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CP VIOLATION IN HEAVY FLAVOR DECAYS: PREDICTIONS AND SEARCH STRATEGIES*

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Abstract

We present a comprehensive analysis of the phenomenology relevant for CP violation in exclusive B and D decays. Some of the CP asymmetries can be calculated rather reliably in the Standard Model, while others contain large uncertainties. We point out that studies of inclusive decays will have to deal with partial cancellations occurring between different exclusive channels. None of the measurements will be easy. Yet a realistic evaluation of the promise the various searches hold out cannot be given at present; first, one has to build a data base on the production and decay properties of heavy flavor decays. We try to identify the required information.

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Introduction

More than twenty years have passes since the discovery of CP violation in the decays of neutral kaons. Despite strenuous efforts, we cannot claim to possess a proven description of this phenomenon, let alone a deeper understanding. We believe this unfortunate situation will not change unless CP violation is found in a different dynamical system.

The weak decays of bottom mesons hold out the promise to allow such a discovery. Some decay modes are expected to exhibit relatively large CP asymmetries. Furthermore, we can have greater confidence in the theoretical treatment of B mesons than of kaons. On the other hand, bottom mesons are much shorter lived than kaons; thus, different methods have to be employed to exhibit possible or even expected CP asymmetries.

A number of papers on this subject have appeared in the literature; nevertheless we think that a comprehensive and updated treatment of these phenomena is called for. By doing so, we hope to clarify some of the considerable confusion that has arisen in parts of the literature. At the same time, we want to suggest further directions in research — both experimental and theoretical — that would be of great benefit in this context.

The paper will be organized as follows: in sect. 1 we present a summary of our analysis on CP violation in bottom and charm decays; the remainder of the paper will be devoted to explaining our findings in more detail. Section 2 contains the phenomenology of those CP asymmetries that involve mixing before we employ the Standard Model to calculate or at least estimate the relevant quantities in sect. 3. In sect. 4 we treat CP asymmetries that can exist even in the absence of mixing, namely those involving final state interactions, and in sect. 5 we list different strategies to search for CP violation in B decays and present numerical estimates. It is obviously premature to give definite numerical predictions when so little is known about branching ratios, *etc.;* therefore, we discuss in sect. 6 how future data on B production and decays will allow us to sharpen our predictions. At this point we also want to stress that although we phrase our analysis for the case of B decays, it is actually a generic treatment that is easily applied to charm decays as well. We will comment on that at the appropriate places.

1. Summary

CPT invariance implies that CP violation can enter only via complex phases. Thus CP asymmetries can show up only if at least two coherent amplitudes contribute to a given process. There are basically two ways in which this can happen: (a) via particle-antiparticle mixing and (b) via final state interactions.

1.1 MIXING AND CP VIOLATION

Mixing describes a situation where the mass eigenstates are a coherent superposition of a particle and its antiparticle.

The time evolution of a meson that was produced as a B° (bottom) or $\overline{B}^\circ$ meson, respectively, at time $t = 0$ is given by [1]

$$|B^\circ(t)\rangle = g_+(t)\,|B^\circ\rangle + \frac{q}{p}\,g_-(t)\,|B^\circ\rangle \tag{1.1}$$

$$\left|\overline{B}^\circ(t)\right\rangle = \frac{p}{q}\,g_-(t)\,|B^\circ\rangle + g_+(t)\left|\overline{B}^\circ\right\rangle \tag{1.2}$$

$$g_\pm(t) = \tfrac{1}{2}\exp\left\{-\tfrac{\Gamma_1}{2}t\right\}\ \exp\{im_1 t\}\ \left(1 \pm \exp\left\{-\tfrac{\Delta\Gamma}{2}t\right\}\ \exp\{i\Delta m t\}\right) \tag{1.3}$$

$$\Delta\Gamma = \Gamma_2 - \Gamma_1\ ; \qquad \Delta m = m_2 - m_1\ ; \qquad \frac{q}{p} = \frac{1-\epsilon}{1+\epsilon} \tag{1.4}$$

$\Gamma_i, m_i, i = 1,2$ are the width and mass of the two mass eigenstates B_i. In the following we will set $\Delta\Gamma = 0$ for convenience. As far as CP violation is concerned, no new features are introduced by $\Delta\Gamma \neq 0$; secondly, for B° mesons one calculates $\Delta\Gamma \ll \Delta m$ with considerable confidence [2] (although this might not hold for D° mesons [3,4]).

Mixing will manifest itself most clearly via decays of neutral bottom or charm mesons that lead to "wrong sign" leptons [1,5]:

$$r_B = \frac{\Gamma(B^\circ \to \ell^+ X)}{\Gamma(B^\circ \to \ell^- X)} \simeq \frac{x^2}{2+x^2} \tag{1.5}$$

$$r_D = \frac{\Gamma(D^\circ \to \ell^- X)}{\Gamma(D^\circ \to \ell^+ X)} \simeq x^2 + y^2 2 \tag{1.6}$$

$$x = \frac{\Delta m}{\Gamma}\ , \qquad y = \frac{\Delta\Gamma}{2\Gamma} \tag{1.7}$$

The detailed time evolution of these ratios will be given later, as well as a method that uses nonleptonic decays to search for mixing.

There are three ways in which CP violation can then show up:

(a) Semileptonic Decays [1]

$$\frac{\Gamma(B^\circ \to \ell^+ X) - \Gamma(\overline{B}^\circ \to \ell^- X)}{\Gamma(B^\circ \to \ell^- X) + \Gamma(\overline{B}^\circ \to \ell^+ X)} = \frac{1 - \left|\frac{p}{q}\right|^4}{1 + \left|\frac{p}{q}\right|^4} \quad . \tag{1.8}$$

(b) Nonleptonic Decays

(i) When f is a final state that is common to both B° and $\overline{B}^\circ$ decays (a property that is then shared by the CP conjugate state f^{CP} one can look for the following asymmetry [6,7]:

$$\frac{\Gamma(B^\circ \to f) - \Gamma(\overline{B}^\circ \to f^{CP})}{\Gamma(B^\circ \to f) + \Gamma(\overline{B}^\circ \to f^{CP})} \simeq \frac{2}{1+|\rho|^2}\,\frac{x}{1+x^2}\,\mathrm{Im}\,\frac{p}{q}\,\rho_f$$
$$\rho_f \equiv \frac{A(\overline{B}^\circ \to f)}{A(B^\circ \to f)} \quad . \tag{1.9}$$

The detailed time evolution that has been integrated over here will be given later.

A simple realization of this scenario is obtained when f is a CP eigenstate as in $B^\circ \to \psi K_s$, $D\overline{D}K_s$, $D\overline{D}$ or in $\overline{D}^\circ \to K_s\pi^\circ$, $K_s\rho^\circ$, K^+K^-; in which case $F^{CP} = \pm f$ holds. Furthermore, the amplitudes $A(\overset{(-)}{B} \to f)$ are not necessarily suppressed by small KM quark mixing angles in that case which leads to very sizeable asymmetries. If f is not a CP eigenstate, *e.g.*, $f = D^+\pi^-$, then the asymmetry will be reduced by small KM angles $|U(b \to u) \times U(d \to c)|$ as we will quantify below.

In electromagnetic or strong processes one always produces bottom mesons in conjunction with antibottom hadrons: $B^\circ \overline{B}$ or $B\overline{B}^\circ$. Therefore, one can measure the asymmetry given in eq. (1.9) only via flavor-tagging the other B decay. This can be achieved most simply by observing direct leptons from semileptonic B decays; thus the asymmetry [eq. (1.9)] gets translated into a difference between the $\ell^+ f$ and $\ell^- f^{CP}$ correlation:

$$\text{asymmetry} = \frac{\sigma(B^\circ \overline{B} + B\overline{B}^\circ \to \ell^+ f\, X) - \sigma(B^\circ \overline{B} + B\overline{B}^\circ \to \ell^- f^{CP}\, X)}{\sigma(B^\circ \overline{B} + B\overline{B}^\circ \to \ell^+ f\, X) + \sigma(B^\circ \overline{B} + B\overline{B}^\circ \to \ell^- f^{CP}\, X)}$$

The details can be found in sect. 2.

(*ii*) If a $B^\circ \overline{B}^\circ$ (or $D^\circ \overline{D}^\circ$) pair is produced in a p wave configuration as it happens for $e^+e^- \to B^\circ \overline{B}^\circ$ (for a one-photon intermediate state), then the process

$$B^\circ \overline{B}^\circ \to f_1\, f_2 \qquad (1.10)$$

can occur only due to CP violation if f_1 and f_2 are CP eigenstates with the *same* CP parity [7,10].

Employing the Standard $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)$ Model, one obtains predictions that are summarized in table 1. Two general features are obvious: some of the asymmetries are expected to be really large, yet the accompanying branching ratios for these exclusive modes are small.

This raises the question whether one can search for such asymmetries in inclusive decays, thus increasing the available statistics. This can be done, but only with considerable care for the following reason: if the final state f

is a CP eigenstate, then the sign of the asymmetry is determined—as will be shown later—by the CP parity of F. Thus an asymmetry in $B^\circ \to K_s + X$ will be equal in size, but opposite in sign to the one in $B^\circ \to K_L + X$. An analogous effect holds when comparing, say, an asymmetry in $B^\circ \to D^+\pi^-$ versus $\overline{B}^\circ \to D^-\pi^+$ with one in $B^\circ \to D^+\rho^-$ versus $\overline{B}^\circ \to D^-\rho^+$. Therefore when one performs an analysis of such decays in an inclusive fashion, one takes the risk that the large asymmetries in the various exclusive channels will cancel each other, at least partially.

It follows, then, that when one sums over different channels to increase statistics, one should not do it blindly. Instead, one has to included information on the CP parities of the states over sums over. This is actually an easier task than it might appear at first sight. As an example, consider the decay chain

$$\begin{array}{lcl} B^\circ \to D^\circ\ M^\circ & \searrow & \\ & & (K_s\, N^\circ)\ M^\circ \\ \overline{B}^\circ \to \overline{D}^\circ\ M^\circ & \nearrow & \end{array} \tag{1.11}$$

It is easy to convince oneself that the CP parity of the state $(K_s\, N^\circ)\, M^\circ$ is positive for all configurations as long as N and M belong to the pseudoscalar, vector or axial vector nonet.

1.2 CP ASYMMETRIES VIA FINAL STATE INTERACTIONS

CP asymmetries can emerge also in the absence of mixing, for which the cleanest scenario is provided by charged B (or D) decays [6,8,9]. When two different amplitudes contribute to the decay of a bottom hadron B into a final state f, one writes for the matrix element

$$\begin{aligned} M_f &\equiv \langle f|\,\mathcal{L}\,|B\rangle \\ &= \langle f|\,\mathcal{L}_1\,|B\rangle + \langle f|\,\mathcal{L}_1\,|B\rangle \qquad (1.12) \\ &= g_1\,M_1 \exp\{i\alpha_1\} + g_2\,M_2 \exp\{i\alpha_1\} \end{aligned}$$

where M_1, M_2 denote the matrix elements for the weak transition operators $\mathcal{L}_1$, $\mathcal{L}_2$ with the weak parameters g_1, g_2 and the strong (or electromagnetic) phase shifts α_1, α_2 factored out.

For the CP conjugate decay $\overline{B} \to f^{CP}$ one then finds

$$\begin{aligned} \overline{M}_f &\equiv \left\langle f^{CP} \middle| \mathcal{L} \middle| \overline{B} \right\rangle \\ &= g_1^*\,M_1 \;\exp\{i\alpha_1\} + g_2^*\,M_2\;\exp\{i\alpha_2\} \end{aligned} \qquad (1.13)$$

and therefore

$$\Gamma(B \to f) - \Gamma(\overline{B} \to f^{CP}) \;\propto\; \mathrm{Im}\; g_1^*\,g_2\;\sin(\alpha_1 - \alpha_2)\,M_1\,M_2\;. \qquad (1.14)$$

From eq. (1.14) one reads off two necessary conditions for such a CP asymmetry to show up:

(*i*) The weak couplings g_1 and g_2 have to possess a relative complex phase. In the Standard Model this implies that the transition rates for such modes are suppressed by small KM angles.

(*ii*) Nontrivial phase shifts $\alpha_1 \neq \alpha_2$ have to be generated from the strong (or electromagnetic) forces.

Condition (ii) does not, in principle, pose a severe restriction. Since the two amplitudes will in general differ in their isospin structure, a nontrivial phase difference will exist. However, as we will see later on, no reliable estimate of condition (ii) can be given.

Basically there are three ways in which condition (i) can be met:

(α) interplay between the two cascade processes [6]

$$\begin{array}{lcl} B^- \to D^\circ \; (K_S \text{ or } K^-) + X^- & & B^- \to \overline{D}^\circ \; (K_S \text{ or } K^+) + X^- \\ \quad\quad \hookrightarrow K_S\, Y & \text{and} & \quad\quad \hookrightarrow K_S\, Y \end{array} \tag{1.15}$$

for which the Feynman diagrams are given in fig. 1;

(β) interplay between quark decay and weak annihilation reactions (see fig. 2) in $B^- \to D^{\circ *} D^-$;

(γ) Penguin contributions generating $B^- \to K^- \rho^\circ$ modes, fig. 3.

Typical guestimates for CP asymmetries in charged B decays are given in table 2. The most reliable estimate can be given for case (α); nevertheless, all the possibilities have to be analyzed. Again, care has to be applied when summing over different channels.

1.3 CP ASSYMETRIES IN D DECAYS

The whole previous discussion can be applied to D decays in an analogous fashion. Thus, one should search for a difference in

- $D^\circ \to K_s\, N$, $N \subset$ pseudoscalar, vector, axial vector nonet,
  or $D^\circ \to K^+ K^-$

- $D^+ \to \pi^+\rho^\circ,\ K^+K^\circ$ or $F^+ \to K_s\,\rho^+$ versus its CP conjugate modes.

No observable effects are predicted by the Standard Model, yet it is consistent with present phenomenology that asymmetries of order .1% up to 1% could show up.

1.4 BUILDING THE DATABASE ON HEAVY FLAVOR STATES

Some of the uncertainties encountered before can be overcome by more experimental data. This is obviously true for the branching ratios of the relevant channels, such as

$$B_d \;\to\; \psi\,K_s\ \psi\,K_s\,\pi's,\ D\overline{D}\,K_s\,D^+D^-,\ p\bar{p}$$

$$B_s \;\to\; \psi\eta,\ \psi\eta',\ \psi\phi,\ F^+F^-$$

$$B^- \;\to\; D^{\circ *}D^-,\ \pi^-\rho^\circ,\ K^-K^\circ\ .$$

Of particular interest are the branching ratios for the channels

$$B^\circ \to D^\circ\,M \to (K_s\,N)_D\,M$$

where N, M are any members of the pseudoscalar, vector or axial vector nonets. As we have already stressed, the CP asymmetries from these channels will add rather than cancel.

There are other decay modes which are of great indirect help in elucidating the underlying decay mechanisms, such as

$$B^- \to \psi\,K^-,\ D\overline{D}\,K^-,\ D\,F^-,\ K^-\pi^\circ \text{ and } \tau_{B^\pm} \text{ versus } \tau_{B^\circ}\ .$$

At the moment, we still have only fairly rudimentary information on B decays; this will have to change.

The strength of $B^\circ - \overline{B}^\circ$ mixing, which is a fascinating phenomenon in its own right, acts as an important input parameter to the size of CP asymmetries. Its detection would be highly valuable for refining the predictions.

Lastly, although e^+e^- annihilation provides the cleanest setting for the production of heavy flavors, one might have to use hadronic collisions to accumulate the required statistics. Very little is known about production specifics of heavy flavors in hadronic collisions. Again, this has to and will change.

The results summarized above will be explained in more detail in the subsequent sections.

2. CP Asymmetries Involving Mixing

2.1 EXCLUSIVE DECAYS To obtain a sufficiently large sample of B (and D) mesons, one has to rely on electromagnetic or even strong production processes. In either case, one has to consider pair production. Three cases should then be distinguished.

(*i*) A neutral meson and its antiparticle are produced:

$$B_d \overline{B}_d, \quad B_s \overline{B}_s$$

with $B_q = (b\bar{q})$. This is the situation encountered just above threshold.

(*ii*) The two neutral mesons are not conjugate to each other:

$$B_d \overline{B}_s \; + \; \text{h.c.} \; .$$

(*iii*) The neutral meson is produced in conjunction with a charged meson or baryon

$$B_q \begin{array}{c}\overline{B}_u \\ \overline{\Lambda}_b\end{array} + \text{h.c.}, \quad , \qquad q = d, s \quad .$$

Subsequently, the bottom hadrons decay. For our later discussion, we distinguish between f° (final states of neutral B mesons) and $f^\pm$ (final states of charged B or Λ decays).

Case (iii) is the simplest to treat since only one meson can oscillate. Using the definitions for the decay amplitudes

$$A(\overset{(-)}{B}{}^\circ \rightarrow f) = \overset{(-)}{A} \tag{2.1}$$

one finds

$$\text{rate}\left[B^\circ(t)\overline{B}_u \rightarrow f^\circ f^+\right] \propto \text{rate}\left(\overline{B}_u \rightarrow f^+\right) \ \exp\{-\Gamma t\}$$
$$\times \left\{(1+\cos\Delta mt)|A|^2 + (1-\cos\Delta mt)\left|\frac{q}{p}\right|^2 |\bar{A}|^2 - 2\sin(\Delta mt)\ \text{Im}\ \frac{p}{q}A\bar{A}^*\right\} \tag{2.2}$$

$$\text{rate}\left[\overline{B}^\circ(t)B_u \rightarrow f^\circ f^-\right] \propto \text{rate}\left(B_u \rightarrow f^-\right) \ \exp\{-\Gamma t\}$$
$$\times \left\{(1+\cos\Delta mt)|\bar{A}|^2 + (1-\cos\Delta mt)\left|\frac{p}{q}\right|^2 |A|^2 + 2\sin(\Delta mt)\ \text{Im}\ \frac{p}{q}A\bar{A}^*\right\} \tag{2.3}$$

Case (ii) presents a more complex situation since both mesons can oscillate, though not into each other. Defining

$$A(\overset{(-)}{B}_d \rightarrow f_d) = \overset{(-)}{A}_d, \qquad A(\overset{(-)}{B}_s \rightarrow f_s) = \overset{(-)}{A}_s \tag{2.4}$$

one obtains

$$\text{rate}\ \left\{\ \left[B_s(t_s)\,\overline{B}_d(t_d) + B_d(t_d)\,\overline{B}_s(t_s)\right] \to f_d f_s\ \right\} \propto \exp\left\{-\Gamma_d\, t_d\right\}\ \ \exp\left\{-\Gamma_s\, t_s\right\}$$

$$\times\ \Bigg\{[1+\cos\,(\Delta m_d t_d + \Delta m_s t_s)]\ \left(|A_s\bar{A}_d|^2 + |A_d\bar{A}_s|^2\right)$$

$$+\ [1-\cos\,(\Delta m_d t_d + \Delta m_s t_s)]\ \left(\left|\frac{p}{q}\right|^2\ |A_s A_d|^2 + \left|\frac{q}{p}\right|^2\ |\bar{A}_s\bar{A}_d|^2\right)$$

$$+\ 2\sin\,(\Delta m_d t_d + \Delta m_s t_s)\ \mathrm{Im}\ \left(\frac{p}{q}\ A_s A_d + \frac{q}{p}\ \bar{A}_s\bar{A}_d\right)\ \left(A_d\bar{A}_s + A_s\bar{A}_d\right)^*\Bigg\} \tag{2.5}$$

Case (i) finally has the added complexity that the two states can mix into each other. Therefore one has to include effects due to quantum statistics and gets accordingly

$$\text{rate}\ \left(B_q(t)\,\overline{B}_q(\bar{t})\big|_{C\,=\,\mp} \longrightarrow f_a\, f_b\right)\ \propto\ \exp\left\{-\Gamma_q(t+\bar{t})\right\}$$

$$\times\ \Bigg\{[1+\cos\ \Delta m_q(t\mp\bar{t})]\,\left|A_a\,\bar{A}_b\ \mp\ A_b\,\bar{A}_a\right|^2$$

$$+\quad [1-\cos\ \Delta m_q(t\mp\bar{t})]\left|\frac{p}{q}\ A_a\,A_b\ \mp\frac{q}{p}\ \bar{A}_a\,\bar{A}_b\right|^2$$

$$+\quad 2\sin\ \Delta m_q(t\mp\bar{t})\ \mathrm{Im}\ \left(\frac{p}{q}\ A_a\,A_b\ \mp\frac{q}{p}\ \bar{A}_a\,\bar{A}_b\right)\left(A_a\,\bar{A}_b\ \mp\ A_b\,\bar{A}_a\right)^*\Bigg\} \tag{2.6}$$

for $B_q\overline{B}_q$ being in a charge conjugation odd ($C\ =\ -1$) or even ($C\ =\ +$) configuration, respectively. The notation is analogous to the one used in previous cases:

$$\overset{(-)}{A}(B_q \to f_{a,b})\ =\ \overset{(-)}{A}_{a,b}\ ,\quad q=d,s\quad . \tag{2.7}$$

The expressions (2.2), (2.3), (2.5), and (2.6) serve as master equations to describe all types of CP asymmetries that can emerge in the presence of mixing. First we discuss the important features qualitatively.

A difference between CP conjugate reaction rates can be produced by two terms in eqs. (2.2)–(2.6):

(a) $|p/q|^2$ could differ from unity:

$$\left|\frac{p}{q}\right|^2 - 1 = \frac{4\mathrm{Re}\,\epsilon}{|1-\epsilon|^2} \tag{2.8}$$

and thus $|p/q|^2 \neq |q/p|^2$. Such a difference would exhibit itself most cleanly in semileptonic B decays. For example, one derives easily from eqs. (2.2),(2.3):

$$\begin{aligned} a_{SL} &= \frac{\sigma\left(B^\circ \overline{B}^+ \to \ell^+ \ldots f^+\right) - \sigma\left(\overline{B}^\circ B^- \to \ell^- \ldots f^-\right)}{\sigma\left(B^\circ \overline{B}^+ \to \ell^+ \ldots f^+\right) + \sigma\left(\overline{B}^\circ B^- \to \ell^- + \ldots f^-\right)} \\ &= \frac{\left|\frac{q}{p}\right|^2 - \left|\frac{p}{q}\right|^2}{\left|\frac{q}{p}\right|^2 + \left|\frac{p}{q}\right|^2} \end{aligned} \tag{2.9}$$

where we have used $|A|^2 = |\bar{A}|^2$ due to CPT invariance. The production of these "wrong sign" leptons of course depends crucially on the strength of mixing; more specifically, the production rate is proportional to $(\Delta m/\Gamma)^2$.

(b) Among the *nonleptonic* modes there are some that are *common* to both B° and $\overline{B}^\circ$ decays. As already stated in sect. 1, they can be CP eigenstates — like ψK_S in B_d decays, but need not be [11]—like $D^-\pi^+$. Setting $|p/q|^2 = 1$

for simplicity and clarity (later we will see that this is actually predicted to hold in the Standard Model), one derives from eqs. (2.2),(2.3):

$$\begin{aligned} A_{NL}(t) &= \frac{\sigma\left(\overline{B}^{\circ}(t)B^{-} \to f^{\circ}f^{-}\right) - \sigma\left(B^{\circ}(t)B^{+} \to (f^{\circ})^{CP}f^{+}\right)}{\sigma\left(\overline{B}^{\circ}(t)B^{-} \to f^{\circ}f^{-}\right) + \sigma\left(B^{\circ}(t)B^{+} \to (f^{\circ})^{CP}f^{+}\right)} \\ &\simeq \frac{2\sin\,\Delta mt\ \mathrm{Im}\ \frac{p}{q}\ A\bar{A}^{*}}{(1+\cos\ \Delta mt)|A|^{2} + (1-\cos\ \Delta mt)|A|^{2}} \end{aligned} \tag{2.10}$$

with $A = A(B^{\circ} \to f^{\circ})$, $\bar{A} = A(\overline{B}^{\circ} \to f^{\circ})$; $(f^{\circ})^{CP}$ denotes, as before, the CP conjugate state of f°.

Two technical remarks might elucidate eq. (2.10):

- Integrating over all decay times t from zero to infinity one finds

$$\langle A_{NL}\rangle_{t} \propto \frac{x}{1+x^{2}}\ , \quad x = \frac{\Delta m}{\Gamma} \tag{2.11}$$

In other words, this CP asymmetry vanishes both in the limit of small mixing ($x \simeq 0$) and large mixing ($x \gg 1$). For $x = 1$ this mixing factor actually reaches its maximal value of $1/2$.

- CP violation resides in the quantity

$$\mathrm{Im}\ \frac{p}{q}\ A\bar{A}^{*} = |\bar{A}|^{2}\ \mathrm{Im}\ \frac{p}{q}\rho_{f}\ , \quad \rho_{f} = \frac{A}{\bar{A}} \tag{2.12}$$

which contains one factor (p/q) coming from mixing, *i.e.*, $\Delta B = 2$ transitions, and another one (ρ_{f}) coming from decays, *i.e.*, $\Delta B = 1$ transitions. It is exactly this combination that is invariant under changes in the phase convention for $\overline{B}^\circ$ states: $\left|\overline{B}^\circ\right\rangle \rightarrow \exp\{-i\alpha\} \left|\overline{B}^\circ\right\rangle$ leads to the simultaneous transformations $(p/q) \rightarrow \exp\{-i\alpha\}\ (p/q)$ and $\rho_f \rightarrow \exp\{i\alpha\}\ \rho_f$, thus leaving $(p/q)\,\rho_f$ invariant as it has to be for a physical observable.

Closely related asymmetries can be obtained from eqs. (2.5),(2.6). The procedure to follow there has the same basic elements as used in eq. (2.10):

- One chooses a decay channel f° that is common to both B° and $\overline{B}^\circ$ mesons.
- One selects a second decay channel which betrays whether the decaying hadron was, at decay time t, a B° or $\overline{B}^\circ$. Semileptonic modes (or decays to charged D mesons) meet this requirement. Thus case (iii) can be illustrated by the following example:

$$
\begin{aligned}
&A_{NL}(t,\bar{t}) \\
&= \frac{\sigma\left[B^\circ(t)\overline{B}^\circ(\bar{t})\Big|_{C=\pm} \longrightarrow f^\circ\ell^-\ldots\right] - \sigma\left[B^\circ(t)\overline{B}^\circ(\bar{t})\Big|_{C=\pm} \longrightarrow (f^\circ)^{CP}\ell^-\ldots\right]}{\sigma\left[B^\circ(t)\overline{B}^\circ(\bar{t})\Big|_{C=\pm} \longrightarrow f^\circ\ell^-\ldots\right] + \sigma\left[B^\circ(t)\overline{B}^\circ(\bar{t})\Big|_{C=\pm} \longrightarrow (f^\circ)^{CP}\ell^-\ldots\right]} \\
&= \frac{2\sin\,\Delta m(t\pm\bar{t})}{|A|^2+|\bar{A}|^2}\mathrm{Im}\,\frac{p}{q}A\bar{A}^*
\end{aligned}
\tag{2.13}
$$

There is one more way in which CP violation can manifest itself: if $B^\circ\,\overline{B}^\circ$ are produced in a relative p ware, *i.e.*, a C odd configuration as it happens in $e^+e^- \rightarrow B^\circ\,\overline{B}^\circ$ for one-photon intermediate states, then the reaction

$$
B^\circ\,\overline{B}^\circ\Big|_{C=-} \rightarrow f_1^\circ\, f_2^\circ
$$

can proceed only via CP violation if f_1, f_2 denote CP eigenstates with the same CP parity. This fact can also be read off from eq. (2.6), using CPT invariance to obtain

$$\bar{A}_i = \exp\{i\phi_i\} \ A_i \ , \quad i = 1,2 \tag{2.14}$$

one derives

$$\begin{aligned} \text{rate} \left[B^\circ(t)\, \overline{B}^\circ(\bar{t}) \Big|_{C=-} \longrightarrow f_1 f_2 \right] \ &\propto \ \exp\{-\Gamma(t+\bar{t})\} \ |A_1|^2 |A_2|^2 \\ &\times \{[1+\cos\,\Delta m(t-\bar{t})]\,[1-\cos(\phi_2-\phi_1)] \\ &+ \ [1-\cos\,\Delta m(t-\bar{t})]\,[1-\cos(\phi_2+\phi_1)] \\ &- \ 2\sin\,\Delta m(t-\bar{t})(\sin\,\phi_1 - \sin\,\phi_2)\} \end{aligned} \tag{2.15}$$

using, as usual, the Standard Model result $|p/q|^2 \simeq 1$ for simplicity.

Again, some technical comments might serve well to elucidate eq. 2.15:

- If there is only one universal phase ϕ, *i.e.*, $\phi_1 = \phi_2 = \phi$, as in a superweak ansatz of CP violation, then one finds

$$\begin{aligned} &\text{rate} \left[B^\circ(t)\, \overline{B}^\circ(\bar{t}) \Big|_{C=-} \longrightarrow f_1 f_2 \right] \\ &\propto \ \exp\{-\Gamma(t+\bar{t})\} \ |A_1|^2 |A_2|^2 \ [1-\cos\,\Delta m(t-\bar{t})]\,(1-\cos\,2\phi) \end{aligned} \tag{2.16}$$

which behaves like $(\Delta m/\Gamma)^2$ for small mixing. If however, $\phi_1 \neq \phi_2$, then one obtains for small mixing

$$\text{rate}\left[B^\circ(t)\,\overline{B}^\circ(\bar{t})\Big|_{C=-} \longrightarrow f_1 f_2\right]$$

$$\propto \exp\{-\Gamma(t+\bar{t})\}\,|A_1|^2\,|A_2|^2\,[1+\cos\Delta m(t-\bar{t})]\,[1-\cos(\phi_2-\phi_1)] + \ldots \tag{2.17}$$

which depends rather little on $\Delta m/\Gamma$.

To summarize our preceding discussion, there are three ways in which CP violation can manifest itself in B° decays as one learns from inspecting the master equations (2.2)–(2.6). The driving force can be:

(*i*) a difference $|p/q|^2 - |q/p|^2 \propto \mathrm{Re}\,\epsilon$, which is measured most directly in semileptonic decays.

(*ii*) an interplay between phases in decay amplitudes and in ϵ leading to a difference between $\Gamma(B^\circ(t) \to f^\circ)$ and $\Gamma[\overline{B}^\circ(t) \to (f^\circ)^{CP}]$;

(*iii*) phases in decay amplitudes producing the reaction $B^\circ(t)\,\overline{B}^\circ(\bar{t})\Big|_{C\,=\,-} \longrightarrow f_1\, f_2$ where f_1, f_2 are CP eigenstates with the *same* CP parity.

2.2 INCLUSIVE STUDIES

So far we have discussed CP asymmetries in exclusive nonleptonic decays. It was already stated in sect. 1 that the branching ratios for the relevant modes are expected to be small, as will be discussed in more detail later on. Then the question arises rather naturally whether one can increase statistics by summing over several channels. We will show now that such semi-inclusive measurements can be done, if great care is applied.

(a) The Problem.

If the final state f° common to both B° and $\overline{B}^\circ$ decays is a CP eigenstate, then its CP parity will determine the sign of the corresponding CP asymmetry:

$$
\begin{aligned}
\rho_{f_t} &= \frac{\left\langle f_\pm \middle| \mathcal{L}(\Delta B = 1) \middle| \overline{B}^\circ \right\rangle}{\left\langle f_\pm \middle| \mathcal{L}(\Delta B = 1) \middle| B^\circ \right\rangle} \\
&= \pm \frac{\langle f_\pm | \, \mathcal{L}^{CP}(\Delta B = 1) \, | B^\circ \rangle}{\langle f_\pm | \, \mathcal{L}(\Delta B = 1) \, | B^\circ \rangle}
\end{aligned}
\tag{2.18}
$$

with

$$CP \, | f_\pm \rangle = \pm \, | f_\pm \rangle \quad .$$

Thus summing over final states f with opposite CP parities will create a strong tendency for cancellations. As an extreme example, if one sums over ψK_S and ψK_L decays of $\overset{(-)}{B}_d$ mesons one is guaranteed to find a vanishing CP asymmetry since

$$
\begin{aligned}
CP \, | \psi K_S \rangle &= - \, | \psi K_S \rangle \\
CP \, | \psi K_L \rangle &= | \psi K_L \rangle \\
BR(\overset{(-)}{B}_d \to \psi K_S) &= BR(\overset{(-)}{B}_d \to \psi K_L)
\end{aligned}
\tag{2.19}
$$

At the same time, the asymmetry in the individual channels $\overset{(-)}{B}_d \to \psi K_S$ and $\overset{(-)}{B}_d \to \psi K_L$ is expected to be large as stated in sect. 1.

The same problem of cancellations in inclusive measurements is encountered when f° is not a CP eigenstate. For a positive asymmetry in, say, $B^\circ \to D^+\pi^-$ versus $\overline{B}^\circ \to D^-\pi^+$ translates into a negative asymmetry in $B^\circ \to D^+\pi^+\pi^\circ$ versus $\overline{B}^\circ \to D^-\pi^+\pi^\circ$ (leaving out subtleties due to final state interactions to which we will return in sect. 4):

$$\begin{aligned} \left\langle D^+\pi^- \middle| \mathcal{L}(\Delta B = 1) \middle| \overline{B}^\circ \right\rangle &= \left\langle D^-\pi^+ \middle| \mathcal{L}^{CP}(\Delta B = 1) \middle| B^\circ \right\rangle \\ \left\langle D^+\pi^-\pi^\circ \middle| \mathcal{L}(\Delta B = 1) \middle| \overline{B}^\circ \right\rangle &= -\left\langle D^-\pi^+\pi^\circ \middle| \mathcal{L}(\Delta B = 1) \middle| \overline{B}^\circ \right\rangle \end{aligned} \tag{2.20}$$

since $CP\,|\pi^\circ\rangle = -\,|\pi^\circ\rangle$. The extent of the cancellation depends of course on the relative branching ratios of the channels.

(b) Possible Solutions

The cleanest way of avoiding these cancellations consists of summing only over identified exclusive modes weighting each contribution with the appropriate sign. This task will probably be less formidable than it appears at first:

- There are wide classes of decay modes that contribute with the same sign to CP asymmetries. For example, one easily determines the CP parity of the final state in

$$\overset{(-)}{B^\circ} \to \overset{(-)}{D^\circ}\ M$$
$$\hookrightarrow K_S\ N$$

to be

$$CP\,[(K_S\,N)\,M] = (-1)^{J_N+J_M}\,CP[M]\,CP[N] \tag{2.21}$$

where J_N, J_M denote the spin of system N and M, respectively. Thus one finds that the combined CP parity is even as long as N, M are any member of the pseudoscalar vector or axialvector nonet.

- MARK III data [12] show that D decays are dominated by two-body final states (when resonances are included). Thus it appears possible (though not certain) that decays like $\overset{(-)}{B^\circ} \to \overset{(-)}{D^\circ} M \to (K_S\, N)\, M$ are actually dominated by final states with even CP parity. Then an inclusive measurement would suffer only mildly from cancellations. This represents one example where more experimental information on B decays will be of crucial help.

(c) D Decays

Suffice it to state at this place that this whole phenomenological analysis can be applied to D° decays in a completely analogous fashion.

3. Predictions from the Standard Model

We will show that three essential parameters, $x = \Delta m/\Gamma$, $\mathrm{Re}\,\epsilon$, $\mathrm{Im}\ (p/q)\,\rho_f$, can be estimated in the Standard Model with considerable, though by no means absolute, confidence for B_d and B_s mesons. The situation is much less promising for D° mesons, as will be explained briefly in the end.

A. Δm

Δm *per se* does not tell us anything about CP violation, although it certainly is a fascinating phenomenon in its own right. Yet a nonvanishing $x = \Delta m/\Gamma$ is required for many CP asymmetries to manifest themselves, as shown in sect. 2.

Since the mass of the B mesons is clearly outside the energy regime where the usual hadronic resonances operate, we can rely on the quark box operator to calculate the effective $\Delta B = 2$ transition operator; the latter is a function of the top mass m_t and various KM mixing angles involving the top quark. None of these parameters is known for sure; yet relying on an ansatz with just three quark-lepton families, we can relate the KM angles involving the top to those involving the bottom quark [19]:

$$\begin{aligned} |U(tb)| &\simeq 1\,, \quad U(ts)| \simeq |U(bc>)| \simeq 0.05 \pm 0.01 \\ |U(td)| &\simeq |U(bu)| < 0.008 \end{aligned} \tag{3.1}$$

Equation (3.1) is based on the information obtained from experimental studies of B decays. Two features should be noted:

- Although $|U(bc)|$ and *a fortiori* $|U(ts)|$ are not well determined one should keep in mind that these uncertainties drop out from $\Delta m/\Gamma\ (B_s)$.
- There exists only an upper limit on $|U(td)|$; thus we will obtain only an upper limit on $\Delta m/\Gamma\ (B_d)$ (unless one insists on the KM scheme to reproduce CP violation as observed in K_L decays).

Intrinsic theoretical uncertainties enter when one undertakes to evaluate the on-shell matrix element $\left\langle B^\circ \middle| \mathcal{L}(\Delta B = 2) \middle| \overline{B}^\circ \right\rangle$, which is usually parametrized as follows:

$$\left\langle \overline{B}^\circ \middle| (\bar{b}q)_{V-A}(\bar{b}q)_{V-A} \middle| B^\circ \right\rangle = \frac{4}{3} R_B\, f_B^2\, m_B^2 \tag{3.2}$$

$R_B = 1$ represents saturation by vacuum insertion [13]:

$$R_B = \frac{\left\langle \overline{B}^\circ \middle| (\bar{b}q)_{V-A}(\bar{b}q)_{V-A} \middle| B^\circ \right\rangle}{\left\langle \overline{B}^\circ \middle| (\bar{b}q)_{V-A} \middle| 0 \right\rangle \left\langle 0 \middle| (\bar{b}q)_{V-A} \middle| B^\circ \right\rangle} \tag{3.3}$$

For the problem at hand one is directly interested in the size of the product $R_B f_B^2$. Nevertheless, we choose to discuss them separately; this corresponds to the usual treatment in the literature. From some theoretical models of the hadronic wavefunctions, one has so far extracted only f_B, and R_B has to be of order one. In principle, at least, f_B, f_D and f_F can be obtained from measurements of $B \to \tau\nu$, $F \to \tau\nu$, $D \to \tau\nu, \mu\nu$ or (with more theoretical bias) from the hyperfine mass splittings $M(B^*) - M(B)$, $M(F^*) - M(F)$, $M(D^*) - M(D)$.

In table 3 we have listed the results on f_{D,F,B_d,B_s} derived from different theoretical approaches. One can add that recent experimental evidence for $M(F^*) - M(F) \sim 150$ MeV would suggest $f_F \sim 200$ MeV.

Table 3 shows a still considerable scattering of theoretical predictions though one should also note that the various predictions have shown a marked tendency over the last few years to converge.

As first shown in ref. [20] (see also ref. [16])

$$R_B = 1$$

under two conditions:

(i) one employs a static quark model obeying manifest translational invariance;

(ii) one relies on a valence quark description for the mesons.

Dropping condition (ii) will lead to [20,16]

$$R_B > 1$$

Giving up condition (i)—as is the case for bag models— yields

$$R_B < 1 \quad .$$

Putting everything together, we estimate as a reasonable guideline

$$R_B\, f_B^2 \sim \begin{cases} 0.01 - 0.02 \text{ GeV}^2 & \text{for} \quad B_d \\ 0.02 - 0.04 \text{ GeV}^2 & \text{for} \quad B_s \end{cases} \tag{3.4}$$

Assuming $m_t \lesssim 40$ GeV, one obtains as typical numbers

$$x(B_d) \ \lesssim\ 0.03 - 0.02\ , \quad \tau(B_d)\ \lesssim\ .1\% - 4\% \tag{3.5}$$

$$x(B_s)\ \sim\ 1 - 4\ , \quad \tau(B_s)\ \sim\ 33\% - 90\% \tag{3.6}$$

indicating sizeable or even large B_s—$\overline{B}_s$ mixing and small, yet possibly observable B_d—$\overline{B}_d$ mixing. On the other hand, at present one cannot make a precise prediction even when m_t is known.

B. Re ϵ

As stated before, the charge asymmetry in $B^\circ\,\overline{B}^\circ \rightarrow \ell^\pm\ell^\pm + X$ is proportional to Re ϵ

$$a_{SL} = \frac{\sigma(B^\circ \overline{B}^\circ \to \ell^-\ell^- + X) - \sigma(B^\circ \overline{B}^\circ \to \ell^+\ell^+ + X)}{\sigma(B^\circ \overline{B}^\circ \to \ell^-\ell^- + X) + \sigma(B^\circ \overline{B}^\circ \to \ell^+\ell^+ + X)} = \frac{\left|\frac{p}{q}\right|^2 - \left|\frac{q}{p}\right|^2}{\left|\frac{p}{q}\right|^2 + \left|\frac{q}{p}\right|^2}$$

$$= \frac{4\left(1+|\epsilon|^2\right)}{\left(1+|\epsilon|^2\right)^2 + 4(\mathrm{Re}\,\epsilon)^2\ \mathrm{Re}\ \epsilon} = \frac{\mathrm{Im}\ \frac{\Gamma_{12}}{M_{12}}}{1+\frac{1}{4}\left|\frac{\Gamma_{12}}{M_{12}}\right|^2} \quad . \tag{3.7}$$

On rather general grounds, one expects a_{SL} to be small: $|\Gamma_{12}/M_{12}|$ is small due to $m_b^2 \ll m_t^2$; more specifically $|\Gamma_{12}/M_{12}| < 0.1$ for $m_t > 35$ GeV. In addition, if one restricts oneself to the Standard Model with just three families, one finds

$$\mathrm{Im}\ \frac{\Gamma_{12}}{M_{12}} \ll \left|\frac{\Gamma_{12}}{M_{12}}\right| \ll 1 \tag{3.8}$$

and thus [22]

$$a_{SL} \sim \begin{cases} 10^{-3} & \text{for } B_d \text{ decays} \\ 10^{-4} & \text{for } B_s \text{ decays} \end{cases} \tag{3.9}$$

which would be too small for observation. Yet one should keep in mind that "New Physics" beyond the Standard Model could yield sizeable contributions to M_{12} while being insignificant for Γ_{12}. Then

$$\mathrm{Im}\ \frac{\Gamma_{12}}{M_{12}} \sim \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|\right) \tag{3.10}$$

could hold leading to

$$a_{SL} \sim \mathcal{O}(1\%) \quad . \tag{3.11}$$

Super–Gravity, Left–Right or Composite Models allow for such scenarios.

C. Im $(p/q)\ \rho_f$

In calculating $\rho_f = [A(B^\circ \to f)]/[A(\overline{B}^\circ \to f)]$ one has to distinguish between f being a CP eigenstate and otherwise.

(i) f is a CP eigenstate.

Two examples are provided by $B_d \to \psi K_S$, $D\overline{D}$, where the latter mode is Cabibbo suppressed. Comparing them with the CP conjugate channels $\overline{B}_d \to \psi K_S$, D^+D^- one realizes, and this holds for all such decays:

- the topologies of the quark diagrams for B_d and $\overline{B}_d$ decays are equivalent;
- phase space and final state interactions are identical.

Therefore the amplitudes for $B_d \to f$ and $\overline{B}_d \to f$ can differ at most by a phase:

$$|\rho_f| = \left|\frac{A(B^\circ \to f)}{A(\overline{B}^\circ \to f)}\right| = 1 \quad . \tag{3.12}$$

Secondly, $|p/q| = 1$ for $\mathrm{Re}\,\epsilon = 0$. Therefore

$$\frac{p}{q}\,\rho_f \simeq \exp\,\{i\phi\} \quad . \tag{3.13}$$

One can consider B_d and B_s decays due to the transitions $b \to c\bar{c}s$, $b \to c\bar{c}d$ and $b \to u\bar{u}d$ and finds

$$\frac{p}{q}\,\rho_f\bigg|_{Bq} = \frac{(U_{bt}\,U^*_{qt})^2}{|U_{bt}\,U^*_{qt}|^2} \left\{ \begin{array}{c} \dfrac{(U_{bc}\,U^*_{sc})^2}{|U_{bc}\,U^*_{sc}|^2} \\[2ex] \dfrac{(U_{bc}\,U^*_{dc})^2}{|U_{bc}\,U^*_{dc}|^2} \\[2ex] \dfrac{(U_{bu}\,U^*_{du})^2}{|U_{bu}\,U^*_{du}|^2} \end{array} \right\} \quad \text{for} \quad \left\{ \begin{array}{c} b\bar{q} \to c\bar{c}s\bar{q} \\ b\bar{q} \to c\bar{c}d\bar{q} \\ b\bar{q} \to u\bar{u}d\bar{q} \end{array} \right\} \quad . \tag{3.14}$$

One technical comment is in order here to avoid a possible confusion: the expressions (3.14) *per se* are not invariant when one changes the phases of the quark fields:

$$d \to d \exp\{i\delta_d\} \quad , \quad s \to s \exp\{i\delta_s\} \tag{3.15}$$

leads to

$$\left.\frac{p}{q}\,\rho_f\right|_{B_d} \to \left.\frac{p}{q}\,\rho_f\right|_{B_d} \exp\{2i(\delta_d - \delta_s)\} \quad .$$

One has to keep in mind however that the relevant quantity is, strictly speaking, $(p/q)\,\rho_f\,F$ where

$$F = \frac{\langle K_S\,|(\bar{d}s)\rangle\,\langle K_S\,|(\bar{s}d)\rangle^*}{|\langle K_S\,|(\bar{d}s)\rangle\,\langle K_S\,|(\bar{s}d)\rangle|} \tag{3.16}$$

Since

$$F \to \exp\{2i(\delta_s - \delta_d)\}\;F$$

under eq. (3.15) one sees that $(p/q)\,\rho_f\,F$ is indeed invariant, as it has to be. Just for simplicity we will suppress F in the following.

It is most convenient to employ the Wolfenstein parameterization [23] of the KM matrix

$$U_{KM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix} \quad . \tag{3.17}$$

The B lifetime requires $A = 1.1 \pm 0.2$ and the upper limit on $U(bu)$ translates into

$$\rho^2 + \eta^2 < 0.5 \quad . \tag{3.18}$$

Insisting on ϵ_K being reproduced in the KM ansatz yields, for $m_t \sim 40 - -60$ GeV,

$$\begin{aligned} \eta &\sim 0.5 \\ |\rho| &\gtrsim 0.5 \end{aligned} \tag{3.19}$$

- This information is obviously not sufficient to lead to precise predictions of $\text{Im}\,(p/q)\,\rho_f$. Yet typical numbers can be obtained when using $\eta = 1/2$, $\rho = 1/2\ [0] - 1/2$:

	$b \to c\bar{c}s,\ b \to c\bar{c}d$	$b \to u\bar{u}d$
$\text{Im}\ \frac{p}{q}\,\rho_f\Big\vert_{B_d}$	$-\frac{2\eta(1-\rho)}{(1-\rho)^2+\eta^2} \sim -1\left[-\frac{4}{5}\right] - \frac{3}{5}$	$-\frac{2\eta\left[\rho-(\rho^2+\eta^2)\right]}{\left[(1-\rho)^2+\eta^2\right](\rho^2+\eta^2)} \sim 0\left[\frac{4}{5}\right]\frac{4}{5}$
$\text{Im}\ \frac{p}{q}\,\rho_f\Big\vert_{B_s}$	$\mathcal{O}(10^{-2})$	$-\frac{2\rho\eta}{\rho^2+\eta^2} \sim -1\,[0]\,1$

These numbers represent a clear tendency for large CP asymmetries in B_d and in those B_s decays that are suppressed by the small mixing angle $U(bu)$. Assembling the various elements one finds for the asymmetry defined in eq. (2.10)

$$\begin{aligned} A^d_{NL}(t) &= \frac{\sigma[\overline{B}_d(t)\,B^- \to f^\circ f^-] - \sigma[B_d(t)\,B^+ \to f^\circ f^+]}{\sigma[\overline{B}_d(t)\,B^- \to f^\circ f^-] + \sigma[B_d(t)\,B^+ \to f^\circ f^+]} \\ &\simeq \sin \Delta m_d t\ \text{Im}\ \frac{p}{q}\,\rho_f \quad . \end{aligned} \tag{3.20}$$

which upon integration over all decay times t gives

$$\left\langle A^d_{NL}\right\rangle_t \simeq \frac{x_d}{1+x_d^2}\ \mathrm{Im}\ \frac{p}{q}\,\rho_f \sim 0.02-0.2 \quad . \tag{3.21}$$

Analogously, one finds for $(b\bar{s}) \to (u\bar{u}d\bar{s})$ transitions

$$\left\langle\ A^s_{NL}\right\rangle_t \simeq \frac{x_s}{1+x_s^2}\ \mathrm{Im}\ \frac{p}{q}\,\rho_f \sim 0.1-0.5 \quad . \tag{3.22}$$

The smallness of Im $(p/q)\,\rho_f$ for $b\bar{s} \to b\bar{c}c\bar{s}$ is easily understood: on the leading level only the second and third family contribute; thus, on this level, there cannot be a CP asymmetry.

(ii) f is not a CP Eigenstate

There are final states common to both B° and $\overline{B}^\circ$ decays which are not CP eigenstates; *e.g.*, [11,24]

$$\overset{(-)}{B}_d \to D^-\pi^+\ , \quad D^+\pi^-$$

$$\overset{(-)}{B}_s \to F^+K^-\ , \quad F^-K^+$$

The diagrams producing $\overset{(-)}{B}_d \to D^\pm\pi^\mp$ are shown in fig. 4. From it one reads off two features that hold in general for such transitions:

- two suppressed KM angles are involved, namely $U(bu)$ together with $U(dc)$ or $U(su)$;
- the topology of the relevant quark diagrams is not the same for B° and $\overline{B}^\circ$ decays. For example, one sees in fig. 4 that the D^+ is produced in a different way in B_d and $\overline{B}_d$ decays. We do not know how to evaluate the

relevant matrix elements from first principles; employing a factorization ansatz, one finds

$$\begin{aligned}\rho_f &= \frac{A(\overline{B}_d \to D^+\pi^-)}{A(B_d \to D^+\pi^-)} \simeq \frac{U(bu)\,U(dc)}{U(bc)}\,\frac{f_D}{f_\pi}\,\frac{m_B^4}{(m_B^2 - m_D^2)^2} \\ &\sim (1\text{–}2)\ \lambda^2\,(\rho - i\eta)\end{aligned} \tag{3.23}$$

where our ignorance concerning the correct value for f_D/f_π is the source of the uncertainty listed above. In addition, there are further uncertainties concerning the validity of the factorization ansatz and the importance of color mixed operators [25]. In any case the ratio ρ_f is not a unit vector in the complex plane nor is $(p/q)\,\rho_f$, when f is not a CP eigenstate, and considerable uncertainties surface.

The analogous procedure yields for B_s decays

$$\begin{aligned}\rho_f &= \frac{A(\overline{B}_s \to F^+K^-)}{A(B_s \to F^+K^-)} \simeq \frac{U(bu)\,U(sc)}{U(bc)\,U(su)}\,\frac{f_F}{f_K}\,\frac{m_{B_s}^4}{(m_{B_s}^2 - m_F^2)^2} \\ &\sim 1.6\,(\rho - i\eta)\end{aligned} \tag{3.24}$$

These estimates for the ratio of matrix elements are entered into the expressions for CP asymmetries in eqs. (2.10),(2.13):

$$\begin{aligned}A^d_{NL}(t) &= \frac{\sigma\left[B_d(t)\,B^+ \to (D^+\pi^-)\,f^+\right] - \sigma\left[\overline{B}_d(t)\,B^- \to (D^+\pi^+)\,f^-\right]}{\sigma\left[B_d(t)\,B^+ \to (D^+\pi^-)\,f^+\right] + \sigma\left[\overline{B}_d(t)\,B^- \to (D^+\pi^+)\,f^-\right]} \\ &\simeq \frac{2\sin\Delta mt}{1+\cos\Delta mt}\,\frac{\lambda^2\,\eta}{(1-\rho)^2+\eta^2}\,\left[2\rho(1-\rho) - (1-\rho)^2 + \eta^2\right] \\ &\sim \frac{\sin\Delta mt}{1+\cos\Delta mt}\,\{0.05,\,-0.03,\,-0.05\}\end{aligned} \tag{3.25}$$

for $\eta = 1/2$, $\rho = \{1/2,\,0,\,-1/2\}$.

$$
\begin{aligned}
A^d_{NL}(t) &= \frac{\sigma\left[B_d(t)\,B^+ \to (D^-\pi^+)\,f^+\right] - \sigma\left[\overline{B}_d(t)\,B^- \to (D^+\pi^-)\,f^-\right]}{\sigma\left[B_d(t)\,B^+ \to (D^-\pi^+)\,f^+\right] + \sigma\left[\overline{B}_d(t)\,B^- \to (D^+\pi^-)\,f^-\right]} \\
&\simeq -\sin\Delta mt\,\frac{2\eta\left[\rho-(\rho^2+\eta^2)\right]}{\left[(1-\rho)^2+\eta^2\right]\,(\rho^2+\eta^2)} \\
&\sim -\sin\Delta mt\,\left\{0,\frac{4}{5},\frac{4}{5}\right\}
\end{aligned}
\tag{3.26}
$$

$$
\begin{aligned}
A^s_{NL}(t) &= \frac{\sigma\left[B_s(t)\,B^+ \to (F^+K^-)\,f^+\right] - \sigma\left[\overline{B}_s(t)\,B^- \to (F^-K^+)\,f^-\right]}{\sigma\left[B_s(t)\,B^+ \to (F^+K^-)\,f^+\right] + \sigma\left[\overline{B}_s(t)\,B^- \to (F^-K^+)\,f^-\right]} \\
&\sim -\sin\Delta mt\;1.6\,\eta \sim \sin\Delta mt
\end{aligned}
\tag{3.27}
$$

Integrating over the decay times t one finally obtains the guestimates shown in table 1.

(*iii*) f Containing a Neutral D Decay

Those final states f that contain the decay products from $\overset{(-)}{D}{}^{\circ}$ mesons present a further subtlety: their classification of being a CP eigenstate or not depends on the decay mode of the neutral D meson. For example, the decay

$$B_d \to D^{\circ}\pi^{\circ}$$

at first sight does not lead to a CP eigenstate. Yet a difference between D° and $\overline{D}^{\circ}$ has to be established via its decays. Therefore the decay chain

$$
\begin{aligned}
B_d \;\to\; & D^{\circ}\;\;\pi^{\circ} \\
& \hookrightarrow K_S + \pi's
\end{aligned}
\tag{3.28}
$$

leads to a CP eigenstate and is thus treated in complete analogy to subsect. 3.C.i. The process

$$B_d \to D^\circ \; \pi^\circ \qquad \qquad (3.29)$$
$$\hookrightarrow K^- + \pi' s$$

on the other hand is treated like the modes discussed in subsect. 3.C.ii.

D. CP Violation in D° Decays

The Standard Model does not predict CP violation in D decays on a level that could be observed. Nevertheless one should search for them in a dedicated fashion.

As has been stated in the literature [3,4], the Standard Model cannot generate $D^\circ - \overline{D}^\circ$ mixing with a strength exceeding .1% appreciably. On the other hand, New Physics in the form of genuine flavor changing neutral currents could produce $D^\circ - \overline{D}^\circ$ mixing on the 1% level. The impact of such New Physics on CP asymmetries would be two-fold:

(*i*) There could be a CP asymmetry in semileptonic decays as measured by $\Gamma(D^\circ \to \ell^- X)$ versus $\Gamma(D^\circ \to \ell^+ X)$; our present knowledge allows for a signal on the 1% level.

(*ii*) Decays of $\overset{(-)}{D^\circ}$ into CP eigenstates f like $K_S \pi^\circ$, $K_S \rho^\circ$, *etc.*, offer the best chance for finding CP violation in nonleptonic decays. One should keep in mind that mixing observables like the rate for $D^\circ \overline{D}^\circ \to \ell^\pm \ell^\pm + X$ depend on $x^2 + y^2$, $x = \Delta m/\Gamma$, $y = \Delta\Gamma/2\Gamma$, whereas the CP asymmetry $\Gamma(D^\circ \to f) - \Gamma(\overline{D}^\circ \to f)$ is proportional just to x! Thus mixing if it is observed on the 1% level can lead to CP asymmetries that are also on the 1% level.

4. Final State Interactions and CP Violation

As explained in sect. 1, CP violation can appear also in the absence of mixing. Other mechanisms have to be invoked then to supply the required two coherent amplitudes. This can happen due to an interplay between two different cascade processes, between quark decay and weak annihilation or due to Penguin operators.

A. INTERPLAY BETWEEN DIFFERENT CASCADES

The (Cabibbo disfavored) decay $B^- \to K_s K + \ldots$ can be produced by two different cascades:

$$\begin{array}{l} B^- \to D^\circ \; (K^- \text{ or } K_S) + \ldots \\ \quad \hookrightarrow K_S + \ldots \end{array} \quad \text{and} \quad \begin{array}{l} B^- \to \overline{D}^\circ \; (K^- \text{ or } K_S) + \ldots \\ \quad \hookrightarrow K_S + \ldots \end{array} \tag{4.1}$$

From the quark diagrams given in fig. 1, one reads off

$$\begin{aligned} A(B^- \to K_s K X_i Y_j) \;\propto\; & U(bc)\, U^*(su)\, U^*(sc)\, U(du)\; A_1 \\ & + U(bu)\, U^*(sc)\, U(sc)\, U(du)\; A_2 \end{aligned} \tag{4.2}$$

with

$$A_1 = \left\langle D^\circ K X_i \right| (\bar{c}b)_{V-A}(\bar{s}u)_{V-A} \left| B^- \right\rangle \left\langle K_s Y_j \right| (\bar{s}c)_{V-A}(\bar{u}d)_{V-A} \left| D^\circ \right\rangle$$

$$A_2 = \left\langle \overline{D}^\circ K X_i \right| (\bar{u}b)_{V-A}(\bar{s}c)_{V-A} \left| B^- \right\rangle \left\langle K_s Y_j \right| (\bar{c}s)_{V-A}(\bar{d}u)_{V-A} \left| \overline{D}^\circ \right\rangle .$$

Therefore

$$\Gamma\left(B^{(\mp)} \to K_s \overset{(-)}{K} X_i Y_j\right) \propto |A_1|^2 \left\{1 + \frac{|U(bu)|^2\,|U(sc)|^2}{|U(bc)|^2\,|U(su)|^2}\,|\bar{\rho}|^2 \right.$$

$$+ \; 2\mathrm{Re}\, \frac{U(bu)\,U(sc)}{U(bc)\,U^*(su)}\ \mathrm{Re}\ \bar{\rho} \tag{4.3}$$

$$\left. (\mp)\ 2\mathrm{Im}\, \frac{U(bu)\,U(sc)}{U(bc)\,U^*(su)}\ \mathrm{Im}\ \bar{\rho}\right\}$$

with $\bar{\rho} = A_2/A_1$. Employing again the Wolfenstein parametrization, one gets

$$\Gamma\left(B^{(\mp)} \to K_s \overset{(-)}{K}\, X Y\right) \propto |A_1|^2\ \left\{1 + \left(\rho^2 + \eta^2\right)\ |\bar{\rho}|^2 + 2\rho\,\mathrm{Re}\,\bar{\rho}\ \mp\ 2\eta\,\mathrm{Im}\,\bar{\rho}\right\}$$

$$\lesssim |A_1|^2\ \left\{1 + \tfrac{1}{2}\,|\bar{\rho}|^2 + \mathrm{Re}\,\bar{\rho}\ \mp\ \mathrm{Im}\ \bar{\rho}\right\} \tag{4.4}$$

$$\frac{\Gamma\left(B^+ \to K_s\,\bar{K}\,X_i\,Y_j\right) - \Gamma\left(B^- \to K_s\,K\,X_i\,Y_j\right)}{\Gamma\left(B^+ \to K_s\,\bar{K}\,X_i\,Y_j\right) + \Gamma\left(B^- \to K_s\,K\,X_i\,Y_j\right)} \lesssim \frac{\mathrm{Im}\ \bar{\rho}}{1 + \frac{1}{2}\,|\bar{\rho}|^2 + \mathrm{Re}\,\bar{\rho}}$$

Final state interactions have to be evoked to generate a nonvanishing imaginary part in the ratio of matrix elements. While we do not know how to estimate $\mathrm{Im}\,\bar{\rho}$ in a reliable way, there exists no reason why it should be particularly tiny. A 10% asymmetry in $\Gamma(B^+ \to K_S\bar{K}X_iY_j)$ versus $\Gamma(B^- \to K_S\bar{K}X_iY_j)$ is an optimistic, but not unreasonable guestimate.

Equation (4.4) was written down for a single exclusive decay mode. Each individual mode will however command only a small overall branching ratio. This raises again the question whether statistics can be gained via summing over different exclusive modes. An analysis analogous to the one given in sect. 2 shows that adding exclusive decays in an indiscriminate way will very likely wash out any asymmetry; on the other hand it is a good idea to analyze

$$\sum_{i,j} \Gamma\left(\overset{(-)}{B}_u \rightarrow K_s \overset{(-)}{K} X_i Y_j\right)$$

where X_i, Y_j are any member of the pseudoscalar, vector or axialvector nonet.

B. INTERPLAY BETWEEN QUARK DECAY AND WEAK ANNIHILATION

The B decays will receive contributions from quark decay and —to a certain degree—from weak annihilation (WA). They can contribute coherently to modes such as

$$B_u \rightarrow D^{\circ *} D^- \tag{4.5}$$

for which the quark diagrams are shown in fig. 2. The KM angles involved certainly do contain the CP violating phase. The question one then has to address concerns the size of the WA contribution.

Recent CLEO data yield for the lifetime ratio of charged and neutral B mesons [26]

$$0.6 < \frac{\tau(B^{\pm})}{\tau(B^{\circ})} < 1.66 \tag{4.6}$$

i.e., a substantially smaller ratio than that for D mesons. This agrees with the general theoretical expectation that WA is much less significant for B than for D decays. Nevertheless, it is possible (though not assured) that WA contributes significantly to certain special channels such as the one in eq. (4.5), even considering the fact that it requires $c\bar{c}$ excitation. Again, the relevant final state interactions are not known. There are two alternative approaches for obtaining a guestimate which use completely different starting points [8]: firstly, one can argue that a rather wide axial resonance with a mass $M_A \sim 4.2$ GeVaffects $B_u \to D^*\overline{D}$ in a significant way; the diametrically opposed view is to rely on asymptotic form factors for "W" $\to D^*\overline{D}$ as suggested by perturbative QCD. Such arguments sketch a scenario where

$$\frac{\Gamma\left(B^- \to D^{\circ *} D^-\right) - \Gamma\left(B^+ \to \overline{D}^{\circ *} D^+\right)}{\Gamma\left(B^- \to D^{\circ *} D^-\right) + \Gamma\left(B^+ \to \overline{D}^{\circ *} D^+\right)} \sim \mathcal{O}(1\%) \tag{4.7}$$

might hold.

C. CP Violation and Penguin Operators

Allowing for Penguin operators, as shown in fig. 3, opens up more possibilities. They can be calculated with some degree of confidence, since they are short distance operators [27], and they possess both a real and an imaginary part. A relevant channel is

$$B_u^- \to K^- \rho^\circ \ . \tag{4.8}$$

One actually obtains large CP asymmetries from a Penguin calculation which includes spectator diagram

$$\frac{\Gamma\left(B^- \to K^- \rho^\circ\right) - \Gamma\left(B^+ \to K^+ \rho^\circ\right)}{\Gamma\left(B^- \to K^- \rho^\circ\right) + \Gamma\left(B^+ \to K^+ \rho^\circ\right)} \sim \mathcal{O}(10\%) \tag{4.9}$$

The drawback is that the branching ratios for such modes are expected to be very small as discussed in more detain in sect. 5.

D. Summary on CP violation in Charged B Decays

(*i*) Very sizeable asymmetries of order 10% could show up in certain decay modes while 1% effects can emerge in others.

(*ii*) The branching ratios for the relevant exclusive modes are expected to be small, *i.e.*, not exceeding .1%. Statistics can be gained by summing over exclusive modes, but this has to be done in a careful and not in an indiscriminate manner.

(*iii*) Estimates on the expected size of the effects are severely affected by our ignorance of the relevant final state interactions. This implies in turn that nil results cannot be interpreted in an unambiguous fashion.

5. Search Strategies

As explained in sec. 3, we can make rather reliable predictions on the size of CP asymmetries such as $[\Gamma(B^\circ \to f) - \Gamma(\overline{B}^\circ \to \bar{f})]/[\Gamma(B^\circ \to f) - \Gamma(\overline{B}^\circ \to \bar{f})]$, in particular when f is a CP eigenstate. Yet one needs the branching ratio for the relevant modes to evaluate the prospects for doing such a measurement.

A. BRANCHING RATIOS FOR EXCLUSIVE MODES

(1) f is a CP Eigenstate

(α) Hidden charm. As explained before, the decay $B^\circ \to \psi K_s$ provides a clean lab to search for CP asymmetries. While the inclusive reaction $B \to \psi + X$

has recently been established with a branching ratio of $\sim 1\%$, no signal has been seen yet for $B \to \psi K$. There exists a direct upper limit [28]

$$BR\left(B_d \to \psi \overline{K}^{\circ}\right) < 0.9\% . \tag{5.1}$$

Yet with a modicum of theory a much more stringent limit can be obtained. From $BR(B^{\pm} \to \psi K^{\pm}) < 0.07\%$ [28] one infers $BR(B_d \to \psi \overline{K}^{\circ}) < 0.07\%$ and thus

$$BR(B_d \to \psi K_s) < 3.5 \times 10^{-4} . \tag{5.2}$$

The equally useful channel $B^{\circ} \to \psi K_s \pi^{\circ}$ suffers from a similar suppression: $BR(B^{\circ} \to \psi K^{*\circ}) \leq 0.5\%$ [28] implies

$$BR(B_d \to \psi K_s \pi^{\circ}) \leq 10^{-3} . \tag{5.3}$$

Theoretically one expects the branching ratios to be close to these upper limits.

Nothing is known, of course, about the corresponding B_s decays; however, one expects similar branching ratios of order 10^{-3}.

(β) Open charm. Experimentally nothing is known about $B \to D\overline{D} + X$. Thus we can offer only guestimates:

$$BR(B_d \to D\overline{D} K_s) \sim .1\% - 2\% \tag{5.4}$$

$$BR(B_s \to F^+ F^-) \sim 3\% \tag{5.5}$$

$$BR(B_d \to D^+ D^-) \sim .5\% \tag{5.6}$$

The last mode is formally Cabibbo suppressed, yet form factor effects could enhance it over naive expectations as it happened for the analogous mode $D^\circ \to K^+K^-$.

(γ) Charm cascade. Cascade decays like

$$\begin{aligned} B^\circ \;\to\; &\overset{(-)}{D}{}^\circ + \pi's \\ &\hookrightarrow K_S + \pi's \end{aligned} \tag{5.7}$$

lead to CP eigenstates. A typical combined branching ratio is (we give a more complete list later on):

$$BR\left(B_d \to \overset{(-)}{D}{}^\circ \rho^\circ \to (K_s W)\rho^\circ\right) \sim 6 \times 10^{-4}$$

(2) f is Not a CP Eigenstate

One of the few known branching ratios is

$$BR(B_d \to D^+\pi^-) \;\sim\; .5\,\% \;. \tag{5.9}$$

Hence, one estimates

$$BR(\overline{B}_d \to D^+\pi^-) \;\lesssim\; \mathcal{O}(10^{-5}) \;. \tag{5.10}$$

Analogously, one predicts

$$BR(B_s \to F^+K^-) \,,$$

$$BR(\overline{B}_s \to F^+K^-) \sim \mathcal{O}(10^{-3}) \tag{5.11}$$

$$BR(B_s \to D^\circ K_s) \sim \mathcal{O}(10^{-3}) \tag{5.12}$$

$$BR(B_s \to D^\circ \phi) \sim \text{few } 10^{-3} \tag{5.13}$$

$$BR(B_d \to D^\circ \pi^\circ) \sim (1-3) \times 10^{-3} \tag{5.14}$$

$$BR(B_d \to D^\circ K_s) \sim 10^{-4} \tag{5.15}$$

$$BR(B^\pm \to D^\pm \overset{(-)}{D}{}^{\circ *}) \sim 10^{-3} \tag{5.16}$$

$$BR(B^\pm \to \pi^\pm \rho^\circ) \,,$$

$$BR(B^\pm \to K^\pm \overset{(-)}{K}{}^{\circ *}) \lesssim 10^{-5} \tag{5.17}$$

B. INCLUSIVE SEARCHES

The guestimates given above lead to one general conclusion: those decay modes that should exhibit CP asymmetries of order 10% possess branching ratios of typically at most 10^{-3}. There are modes with somewhat larger branching ratios—yet there the expected asymmetry does not exceed 1% in an appreciable fashion. Since searches for such effects pose very difficult problems, one would like to increase statistics by studying inclusive processes.

It has already been stated that an indiscriminate summation over exclusive modes—such as $B \to \psi + X$— will wash out an asymmetry, since its sign will be different for different channels.

If the final state f is a CP eigenstate, then the sign of the asymmetry will depend on the CP parity of f. Therefore we give not only the expected branching ratios for the relevant modes in table 4, but also their CP parities.

This table shows what was said before. Although the overall branching ratios for the relevant exclusive channels do not exceed 10^{-3} (apart from some exceptions), there are a large number of such channels. Thus one can entertain the idea of summing the contributions from different modes; this has to be done with care, keeping track of the CP parities of the final states. In particular, one should not sum over modes without discriminating K_s against K_L.

Very similar considerations have to be applied when f is not a CP eigenstate. The CP asymmetry expected for $\overset{(-)}{B}_d \to D^{\pm}\pi^{\mp}$ changes sign when $\overset{(-)}{B}_d \to D^{\pm}\pi^{\mp}\pi^{\circ}$ is considered. Therefore, a blind summation over final states is likely to destroy a signal.

6. Building the Data Base

Considering how little we know for certain about B decays, it is amazing that we can make all these predictions discussed before. It would be clearly preposterous to treat them as more than guide lines for further studies. Obtaining more experimental information on heavy flavor physics in the future will allow us to refine our predictions in due course. In the following we will list the kind of information needed to achieve such a refinement.

A. BRANCHING AND LIFETIME RATIOS

A few exclusive B decays have been seen, so in the future one can hope to identify many more. Of particular interest are, as explained before,

$$B_d \to \psi K_s,\ \psi K_s \pi^\circ,\ \psi K_s \pi^+\pi^-,\ D\overline{D}K_s,\ D\overline{D}\, D^*\overline{D}^*,\ D^\circ\pi^\circ,\ D^\circ\rho^\circ\ . \tag{6.1}$$

No B_s decay has been identified. In the present context it would be of most direct interest to find

$$B_s \to \psi\eta,\ \psi\eta',\ \psi\phi,\ F^+F^-,\ D^\circ\phi\ . \tag{6.2}$$

There are other decay modes that are not directly relevant for CP asymmetries, but can shed light on the underlying decay mechanism and the impact of final state interactions:

(*i*) observing $B \to K\pi$ would shed light on the significance of Penguin operators [27];

(*ii*) comparing $B^\circ \to D^\circ\,\pi^\circ,\ D^\circ\,\rho^\circ$ with $B^\circ \to D^+\pi^-,\ D^+\rho^-$ will tell us something about the impact of final state interactions.

(*iii*) Most authors argue that $B^\pm$, B_d and B_s lifetimes should differ very little from each other, since weak annihilation reactions are expected to be relatively small (unlike in the D case). It would be very useful to test this either by measuring $\tau(B^\pm)$ versus $\tau(B_d)$ versus $\tau(B_s)$ directly or via $BR(B^\pm \to \ell + X)$ versus $BR(B_d \to \ell + X)$ *etc.*, or via $BR(B^\pm \to \psi K^\pm)$ versus $BR(B_d \to \psi\overline{K}^\circ)$ or $BR(B^\pm \to \psi K^{\pm *})$ versus $BR(B^\circ \to \psi\overline{K}^{\circ *})$.

B. B° — $\overline{B}^\circ$ MIXING

Our predictions on the strength of B°—$\overline{B}^\circ$ mixing can be sharpened considerably once the top quark mass is known to within a few GeV. It would yield a decent prediction for $\Delta m(B_s)$; to do the same for $\Delta m(B_d)$ requires a determination of $U(bu)$.

Of course—and preferably—the strength of B°—$\overline{B}^\circ$ mixing can be measured, either via like-sign dileptons or the forward-backward asymmetry of bottom jets in e^+e^- annihilation , or via searching for $B^\circ \overline{B}^\circ \rightarrow DD + X$, $DF + X$, $FF + X$.

C. PRODUCTION CROSS SECTIONS FOR HEAVY FLAVORS

The cleanest environment is provided by e^+e^- annihilation on or off the Z° resonance. Besides direct production of charm and bottom, one should study also the option of indirect production; in particular, top or even toponium decays could serve as B factories.

Hadronic collisions offer much higher yields. Then the question arises whether and how these large yields can be harnessed to study delicate phenomena like B°—$\overline{B}^\circ$, D°—$\overline{D}^\circ$ mixing and CP violation in these systems. Much more experimental as well as theoretical work is needed before a meaningful answer can be given.

Photoproduction of heavy flavors represents a situation between the two extremes sketched above and holds a great promise for the future.

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Table 1. CP Asymmetries in B° Decays

Decay Modes	Estimated Branching Ratio	CP Asymmetry
A. Semileptonic		
$B_d\overline{B}_d \to \ell^{\pm}\ell^{\pm} X$	$\lesssim 4 \times 10^{-4}$	10^{-3}
$B_s\overline{B}_s \to \ell^{\pm}\ell^{\pm} X$	$10^{-3} - 10^{-2}$	10^{-4}
B. Nonleptonic		
CP PURE FINAL STATE		
$B_d \to \psi K_s$	$\sim 5 \times 10^{-4}$	$0.02 - 0.2$
$\to \psi K_s \pi^\circ$	10^{-3}	$0.02 - 0.2$
$\to \psi K_s + X$	5×10^{-3}	$0.02 - 0.2$
$\to D\overline{D} K_s$	$(0.1 - 2) \times 10^{-2}$	$0.02 - 0.2$
$\to D\overline{D}$	5×10^{-3}	$0.02 - 0.2$
$B_s \to \psi \phi$	10^{-3}	$10^{-3} - 0.01$
$\to F^+F^-$	3×10^{-2}	$10^{-3} - 0.01$
$\to \psi\eta$	5×10^{-4}	$10^{-3} - 0.01$
$B_d \to K_s \pi$	$\leq 10^{-4}$	$0.1 - 0.5$
$\to K_s \rho$	10^{-4}	$0.1 - 0.5$
CP MIXED FINAL STATE		
$B_d \to \sum_{i,j} (K_s N_j)_{D^\circ} M_j$	$\sim \mathcal{O}(1\%)$	$0.02 - 0.2$
$N_i, M_j \in \{pseudoscalar\ vector,\ axialvector\ nonet\}$		
NON-CP FINAL STATE		
$B_d \to D^+\pi^-$	1%	$10^{-3} - 0.01$
$\to D^\circ K_s$	$\mathcal{O}(10^{-3})$	$10^{-3} - 0.01$
$B_x \to F^+K^-$	$\mathcal{O}(10^{-3})$	$0.1 - 0.5$

Table 2. CP Asymmetries in Charged B Decays

Decay Modes	Estimated Branching Ratio	CP Asymmetry
A. Interference between different cascades: $B^{\pm} \rightarrow \sum_{i,j} (K_s N_i)\, K M_j$	$\sim \mathcal{O}(0.1\%)$	$10^{-3} - 0.01$
B. Interference between quark decay and WA: $B^{\pm} \rightarrow D^{\circ *} D^{\pm}$	$\sim \mathcal{O}(0.1\%)$	$10^{-3} - 0.01$
C. Penguin processes: $B^{\pm} \rightarrow K^{\pm} \rho^{\circ}$	10^{-4}	$0.01 - 0.1$

Table 3. **Theoretical Estimates of Meson Decay Constants (in MeV)**

$f_\pi = 133$ MeV	f_D	f_F	f_{B_d}	f_{B_s}
QCD Sum Rules [14]	200		190 ± 30	210 ± 30
Nonrelat. Potentials [15]	150	210	125	175
Relat. Potentials [16]	150–230	190–390	100–190	140–230
MIT Bag Models [17]	100–170	~ 190	70–150	140–200
$M(1^{--}) - M(0^{-+})$ [18]	112-200		70–130	

Table 4

Mode	Expected Branching Ratio	CP Parity
$B_d \to \psi\, K_s$	few $\times 10^{-4}$	$-$
$\to \psi\, K_s \pi^\circ$	$\mathcal{O}(10^{-3})$	$+$
$\to D\overline{D}$	few $\times 10^{-3}$	$+$
$\to D\overline{D}\, K_s$	$10^{-3} - 2 \times 10^{-2}$	$+$ (mostly)
$B_s \to \psi\, \eta$	few $\times 10^{-4}$	$+$
$\to \psi\, \phi$	$\mathcal{O}(10^{-3})$	$+$ (mostly)
$\to F^+F^-$	$\sim 3\%$	$+$
$B_d \to D^\circ\, \pi^\circ \to f^D \pi^\circ$	$\sim 1\% \times BR(D^\circ \to f^D)$	$-(-1)^{CP_{f_D}}$
$\to D^\circ\, \rho^\circ \to f^D \rho^\circ$	$\sim \mathcal{O}(1\%) \times BR(D^\circ \to f^D)$	$-(-1)^{CP_{f_D}}$

Mode	Measured Branching Ratio	CP Parity
$D^\circ \to K_s\, \rho^\circ$	0.7%	$-$
$\to K_s\, \omega$	2%	$-$
$\to K_s\, \phi$	0.5%	$-$
$\to K_s\, \pi^\circ$	1.1%	$-$
$\to K_s\, \eta$	0.9%	$-$
$\to \rho\pi$	$\sim 1\%$	$-$
$\to K^+K^-$	0.6%	$+$
$\to \rho\rho$	$\sim 1.5\%$	$+$

Figure Captions

Fig. 1. Two cascade diagrams contributing to $B^- \to D^\circ K \to K_S K + \ldots$.

Fig. 2. Quark decay and weak annihilation contribution to $B^- \to D^{\circ *} D^-$.

Fig. 3. Penguin contributions to $B^- \to \delta\pi, K^* K$.

Fig. 4. Decay diagrams for $\overset{(-)}{B}_d \to D^+$, $\pi^- D^- \Pi^+$.

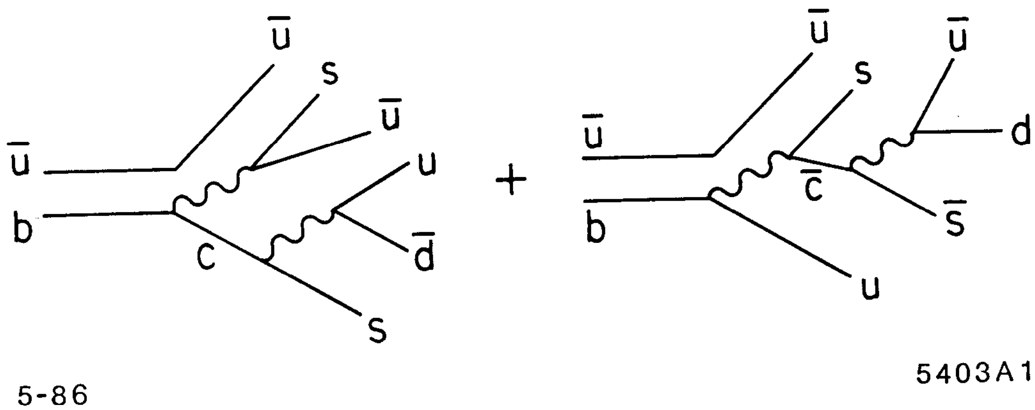
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Fig. 1

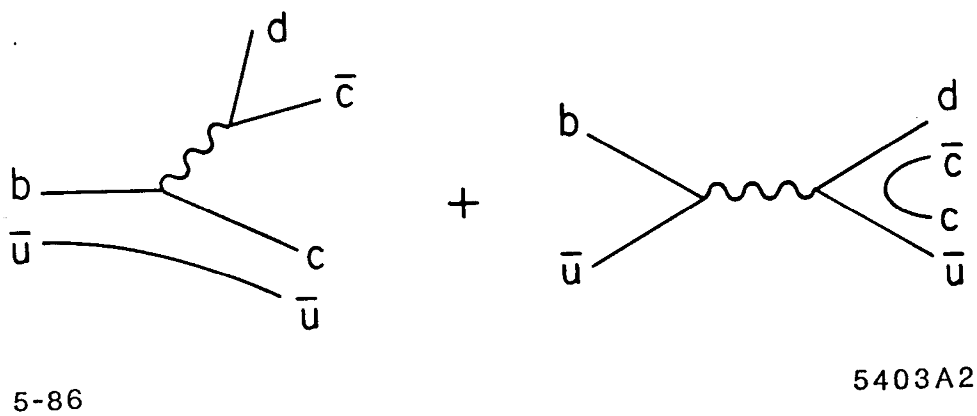

Fig. 2

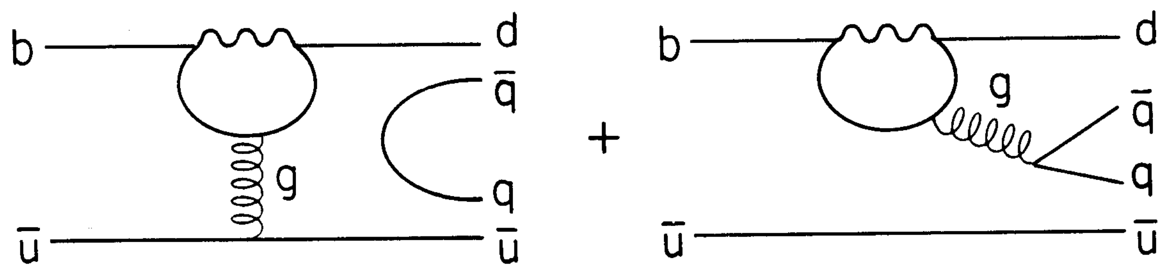

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Fig. 3

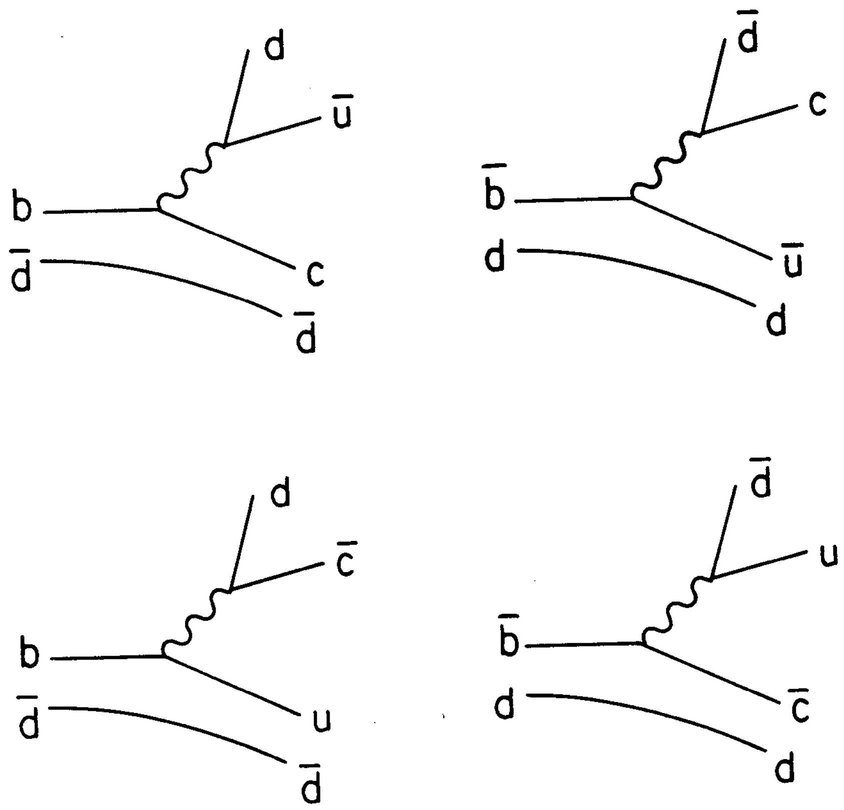

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Fig. 4