SLAC – PUB – 3954
May 1986
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Note on the Baryon Asymmetry Thermalized by the Anomalous Electroweak Processes*

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ABSTRACT

We comment on the thermalized cosmological baryon asymmetry in the presence of the anomalous electroweak interactions and correct the value given by Kuzmin *et al.*

submitted to Physics Letters B

⋆ Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.
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Recently the anomalous electroweak effects due to the instanton like contributions draw attention in connection with the baryon asymmetry of the universe. Kuzmin *et al.* [1] claimed that at enough high temperature of the order of 200 Gev, the anomalous processes give a competing rate against the expansion rate of the universe. It means that the baryon number non-conserving reactions take part in up to that temperature to bring about the complete thermal equilibrium. They referred to a master equation for the total baryon and lepton numbers,

$$\frac{dB}{dt} = \frac{dL}{dt} = -\tau^{-1}(B+L) \ , \tag{1}$$

and concluded that the thermalized baryon number at $T_c \sim 200\text{Gev}$ is given by

$$\langle B \rangle = \frac{1}{2}(B_{\text{in}} - L_{\text{in}}) \ , \tag{2}$$

where B_{in} (L_{in}) are primordial baryon (lepton) asymmetries produced by any other non-equilibrium processes until that time. Using Eq.(2), Fukugita and Yanagida [2] noted that, for getting the present baryon asymmetry, B_{in} is not necessarily to be produced by, say, the usual GUT scenario, supposing that L_{in} can be generated via heavy Majorana neutrino decay products.

In this note, we comment on the formulae above. We claim that the final thermalized baryon number, Eq.(2), should read

$$\langle B \rangle = \frac{1}{4}(B_{\text{in}} - L_{\text{in}}) \ . \tag{3}$$

In order to clarify the origin of the factor appearing above, we take a case with $\text{SU}(N_c)$ color N-plet quarks whose baryon number is $1/N_c$.

First of all, we note that, if the thermal equilibrium is realized, then one does not have to consider any master equations. Instead, one can immediately write down the general canonical distribution formula which is assured by an infinite heat bath:

$$P(n) \propto \exp(-\mu_i Q_i) \ , \tag{4}$$

where Q_i are conserved additive quantum numbers of the system, and μ_i are the corresponding generalized chemical potentials which may be positive or negative. In our case, the conserved quantities are the energy and the B – L quantum number which is respected even by the anomalous interactions. The partition function is expressed as

$$Z = \sum \exp(-\beta E - \mu_{\rm B-L} n_{\rm B-L}) \ , \tag{5}$$

$$n_{\rm B-L} = \sum_i^{N_c} \frac{1}{N_c} n_{\rm q}^i \ - \ n_{\rm l} \ , \tag{6}$$

where $n_{\rm q}^i$ ($n_{\rm l}$) is the difference between the numbers of particles and antiparticles for quarks (leptons).

To see the thermal expectation value of n_a's ($a = i$ for quarks, $a = N_c + 1$ for leptons),

$$\langle n_a \rangle = \frac{1}{Z} \sum n_a \exp(-\beta E - \mu_{\rm B-L} n_{\rm B-L}) \ , \tag{7}$$

we expand it with respect to $\mu_{\rm B-L}$. For a small symmetry breaking where the ratio of $\langle n_a \rangle$ to the entropy $\ll 1$, the lowest order estimate will give a fairly good approximation:

$$\langle n_a \rangle \sim \langle n_a \rangle_0 - \mu_{\rm B-L}(\langle n_a n_{\rm B-L} \rangle_0 - \langle n_a \rangle_0 \langle n_{B-L} \rangle_0) \ , \tag{8}$$

where $\langle \ \rangle_0$ means an expectation value with $\mu_{\rm B-L} = 0$ in Eq. (7). Due to the CPT invariance of the theory [3], $\langle n_a \rangle_0$ vanishes exactly. Neglecting the off diagonal parts and quark-lepton differences of the two body correlation, since they come from higher loop effects and/or finite mass effects, we find that

$$\left\langle n_{\mathrm{q}}^{i}\right\rangle = -\frac{1}{N_c}\left\langle n_{\mathrm{l}}\right\rangle = -\frac{1}{N_c}\mu_{\mathrm{B-L}}\left\langle n_l^2\right\rangle_0 \ , \tag{9}$$

and therefore

$$\langle B\rangle = -\frac{1}{N_c}\langle L\rangle \ . \tag{10}$$

This indicates that

$$\langle B\rangle = \frac{1}{N_c+1}\langle B-L\rangle = \frac{1}{N_c+1}(B_{\mathrm{in}}-L_{\mathrm{in}}) \ , \tag{11}$$

which gives Eq.(3) for $N_c = 3$.

Now we check the above formula by deriving the corresponding master equation for the anomalous processes. A general form of the master equation is written as

$$\begin{aligned}\frac{dn(p_1)}{dt} = \sum_{mn}\int\prod_1^m dp\prod_1^n dp'\{\Gamma(p'\to p)\prod_1^n n(p')\prod_1^m(1\pm n(p))\\ -\Gamma(p\to p')\prod_1^m n(p)\prod_1^n(1\pm n(p'))\} \ ,\end{aligned} \tag{12}$$

where $n(p)$ is the particle density of a state defined by a set of quantum numbers p ; Γ is a rate of the process; and $\pm$ is taken as $+$ for bosons and $-$ for fermions. Due to the unitarity, the fixed points of the above equation are given by the type of solutions in Eq.(4), whether the microscopic T-invariance holds or not [4]. (Of course this guarantees the consistency between the distribution(4) and the master equation (12).) Let us expand the equation (12) around the fixed point (4), introducing a deviation via defining different chemical potentials for each fermion. Getting a linearized master equation for chemical potentials, we transform it into that for $\langle n_a\rangle$ by using the linear relation in Eq.(9)[5]:

$$\frac{d\langle n_a\rangle}{dt} = M_{ab}\langle n_b\rangle \ , \tag{13}$$

where the matrix M_{ab} is (N_c+1) dimensional matrix.

Taking a common $\langle n_a \rangle$ value for quarks of every color, the only interactions contributing to the master equation is those of violating B and L conservation, since, for any other interactions, the trial function used above is already their zero mode function. In the lowest order approximation, the anomalous interactions has an exchange symmetry among each kind of fermions, because they do not discriminate the fermion species, quarks or leptons. Therefore the driving matrix in Eq.(13) can be approximated such that

$$M_{ab} = C_1 \delta_{ab} + C_2 u_{ab} \ , \tag{14}$$

where $u_{ab} = 1$ for all a , b. The above matrix is easily diagonalized. Noticing that the anomalous processes conserve N_c quantities, $n_{\rm q}^i - n_{\rm l}$ $(i = 1, N_c)$, we conclude that C_1 must vanish and M has only one non-zero eigenvalue whose eigenmode $n_{\rm NZ}$ is defined by

$$n_{\rm NZ} = \sum_i^{N_c} n_{\rm q}^i + n_{\rm l} \ , \tag{15}$$

that is, the sum for all relevant fermions. Hence the correct master equation should read

$$\frac{d}{dt} \langle N_c B + L \rangle = -\tau^{-1} \langle N_c B + L \rangle \ , \tag{16}$$

instead of Eq. (1). This equation leads to exactly the same baryon asymmetry as is given by Eq.(11).

In conclusion, the additional factor in Eq.(11) comes from the fact that the particle excess of a fermion depends on its charge of a conserved quantum number (which is B − L in this case) . Only the conserved quantities determine the canonical distribution and the charge gives the effective chemical potential for each component.

The author would like to thank K. Seo and J. Frieman for discussions, and the Japan Society for the Promotion of Science for the financial support.

REFERENCES

1. V.A.Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. 155B (1985) 36.

2. M. Fukugita and T. Yanagida, preprint RIFP-641.

3. S. Dimopoulos and L. Susskind, Phys. Rev. D18 (1978) 4500.

4. B. Toussaint, S. B. Treiman, F. Wilczek and A. Zee, Phys. Rev. D19 (1979) 1036; S. Weinberg, Phys. Rev. Lett. 42 (1979) 850.

5. S. B. Treiman and F. Wilczek, Phys. Lett. 95B (1980) 222.