \sqrt{G} f(\phi, G_{\mu\nu}) \tag{1}$$

There are other massless fields in the theory, for simplicity we have set their vacuum expectation values (vev) to zero. A standard scaling argument[16,17] shows that f must be invariant under a constant shift in ϕ, hence it may involve derivatives of ϕ but not ϕ itself. The lowest order contribution to f is given by[10,17]

$$f^{(0)}(G, \phi) = R - (D\phi)^2 - 2D^2\phi \tag{2}$$

where R is the scalar curvature and D denotes the covariant derivative. ϕ independent contribution to f has been calculated by Gross and Witten[2] to order α'^3. It was shown that the order α' and α'^2 contributions vanish, whereas the order α'^3 contribution is non-zero on a general manifold. Let us call this contribution $\alpha'^3 Y$. For the time being we shall carry out our analysis by replacing f by $f^{(0)} + \alpha'^3 Y$ in Eq.(1). Later we shall argue that higher order ϕ dependent contribution to f does not change our conclusion. With this action the equations of motion for $G_{\mu\nu}$ and ϕ may be written as,

$$[R_{\mu\nu} - D_\mu D_\nu \phi + \alpha'^3 W_{\mu\nu}] - \tfrac{1}{2} G_{\mu\nu}[R - 2D^2\phi - (D\phi)^2 + \alpha'^3 Y] = 0 \tag{3}$$

$$R - 2D^2\phi - (D\phi)^2 + \alpha'^3 Y = 0 \tag{4}$$

ignoring terms of order $\alpha'^3 D\phi$. These terms will turn out to be of order α'^6 in the background we shall consider, and hence will not affect the equations of motion to order α'^3. $W_{\mu\nu}$ denotes the variation of Y with respect to $G^{\mu\nu}$ and was calculated in Ref.[3]. It is best expressed in complex coordinates, so we introduce holomorphic and anti-holomorphic coordinates z^i and $\bar{z}^i$ respectively on the manifold. Since Y is already accompanied by a factor of α'^3, and since our background metric will differ from a Ricci-flat Kahler metric only at order α'^3, we may substitute for Y and $W_{\mu\nu}$ their values for a Ricci-flat Kahler metric. It was shown that for a Ricci-flat Kahler metric Y vanishes and $W_{\mu\nu}$ has the form:

$$W_{\mu\nu} = \tilde{W}_{\mu\nu} - D_\mu D_\nu Q \tag{5}$$

$$\tilde{W}_{ij} = \tilde{W}_{\bar{i}\bar{j}} = 0$$

$$\tilde{W}_{i\bar{j}} = 2D_i D_{\bar{j}} Q \tag{6}$$

where Q is a scalar proportional to the Euler density. Both the equations (3) and (4) are satisfied to order α'^3 if,

$$R_{i\bar{j}} + 2\alpha'^3 D_i D_{\bar{j}} Q = 0 \tag{7}$$

$$R_{ij} = R_{\bar{i}\bar{j}} = 0 \tag{8}$$

$$\phi = -\alpha'^3 Q \tag{9}$$

It was shown in Ref.[6] that there always exists a Kahler metric satisfying equations (7) and (8) on a Calabi-Yau manifold. Eq.(9) is a new equation. The reader may be puzzled by the fact that the vanishing of the σ-model β-function, which in this case is given by $R_{\mu\nu} - D_\mu D_\nu \phi + \alpha'^3 W_{\mu\nu}$, requires a non-vanishing ϕ, since in the analysis of Ref.[6] we did not need any dilaton field for the vanishing of the β-function. In fact we implicitly had to set the dilaton field to zero, since a non-zero vev of the dilaton field would introduce unwanted contribution to the

β-function of the form $D_i D_j \phi$ and $D_{\bar{i}} D_{\bar{j}} \phi$ in the analysis of Ref.[6]. The point is that the dilaton field which had to be set to zero in Ref.[6] is related to the one that appears in this paper by a local field redefinition of the form $\phi' = \phi + \alpha'^3 Q$.* Indeed, ϕ' must vanish in order to have vanishing β-function. Our result implies that if we had calculated the central charge in the scheme of Ref.[6] with a vanishing dilaton field ($\phi' = 0$), it would be proportional to $R + 2\alpha'^3 D^2 Q$ to order α'^3 and would vanish whenever Eqs.(7) and (8) are satisfied.

We shall now briefly comment on the inclusion of higher order ϕ dependent terms in f. Since these terms involve derivatives of ϕ, they are of order α'^4 or higher when evaluated in the background given in Eqs.(7)-(9), and may almost always be ignored. The only exception are the terms linear in ϕ, since they may give a ϕ independent contribution to the dilaton field equation (Eq.(4)). Now, if the equations of motion correspond to the criteria for conformal invariance of the σ-model in a curved two dimensional background, then terms inside each of the square brackets in Eq.(3) must vanish separately. (The reason is that the σ-model β-functions cannot involve explicit factor of $G_{\mu\nu}$ [20]). This will give us too many equations for them to be interpreted as the criteria for vanishing of the β-function and the correction to the central charge in the σ-model, unless Eq.(4) follows from the two equations obtained from Eq.(3). Since we have argued that Eq.(3) does not get affected by the presence of the higher order ϕ dependent terms in f to this order, Eq.(4) must also remain unaffected by the presence of such terms. Hence we conclude that if the string equations of motion indeed correspond to the vanishing of the σ-model β-functions and the correction to the central charge, then the ϕ dependent terms in f cannot affect our conclusion.

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* Alternatively, the term $D_\mu D_\nu Q$ in the σ-model β-function may be absorbed by a renormalization scale dependent redefinition of the bosonic fields X^μ of the σ-model.[18] . These are in fact equivalent descriptions. In flat space-time, the presence of the dilaton field corresponds to the addition of a term proportional to $\partial_\alpha \partial_\beta \phi - g_{\alpha\beta} \partial^2 \phi$ to the two dimensional energy-momentum tensor.[12,14,19] The presence of this term modifies the transformation law of the fields X^μ under a scale transformation. The same effect is achieved by a scale dependent redefinition of the σ-model fields X^μ.

We conclude our discussion by considering the two possibilities which may occur at higher orders in the perturbation theory:

a)The central charge of the virasoro algebra may remain unrenormalized to all orders in the perturbation theory. The central charge of a two dimensional field theory is independent of the coordinates of the internal manifold when the σ-model is conformally invariant in flat space-time.[10,14] This fact may prove particularly useful in proving the above result. This result would be consistent with the general arguments of Ref.[21] showing the stability of the Calabi-Yau vacuum.

b) It may turn out that the central charge of the Virasoro algebra receives non-vanishing contribution beyond order α'^4 on a general Calabi-Yau manifold. Most of the Calabi-Yau manifolds are, however, parameterized by several continuous parameters, and we may expect the central charge to depend on these parameters. Since the central charge is a constant, one would expect that there will be a subspace of this parameter space where the correction to the central charge vanishes. This will tell us that not all Calabi-Yau manifolds but only a subset of those are candidates for string compactification.

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