SLAC – PUB – 3942
April 1986
(A)

CIRCULAR MACHINE DESIGN TECHNIQUES AND TOOLS*

Roger V. Servranckx
University of Saskatchewan
Saskatoon, Saskatchewan, Canada S7N-0W0

and

Karl L. Brown
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

This report is based on a series of two lectures on Charged Particle Optics presented by the first author at the SLAC Accelerator Physics Summer School in July 1985 and on presentations made by both authors at the Second International Conference on Charged Particle Optics in Albuquerque, New Mexico, May 19-23, 1986.

Presented at the Second International Conference on Charged Optics, Albuquerque, New Mexico, May 19-23, 1986

* Work supported in part by the Department of Energy, contract DE-AC03-76SF00515 and by the National Sciences and Engineering Research Council of Canada.

1. Summary and Introduction

This report attempts to outline some of the basic optics principles involved in the design of circular machines such as Alternating Gradient Synchrotrons, Storage and Collision Rings, and Pulse Stretcher Rings. It is a collection of notes and comments gathered over several years by the authors from the publications of, and conversations with, many experts in the Accelerator Design field. The information and the references provided are representative but not exhaustive of the wealth of information available.

The primary intent of the report is to give novices a starting point in their design efforts. It is assumed that the choice of the basic parameters needed for the physics goals of the machines have been specified before this exercise begins, although in practice it is recognized that several design iterations are usually necessary before a final machine design is formulated.

We first define the typical problems facing a designer and then review the main references and computational tools that are presently available to tackle the problems.

Chapter three identifies some problems met in accelerator design. Chapter four provides the basic mathematical formulae needed for the understanding of the following chapters. Chapter five gives a short presentation of some basic optical modules used in design. Chapters six and seven present solutions to some first-order and second-order problems respectively .

2. The Design of Circular Machines

In this chapter we define the principal problems facing the designer and introduce the tools that are available to solve these problems. However, before delving into the description of these problems , we first present a general schematic structure, characteristic of a typical circular machine.

Figure 1 shows such a schematic structure. The dashed line sketches the beam envelope and the full line sketches the dispersion function (the η function: the trajectory followed by an off-momentum reference particle). For simplicity we represent the machine (or perhaps a section of a machine) on a straight line instead of on a curve.

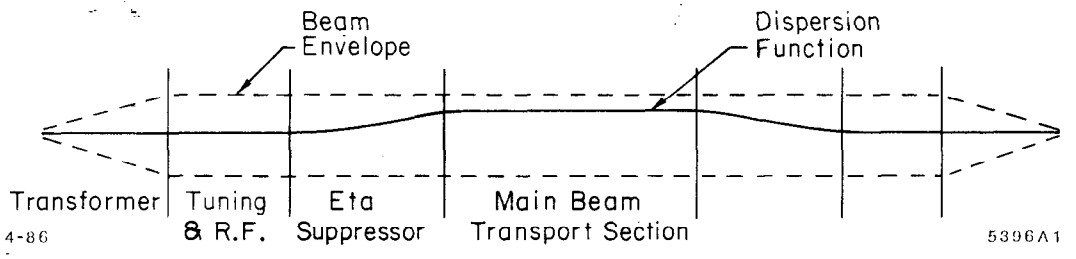

Figure 1. Schematic Structure of a Circular Machine

The central part of the machine, containing most of the bending dipole magnets, is designed to keep the size of the beam as constant as possible. The dipole and quadrupole components in this section generate a dispersion (η function) of the beam which is also more or less held constant.

Some parts of the machine may need to be dispersion free. To achieve this, the central section is surrounded by two special sections which suppress the dispersion. In the literature, these are called 'dispersion suppressors'.

The dispersion free zones can be used for a variety of functions, for example, to tune the machine or to locate the RF Cavities. These sections can also be beam spin manipulators or beam diagnostic sections, etc.

The two end sections, shown in figure 1 are transformers (magnifying or demagnifying sections) which change the transverse size of the beam from its 'normal size' in the main bending regions to the required size at some particular point (an interaction point, a light source point, etc.).

2.1 THE BASIC PROBLEMS

First we shall talk of the 'optics' order of the problem. The order is related to the particular coordinate system chosen to describe the particle motion. For our purposes we choose the coordinate system adopted in the TRANSPORT formalism.[1] This definition of order can also be found in other references.[2] [3]

Zero-Order Optics

We define zeroth-order optics as that which specifies the coordinates of the reference trajectory in a machine. The physical layout of a machine must guarantee closure and continuity of the tangent along the reference trajectory. Thus the problem of zeroth-order optics is a problem in geometry. It is, nevertheless, very important to the success of a machine and is intimately tied to the surveying tolerances needed in positioning of all of the magnets in the machine lattice.

First-Order Optics

The first-order optics problems constitute the beginning of a machine lattice design. One of those problems is the stable guidance of the beam as it makes many revolutions around the machine. Besides determining the conditions which provide first-order stability, it also involves determining the proper tunes to avoid resonance blow-up of the circulating beam. Other problems are related to meeting the needs for the special achromatic and collision regions, and satisfying the conditions required for special functions of the planned machine (as for example controlling the magnitude of the dispersion function in collision rings and synchrotron radiation sources).

Second-Order Optics

For a very wide range of the choice of the machine parameters, the particle motion, when restricted to first-order, is theoretically stable to all amplitudes. The Dynamic Aperture, defined as the maximum amplitude for which the particles remain stable, is a problem that first appears in second-order optics. In some machines (e.g. those using resonant extraction) the second-order behaviour is intrinsic to the functions to be achieved as opposed to being just a perturbation to the desired first-order functions.

Optics orders greater than 2

Optics of order greater than 2 is required in the study of general stability, the determination of beam sizes, the analysis of machine sensitivity to errors, and to coherent and incoherent perturbations.

2.2 Some Mathematical Tools Available

In this section we review some of the more common mathematical tools used by machine designers and make some comments on their relative utility.

The First-Order Matrix Formalism

The linear motion of particles in a circular machine is most easily described by a matrix formalism. This formalism is a direct consequence of the theory of second order linear differential equations. The first reference for the basics is that provided by Courant and Snyder[3] in their classic paper in 1958. A more recent presentation of linear optics is given by Brown and Servranckx in reference.[2]

Second-Order Matrix Formalism

A word of caution is needed here. Second Order in this context applies to the level of approximation with which the differential equations of motion are

solved, and must not be confused with the order of the differential equations of motion which is always 2.

Matrix theory is basically a tool applicable to studying linear phenomena (meaning Linear Differential Equations). K.L. Brown introduced a notational technique that used the matrix formalism for the description of second and higher order approximations to the solutions of the differential equations of motion.[4] From this the TRANSPORT program was evolved and the most recent versions of TRANSPORT have now been extended to include third order optics. This will be reported on in this conference by David C. Carey.[5]

Hamiltonian Approach

Readers familiar with the work of Courant and Snyder and of Brown, know that the tools described in the two previous sections apply best to the description of the motion in individual magnetic elements. The Hamiltonian approach to study circular machines tends to concentrate on the global behaviour of the machine. Two references[6] [7] to the Hamiltonian methods are provided for the reader in the reference section of this paper as an introduction to the subject.

Lie Algebraic Formalism

The general Hamiltonian formulation which describes the motion of charged particles satisfies a complex mathematical condition called the symplectic condition. This mathematical structure leads to the introduction of an Operator Algebra named after Sophus Lie. This Lie Algebra has now been introduced as a practical tool for the study of machine theory by the work of A. Dragt[8] and his collaborators. One of the great successes of this approach is its more compact description of higher order effects and the potential of deriving general theorems in third- and higher order optics as guidelines to optical solutions of complex problems.

2.3 COMPUTATIONAL TOOLS

We provide here a list of some of the more commonly used computer programs available to the designer. The list is not exhaustive and does not, for example, contain programs recently developed to simulate Beam-Beam and Space Charge effects.

AGS[9]	A General Design Program
BBI[10]	For Studying Bunched Beam Instabilities
COMFORT[11]	A First Order Machine Control Program
DIMAD[12]	A General Design Program

HARMON[13]	A Program for Harmonic Analysis Studies
MAD[14]	A General Design Program
MARYLIE[15]	A General Design Program
PATRICIA[16]	A General Design Program
PETROC[17]	A General Design Program
SYNCH[18]	A General Design Program
TRANSPORT[1]	A General Design Program
TURTLE[19]	A General Tracking Program

The following comments are intended to help the novice in the choice of the appropriate programs to use for designing each aspect of a circular machine. Some programs cover a wide range of tasks while others are specialized to accomplish specific ones. Our classification does not reflect all of the capabilities of a given program as we do not have extensive experience with every program listed. The comments pertain to the version of the programs generally available to the public. Some programs are presently undergoing major revisions and almost all of them are in a continuous evolutionary development. The best advice we can give is to contact the authors to obtain more accurate and detailed information about the current status and capabilities of their particular computer program.

International Standard Input Format

In 1984, a group of program authors and users met at SLAC to discuss the possibility of defining a Standard Input Format. The result was the introduction of an input format following closely that of the program MAD.[20]

The following programs are now fully compatible with the International Standard Input Format: DIMAD, HARMON, MAD, TRANSPORT and TURTLE. Work is also in progress to make PATRICIA and MARYLIE compatible. Hopefully others will follow in this international effort to standardize the input format. We consider this to be very high priority item because of the potential to save time and avoid mistakes.

Matrix Analysis

Given the first and second order matrix representing a closed machine, one can derive most of the fundamental properties of the particle motion around the machine : for example, the tunes, chromaticities, beta functions, and dispersion function values at the closing point (end point of the matrix) of the machine.

All of the programs listed above, with the exception of BBI, HARMON, and TURTLE, have some form of matrix analysis capabilities.

AGS, COMFORT, PATRICIA, PETROC, and SYNCH do this analysis to first order. DIMAD, MAD, and MARYLIE do the analysis to second order, and TRANSPORT to third order.

Some versions of MARYLIE can treat higher order for special systems.

Some programs (AGS and PATRICIA) treat off-momentum analysis in an exact way even though the study is limited to first order matrices. However AGS and PATRICIA cannot correctly simulate combined function magnets which have dipole, quadrupole, and sextupole components present in the same element.

Tracking Analysis

DIMAD, MAD, MARYLIE, PATRICIA, and TURTLE all have options to study particle motion by tracking particles through the individual elements defining a machine. This enables one to analyse the effects of higher-order optics coupling terms between distinct elements. The analysis provided varies greatly from program to program and the interested reader is invited to consult the respective users' guides to determine the advantages and disadvantages of each program.

PATRICIA treats the elements in the 'kick' approximation and consequently provides symplectic tracking.

DIMAD, MAD, and MARYLIE also provide a general tracking mode in which the transforms are symplectic to the order of the approximation. They each provide a slightly slower tracking option which is symplectic to all orders.

Misalignment Errors

DIMAD, PETROC, TRANSPORT, and TURTLE can impose misalignment and field errors on the elements and thus provide information about the behaviour of machines with errors. DIMAD and PETROC also offer orbit correction schemes and a varying degree of machine behaviour analyses with errors and corrections present.

Special Analysis

HARMON and PATRICIA provide useful harmonic analysis of the machine structure. HARMON uses the harmonic analysis to optimise the sextupole distribution so as to reduce the effect of 'user selected' resonances.

BBI is a special program providing insight into coherent and incoherent bunched beam interactions with the environment (the vacuum chamber) and

between adjacent bunches.

3. Typical Problems in Accelerator Design

In this chapter we list some of the more common problems faced by the machine designer

3.1 Global Value Problems

These problems affect the control of parameters which are characteristic of the complete closed circular machine.

1) Tune adjustment in both transverse directions.
2) Chromaticity adjustments in both transverse directions.
3) Control of momentum compaction.

3.2 Local Value Problems

1) Control and adjustment of the chromatic dispersion.
2) Control and adjustment of beta function values and their chromatic dependence.
3) Special function adjustment : extraction schemes like $\frac{1}{2}$-resonance and $\frac{1}{3}$-resonance schemes, cooling schemes, spin polarisation modification and preservation.

4. Basic Mathematical Formulae

In this chapter we present simple basic formulae useful for a first draft of a design. More detailed and complete formulae may be found in references.[21] [2] [22]

4.1 Zeroth-Order Coordinates

To determine the correct layout and closure conditions of a machine one needs to define two coordinate systems : X, Y, Z is the set of absolute coordinates to which all points are referred, and x, y, z is a set of local coordinates attached to each element of the machine. The local coordinate system is attached to the nominal reference orbit of the machine. The nominal reference orbit is the trajectory followed by an ideal particle having the ideal momentum.

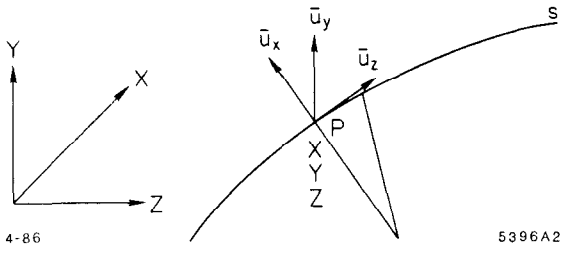

Figure 2. Reference Coordinate Systems

Figure 2 illustrates these two coordinate systems. O is the center of curvature of the trajectory followed by the nominal reference particle. To keep track of the possible space rotation of the plane of the trajectory one needs to define the local reference system by its three unit vectors $\overline{u}_x, \overline{u}_y, \overline{u}_z$. The layout of a machine is then completely determined by the coordinates X, Y, Z of the point P and the components of the three unit vectors $\overline{u}_x, \overline{u}_y, \overline{u}_z$ given at the entrance and exit of every element of the machine. $\overline{u}_z$ is the unit vector tangent to the trajectory, $\overline{u}_x$ is the unit vector perpendicular to the trajectory situated in the plane of the trajectory and $\overline{u}_y$ completes the right-handed set of unit vectors.

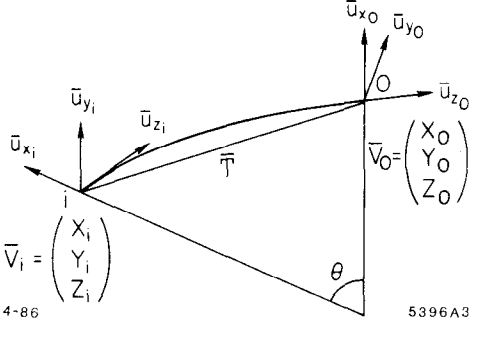

Figure 3. Coordinate Transformation in an Element

Figure 3 illustrates the relationship between the entrance and exit local coordinate systems in any element. Let

$$\overline{V} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \qquad \text{and } U = (\overline{u}_x, \overline{u}_y, \overline{u}_z) \tag{4.1}$$

Then the following formulae apply :

1) For a straight element (no curvature)

$$\overline{V}_o = \overline{V}_i + \overline{V}_l \qquad \text{and} \qquad U_o = RU_i \tag{4.2}$$

where $\overline{V}_i$, $\overline{V}_o$ are the absolute coordinate vectors at the entrance and exit points respectively, $\overline{V}_l$ is the length vector of the straight element, U_i, U_o are the matrices of the unit vectors at entrance and exit points and R is the matrix representing a possible rotation around the longitudinal axis.

2) For a curved element

With the notation indicated in figure 3 we have

$$\overline{V}_o = \overline{V}_i + \overline{T} \qquad \text{and} \qquad U_o = R_1^{-1} R R_1 U_i \tag{4.3}$$

where $\overline{T}$ is the chord vector spanning the element, R_1 is a rotation around the entrance longitudinal axis describing bending elements which bend the beam out of the reference horizontal plane. and where R is a rotation of angle θ around the y axis.

With these basic equations one can keep track of the closure of the basic reference orbit. Programs that have a layout display capability (e.g: TRANSPORT, MAD, DIMAD etc.) list a complete table of coordinates and orientation angles of the local coordinate system. This table is essential to surveyors and design engineers during the construction phase.

4.2 First-Order Formulae

In the local coordinate system a particle can be adequately represented by the vector :

$$\overline{v} = (x, x', y, y', -ct, \delta) \tag{4.4}$$

where δ is defined as : $\delta = \frac{(p-p_0)}{p_0}$ and p is the momentum of the particle and p_0 is the reference momentum (momentum of the ideal particle).

To first order this vector will vary linearly through any given element and also through any section of the full machine.

$$\overline{v}_0 = M\overline{v}_i \qquad \text{where } M \text{ is a } 6 \times 6 \text{ matrix} \tag{4.5}$$

and $\overline{v}_i$ and $\overline{v}_0$ represent the vector $\overline{v}$ at the entrance and exit of an element. M is called the transfer matrix of the element.

Restricting the motion to one plane with dispersion one obtains :

$$\overline{v} = (x, x', \delta) \qquad \text{and} \qquad M = \begin{pmatrix} c & s & d \\ c' & s' & d' \\ 0 & 0 & 1 \end{pmatrix} \tag{4.6}$$

We have adopted the notation of reference.[2] c and s are the cosine- and sine-like functions, d is the dispersion term. c', s' and d' are the derivatives of these functions with respect to the longitudinal coordinate.

When M describes stable motion (abs(trace(M)) < 2) then it can be written as :

$$M = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu & d \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu & d' \\ 0 & 0 & 1 \end{pmatrix} \tag{4.7}$$

with the condition : $|M| = 1$, from which we obtain :

$$\beta\gamma - \alpha^2 = 1 \tag{4.8}$$

The parameters μ, β, γ and α defined here for a full machine can be defined as local parameters at every point around the machine (see for example reference[2]). With this definition the particle motion (restricted to one transverse plane is described by :

$$x(s) = \sqrt{\epsilon\beta(s)}\cos(\mu(s) + \phi) \tag{4.9}$$

where ϵ is the emittance parameter and ϕ the initial phase.

The fundamental relation between μ and β is :

$$\mu(s) = \int_0^{\mathbf{s}} \frac{d\sigma}{\beta(\sigma)} \tag{4.10}$$

The variable **s** is the longitudinal coordinate measured along the reference orbit. The global tune of the closed machine (of length L) is defined by :

$$\nu = \frac{\mu(L)}{2\pi} \tag{4.11}$$

The strength of a magnetic mutipole element is defined by :

$$K_n = \frac{1}{B\rho}\frac{B_0}{a^n} = \left(\frac{1}{B\rho}\right)\left(\frac{1}{n!}\right)\frac{\partial^n B_y}{\partial x^n}\bigg|_{x=y=0} \tag{4.12}$$

where $B\rho$ is the particle rigidity, B_0 is the magnetic induction at the pole tip of the multipole magnet, a is the radial aperture of the multipole.

With this definition the tune is also given by the integral:

$$\nu = \frac{1}{4\pi}\int_0^L \beta(\sigma)K_1(\sigma)d\sigma \tag{4.13}$$

and the variation of the tune with momentum (with δ) is :

$$\frac{d\nu}{d\delta} = -\frac{1}{4\pi}\int_0^L \left[K_1(\sigma) - 2K_2\eta(\sigma)\right] d\sigma \tag{4.14}$$

The above formulae are the basic minimum needed for machine design. For more detail the reader should consult the references.[22] [21] [2] For formulae involving second order properties the reader is well to look up the tables I, II and III of reference.[2]

5. Building Blocks for Machine Design

In this chapter we introduce the reader to basic optics building blocks that can be used to formulate an initial machine design. We give only the elementary properties of these blocks and their transfer matrices only in one transverse plane.

Although standard building blocks (modules) may not be the final answer to a particular design, they allow one to quickly get an 'existence proof' that a design is possible and to make cost estimates for such a design. Refining the design for cost and physics optimization can come later.

5.1 Drifts or Field Free Spaces

In field free spaces, the slope of particles remains constant and the transverse coordinate is proportional to the distance travelled L. So the transfer matrix is :

$$M = \begin{pmatrix} 1 & L & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5.1}$$

5.2 Quadrupoles in the thin lens approximation

In quadrupoles, the transverse coordinate is constant and the slope variation is proportional to the strength of the quadrupole (the reciprocal of the focal length f). Its matrix, in the thin lens approximation is

$$M = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5.2}$$

5.3 DIPOLES IN THE THIN LENS APPROXIMATION

A wedge dipole with a bending angle α and radius of curvature ρ has the following matrix for the motion in its bending plane :

$$M = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{\sin\alpha}{\rho} & 1 & \sin\alpha \\ 0 & 0 & 1 \end{pmatrix} \tag{5.3}$$

If the deflection angle α is small and the radius of curvature is much greater than 1 , the matrix can be further approximated as:

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \alpha \\ 0 & 0 & 1 \end{pmatrix} \tag{5.4}$$

5.4 THE FODO ARRAY WITHOUT DIPOLES

The simplest and most common stable building block used in circular machine design is the FODO array. Figure 4 illustrates its symmetric setup with equal focusing strengths in both planes. More information can be found in references.[2] [23]

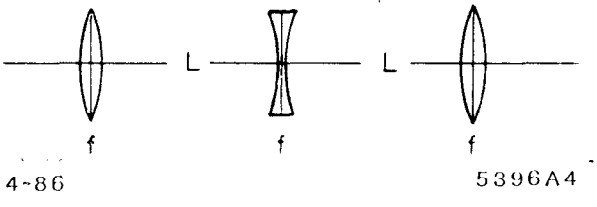

Figure 4. Symmetric Setup of the FODO Array

Using the transfer matrices defined in the previous sections, we obtain, after matrix multiplication, the following transfer matrix for the FODO array (from the middle of the first focusing quadrupole to the middle of the second focusing quadrupole):

$$M = \begin{pmatrix} 1 - \frac{L^2}{2f^2} & 2L(1 + \frac{L}{2f}) & 0 \\ -\frac{L}{2f^2}(1 - \frac{L}{2f}) & 1 - \frac{L^2}{2f^2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5.5}$$

The phase advance per cell μ, derived by comparing the above equation with equation (4.7), is:

$$\cos\mu = \left(1 - \frac{L^2}{2f^2}\right) \qquad \text{or} \qquad \sin\left(\frac{\mu}{2}\right) = \frac{L}{2f} \tag{5.6}$$

similarly, the beta function value at entrance and exit is :

$$\beta = 2L\frac{1 + \sin(\frac{\mu}{2})}{\sin\mu} \tag{5.7}$$

The ratio of the maximum and minimum values for beta (occuring at the center of the focusing and defocusing quadrupole positions) is :

$$\frac{\beta_{\max}}{\beta_{\min}} = \frac{1 + \sin(\frac{\mu}{2})}{1 - \sin(\frac{\mu}{2})} \tag{5.8}$$

5.5 THE FODO ARRAY WITH BENDING DIPOLES

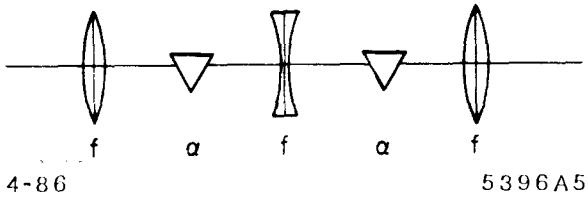

Figure 5. Symmetric Setup of the FODO Array with Dipoles

Consider a FODO array in which each drift now contains a dipole magnet as illustrated in figure 5. If the focusing effect of the dipole magnets is neglected and if the bending angle α, of each dipole magnet, is small, the transfer matrix of the FODO cell now becomes:

$$M = \begin{pmatrix} 1 - \frac{L^2}{2f^2} & 2L(1 + \frac{L}{2f}) & 2\alpha L + \alpha\frac{L^2}{2f} \\ -\frac{L}{2f^2}(1 - \frac{L}{2f}) & 1 - \frac{L^2}{2f^2} & 2\alpha - \frac{\alpha L}{2f} - \frac{\alpha L^2}{4f^2} \\ 0 & 0 & 1 \end{pmatrix} \tag{5.9}$$

The matched dispersion characterised by the vector $(\eta, \eta', 1)$ is defined by the eigenvector equation:

$$M \begin{pmatrix} \eta \\ \eta' \\ 1 \end{pmatrix} = \begin{pmatrix} \eta \\ \eta' \\ 1 \end{pmatrix} \tag{5.10}$$

The solution of which, for figure 5 is :

$$\eta = \alpha L(4\frac{f^2}{L^2} + \frac{f}{L}) \qquad \text{and} \qquad \eta' = 0 \tag{5.11}$$

It is worth noting here that the eta-vector $(\eta, \eta', 1)$ is an eigenvector of the Matrix M with eigenvalue 1. Observe that its third coordinate (arbitrarily chosen) is 1. Since M is a 3×3 matrix, it has three eigenvalues and eigenvectors. M being the matrix of a stable motion, the other two eigenvalues are imaginary and the third coordinate of the corresponding eigenvectors is zero. For more detail see reference.[3]

Written in terms of the phase advance, μ, the matched dispersion is equal to :

$$\eta = \alpha L\left(\frac{2 + \sin(\frac{\mu}{2})}{2\sin^2(\frac{\mu}{2})}\right) = \alpha L\left(\frac{1 + \frac{1}{2}\sin(\frac{\mu}{2})}{\sin^2(\frac{\mu}{2})}\right) \tag{5.12}$$

This last formula shows that η is a decreasing function of the cell phase advance μ. Practically this decrease is not very significant above 135 degrees of phase advance.

6. Some First-Order Problems and Their Solution

In this chapter we analyse three representative first-order design problems and present typical approaches to their solutions. These solutions are not unique, sometimes higher-order considerations will point to different approaches. The three problems considered are the creation of achromatic transport sections, matching η function values and matching beta function values, all of which are fundamentally important to circular machine design.

As is seen from the matrix for a FODO array, dipoles always introduce dispersion. However, some sections of the machine perform better if they are dispersion free, such as : the interaction regions, the RF station sections, and the injection and extraction regions.

6.1 First-Order Achromatic Sections

For both linear and higher order optics reasons, we have found it useful to incorporate achromatic modules as building blocks in circular machine designs.

A General Theorem for a First-order Achromatic Module

Consider a cell C whose 3×3 matrix is:

$$R = \begin{pmatrix} M & \overline{w} \\ 0 & 1 \end{pmatrix} \qquad \overline{w} = \begin{pmatrix} d \\ d' \end{pmatrix} \tag{6.1}$$

where M is the 2×2 tranfer matrix and $\overline{w}$ is the dispersion vector.

A lattice of n identical repetitive cells is achromatic to first order if and only if

$$M^n = I$$

or

Each cell has is achromatic, or $\overline{w} = 0$

Proof The matrix T of the set of n cells is :

$$T = R^n = \begin{pmatrix} M^n & M^{n-1}\overline{w} + M^{n-2}\overline{w} + \cdots + \overline{w} \\ 0 & 1 \end{pmatrix} \tag{6.2}$$

The dispersion vector of this matrix can be written :

$$\bar{d} = (M^n - I)(M - I)^{-1}\bar{w} \tag{6.3}$$

and so $\bar{d}$ will be zero if either $\bar{w}$ is zero or if $M^n - I$ is zero.

Application of First-order Achromatic Module

Straight sections (containing no dipoles) are achromatic to first-order and are (on their own) dispersion free. If straight sections are joined by achromatic curved sections then the dispersion function will remain zero in the straight sections and dispersion is confined internally to the curved sections. Any change in the quadrupole settings of the straight sections will have no effect on the dispersion function (to first order).

6.2 η SUPPRESSION OR η MATCHING

Consider an achromatic curved sections made of $2n$ identical cells C as described in the previous section. Suppose that the cells C are symmetric FODO cells as shown in figure 4.

The dispersion function variation along the achromat is illustrated in figure 6 . Note that the maximum dispersion occurs at the midpoint and that the dispersion function is not repetitive from cell to cell and so is not equal to the matched dispersion of the cell as defined in the FODO array section.

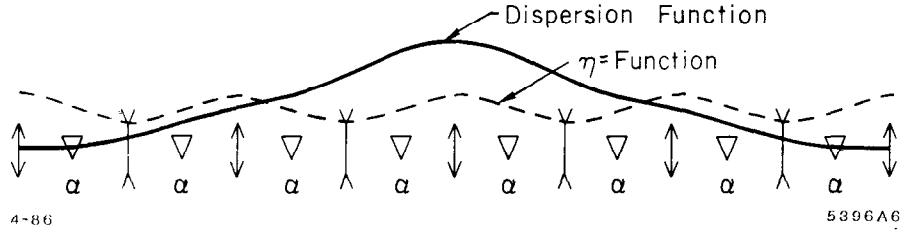

Figure 6. Dispersion Function in an Achromat of 4 Cells

In some applications it is important to match the dispersion to the cell in order to reduce the average value of the dispersion function.

We will now show how this can also be achieve in a modular way.

Consider the curved achromatic section described earlier. The 3×3 transfer matrix of the first half of the section is :

$$M = \begin{pmatrix} -I & \bar{d} \\ 0 & 1 \end{pmatrix} \quad , \quad \bar{d} = \begin{pmatrix} d \\ d' \end{pmatrix} \tag{6.4}$$

where I is the 2×2 unity matrix and $\bar{d}$ is a two component vector.

Straightforward matrix multiplication shows that the matrix M has the following eigenvector :

$$\begin{pmatrix} \frac{\bar{d}}{2} \\ 1 \end{pmatrix} \tag{6.5}$$

because the following identity is verified :

$$M \begin{pmatrix} \frac{\bar{d}}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{\bar{d}}{2} \\ 1 \end{pmatrix} \tag{6.6}$$

From this expression, and using the eigenvector equation (5.10) defining the η function, we obtain the following relation between η and the dispersion at the mid-point:

$$\bar{\eta} = \frac{\bar{d}}{2} \tag{6.7}$$

Note: this theorem is valid even if the cells are not symmetric. The only condition needed is that there exists a 'mid' point where the transfer matrix from the origin is $-I$.

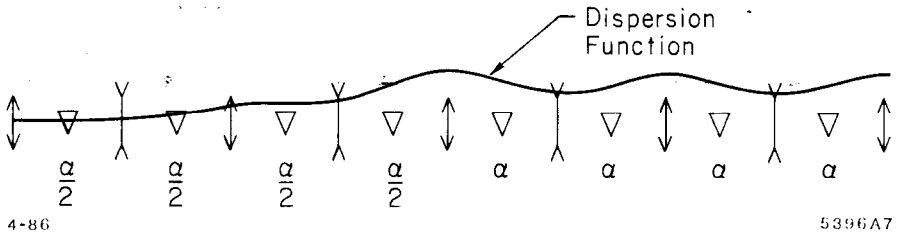

Figure 7. Simple Eta Matching of Achromatic Sections

Conclusion : If the above achromatic section is preceded by a half achromat with half strength dipoles , then the dispersion function will be matched in the cells of the main achromat as shown in figure 7.

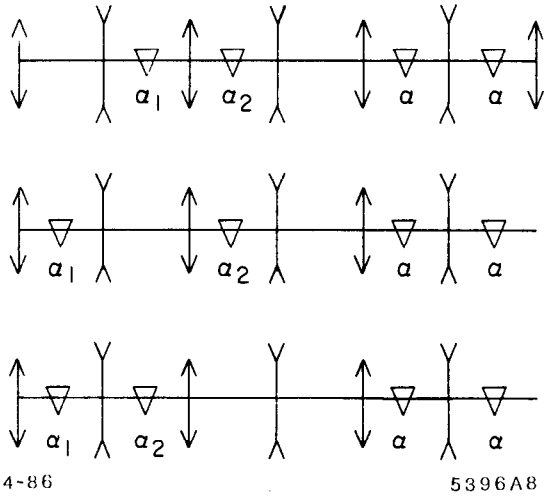

Figure 8. Alternate Eta Matching of Achromatic Sections

This result is valid only to the approximations made in the FODO section: i.e. the bending angle α is small and the focusing properties of the dipole magnets could be neglected. When these approximations cannot be made, the previous result only provides an approximate solution to the η matching problem. However this solution can be refined with computer fitting.

Clearly it may not be desirable, for budgetary reasons and sometimes for physics reasons, to have the four cells of the η matching section occupied by half strength magnets. It is also possible to design matching units with two unoccupied cells and two cells with dipoles. Such a case is illlustrated in figure 8.

A more detailed analysis of alternate dispersion matching systems can be found in reference.[24]

6.3 Beta Matching and Beam Transformers

In most modern circular machines there is the need to manipulate the size of the beam. Generally this beam size manipulation or 'beta-matching' can be achieved by the use of transformers. We illustrate here the use of one such transformer : the half wave length telescopic system. For more details on transformers the reader should consult references.[23] [2]

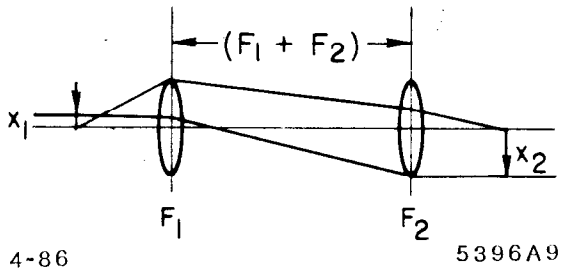

Figure 9. Schematic Layout of a Telescope

The principle of the telescopic system is illustrated, in one dimension, in figure 9, where F_1 and F_2 are the focal lengths of the lenses. The transfer matrix is simply :

$$R = \begin{pmatrix} -\frac{F_2}{F_1} & 0 \\ 0 & -\frac{F_1}{F_2} \end{pmatrix} \tag{6.8}$$

which shows that the system has a magnification M equal to $-\frac{F_2}{F_1}$.

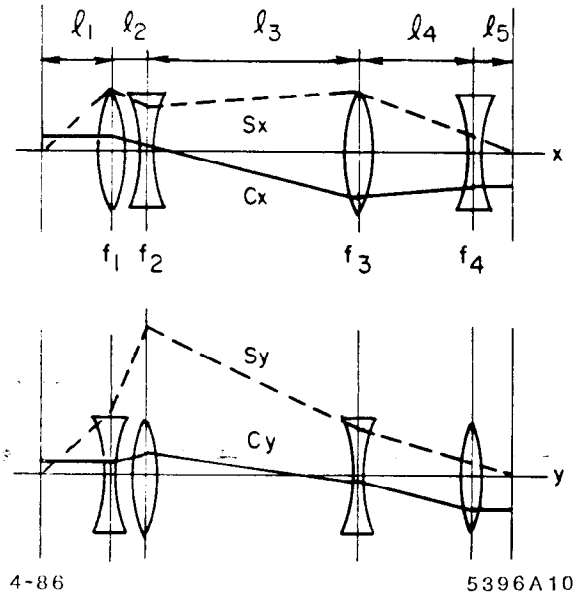

Figure 10. A Two Dimensional Telescopic Transformer

A practical module for a two dimensional telescopic transformer is shown in figure 10.

Each lens in figure 9 has become a doublet in figure 10. They could also have been replaced by a triplet or any other equivalent module. The choice of the parameters l_i and f_i is determined by computer fitting to obtain the following transfer matrix :

$$R = \begin{pmatrix} -M_x & 0 & 0 & 0 \\ 0 & -\frac{1}{M_x} & 0 & 0 \\ 0 & 0 & -M_y & 0 \\ 0 & 0 & 0 & -\frac{1}{M_y} \end{pmatrix} \tag{6.9}$$

Extensive use of telescopic systems is made in the Arcs and the Final Focus System of the SLC project[25] at SLAC.

7. Some Second-Order Problems and Their Solution

In this chapter we present three second-order problems and their solution: the principle of chromatic corrections, a method of control of higher order aberrations, the construction of a second order achromat.

7.1 Principle of Chromatic Corrections

Consider, as indicated in figure 11, the simplified motion of a particle in a cell with a phase advance of 2π.

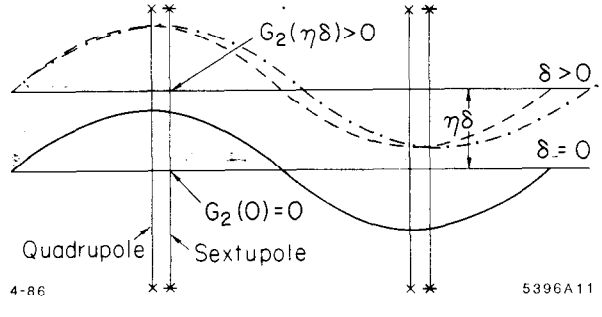

Figure 11. Principle of Chromatic Correction

The solid lines represent the reference orbit and the trajectory of one particle having the reference momentum ($\delta = \frac{p-p_0}{p_0} = 0$). Let us assume moreover that this cell is within a module where the η function is matched. We show, again in simplified form, the reference orbit of momentum δ as a straight line displaced from the reference orbit of momentum 0 by the amount $\eta\delta$. The following formula show the variation of the quadrupole strengths with momentum :

$$K_1 = K_{10} - K_{10}\delta \tag{7.1}$$

As a consequence, particles with positive momentum will have a longer period of oscillation. The trajectory of such a particle is indicated by the line $- \cdot - \cdot$

Suppose we introduce sextupole magnets and, for simplification, we place them at the quadrupole locations. Let us denote by x the transverse coordinate. The normalised field of the sextupoles and its local gradient are given by :

$$\frac{B}{B\rho} = K_2 x^2 \qquad \text{and} \qquad G_2 = 2K_2 x \tag{7.2}$$

So the local gradient, introduced by these sextupoles, around the 0 momentum reference orbit is zero and around the reference orbit for momentum δ is :

$$G_2 = 2K_2\eta\delta \tag{7.3}$$

If this added gradient cancels exactly the term $K_{10}\delta$ of K_1 then the off momentum particle will follow a trajectory having the same period as that of the reference momentum. This is indicated by the dashed line.

The above considerations together with formula (4.13), show that the variation of the tune with momentum (also called chromaticity) satisfies the relation (4.14):

$$\frac{d\nu}{d\delta} = -\frac{1}{4\pi}\int_0^L \beta(\sigma)[K_1(\sigma) - 2K_2(\sigma)\eta(\sigma)]d\sigma \tag{7.4}$$

This procedure is universally used to correct the second order and sometimes the higher order dependence of the tune on momentum.

7.2 THE -I PRINCIPLE

Consider, as schematically indicated in figure 12, two points A and B separated by a module with a transfer matrix equal to -I. A particle arriving at point A with coordinate x_A and slope x'_A will be at B with coordinate and slope of equal value but opposite sign. Suppose at A we subject the particle to an angle kick K. Between A and B the trajectory is deformed as indicated by the dashed line. The angle difference between the full line and the dashed line at B is -K. If we impose to the particle at B a kick equal to K, the motion outside the interval AB is undisturbed.

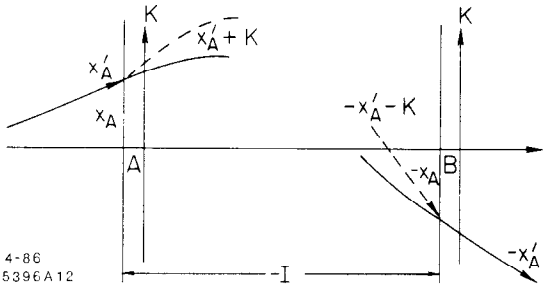

Figure 12. Illustration of the -I Principle

An example of such a situation, for all particles, is the case where two sextupoles of equal strengths are placed at points A and B and the transfer matrix between both points is -I.

The same compensation would be achieved with any pair of equal even order elements (eg: dipoles, decapoles etc.)

For elements with an odd symmetry (eg: quadrupoles, octupoles etc) in their transverse field, compensation is achieved if the strengths of the elements are equal with opposite signs.

If the transfer matrix between A and B is not -I , but has some magnification, and the phase shift is 180 degrees a generalized form of the -I principle applies. Pairs of elements can still be placed 180 degrees but the ratio of the kicks produced by the elements is equal to the magnification.

Any departure of the magnetic fields from the linear configuration creates geometric aberration. The above considerations show that the geometric aberrations created by any even order non linear field distribution can be cancelled to second order if the perturbing fields of equal sign and value are placed -I apart.

7.3 The Second Order Achromat Principle

Consider a first order achromat made of four cells, as illustrated in figure 13.

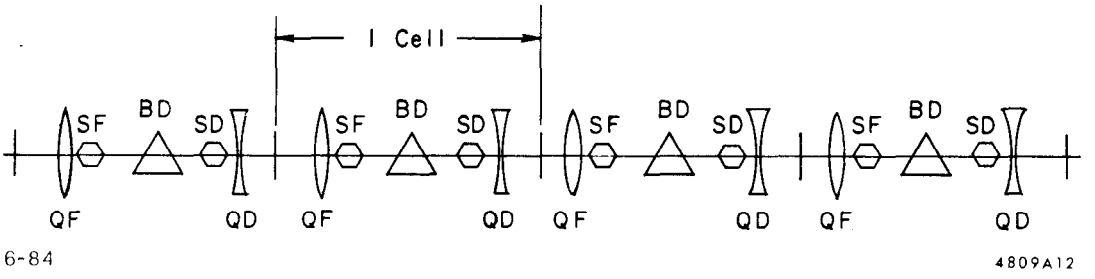

Figure 13. A Four-Cell Second-Order Achromat

QF,QD are focusing and defocusing quadrupoles and BD are the bending dipoles. Second-order chromatic correction of this module can be achieved by placing sextupoles as indicated in figure 13. Because the sextupoles are placed exactly -I apart their contribution to second-order geometric aberrations vanishes. If the strengths of the sextupoles SF and SD are adjusted to reduce one chromatic term in each plane to zero, then all second order chromatic terms vanish with the exception of the term describing the second order momentum dependence of the path length. More information on the Second Order Achromat can be found in references.[26] [27] [29]

In figure 14, we show a slightly modified version of the achromat. Notice that the sextupoles in each plane have been separated into families SF1, SF2 and SD1, SD2 , each pair still being separated by a -I unit. This module will be referred to as a pseudoachromat when the strengths of elements in family 1 is different from the strengths of family 2.

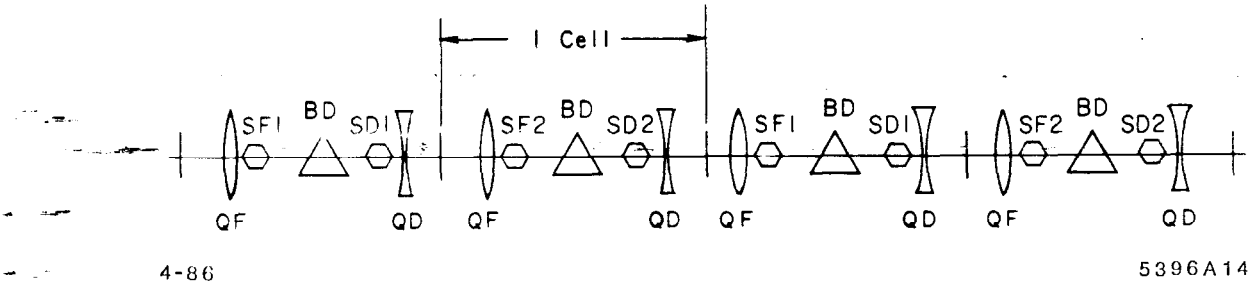

Figure 14. The Pseudoachromat Unit

When a pseudoachromat is inserted into a machine lattice, the strengths of the elements in families 1 and 2 can be adjusted independently to control the chromatic parameters of the machine(eg: the momentum dependence of the tune of the machine and of the beta function at a chosen point). The pseudoachromat is no longer achromatic to second order but the terms describing the second order geometric aberrations still vanish. because of the pairing of the sextupoles.

If one were using six 60-degree cells to build the pseudoachromat then three distinct sextupoles families are available in each transverse plane. In this case three distinct chromatic parameters could be controlled by the sextupole families. More detail about the use of the pseudo-achromat can be found in reference.[28]

8. Conclusion

In this report, we have outlined basic optics principles and have shown some applications to circular machine design. We have pointed to solutions using modular structure for the machine. These solutions must be considered to be a first step in the design process. Higher order effects, misalignement and field errors, collective effects will force revisions and modificatrions to the design.

As an illustration of the usefulness of this approach, let us mention some existing designs, presently under construction or considered for construction, that have used one or more of the ideas presented in this paper :

SLC[25]	SLAC	Stanford
LEP[30]	CERN	Geneva
EROS[31]	SAL	Saskatoon
CEBAF[32]	SURA	Newport News
PSR Ring[33]	BATES	MIT

More detail about the use of these modules in the above projects can be found in reference.[34]

REFERENCES

1. K.L. Brown, D.C. Carey, Ch. Iselin and F. Rothacker, TRANSPORT, A Computer Program for Designing Charged Particle Beam Transport Systems, SLAC 91 (1973 rev.), NAL 91, and CERN 80-04.
2. K.L. Brown and R.V. Servranckx, First and Second Order Charged Particle Optics, SLAC-PUB-3381 (July 1984).
3. E.D. Courant and H.S. Snyder, Theory of the Alternating Gradient Synchrotron, Annals of Physics, Vol. 3, 1-48 (1958).
4. K.L. Brown, A First and Second Order Matrix Theory for the Design of Beam Transport Systems and Charged Particle Spectrometers, SLAC Report No. 75, or Advances in Particle Physics. 1, 71-134 (1967).
5. D.C. Carey. Presentation at this conference.
6. J.S. Bell, 'Hamiltonian mechanics', CERN Accelerator School Course on Advanced Accelerator Physics, Oxford, September 1985.
7. E.D.Courant, R.D.Ruth, W.T.Weng. Stability in Dynamical Systems. SLAC-PUB-3415. August 1984.
8. A. Dragt. Lectures on Nonlinear Orbit Dynamics. 1981 Summer School on The Physics of High Energy Particle Accelerators, Fermi National Accelerator Laboratory, July 1981. AIP Conference Proceedings No. 87, page 147.
9. E. Keil et al, AGS, The ISR Computer Program for Synchrotron Design, Orbit Analysis and Insertion Matching. CERN 75-13 (1975).
10. M. Gygi-Hanney, A. Hofmann, K. Huebner, B. Zotter, BBI, Bunched Beam Instabilities in High Energy Storage Rings. CERN, Geneva.
11. M.D. Woodley, M.J. Lee, J. Jaeger, and A.S. King, COMFORT, Control of Machine Functions and Transport Systems. SLAC PUB 3086 (1983).
12. R.V. Servranckx, K.L. Brown, Lindsay Schachinger, and David Douglas, User's Guide to the Program DIMAD, SLAC-Report-285 (May 1985).
13. Martin H.R. Donald, Chromaticity Correction in Circular Accelerators and Storage Rings Part 1: A User's Guide to the Program HARMON. PEP Note 311. Stanford Linear Accelerator Center (1979).
14. C. Iselin. The MAD program. Lecture notes in Physics 215. Computing in Accelerator Design and Operation. Springer Verlag 1984.

15. A. Dragt, L. Healy, F. Neri, R. Rune, E. Forest, and D. Douglas, MARYLIE, A Program for Non Linear Analysis of Accelerator and Beam Line Lattices. Proceedings of the 1985 Particle Accelerator Conference, Vancouver (1985).

16. Helmut Wiedemann, User's Guide to the Program PATRICIA. PEP Technical Memo 230. Stanford Linear Accelerator Center (1981) and Y.Kamiya, Some Additions to PATRICIA. PEP Technical Memo 254 (1983).

17. G. Guignard and Y. Marti, PETROC Users' Guide, CERN/ISR-BOM-TH/81-32. November 1981. PETROC is the CERN version of the program PETROS written at DESY.

18. A. Garren, A. Kenney, E. Courant, M. Syphers. A User's Guide to SYNCH. Fermilab report to be published.

19. D.C. Carey, K.L. Brown, and Ch. Iselin, TURTLE, a Computer Program for Simulating Charged Particle Beam Transport Systems, Including Decay Calculations, SLAC-Report-246 (March 1982).

20. D.C. Carey and F. Christoph Iselin, A Standard Input Language for Particle Beam and Accelerator Computer Programs, Proceedings of the 1984 Summer Study on the Design and Utilization of the Superconducting Super Collider, June 23-July 13, 1984, Snowmass, Colorado, pages 389-391.

21. C. Bovet et al, A Selection of Formulae and Data Useful for the Design of A. G. Synchrotrons, CERN/MPS-SI/Int. DL/70/4 April, 1970.

22. G. Guignard. Selection of Formulae Concerning Proton Storage Rings. CERN-77-10, June 1977.

23. K.L.Brown. Presentation at this conference.

24. B.Autin, Introduction to beam optics. AIP Conference Proceedings #127. Physics of High Energy Particle Accelerators (BNL/SUNY SummerSchool 1983).

25. SLAC Linear Collider Conceptual Design Report, SLAC-Report-229, June 1980.
and
SLC Design Handbook, Stanford Linear Accelerator Center, Stanford University, December 1984.

26. K.L. Brown, A Second-Order Magnetic Optical Achromat, SLAC PUB 2257 (February 1979), and IEEE Transactions on Nuclear Science, Vol NS-26, No. 3, June 1979, p. 3490-3492.

27. D.C. Carey, Why a Second-Order Magnetic Optical Achromat Works, FERMILAB PUB 79/61-EXP, 2042, and Nucl. Instrum. and Methods 189(1981)365-367.

28. K.L. Brown and R.V. Servranckx, Chromatic Corrections for Large Storage Rings, the 11th Int. Conf. on High Energy Accelerators, CERN, Geneva, Switzerland, 1980, p. 656-660.

29. K.L. Brown and R.V. Servranckx, A Magnifying Magnetic Optical Achromat, SLAC PUB 2789 (August 1981). and Nucl. Instrum. and Methods 203(1982) 73/79.

30. LEP DESIGN REPORT, CERN-LEP/84-01

31. J.C. Bergstrom, H.S. Caplan, R.V. Servranckx and B.E. Norum. The Saskatoon Project: SORE(EROS) A Pulse Stretcher for the Saskatchewan 300 Mev Linac. IEE Transactions on Nuclear Science, vol NS 30 August 1983. Santa Fe Accelerator Conference.

32. The CEBAF project : A Proposal for a National Electron Accelerator Laboratory, SURA October 1982
and
B.NORUM et al. : CEBAF : a High Energy High Duty Factor Electron Accelerator for Nuclear Physics. Nuclear Instruments and Methods in Physics Research. Vol B 10/11 1985 p. 337.

33. Pulse Stretcher Ring Proposal for a CW Upgrade of the Bates Linear Accelerator Center, MIT, June 1984.

34. K.L. Brown and R.V. Servranckx, Applications of the Second-Order Achromat Concept to the Design of Particle Accelerators. 1985 Particle Accelerator Conference, Vancouver, BC, Canada, May 13-16, 1985, pages 2288-2290.