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CAN A HEAVY HIGGS FIELD BECOME LIGHT AT LOW ENERGIES?*

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ABSTRACT

We study a perturbative evolution of the mass parameter of the Higgs fields in a supersymmetric gauge theory with N_c colors and $2N_f$ chiral superfields which interact with the Higgs fields via the Yukawa-type interactions. In general this mass parameter decreases at most one order of magnitude from energy scales $\Lambda_{MC} \sim 10^{15}\ GeV$ to $\Lambda_{HC} \sim 10^{3}\ GeV$. However, in a specific case when the beta function for the gauge coupling is small and the gauge coupling is relatively large, this mass parameter can decrease by many (3 to 5) orders of magnitude. We comment on the relevance of such a scenario for the tree level fine-tuning problem in the softly broken supersymmetric grand unified theories. In another context this may be relevant for the study of dynamically generated fermionic masses in a theory based on underlying preonic dynamics.

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In this note we would like to point out certain features of the perturbative evolution of the mass parameter of the Higgs fields. In particular, we would like to present a mechanism which ensures that at low energies $\Lambda_{HC} \sim 10^3$ GeV the mass parameter is many (3 to 5) orders of magnitude smaller than it is at large scale $\Lambda_{MC} \sim 10^{15}$ GeV.

Because of the well known gauge hierarchy problem in ordinary gauge theories at the loop level we confine our analysis to the supersymmetric gauge theory. The model, which has all the general features we want to exhibit, is based on $SU(N_c)$ gauge symmetry. The theory has $2N_f$ massless chiral superfields $\psi_a^L, (\psi_a^R)^c$ with $a = 1, \ldots, N_f$, which transform under $SU(N_c)$ as $\underline{\mathrm{N}}_c$ and $\underline{\overline{N}}_c$, respectively. One has also Higgs superfields ϕ_{ab}^I and ϕ_{ab}^{II} with $a, b = 1, \ldots, N_f$, which are singlets under the gauge group but they transform as $(\overline{N}_f, N_f)$ and $(N_f, \overline{N}_f)$ respectively under the global $SU(N_f)_L \times SU(N_f)_R$ symmetry.[♮1] In the superpotential ψ's and ϕ's couple in the following schematic Yukawa-type interactions:

$$W_h = h \sum_{a,b=1}^{N_f} \phi_{ab}^I \psi_a^L (\psi_b^R)^c \ . \tag{1}$$

There is also the self-interaction term for ϕ's:

$$W_m = m \sum_{a,b=1}^{N_f} \phi_{ab}^I \phi_{ab}^{II} \tag{2}$$

which yields the supersymmetric mass parameter m for the ϕ^I and ϕ^{II} fields.

We shall study the evolution of the parameters of the theory within perturbation theory. The one-loop renormalization group equations (RGE's) are of the

♮1 For the sake of simplicity, we chose the global flavor symmetry to be preserved.

following form:

$$\frac{dg}{dt} = -\frac{b}{2}\, g^3 \tag{3}$$

$$\frac{dh}{dt} = (Ah^2 - Bg^2)h \tag{4}$$

$$\frac{dm^2}{dt} = Ch^2m^2 \ . \tag{5}$$

Here $t = \frac{1}{16\pi^2}\ \ln \frac{E}{\Lambda_{MC}}$. Parameters b, A, B and C depend only on N_c and N_f and are of the following form:

$$b = 2(3N_c - N_f) \tag{6}$$

$$A = N_c + 2N_f,\ \ B = 2\ \frac{(N_c^2 - 1)}{N_c},\ \ C = 2N_c \ . \tag{7}$$

The RGE's (3-5) can be solved giving the following result:

$$g^2 = \frac{g_0^2}{1 + bg_0^2 t} \tag{8}$$

$$h^2 = \frac{\mathcal{C} g^2}{1 - \left(1 - \frac{\mathcal{C} g_0^2}{h_0^2}\right)\left(\frac{g_0^2}{g^2}\right)^{\mathcal{D}}} \tag{9}$$

$$m^2 = m_0^2 \left\{ \frac{g_0^2}{g^2} \left[\frac{\mathcal{C}\ \frac{g_0^2}{h_0^2}}{1 - \left(1 - \frac{\mathcal{C} g_0^2}{h_0^2}\right)\left(\frac{g_0^2}{g^2}\right)^{\mathcal{D}}} \right]^{1/\mathcal{D}} \right\}^{\mathcal{C}C/b} \tag{10}$$

where

$$\mathcal{C} \equiv \frac{B - \frac{b}{2}}{A} \tag{11}$$

$$\mathcal{D} \equiv \frac{B - \frac{b}{2}}{\frac{b}{2}} \ . \tag{12}$$

The subscript 0 denotes the value of the parameters at Λ_{MC}. The following general infrared behavior of the parameters can be observed:

(i) The gauge coupling constant g becomes strong at low energies for $b > 0$ or $N_c > 3N_f$. Note also that any additional chiral superfields would make b smaller.

(ii) The Yukawa coupling h follows the evolution of g in the infrared regime. From Eq. (9) one sees that h^2 approaches the infrared "fixed point"[1,2] Cg^2. Thus, when g becomes strong at low energies h is strong as well (see Fig. 1 for the evolution of g and h). From (9) one sees that the larger the initial value h_0 the "sooner" the infrared fixed point is reached. Note also that in the case when $b \to 0$, C is largest; i.e., $C \to 2$, when $N_c \gg N_f$ (see eq. (11)).

(iii) From the RGE (5) one sees that m always decreases as E decreases, and it remains positive. From the solution (10) one also sees that, in general, as long as the coupling constants are within the perturbative regime, i.e., $\{N_c g^2, N_c h^2\} < \mathcal{O}(8\pi^2)$, the mass parameter m is at most one order of magnitude smaller than m_0 (see fig. 2, case (i) for a generic case). Thus, in general one does not have a scenario in which the infrared value of the mass parameter is many orders of magnitude small than m_0.

Is there a possibility that for a specific choice for initial values of the parameter one can actually achieve $m/m_0 \ll 10^{-1}$ and still remain within perturbation theory? Though this looks highly implausible, there seems to be a viable possibility. First one should achieve $b \ll N_c$ and $\{N_c g_0^2, N_c h_0^2\} \lesssim \mathcal{O}(8\pi^2)$ so that a relatively strong gauge coupling constant evolves slowly over a large range of the energy scales from $\Lambda_{MC} \sim 10^{15}\ GeV$ to $\Lambda_{HC} \sim 10^3\ GeV$. On the other hand, one should also have $C \gtrsim O(1)$ (see eq. (11)) so that h^2 approaches a relatively large infrared "fixed point", which is in this case $Cg^2 \gtrsim O(1)g^2$. However, all those constraints do not seem to be satisfied at the same time. Namely, as $b \to 0$, i.e., $N_f \to 3N_c$, one has $C \to \frac{2}{7} \ll 1$ i.e., h^2 is not large enough to achieve $m/m_0 \ll 10^{-1}$.

One therefore needs additional chiral superfields which couple to the gauge fields weakly or not at all to ϕ's. Such fields would contribute to a smaller value of b but would not change the value of C. In the case of $b \ll 1$ and g_0 being relatively strong one should also take into account the two-loop correction to the gauge coupling beta function.[♯2]

Therefore, if one has, for example, a situation with $g_0^2 = 3$, $h_0^2 = 4$, $\beta_g \equiv -(\frac{b}{2}\ \tilde{g}^2 + \frac{b_2}{2}\ \tilde{g}^4)g \sim 0.1\tilde{g}_0^2 g_0$, $N_c = 8$, $N_f = 6 < N_c$,[♯3] and thus $C \sim 4/5$, one can achieve $m/m_0 \sim 10^{-4}$ for $\Lambda_{HC}/\Lambda_{MC} \sim 10^{-12}$ (see fig. 2), case (ii). Here $\tilde{g}^2 = g^2/(16\pi^2)$. The above choice of parameters is a representative case and does not involve fine tuning. Actually, for a more optimal choice with $N_c \gg N_f$, the ratio m/m_0 can be another one or two orders of magnitude smaller. In addition this choice of parameters is still within the perturbation theory; namely, the two-loop corrections for the beta functions for h and m (see eqs. (4,5)) may yield a 10% correction only.[♯4]

♯2 Note that for $b = 0$, i.e., when the one-loop beta function for the gauge coupling is zero, the two-loop beta function is positive and $\beta_g \simeq [6(N_c^2 - 1)\tilde{g}^4]g$ [3].

♯3 Note that in the case of small β_g, $C \sim B/A$ (see eq. (11)). C approaches its maximum value, i.e., $C \to 2$, when $N_c \gg N_f$. Thus the above choice for N_f is not optimal.

♯4 Presently one does not have good guidelines for the evolution of the parameters for such a theory in the nonperturbative region. Within the lattice gauge theories the evolution of the gauge coupling without fermions has been studied [4]. Also, the lattice gauge theories with scalars and gauge fields have been examined in a particular context [5]. However, evolution of g, h and m has not been studied yet.

Now, we would like to suggest the examples of the elementary particle interactions where such a scenario may be of relevance.

01 SOFTLY BROKEN SUPERSYMMETRIC GRAND UNIFIED THEORY

Although in such theories the gauge hierarchy problem is solved at the loop level, one still has to fine-tune the value of the mass parameter for the Higgs fields which break $SU(2)_L \times U(1)_Y$. In general, the supersymmetric mass parameter, i.e., the equivalent of parameter m in Eq. (2), of such Higgs fields is of the order of the scale where the unified gauge group is broken. The mass parameter has to be fine-tuned to be of the order of the scale at which $SU(2)_L \times U(1)_Y$ is broken. The above suggested scenario[♯5] would then provide a way to lower the value of this mass parameter by 3 to 5 orders of magnitude. This, of course, would not solve the tree-level gauge hierarchy problem, but it would at least make it less severe. Such a scenario may also be utilized in a theory with an intermediate scale $\Lambda \sim (10^3 \text{ to } 10^5)$ GeV where in principle one might achieve the breaking of $SU(2)_L \times U(1)_Y$ without excessive fine tuning.

♯5 This means that in such a theory, one should have a gauge group $SU(N_c)$ with a fairly large gauge coupling and chiral superfields which transform nontrivially under $SU(N_c)$ and couple to the Higgs fields which break $SU(2)_L \times U(1)_Y$ via the strong Yukawa-type interaction. In principle, the $SU(3)$ of QCD and one or two additional flavors with a large Yukawa coupling could have taken such a role. However, the large magnitude of the QCD gauge coupling would not have been compatible with experiment and the idea of grand unification.

Note also, that in the case of a softly broken grand unified theory, one should study the evolution of the softly broken mass parameters,too.

02 THEORIES BASED ON UNDERLYING PREONIC DYNAMICS

Such a scenario may also be important in a preonic theory where at the level of composite fields, one has a Yukawa type interaction between ϕ's and ψ's.

This interaction can contribute to a dynamical generation of ordinary fermionic masses via formation of $\langle\overline{\psi}\psi\rangle \sim \mathcal{O}(\Lambda_{HC}^3)$ condensates [6,7]. Such an interaction may provide a way of generating a desired fermionic mass hierarchy. Unfortunately, the mass of the scalar fields is usually of the order of the compositness scale $\Lambda_{MC} \gg \Lambda_{HC}$. It has been shown [7] that in this case the contribution from the ϕ exchange to the effective potential for $\langle\overline{\psi}\psi\rangle$ condensates is proportional to $h^2/(16\pi^2) \times \Lambda_{HC}^2/m^2$ which in general is much smaller than one. However, if $h(\Lambda_{HC})$ is relatively large and $m(\Lambda_{HC}) \ll m(\Lambda_{MC})$ this contribution need not be negligible and thus it can be responsible for providing a desired fermion mass hierarchy [6].

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FIGURE CAPTIONS

1. Evolution of the gauge (solid) and Yukawa (dots) coupling for (i) $g_0^2 = 0.15$, $h_0^2 = 0.30$, $N_0 = 8$, $N_f = 6$, $b/2 = 18$ and (ii) $g_0^2 = 3$, $h_0^2 = 4$, $N_c = 8$, $N_f = 6$, $\beta_g \approx 0.1\ \tilde{g}_0^2 g_0$. Here $\tilde{g}_0^2 = g^2/(16\pi^2)$.

2. Evolution of the Higgs field mass parameter with the parameters (i) and (ii) of Fig. 1.

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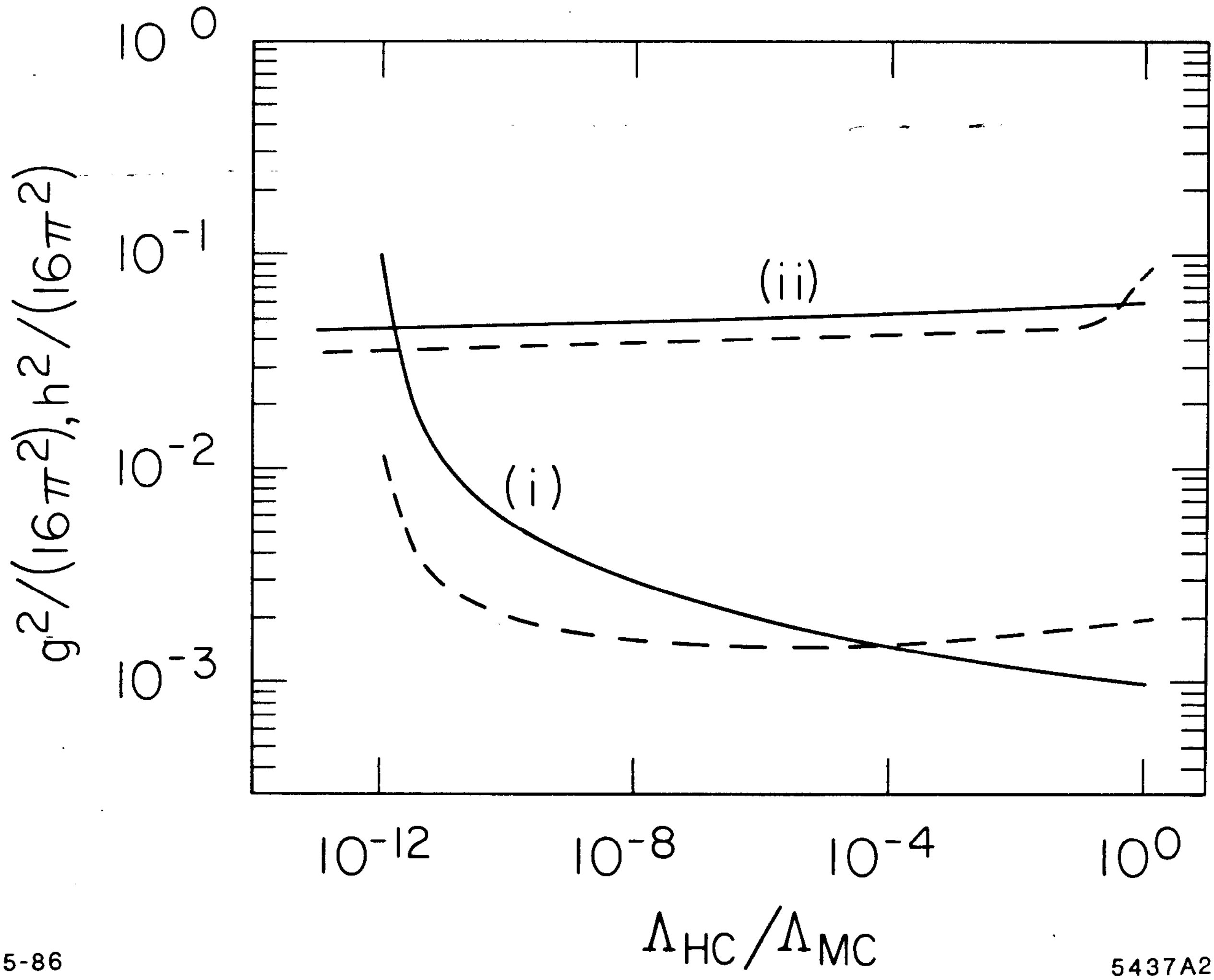

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Fig. 1

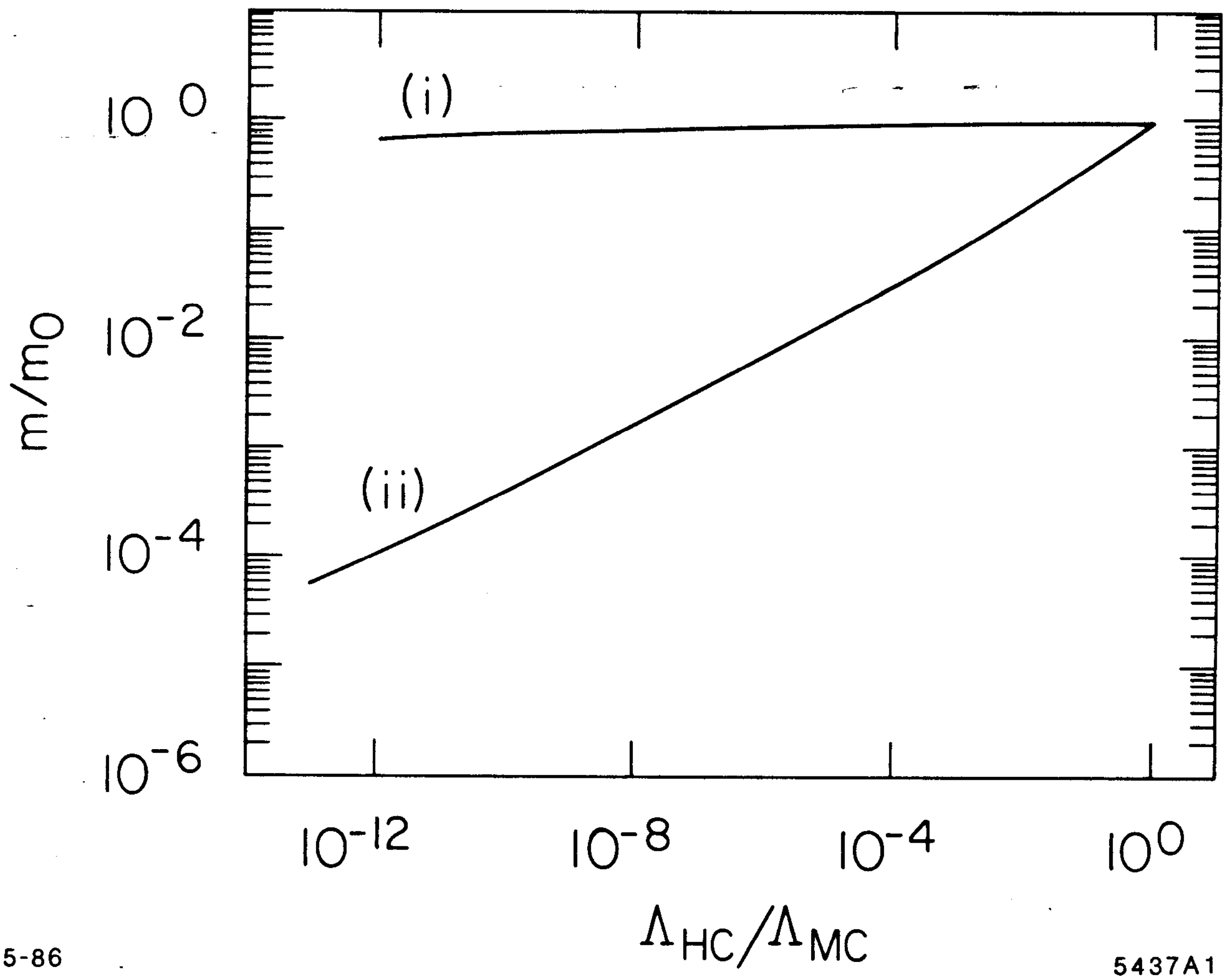

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Fig. 2