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PHENOMENOLOGY OF AN EXTRA NEUTRAL GAUGE BOSON IN ELECTRON-POSITRON COLLISIONS*

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ABSTRACT

Models with an extra neutral gauge boson (Z') are discussed. We review present limits on the properties of such a Z' which follow from requiring that the Z^0 mass not be shifted excessively through mixing, from neutral current experiments, and from the structure of the mass matrix which follows from the Higgs content of particular theories. We then examine what extensions in sensitivity can come from electron-positron annihilation experiments at the Z peak, with emphasis on the importance of cross-section measurements for final states involving lepton or quark pairs, followed by asymmetry measurements with polarized beams. Finally, we consider the additional information provided by asymmetry measurements in electron-positron collisions at energies above the Z.

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1. Introduction

Over the years there have been various theoretical motivations for enlarging the electroweak gauge group beyond the $SU(2) \times U(1)$ of the standard model. Attempts at grand unification of the electroweak with the strong interactions generally lead to gauge bosons beyond those in the standard model, as do left-right symmetric theories.

In addition, the recent advent of superstring theories[1] has given a further boost to interest in this possibility, since the combined low energy gauge group will generally be larger than $SU(3)_C \times SU(2)_L \times U(1)_Y$ in these theories.[2,3] More particularly, superstrings have revived interest in grand unified theories based on the exceptional groups, especially E_6.

Concurrently, from the experimental side, excitement about the range of new physics possibilities accessible at electron-positron colliders operating near 100 GeV center-of-mass energies has rekindled as the SLC and LEP grow closer to operation. The presence of additional neutral gauge bosons may well be one of the "easier" varieties of physics beyond the standard model to detect.

Most attention has been concentrated on the phenomenological implications of having the electroweak gauge group at low energies be $SU(2) \times U(1) \times U(1)$. This not only has the merit of simplicity as a sort of generic extension of the standard model, but was an early favorite arising from superstrings. It is by no means the only possibility[3,4] even within the framework of early scenarios for the derivation of the effective low energy theory from the theory at the Planck scale. We shall concentrate on this case here as well, more because of its definiteness and simplicity as an illustrative example than because it is in any way preferred by an uncertain ancestry connected to superstrings.

The presence of an extra neutral gauge boson, Z', will generally entail mixing with the Z of the standard model. The resulting physical states then will be mixtures of the initial Z and Z'. In particular, the physical Z will have an altered mass and altered couplings when compared to expectations based upon the standard model.

The constraints that the measured versus expected Z mass, the neutral current data, and the Higgs structure (and therefore structure of the $Z - Z'$ mass matrix) of superstring inspired models impose on the Z' mass and its mixing with the Z have been examined in a number of previous works.[5-8] In various combinations in different papers, these constraints have been used to limit the allowed domain of Z' parameters in specific models.

There has also been, both previous to and concurrent with superstrings, a good deal of study of the effects of a Z' upon electron-positron annihilation cross-sections and asymmetries.[9-17] Some of the work on electron-positron annihilation has been done without considering the constraints[5-8] already pre-existing from other experimental information. In this paper we first review these constraints as they presently limit the range of phenomenological possibilities. We also show the further restrictions on the domain of parameters for a Z' which may well exist from outside electron-positron annihilation experiments by the time high statistics data are available at the Z peak.

Then we examine what can be learned from the magnitude of the cross-section for annihilation into lepton and quark pairs at the Z peak. With the concentration on sophisticated experiments, it has been overlooked that this simple information, available at a relatively early stage of experimentation at the Z peak, will already further limit the range of allowed Z' masses and mixing angles in a significant way. With this as background, we consider what can be learned with polarized beams at and above the Z peak. Here we make no claim to uniqueness, as in one theory or another much of this work has also been done by others.[9-13,15-17] However, we put the knowledge to be gained with polarized beams into the same format of Z' mass and mixing angle, and so put this in the proper perspective of what is already known from other experiments.

The next section deals with the models under consideration: the respective electroweak couplings of the Z and Z', their mass matrix, and corresponding mixing. Section 3 treats the existing limits on such models. The section that follows begins with a treatment of what can be learned by electron-positron annihilation measurements at the Z peak without having polarized beams. The further restrictions that can be made using polarized beams follow at the end of that section. Finally, in Section 5 we examine the possibilities of learning additional information, particularly in the case where there is little or no mixing between the Z and Z', by doing experiments in the energy region above the Z. The combination of these measurements is found to provide a very sensitive indication of new gauge bosons up to energies of several hundred GeV.

2. Preliminaries

The Lagrangian describing the interaction of the neutral gauge bosons of an electroweak theory containing a Z' with the corresponding currents can be cast in the form

$$\mathcal{L} = e\, A_\mu\, J^\mu_{em} + \frac{e}{\sin\theta_W \cos\theta_W} Z_\mu\, J^\mu_Z + \frac{e}{\cos\theta_W} Z'_\mu J^\mu_{Z'}, \tag{1}$$

with the last term being new and giving additional neutral current effects. The weak charges to which the ordinary Z couples can be read off from the second term:

$$Q_Z = \frac{e}{\sin\theta_W \cos\theta_W}(I_{3L} - (Q_{em}/e) sin^2\theta_W). \tag{2}$$

Similarly, the weak charges of quarks and leptons which the Z' "sees" are contained in $J^\mu_{Z'}$ and are well-defined in a particular model.

As noted in the Introduction, we shall primarily concentrate in this paper on the case where the gauge boson Z' is coupled to a specific current that grew out of work on superstrings.[2–6] The situation is most clearly analyzed in terms of a much earlier topic, the breaking of E_6 as a group for grand unification.

The group E_6 has rank 6, two greater than the standard model or the group $SU(5)$ which is the smallest grand unified group that contains the standard model. It is convenient then to consider the breaking pattern:

$$E_6 \to SO(10) \times U(1)_\psi \tag{3}$$

and then,

$$SO(10) \to SU(5) \times U(1)_\chi, \tag{4}$$

where we have labelled the $U(1)$'s in a now standard manner.[18] If the $SU(5)$ contains the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$, then any extra $U(1)$ from the breaking of E_6 must be a combination of $U(1)_\psi$ and $U(1)_\chi$. The corresponding Z' will be a combination of Z_ψ and Z_χ which is defined by

$$Z'(\theta_{E_6}) = Z_\psi \cos\theta_{E_6} + Z_\chi \sin\theta_{E_6} \tag{5}$$

In the particular case of superstring theories broken by Wilson loops to a rank 5 group, a special $Z(\theta_{E_6})$ is specified:

$$Z_\eta = -\sqrt{5/8}\; Z_\psi + \sqrt{3/8}\; Z_\chi. \tag{6}$$

It is this Z_η that we shall be considering primarily in this paper, but we shall at various places consider what would happen to the quantity under discussion if the Z' were Z_ψ or Z_χ, as well as other intermediate possibilities.

The couplings of Z_ψ, Z_χ, and therefore Z_η follow from pure group theory and are given[19] in Table I. There one finds not only the couplings to the known fermions which comprise the 10 plus $\bar{5}$ representations of $SU(5)$, but to the "exotic" fermions which make up the full 27 dimensional representation of E_6. Note that because of the breaking pattern in Eqs. (3) and (4), any $Z'(\theta_{E_6})$ has the same coupling to each member of a given $SU(5)$ multiplet. The Z has different couplings generally to different members of an $SU(5)$ multiplet, but since it couples like a generator (or more exactly, a linear combination of generators), the sum over an $SU(5)$ multiplet of the Z charges vanishes. Therefore, also

$$\sum_{SU(5)\ multiplet} Q_Z Q_{Z'(\theta_{E_6})} = 0. \tag{7}$$

The width of the Z' is now determined. In Table II we give the total width and the branching fractions for decays into $u\bar{u}$, $d\bar{d}$, and e^+e^-. In each case we consider the possibility that none or all of the "exotic" fermions in the 27 dimensional representations of E_6 which contain the known quarks and leptons are light enough to be decay products of the Z'.

The physical Z and Z' bosons will not be the states which we have been discussing till now, but a mixture since the presence of an extra neutral gauge boson will generally entail mixing with the Z of the standard model. The two channel mass matrix has the form

$$M^2 = \begin{pmatrix} M_Z^2 - iM_Z\Gamma_Z & \delta M^2 \\ \delta M^2 & M_{Z'}^2 - iM_{Z'}\Gamma_{Z'} \end{pmatrix} \tag{8}$$

and for δM^2 small will be diagonalized by a rotation through an angle

$$\theta_{MIX} \approx \frac{\delta M^2}{M_{Z'}^2 - M_Z^2}. \tag{9}$$

The physical Z mass will be shifted downward from its "bare", standard model value, just as the Z' will be shifted upward:

$$\Delta M_Z \approx \frac{M_Z^2 - M_{Z'}^2}{2M_Z}\, \theta_{MIX}^2. \tag{10}$$

In a given theory, the Higgs content gives restrictions on the elements of the mass matrix in Eq. (8). These restrictions have been formulated in the general case by Cvetič and Lynn.[18] In the particular case of Z_η, written in terms of vacuum expectation values, the mass matrix has the form[20]

$$M^2 = M_Z^2 \begin{pmatrix} 1 & \sin\theta_W \frac{v_2^2-4v_1^2}{3(v_1^2+v_2^2)} \\ \sin\theta_W \frac{v_2^2-4v_1^2}{3(v_1^2+v_2^2)} & \sin^2\theta_W \frac{16v_1^2+v_2^2+25x^2}{9(v_1^2+v_2^2)} \end{pmatrix} \tag{11}$$

Such a form imposes additional correlations between θ_{MIX}, and the physical values of $M_{Z'}^2$ and M_Z^2.

The charges of the physical Z are therefore changed from those of the standard model through the rotation which is necessary to diagonalize the mass matrix in Eq. (8):

$$Q_{Z_{physical}} = Q_Z \cos\theta_{MIX} - Q_{Z'(\theta_{E_6})} \sin\theta_{MIX}. \tag{12}$$

The partial widths of the Z are correspondingly altered, with changes which are linear in θ_{MIX} for small mixing. The same effect is not so obvious if one thinks in terms of the diagonalized mass matrix and extracting the width by looking at its imaginary part. However, if the (mass dependent) imaginary parts of the off-diagonal elements of the original mass matrix (for channels open to both the Z and Z') and energy-dependent widths are taken into account, then the same result is recovered.

Although present in the partial widths, the term linear in θ_{MIX} in the total width of the Z vanishes. The reason is that it involves a sum over $Q_Z Q_{Z'(\theta_{E_6})}$ and, as seen in Eq. (7), this sum is zero when taken over all members of an $SU(5)$ multiplet. The known quarks and leptons in each generation completely fill two such multiplets, and the other "exotic" fermions of the 27 of E_6 fill other $SU(5)$ multiplets. So in either case the change in the total width of the Z is quadratic rather than linear in θ_{MIX} for small mixing.

3. Current Limits

The constraints which the measured as compared to expected (in the standard model) Z mass, the neutral current data, and the Higgs content of superstring models impose have been examined separately or in combination in a number of papers.[5-8] These serve to limit the values of the Z' mass and mixing angle and it is useful to briefly review these, if only to see what remains for electron-positron experiments to do.

We concentrate on Z_η. For the constraint provided by the measured mass of the Z, we have taken a combination of the present statistical and systematic errors as indicating agreement with theory to within 3 GeV and plotted it as the dash-dot curve in Figure 1. Durkin and Langacker[7] have reanalyzed the neutral current data in this context and we plot their boundary of the allowed region as the dashed curve.[21] The mass matrix in Eq. (11) gives the region bounded by the solid curve. As shown in Figure 1, the region of masses allowed for a Z' which has (unmixed) gauge couplings corresponding to Z_η starts at about 130 GeV and the allowed mixing angles obey $|\theta_{MIX}| \lesssim 0.1$. For Z' masses up to several times the Z mass, it is the neutral current data and/or the measured value of the Z mass being consistent with the standard model value which provide more stringent constraints than the Higgs content. The surprisingly low mass value allowed for the Z' is due to the small (compared to the Z) couplings to ordinary fermions of the Z_η.

In the following we take the inner (allowed) region from Figure 1 and use it as a reference curve with respect to which we can see the improvement in the bounds obtainable from future experiments. For example, in Figure 2 we show the boundary curves obtainable from measuring the W mass (relative to the Z) with an error of 500 MeV (curve 1) and of 64 MeV (curve 2). We regard the former as likely attainable in the next generation of hadron collider experiments and the latter as a possible ultimate accuracy.[22] Particularly in the latter case the region of parameter space allowed for the Z' has shrunk considerably. Note that these limits are relevant to the case where there are only additional Z''s; if there are additional W''s as well, they generally mix with the W, adding additional parameters, and removing the connection between the observed W mass and the unmixed Z mass.

4. Limits from Measurements at the Z Peak

With the results of the last section as background, we now direct our attention to electron-positron annihilation at the peak of the Z. We begin with the most straightforward measurements, namely the shift in the mass and width of the Z and the cross section for production of fermion-antifermion pairs at the peak.

Using the equations given in Section 2, we calculate the results shown in Figure 3 for the change in the mass and total width of the Z and the cross-sections for particular final-state fermion pairs in electron-positron annihilation at the (mixed) Z peak as a function of θ_{MIX} when we are considering a Z_η. The mass shift was treated in Sections 2 and 3; it depends on both the mass of the Z' and the mixing angle (we have taken $M_{Z'} = 200$ GeV). The other changes occur because of the altered couplings of the physical Z due to mixing with the Z'. Therefore they depend essentially only on the mixing angle with the Z as long as the Z' is many widths away from the Z.

The shift in the total width is very small and will likely be within measurement systematic errors. This is expected on the basis of Eq. (7) through cancellations of the first order terms in θ_{MIX} when the sum over modes includes all members of an $SU(5)$ multiplet.

This is not true for the cross-section for individual fermion-antifermion final states which is proportional to the partial width of the Z into these particular channels and to $\Gamma_{e^+e^-}$. There are changes of roughly 10% for variations of θ_{MIX} by ± 0.1. Such a change should be significant, particularly for $e^+e^- \to Z \to e^+e^-$ (or equivalently, $e^+e^- \to Z \to \mu^+\mu^-$), where a 3% measurement of the cross-section seems possible.[23] When translated into a limit on θ_{MIX} this corresponds to a one sigma limit of ± 0.08.

The accuracy with which quark-antiquark cross sections can be measured is less than that for mu pairs; 10% is probably a fair estimate[23] for $b\bar{b}$ (isolatable through semileptonic decays), which is the same as $d\bar{d}$ or $s\bar{s}$. The cross sections for $u\bar{u}$ and $c\bar{c}$ can then be obtained by subtraction from the total of all hadronic decays. Because of the decreased accuracy of measurement, these generally provide less of a constraint than the more accurately measured muon pair cross-section, even though the change in the latter due to mixing with Z_η is smaller.

Note also that mixing with Z_η produces a characteristic pattern where the cross section for $\mu^+\mu^-$ and $d\bar{d}$ increases when that for $u\bar{u}$ decreases and *vice versa.* The couplings of each $Z_{\theta_{E_6}}$ are different and produce correspondingly different patterns.

This is illustrated in a different way in Figure 4, where the cross section at the Z peak for annihilation into muon pairs is shown as it depends both on θ_{MIX} and on θ_{E_6}. Depending on which Z' is chosen, one gets an increased or decreased cross section from the value one would have with no Z' (shown by the dotted line). Note that the particular case of a Z_η gives a nearly minimal effect for this particular cross section. Choosing instead Z_ψ or Z_χ for our Z' would have produced much more dramatic effects in the muon pair cross-section and correspondingly better limits on θ_{MIX}. For example, we would have been able to limit $|\theta_{MIX}| \lesssim 0.04$ if the Z' was taken as Z_ψ.

There is a small front-back asymmetry at the Z in the standard model. Mixing with a Z' alters its magnitude as has been calculated in detail elsewhere.[11–13,16,17] In Figure 5 we show the limits placed on $M_{Z'}$ and θ_{MIX} by measurements at the Z peak with 10^4, 10^5, and 10^6 produced Z's. The limits are almost independent of $M_{Z'}$; the slight bending of the curves bounding the allowed region for the lowest Z' masses is due to finite width effects of the Z' (calculated with only decays into non-exotic fermions).

It is seen that this measurement is unlikely to add much to the limits which will be available from other measurements in a similar time period. Measurements with quarks in the final state are difficult because of the small samples of potential events after cuts to isolate a quark rather than an antiquark, and are complicated by $B - \bar{B}$ mixing.[24]

Finally we turn to the information available from experiments performed with a longitudinally polarized electron beam. With such a beam, we can form the asymmetry

$$A_{POL} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \frac{2\ v_e\ a_e}{|v_e|^2 + |a_e|^2}, \tag{13}$$

where σ_R and σ_L are the cross-sections (integrated over final angles for any particular final state or sum of final states) for right- and left-handed incident electrons, respectively, and v_e and a_e are the vector and axial-vector couplings of the Z to electrons. The second equality in Eq. (13) holds only at the peak of the Z. With $\sin^2\theta_W = 0.22$, the polarization asymmetry has a value of about -0.24. More importantly, since v_e is close to zero because of the particular value of $\sin^2\theta_W$ that Nature has chosen, A_{POL} is very sensitive to deviations from the standard model; in particular it is sensitive to changes in couplings from small admixtures of a Z' in the Z.[25] Again, note that because one is looking for changes in the couplings of the Z from the standard model, one is sensitive to the value of θ_{MIX} and not to that of $M_{Z'}$.

It is possible to entertain the idea of looking at decays of the Z into particular quark-antiquark channels with a polarized beam. However, it will be very difficult to get the requisite accuracy because of difficulties in identification of a particular quark and the great loss of statistics inherent in making the very restrictive cuts on the data necessary to isolate a particular channel.

Figure 6 shows A_{POL} at the peak of the Z as it depends both on θ_{MIX} and on θ_{E_6}. The effects of mixing are large, particularly for Z_η. They are almost non-existent for Z_ψ, for it has purely axial-vector couplings to electrons and its admixture does not change the vector coupling of the Z to electrons (to which A_{POL} is most sensitive) in lowest order.

The corresponding limitations on M_{Z_η} and θ_{MIX} are shown in Figure 7. Even with 10^4 Z's and a 5% systematic uncertainty in the polarization of the beam, the allowed region is as small as can be ascertained by the other measurements we have discussed. With 10^6 Z's and a 1% systematic uncertainty, one will be able to bound $|\theta_{MIX}| \lesssim 0.01$!

5. Limits from Measurements Above the Z

We have just seen that fairly tight restrictions can be placed on θ_{MIX} from various measurements at the Z peak. However, there is still the possibility that $\theta_{MIX} = 0$ or very nearly so. Then the Z is just that of the standard model, and there is no effect worth speaking about at $\sqrt{s} = M_Z$.

But there still are dramatic effects off the Z peak, particularly at somewhat higher energies. Even when θ_{MIX} is non-zero it is interesting to look at electron-positron collision energies other than at the Z peak to see what is the relative sensitivity to a Z'.

Figure 8 shows the front-back and polarization asymmetries as a function of $\sqrt{s}$ for several Z' masses and values of θ_{MIX} and θ_{E_6}. For a Z_η at 150 GeV and $\theta_{MIX} = -0.2$, just near the boundary of what is allowed by current experiments (see Figure 1), Figures 8a and 8d show that there are large deviations from what one would expect without a Z' both above and below the Z. Even if $\theta_{MIX} = 0$ the polarization asymmetry starts to deviate significantly from the standard model at $\sqrt{s} \sim 110$ GeV.

Figures 8b and 8e show that if there is appreciable mixing, there are noticeable deviations in the longitudinal polarization asymmetry starting at $\sqrt{s} \sim 110$ GeV even if the mass of a Z_η is as high as 300 GeV. If $\theta_{MIX} = 0$ there are still 10% changes in A_{POL} 15 GeV above and below the Z. However, the absolute value of A_{POL} and the cross-section below the Z are so small that there will be no statistical significance to a measurement there. The deviations for the front-back asymmetry are much smaller (less than about 1% in this case). Because one must identify a final fermion and distinguish it from the corresponding antifermion, adequate statistical power for a significant measurement of the front-back asymmetry appears to be an insuperable problem away from the Z peak.

In general, even for A_{POL}, one will be statistics rather than systematics limited (say, by uncertainty in the beam polarization) when doing measurements off the Z peak. The same integrated luminosity that produces 10^6 Z's at the peak, will give a $\sim 3\ \sigma$ deviation in the polarization asymmetry from the standard model value at $\sqrt{s} \sim 110$ GeV due to the presence of a Z_X at 200 GeV with $\theta_{MIX} = 0$ (see Figure 8f). Changing the mixing angle to -0.03 makes for a $\sim 6\ \sigma$ effect and it remains near 3 σ for the same mixing angle if, in this favorable case, the mass is raised to 400 or 600 GeV.

In summary, using the extra neutral gauge bosons accompanying the breaking of the grand unification group E_6 down to the standard model as examples, we have seen in this paper how a Z' could affect electron-positron annihilation experiments. In general the massive physical neutral gauge bosons will be mixtures of the Z of the standard model and the other neutral gauge bosons. This admixture changes the couplings of the Z from those of the standard model. Accurate measurements of the cross-section at the Z peak will already provide additional constraints on the properties of a Z'. Even more sensitive to these changed couplings is the longitudinal polarization asymmetry; it can be used to limit $\theta_{MIX} \lesssim 0.01$, given foreseeable systematic and statistical errors. But even if $\theta_{MIX} = 0$, measurements off the Z peak involving the front-back or polarization asymmetry can give decisive evidence for a Z'.

The combination of measurements at the Z and above it is a very powerful indicator of the presence of extra neutral gauge bosons. It should be possible, using these experiments in combination, to rule out (or find evidence for!) the presence of a Z' up to masses several times that of the Z.

Table I

Charges of the Z_χ, Z_ψ, and Z_η to fermions in the 27 dimensional representation of E_6 (from Ref. 7). The D is a charge $-e/3$ quark; the N an $SU(2)_L$ singlet, neutral lepton; and the E_0, E^- an $SU(2)_L$ doublet of leptons. The coupling is related to the charge by a factor of $\sqrt{5/3}(e/\cos\theta_W)$.

$SO(10)$	$SU(5)$	$2\sqrt{10}Q_\chi$	$\sqrt{24}Q_\psi$	$2\sqrt{15}Q_\eta$
16	$10(u,d,\bar{u},e^+)$	-1	1	-2
	$\bar{5}(\bar{d},\nu,e^-)$	3	1	1
	$1(\bar{N})$	-5	1	-5
10	$5(D,\bar{E}^0,E^+)$	2	-2	4
	$\bar{5}(\bar{D},E^0,E^-)$	-2	-2	1
1	$1(S^0)$	0	4	-5

Table II

Total widths and branching ratios of the Z_χ, Z_ψ, and Z_η to fermion pairs. The widths are in units of $10^{-3}\ M_{Z'}$. The widths and branching ratios in parentheses are with all the decays into pairs of exotic fermions included.

	$\Gamma(Z' \to \text{all})$	$BR(e^+e^-)$	$BR(u\bar{u})$	$BR(d\bar{d})$
ψ	4.9 (23.)	4.4% (.93%)	13% (2.8%)	13% (2.8%)
χ	11 (23.)	6.1% (2.8%)	3.6% (1.7%)	18% (8.3%)
η	5.8 (23.)	3.7% (.93%)	18% (4.4%)	11% (2.8%)

FIGURE CAPTIONS

1. Constraints on the mass and mixing angle of a possible Z_η following from $\Delta M_Z \leq 3$ GeV (region bounded by the dash-dot curve), neutral current data and the gauge boson masses (region bounded by the dashed curve, from Durkin and Langacker, Ref. 7), and following from the Higgs content in superstring theories (region bounded by the solid curve).

2. Constraints on the mass and mixing angle of a Z' provided by measurement of the mass of the W (relative to that of the Z) to an uncertainty of 500 MeV (region bounded by curve 1) and of 64 MeV (region bounded by curve 2). The dotted curve is the boundary of the allowed region from Figure 1.

3. Change in the mass, width, and peak cross-sections for $e^+e^- \to \mu^+\mu^-, u\bar{u}$ and $d\bar{d}$ at the Z (in units of $\sigma_{pt} = 4\pi\alpha^2/3s$) as a function of θ_{MIX} for mixing with Z_η.

4. The value of the cross-section for $e^+e^- \to \mu^+\mu^-$ at the Z (in units of $\sigma_{pt} = 4\pi\alpha^2/3s$) as a function of θ_{MIX} and θ_{E_6}. The dotted line gives the cross-section level when no Z' is present.

5. Region allowed for the mass and mixing angle of a Z' provided by measurements of the front-back asymmetry in $e^+e^- \to \mu^+\mu^-$ and their agreement within one sigma with the value expected in the standard model with 10^4 Z's (region bounded by curve 1), 10^5 Z's (region bounded by curve 2), and 10^6 Z's (region bounded by curved 3). The dotted line is the boundary of the allowed region from Figure 1.

6. The longitudinal polarization asymmetry (see text) at the peak of the Z as it depends on θ_{MIX} and θ_{E_6}. The dotted line is the value of the asymmetry in the standard model with $\sin^2\theta_W = 0.22$ and no Z' present.

7. Boundaries of the allowed region of Z' masses and values of θ_{MIX} from measurements of the longitudinal polarization asymmetry with 10^4 Z's and a 5% systematic error in knowledge of the polarization of the beam (solid

curve); 10^5 Z's and a 3% error (dashed curve); and 10^6 Z's and a 1% error (dash-dot curve). The boundaries are one sigma limits on the deviation of A_{POL} from the "prediction" of the standard model with no Z'. The dotted curve is the allowed region from Figure 1 for comparison.

8. The front-back asymmetry, A_{FB}, and the longitudinal polarization asymmetry, A_{POL} for $e^+e^- \to \mu^+\mu^-$ as a function of $\sqrt{s}$ for: (a) and (d) $M_{Z_\eta} = 150$ GeV and $\theta_{MIX} = 0$ (dashed curve) and -0.2 (solid curve); (b) and (e) $M_{Z_\eta} = 200$ GeV, $\theta_{MIX} = -0.15$ (solid curve) and $M_{Z_\eta} = 295$ GeV, $\theta_{MIX} = -0.05$ (dashed curve); (c) and (f) $M_{Z_\chi} = 200$ GeV, $\theta_{MIX} = -0.1$ (solid curve) and $\theta_{MIX} = 0$ (dashed curve). The dotted curve is in all cases the expectation without a Z'.

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19. See, for example, J. L. Rosner, Comm. Nucl. Part. Phys. 15, 195 (1986); P. Binétruy *et al*, Nucl. Phys. B273, 501 (1986); D. London and J. L. Rosner, University of Chicago preprint EFI 86-22, 1986 (unpublished).

20. See, for example, P. Binétruy *et al*, Ref. 19; Barger *et al*, Ref. 8; or Cvetič and Lynn, Ref. 13.

21. A similar region was found by V. Barger *et al*, Ref. 8.

22. The value of 64 MeV is obtained by combining in quadrature an error on the "theoretical" value of M_Z of 61 MeV (from an optimistic future error on the measured value of M_W of 75 MeV) with an optimistic error on the experimental value of M_Z measured at LEP of 20 MeV.

23. G. Feldman, private communication.

24. I. Bigi and M. Cvetič, SLAC preprint SLAC-PUB-3966, 1986 (unpublished).

25. The sensitivity of the polarization asymmetry has been noted and analyzed by Bélanger and Godfrey, Ref. 11, Angelopoulos *et al.*, Ref. 12, Cvetič and Lynn, Ref. 13, and Ader *et al.*, Ref. 17.

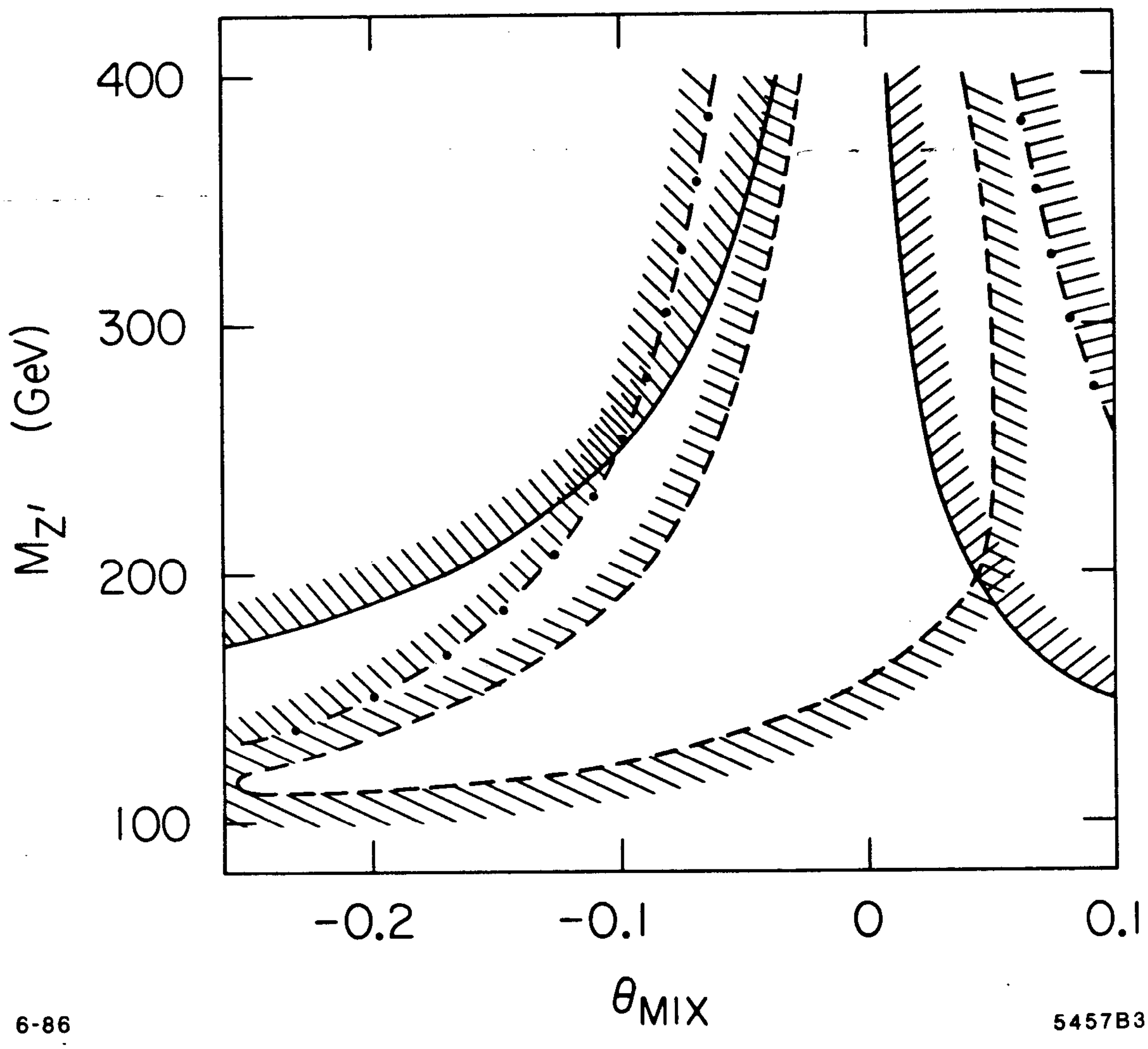


Fig. 1

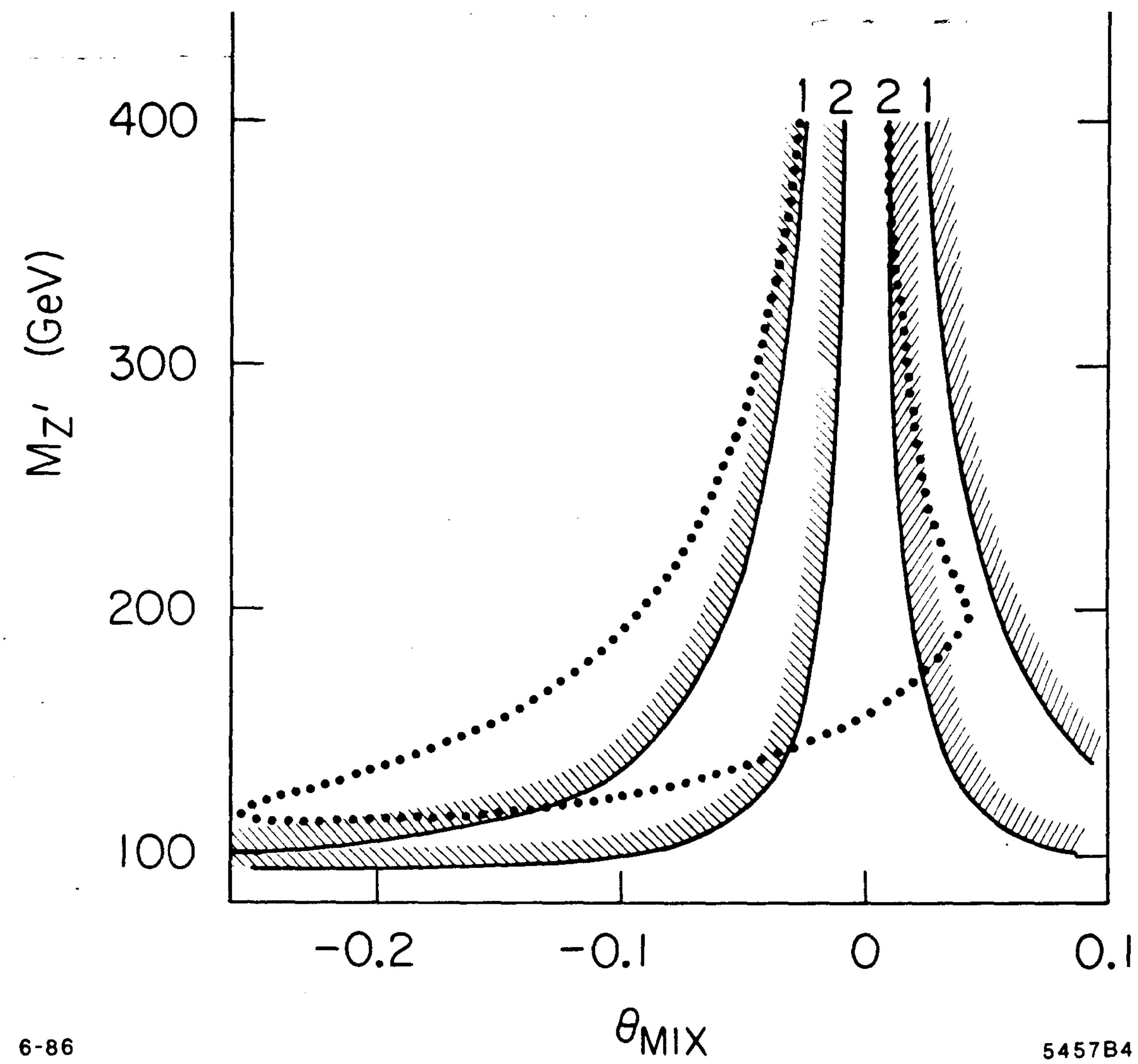


Fig. 2

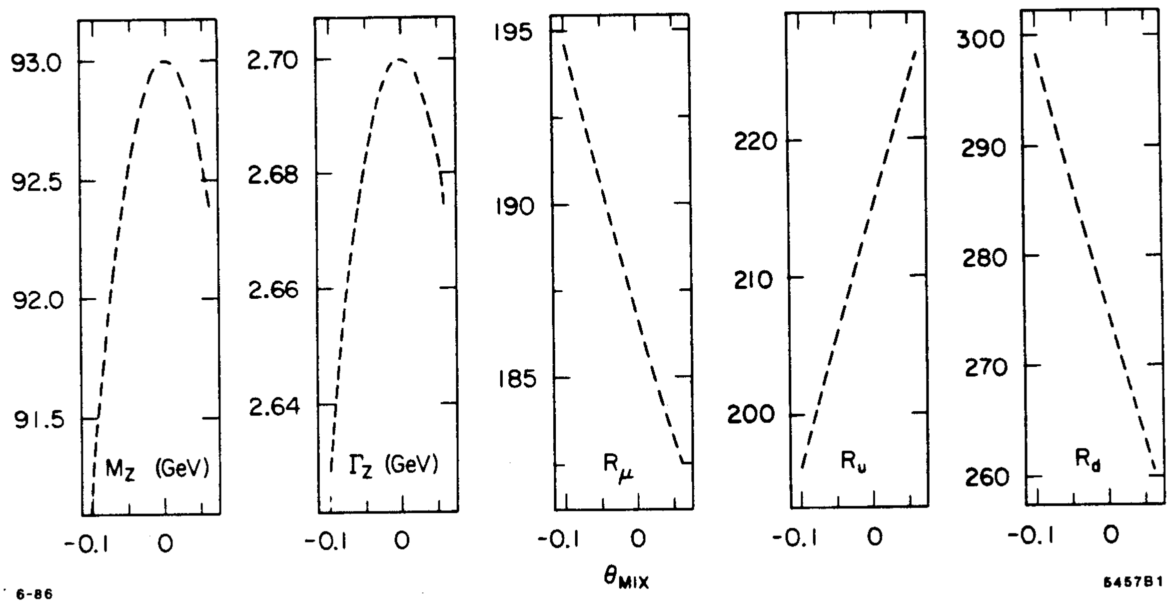

93.0
92.5
92.0
91.5
M_Z (GeV)
2.70
2.68
2.66
2.64
Γ_Z (GeV)
195
190
185
R_μ
220
210
200
R_u
300
290
280
270
260
R_d
-0.1
0
θ_{MIX}
6-86
5457B1

Fig. 3

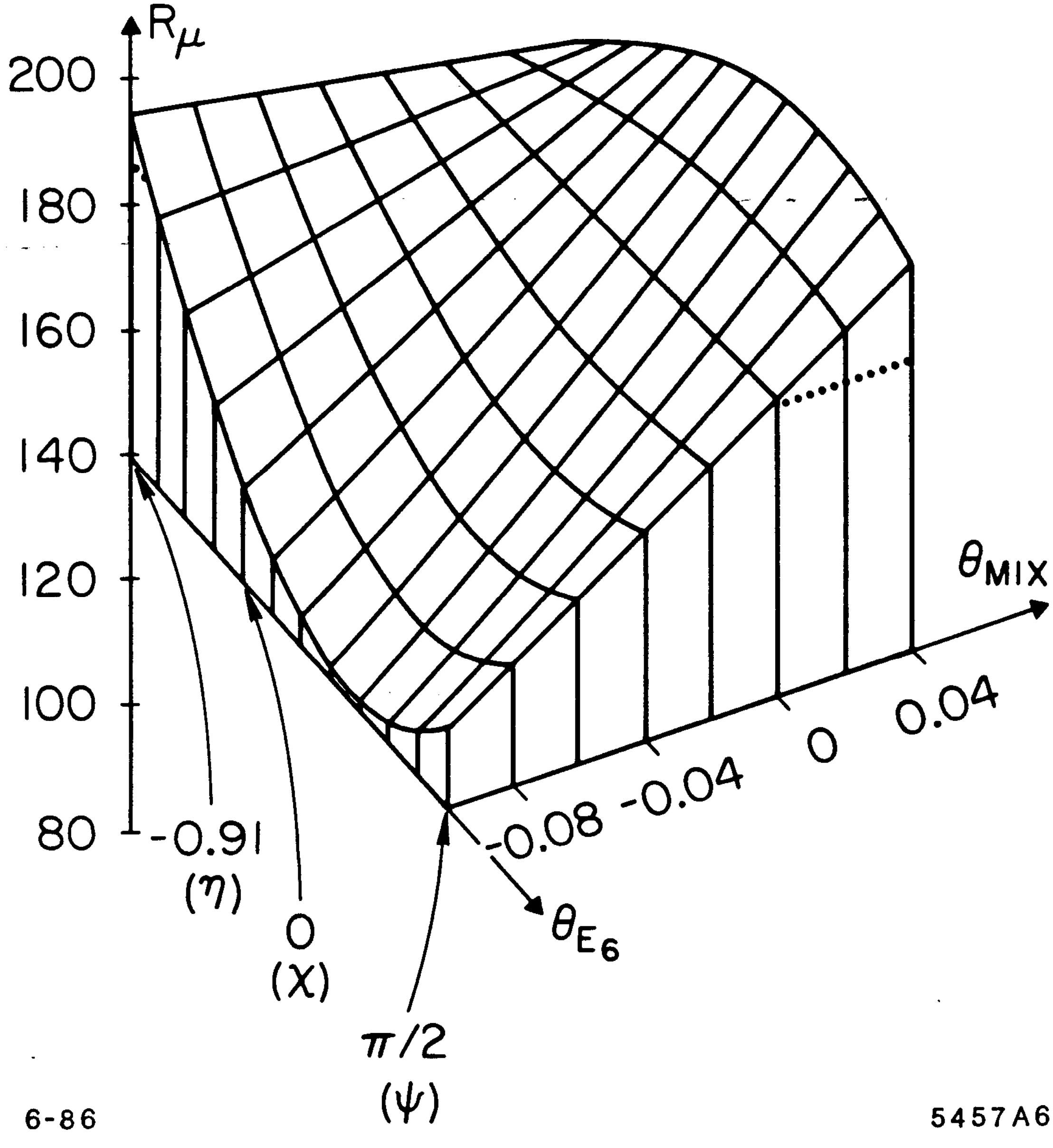

R_μ
200
180
160
140
120
100
80
-0.91
(η)
0
(χ)
$\pi/2$
(ψ)
θ_{E_6}
θ_{MIX}
-0.08
-0.04
0
0.04
6-86
5457A6

Fig. 4

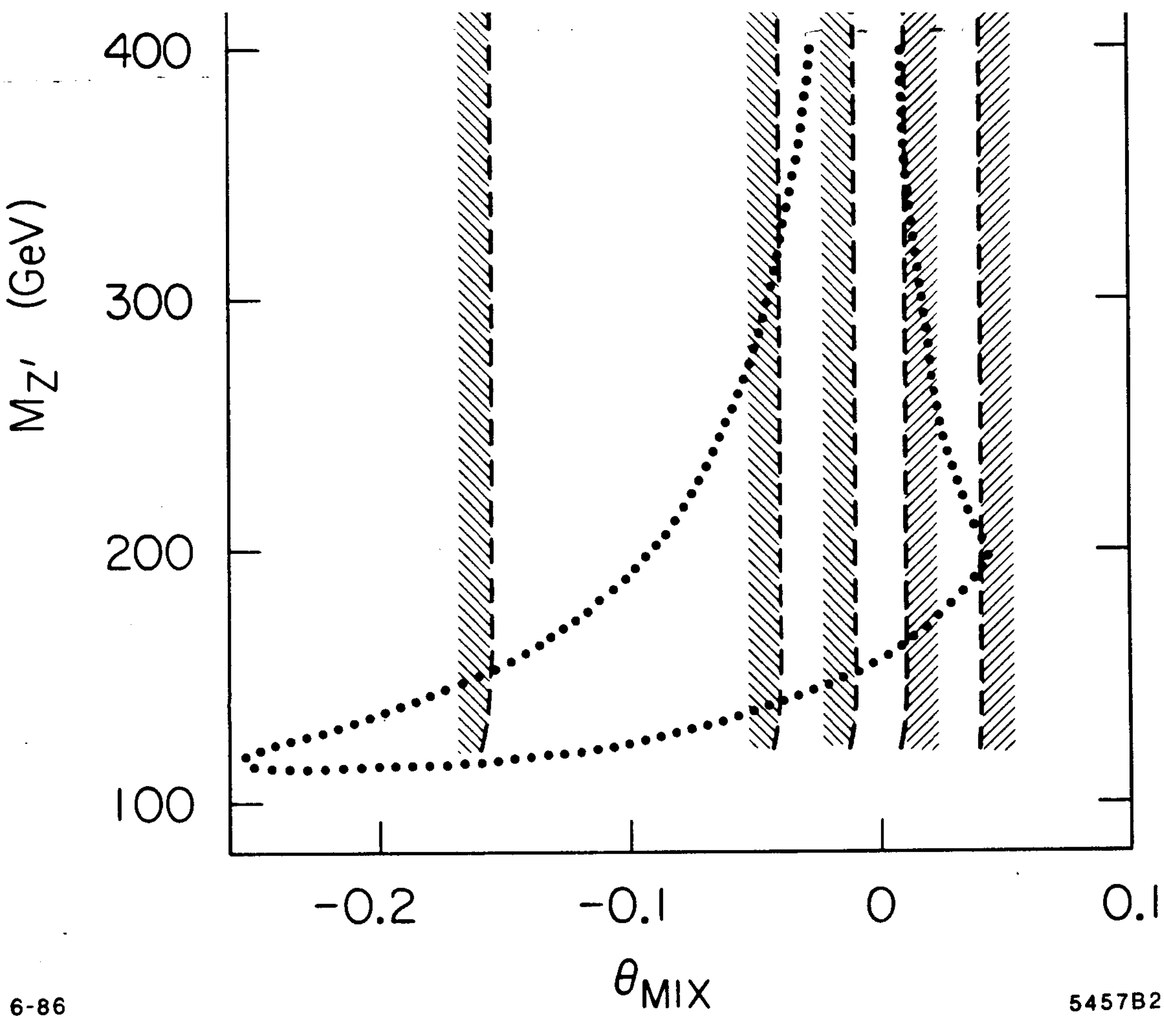
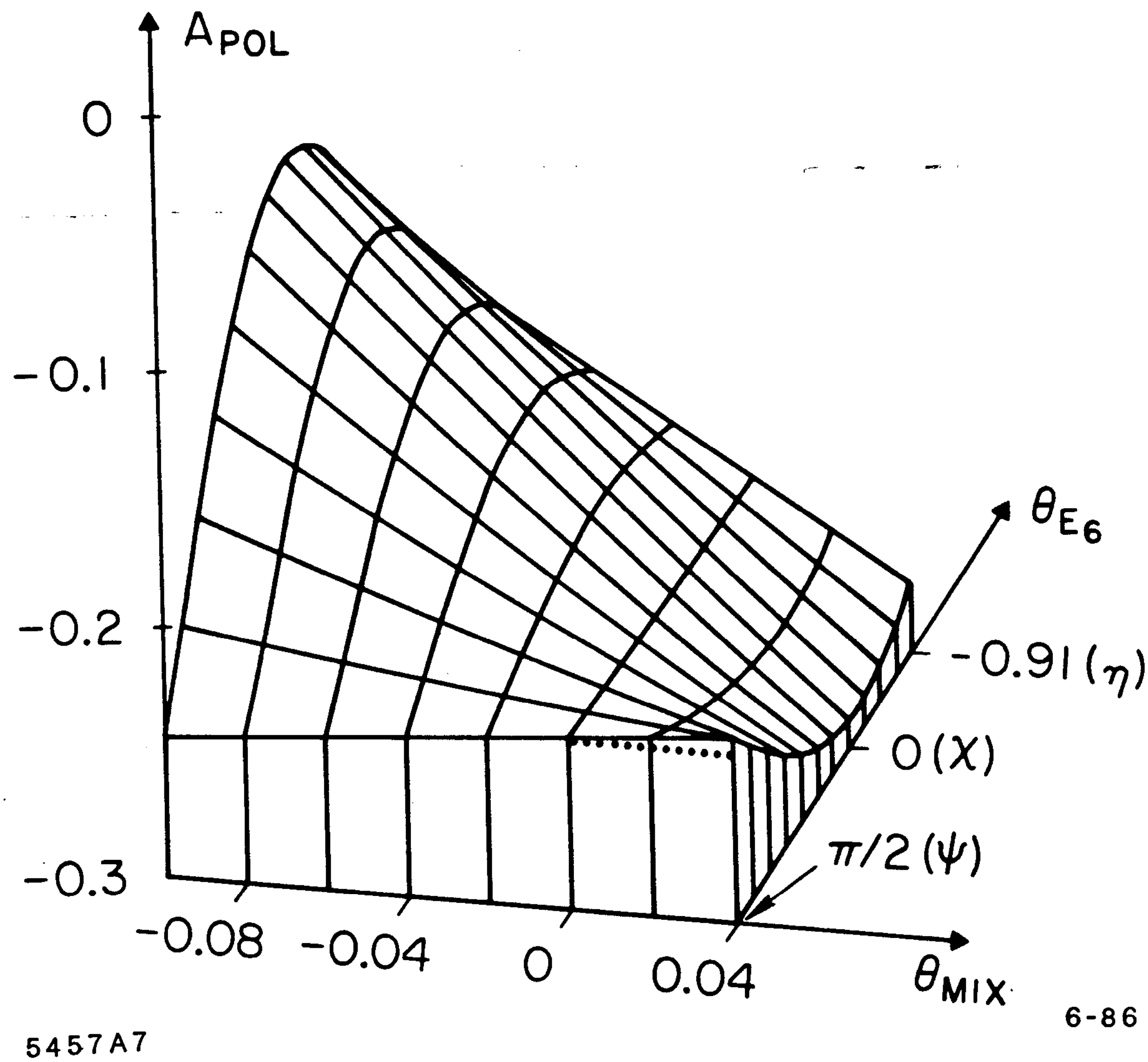


Fig. 5



A_{POL}
0
−0.1
−0.2
−0.3
θ_{E6}
−0.91 (η)
0 (χ)
$\pi/2$ (ψ)
−0.08
−0.04
0
0.04
θ_{MIX}
6-86
5457A7

Fig. 6

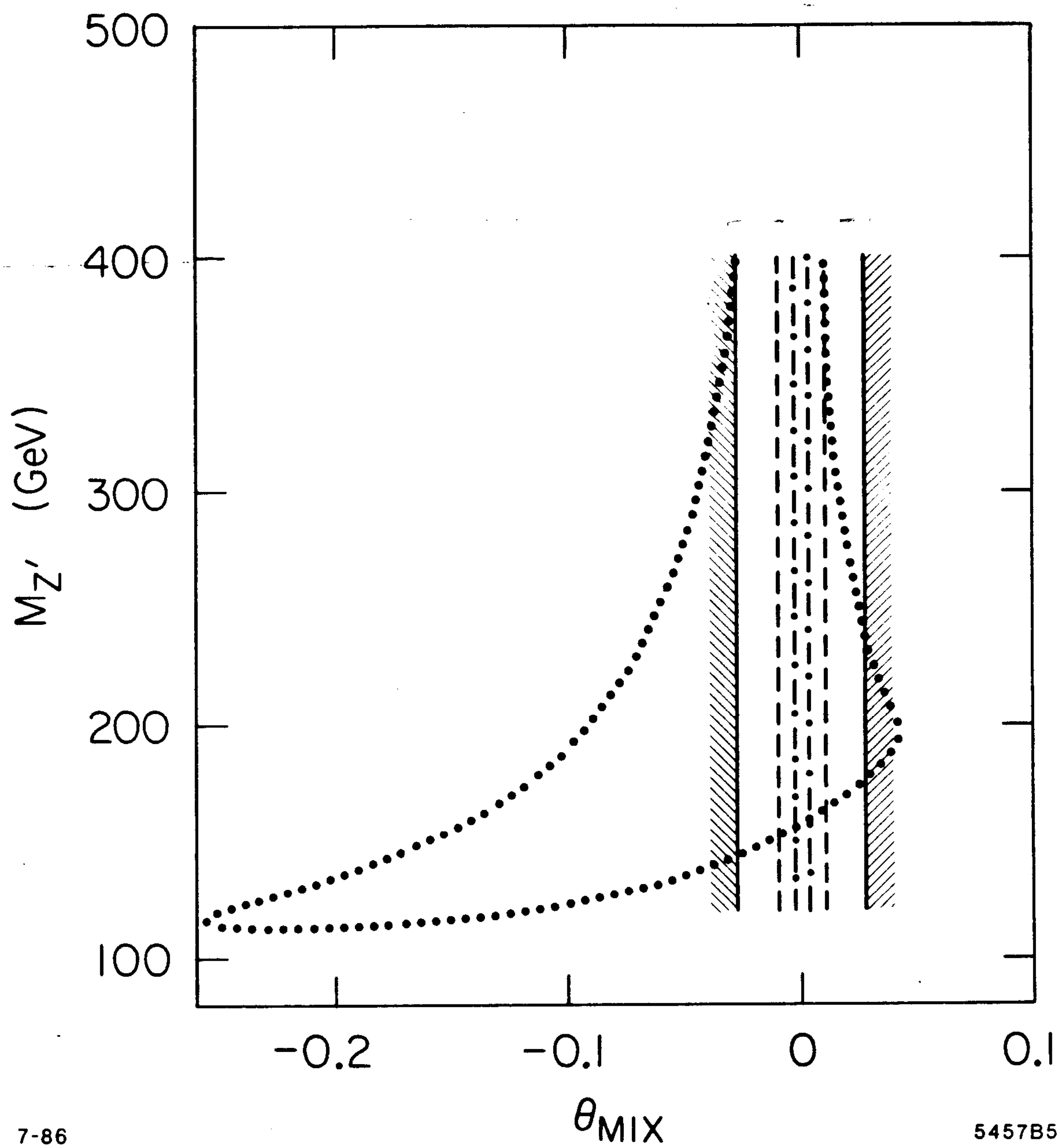
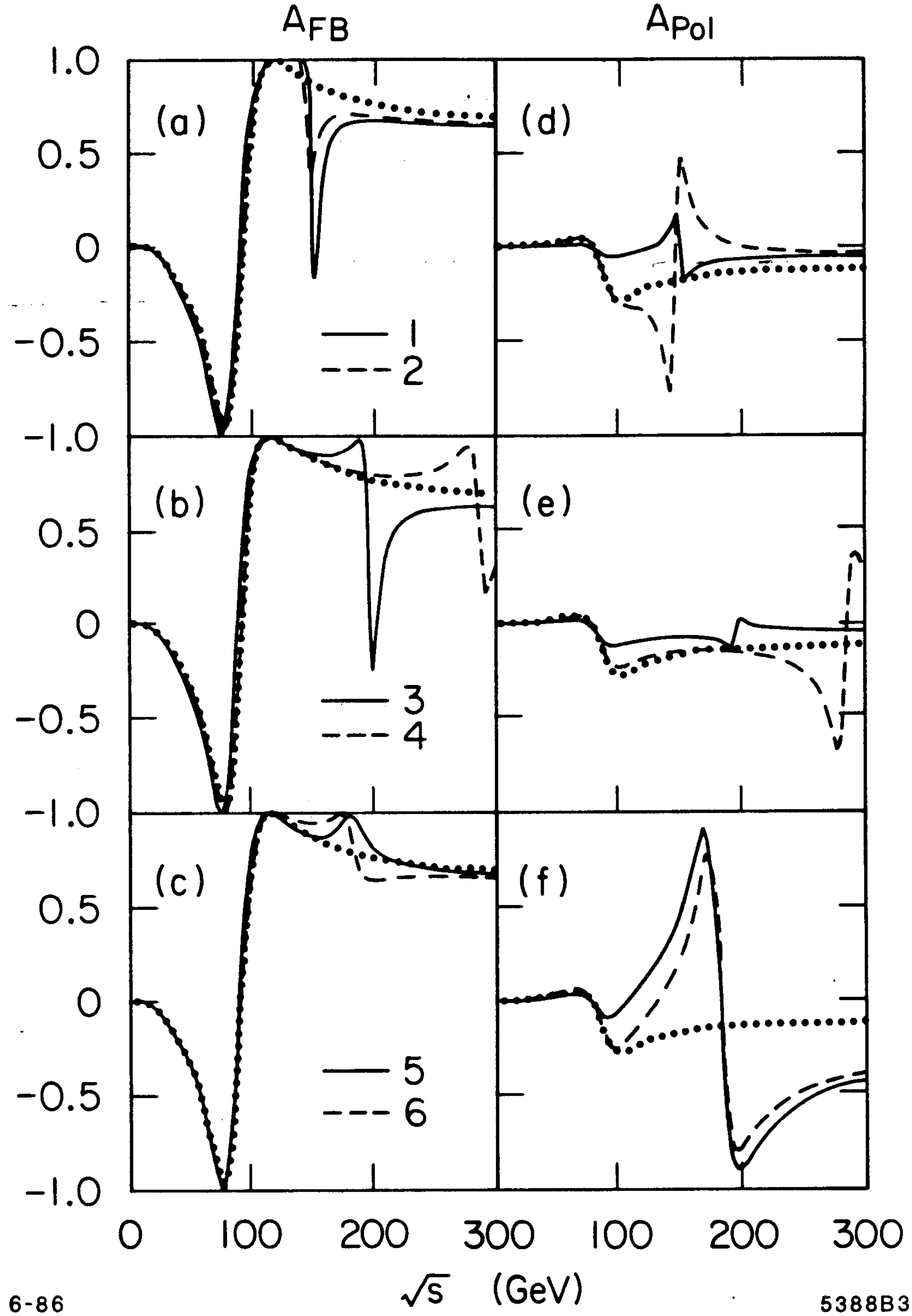


Fig. 7



A_{FB}
A_{Pol}
1.0
0.5
0
−0.5
−1.0
(a)
(b)
(c)
(d)
(e)
(f)
1
2
3
4
5
6
0
100
200
300
$\sqrt{s}$ (GeV)
6-86
5388B3

Fig. 8