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DYNAMICAL BREAKDOWN OF FLAVOR AND SCALAR EXCHANGE*

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ABSTRACT

We study a dynamical breakdown of the flavor symmetry in the presence of the flavor symmetric Yukawa-type interaction between chiral fermions and scalars. We evaluate the properly renormalized effective potential for the two-body fermionic condensates. The form of the effective potential indicates that flavor-symmetry cannot be broken dynamically due to the scalar exchange.

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1. Introduction

Understanding of dynamical breakdown of the flavor along with the chiral symmetry breaking is an important question. Solution to this problem may provide at least a partial answer to the fermion mass hierarchy problem, which until now still remains an important open question.

In vector-like theories strong statements [1, 2] have been made about the dynamical breakdown of the flavor symmetry. Vector-like theories are CP conserving gauge theories with *no* interaction between scalars and fermions. First Coleman and Witten [1] showed that in $N_C \to \infty$ chromodynamics the chiral symmetry of N_f flavors $U(N_f)_L \times U(N_f)_R$ must be broken down to the diagonal $U(N_f)_{L+R}$, i.e., if chiral symmetry is broken it is broken in such a way as *to preserve* the flavor symmetry. This claim has been generalized to *any* vector-like theory by Vafa and Witten [2]. They proved that in a vector-like theory dynamical breakdown of the flavor symmetry *cannot* take place.

The Vafa-Witten constraint is very restrictive because aesthetic arguments almost force us to assume that the flavor symmetries should be broken spontaneously. Theories [3, 4] based on underlying composite field dynamics in general therefore face a stumbling block of the Vafa-Witten constraint. Namely a vector-like theory as a primordial preonic force is ruled out.

However, this constraint need not apply to the case where there is an interaction between fermions and scalars. It is therefore important to study such theories, because they may shed a new light on the spontaneous (dynamical) breakdown of the flavor symmetry. An obvious aesthetically appealing extension of the vector-like theories [4], is to the super-symmetric vector-like theories, because in such theories scalars emerge in a natural and compelling manner.

It has been shown [5] that the Vafa-Witten constraint does not apply to the supersymmetric version of the vector-like theories.

Spontaneous breakdown of the flavor symmetry has also been studied [6] in a particular model with scalar fields $\Phi_{ab}(a,\ b=1,\ 2,\ \ldots,\ N_f)$ and chiral fermions $\psi_a^{L,R}$

$(a=1,\ 2,\ \ldots,\ N_f)$ which interact via the universal Yukawa interaction:

$$\mathcal{L}_Y = h\left(\sum_{a,\ b=1}^{N_f} \Phi_{ab}\ \bar{\psi}_a^L \psi_b^R + \text{ h.c.}\right) \tag{1}$$

On the other hand the self-interaction of Φ_{ab}'s is governed by the most general renormalizable Higgs potential:

$$V = m_\phi^2\ tr\Phi^\dagger\Phi\ +\ \lambda_1\ tr\Phi^\dagger\Phi\Phi^\dagger\Phi\ +\ \lambda_2\left(tr\Phi^\dagger\Phi\right)^2 \tag{2}$$

which respects the global $SU(N_f)_L \times SU(N_f)_R \times U(1)_F$ symmetry of the theory. Nonzero vacuum expectation values (VEV's) of Φ_{ab}'s induce masses for fermions. It has been shown [6] that there are two such minima of the potential. The first one preserves the flavor symmetry:

$$\langle\Phi\rangle = \begin{pmatrix} \kappa & & & \\ & \kappa & & \\ & & \ddots & \\ & & & \kappa \end{pmatrix}, \quad \kappa = \sqrt{\frac{-m_\phi^2}{2(N_f\lambda_2+\lambda_1)}} \tag{3a}$$

with

$$\left\{-m_\phi^2,\ N_f\lambda_2+\lambda_1,\ \lambda_1\right\}\rangle 0 \tag{3b}$$

Thus, the breaking pattern is $SU(N_f)_L \times SU(N_f)_R \times U(1)_F \rightarrow SU(N_f)_{L\,+\,R} \times U(1)_F$.

The other solution *breaks* the flavor symmetry spontaneously:

$$\langle \Phi \rangle = \begin{pmatrix} o & & & & \\ & o & & & \\ & & \ddots & & \\ & & & o & \\ & & & & \kappa \end{pmatrix}, \quad \kappa = \sqrt{\frac{-m_\phi^2}{2(\lambda_1+\lambda_2)}} \tag{4a}$$

with

$$\left\{-m_\phi^2,\ \lambda_1+\lambda_2,\ -\lambda_1\right\} \rangle 0 \tag{4b}$$

Thus, the flavor symmetry $SU(N_f)_L \times SU(N_f)_R \times U(1)_F$ can be spontaneously broken down to $SU(N_f - 1)_L \times SU(N_f - 1)_R \times U(1)_F$ so that only a fermion of one flavor acquires a mass while others still remain massless.[♯1] All the other possibilities turn out to be a saddle point.

This is an interesting observation which explicitly shows that the flavor symmetry can be broken spontaneously via the Higgs-type mechanism. However, one does not get a handle on the question of which vacuum is preferable. In principle, parameters of the Higgs potential are free; their value could possibly emerge from an underlying dynamics which is responsible for the formation of scalar fields.

In this paper we would like to concentrate on another aspect of the flavor symmetry breaking: the *dynamical* breakdown of the flavor symmetry in a vector-like theory and the Yukawa-type interaction (1) between scalars and fermions. We would like to see whether there is an indication that condensates $\langle\ \overline{\psi}_a^L\ \psi_b^R\ \rangle \neq 0$ which break flavor symmetry dynamically can be formed.

♯1 In an attempt [6] to reproduce a realistic fermion mass matrix for the case of four flavors, the soft symmetry breaking terms were introduced in the Higgs potential. These terms broke $SU(4)_L \times SU(4)_R \times U(1)_F$ down to $SU(2)_L^{\ell+\mu} \times SU(2)_R^{\ell+\mu} \times U(1)_F$ and allow for realistic fermionic masses and mixing angles.

We employ the formalism of Cornwall, Jackiw and Tomboulis [7] which allows one to write an effective potential (free energy) for the two body condensates in the presence of all the interactions of the theory. We are aware of the conceptual problems of this formalism [8, 9]: the solution for the two-body condensates is not stationary and its noninteracting part is not bound from below [9]. The formalism has also a limited use because the two body condensates are treated non-perturbatively, while the vertex corrections are treated order by order in the coupling constant expansion. However, this method has proven useful in studying chiral symmetry breaking; it gave an indication for the existence of chirally unstable vacua at the one-loop level in a vector-like theory. Also, in the $N_C \to \infty$ limit it showed that flavor symmetry cannot be broken.

Our approach is therefore the following: we shall study the form of the effective potential for the two-body condensates $\langle \overline{\psi}_a^L \psi_b^R \rangle$ which arise when in addition to the gauge interaction there is also the Yukawa-type interaction (1). We would like to see whether there is an indication for dynamical breakdown of flavor in this case. We shall come to certain conclusions by studying the symmetry structure and the sign in front of certain terms in the effective potential.[♯2]

The paper is organized as follows: For the sake of completeness, we present the formalism for the two body condensates and discuss the results obtained in the vector-like theories in Sect. 2. In Sect. 3 we specify the particle content and the type of the theory we shall study. In Sect. 4 we evaluate the effective potential; we concentrate on the proper renormalization procedure and show that physical

♯2 Note that signs and symmetry structure of particular terms in the effective potential are fixed, unlike in the Higgs potential where they can assume any sign.

conclusions remain the same in both cases, i.e., in the case of the renormalizable theory as well as in an effective theory with the cut-off parameter. Also the large N_f limit is mentioned. Conclusions are given in Sect. 5.

2. Formalism

In this Section, we recapitulate the formalism for the case of a vector-like theory, developed in Ref. [7] and is pedagogically very nicely presented in Ref. [8] One would like to evaluate the free energy for the non-local two-body operator for the fermionic fields:

$$\langle\psi(x)_a\bar{\psi}(y)_b\rangle_K = \int \frac{d^4p}{(2\pi)^4}\exp[-ip(x-y)]S_{ab}(p) \tag{5}$$

with

$$S_{ab}(p) = \left[\frac{i}{\not{p}+\Sigma(p^2)}\right]_{ab} \tag{6}$$

For the sake of simplified notation, the vector notations for the space-time indices is suppressed. Also the equations are written in Euclidean space. The non-local source $K(x,\ y)$ is chosen so that the operator $S_a^b(p)$ corresponds to the physical (nontrivial) propagator of the theory with $\Sigma_{ab}(p^2)$ denoting the dynamically induced fermionic mass matrix element for flavors $a,\ b$. The free energy (effective potential) Γ as a function of S assumes the following functional form:

$$\Gamma = -tr\ell nS^{-1} + tr[S^{-1} - \not{\partial}]S - (\text{diagrams}) \tag{7}$$

Here the integration over momenta is implied and trace implies the trace over the flavor <u>and</u> space time indices. The term (diagrams') includes all the two-particle irreducible vacuum diagrams presented on Fig. 1.[♯3] The free energy Γ gives the proper equation of motion, i.e., by taking a stationary point of the functional derivative of Γ with respect to S, one recovers the gap-equation [10]:

$$\frac{\delta\Gamma}{\delta S} = -S^{-1} + \not{\partial} + \text{(diagrams}') = 0 \tag{8}$$

The term (diagrams') is presented on Fig. 2.

The method takes into account corrections to the mass $\Sigma(p^2)$ of the fermionic propagator to *all* orders while the corrections to the coupling constant renormalization are taken perturbatively order by order in the coupling constant expansion. Thus, any truncation in the coupling constant expansion is inconsistent. We are aware of this problem,[♯4] however, even the results of such an inconsistent expansion can shed a light on the nature of dynamically generated mass for the fermions. For example, by calculating the contribution to the effective potential due to the one gauge boson exchange one can argue that there is a tendency to destabilize the chirally symmetric vacuum when [8]:

$$\frac{3(N_c^2-1)g^2}{8N_c\pi^2} \gtrsim 1 \tag{9}$$

However, the value of g^2 should be so large that the next correction proportional to g^4 is as big as the previous one, thus putting the validity of the truncation in question.

♯3 Calculations are performed in the Landau gauge, which in the case of $U(1)$ gauge symmetry keeps the condensates $\langle\psi(x)\bar{\psi}(y)\rangle$ locally gauge invariant. Note, that in Landau gauge, corrections to the wave function renormalization are zero. Also, if one studies condensates $\langle\psi(x)\bar{\psi}(y)\rangle$ which break global gauge invariance one should include the gauge boson propagators with the nonzero mass for the gauge bosons. However, this is not our case.

♯4 There is also a conceptual problem which arises because the condensate (5) is nonlocal in time, and thus the solution is *not* stationary. Also, the effective potential is not bound from below. This can be remedied [9] by choosing a source local in time, but then the Lorentz covariance is sacrificed.

One can also obtain [11] a qualitative form for $\Sigma(p^2)$ as a function of momentum p (see Fig. 3): for nonzero $\Sigma(p^2)$ one can show that $\Sigma(p^2) \to$ const. as $p \to 0$ and $\Sigma(p^2)$ should falls-off faster than $1/p^2$ as $p \to \infty$ in order for the solution to be consistent with the gap equation.

Another interesting observation is that in the presence of the one gauge boson exchange Γ can be written as $Tr_f F(\Sigma^\dagger \Sigma)$. Here Tr_f denotes a trace over the flavor indices and F is a general function of $\Sigma^\dagger\Sigma$. This implies that, if there is a minimum with *one* nonzero Σ, e.g., $\Sigma_{aa} = \kappa \neq 0$, then all the other Σ's should have the same value, i.e., $\Sigma_{bb} = \kappa$ with $b = 1,\ 2,\ \ldots\ ,\ N_f$.[♯5] In other words, if chiral symmetry is broken ($\kappa \neq 0$) it is broken in a way as to preserve flavor.[♯6]

In the following chapters we shall see how the nature of the effective potential could be changed in the presence of the Yukawa-type interaction between scalars in fermions.

3. The Model

The theory we shall study is the one with chiral fermions ψ_a^L, ψ_b^R $(a,\ b = 1,\ \ldots,\ N_f)$ and elementary scalar fields Φ_{ab} $(a,\ b = 1,\ \ldots,\ N_f)$, in the presence of the gauge interactions with $SU(N_c)$ gauge group, Yukawa-type interactions (2) and scalar field self-interaction (3). These fields transform under the global flavor symmetry $SU(N_f)_L \times SU(N_f)_R$ and the gauge symmetry

♯5 Recall that in the Higgs potential (2) the similar solution (3a) for the VEV of Φ can emerge when $\lambda_2 \equiv 0$. Note also, that when $\lambda_2 = 0$, solution (3a) is the only allowed one, i.e., solution (4a) does not exist.

♯6 Note that in the limit $N_c \to \infty$ and $g^2 N_f =$ const. certain graphs of Fig. (1) are absent and Γ can be again written as $Tr_f F(\Sigma^\dagger\Sigma)$, even when *all* the diagrams are included. This is the essential argument in the proof of flavor preserving solution in $N_c \to \infty$ case presented in Ref. [1].

$SU(N_c)$ in the following way:[♯7]

$$\psi^L \sim (N_f,\ 1,\ N_c) \tag{10}$$

$$\psi^R \sim (1,\ N_f,\ N_c) \tag{11}$$

$$\Phi \sim (\overline{N}_f,\ N_f,\ 1) \tag{12}$$

The choice of the particle content and the interactions is an aesthetically appealing minimal extension of the vector-like theory.

The interactions respect the global flavor symmetry. The gauge interactions are included because they are a source of the chiral symmetry breaking, while Yukawa-type interactions may be a source of the flavor symmetry breaking. The chosen representation is quite general. A choice for a more complicated representation of $\psi^{L,\,R}$ under $SU(N_c)$ adds only to a technically more complicated calculation. The case when Φ is not a singlet under $SU(N_c)$ is not interesting for our particular choice because one would like to preserve the gauge symmetry. This cannot be achieved if Φ is not a singlet under $SU(N_c)$. As we shall show once $\langle\psi(x)\bar{\psi}(y)\rangle$ is nonzero the $\langle\phi\rangle$ is also necessarily nonzero, and $SU(N_c)$ is dynamically broken.

Also, had Φ been a singlet under the global-flavor group the effective potential *could again* have been written as $Tr_f\ F(\Sigma^\dagger\Sigma)$ with F being a general function of $\Sigma^\dagger\Sigma$ and flavor symmetry cannot be broken dynamically.

♯7 When the global flavor symmetry is dynamically broken one encounters Goldstone bosons. However, in a realistic model the Goldstone boson may acquire soft mass due to the explicit global symmetry breaking terms [12].

Another comment is in turn. In our approach we would like to study the break-down of the flavor symmetry driven primarily by the formation of the condensates $\langle\bar{\psi}^L_a\psi^R_b\rangle \neq 0$, i.e., dynamical breakdown of the flavor symmetry in the presence of interactions, and not by the nonzero VEV's of $\langle\Phi_{ab}\rangle \neq 0$. However, once $\langle\bar{\psi}^L_a\psi^R_b\rangle \neq 0$, one also induces nonzero $\langle\Phi_{ab}\rangle$ with the relation:[♯8]

$$\langle\Phi_{ab}\rangle = \frac{h}{m^2_\phi}\,\langle\bar{\psi}^L_a\psi^R_b\rangle + \dots \quad . \tag{13}$$

Also, in a realistic theory scalar fields Φ_{ab}'s can induce flavor changing neutral currents (FCNC). Thus, the assumption:

$$m^2_\phi > h^2\langle\bar{\psi}^L_a\psi^R_b\rangle \tag{14}$$

ensures that the breakdown of the global symmetry is primarily dynamic via formation of $\langle\bar{\psi}^L_a\psi^R_b\rangle$ condensates *and* the flavor changing neutral currents are suppressed.

The presented theory is rather general and the one which has a chance to allow for the ***dynamical*** breakdown of the flavor symmetry. Namely, in addition to the terms of the type $Tr_f F(\Sigma^\dagger\Sigma)$ one induces terms of the type $Tr_f G_1(\Sigma^\dagger\Sigma)\times Tr_f G_2(\Sigma^\dagger\Sigma)$ which arise due to the Φ_{ab} exchange (see Fig. 4). Here F, G_1 and G_2 are general functions of $\Sigma^\dagger\Sigma$. Such terms have a chance of breaking flavor dynamically.[♯9] However, only the explicit calculation can show whether the symmetry structure and the sign of particular terms can indicate which breaking pattern is preferable.

♯8 In the case of $h\langle\Phi_{ab}\rangle \ll \Sigma_{ab}$, the terms in the scalar self-interaction (2) proportional to λ_1 and λ_2 can be neglected.

♯9 Recall that the case of the Higgs potential (2) the solution (4) which breaks flavor spontaneous emerged only for $\lambda_1+\lambda_2 > 0$ and $\lambda_1 < 0$, i.e., the term $\lambda_2(tr\Phi^\dagger\Phi)^2$ should necessarily be present.

4. Calculation of the Effective Potential

In this section we shall evaluate the effective potential for the two body fermionic condensates for the theory presented in Sect. 3. We shall include diagrams due to the 1 - gauge boson exchange and due to 1 - scalar exchange. Those are the contributions proportional to g^2 and h^2, only.[♯10]

Let us first present Γ with the Ansatz (6) for the fermionic propagator:[♯11]

$$\Gamma_0 = \frac{1}{4\pi^2} \int_0^\infty dp\, p^3\, Tr_f \left\{ -\ell n[p^2 + \Sigma^2(p^2)] \; + \; \frac{2\Sigma^2(p^2)}{[p^2 + \Sigma^2(p^2)]} \right\} \tag{15}$$

$$\Gamma_{1-GB} = -\frac{3(N_c^2-1)g^2}{64 N_c \pi^4} \int_0^\infty \int_0^\infty dpd\, k \, \frac{p^3\, k^3}{\text{Max}\ (p^2,\ k^2)} \; Tr_f \left\{ \frac{\Sigma(p^2)\Sigma(k^2)}{[p^2 + \Sigma^2(p^2)]\ [k^2 + \Sigma^2(k^2)]} \right\} \tag{16}$$

$$\Gamma_{1-SC} = -\frac{h^2}{32\pi^4} \int_0^\infty \int_0^\infty dpd\, k\, p^3 k^3\, Tr_f \frac{1}{[p^2 + \Sigma^2(p^2)]}\, Tr_f\, \frac{1}{[k^2 + \Sigma^2(k^2)]} \sum_{\ell=0}^{\infty} \frac{(2\ell+1)!!}{(2\ell+2)!!(\ell+2)} \left(\frac{2kp}{k^2 + p^2 + m_\phi^2} \right)^{2\ell+2} \tag{17}$$

with

$$\Gamma = \Gamma_0 + \Gamma_{1-GB} + \Gamma_{1-SC} \tag{18}$$

Here Tr_f denotes the trace over the flavor indices. The following comments, some of them already mentioned in Sect. 2, are in turn

♯10 As already argued this truncation is not justified, however, it may give an indication which symmetry pattern is preferable. Also, the chosen truncation to the leading order in g^2 *and* h^2 allows for the possibility that both g^2 and h^2 are comparable in magnitude.

♯11 We shall assume that Σ is a real matrix in the flavor space, i.e., we shall not study dynamical breakdown of CP.

(i) From (15) one sees that Γ_0 is not bound from below, however it does have a local minimum at $\Sigma = 0$.

(ii) The value of $\Sigma(p^2)$ should fall-off faster than $1/p^2$ in order for Γ_{1-GB} to be finite.

(iii) One can also explicitly see that Γ_0 and Γ_{1-GB} can be written as $Tr_f F(\Sigma^2)$, with F being a general function of Σ^2, thus allowing *only* for the flavor symmetric solution.

(iv) We can also observe that a term in Γ_{1-GB} proportional to $\Sigma\Sigma$ is negative, thus indicating that the chirally symmetric vacuum with $\Sigma \equiv 0$ can be unstable. On the other hand, the terms in Γ_0 and Γ_{1-SC} proportional to $\Sigma\Sigma$ are positive. Thus, in order to allow for the chiral symmetry breaking, gauge interactions should necessarily be present.

(v) Γ_{1-SC} can be written as $[Tr_f G(\Sigma^2)]^2$. Here G is a function of Σ^2. This allows for the possibility that the flavor symmetric vacuum is unstable. However, the expansion in terms of Σ^2 shows that the term proportional to $(Tr_f\Sigma^2)^2$ has the negative sign. Also this terms is damped by a factor Λ_{HC}^4/m_ϕ^4 and a factor $1/N_f$ relative to the term proportional to $Tr_f\Sigma^4$. Here Λ_{HC} is a scale at which the nonzero value for Σ falls-off. (See Fig. 3). As justified in Sec. 3, $\Lambda_{HC} < m_\phi$. Therefore there is *no* tendency to destabilize the flavor symmetric vacuum.

(vi) The important observation is that Γ_{1-SC} is logarithmically divergent. In an effective theory with the cut-off parameter Λ, with Λ being the compositness scale of the fermions $\psi_a^{L,\,R}$ and/or scalar fields Φ_{ab}, Γ_{1-SC} is finite. However the question of the underlying dynamics and what its contribution to the effective potential might be, remains unanswered.

Therefore, if one neglects the additional effects of the underlying dynamics, one can claim that in the theory with the cut-off parameter, there is an indication that the flavor-symmetric vacuum is stable.

However, our main goal is to study a renormalizable theory and see whether in such a theory there is an indication for the dynamical breakdown of flavor.

4.1 Renormalizable Theory

In order to obtain the proper-finite form of the free energy Γ the bare unrenormalized fermionic propagator S should be reexpressed in terms of the renormalized propagator S_R. One has to take into account the wave function renormalization effects, only. Note that the theory does not have any bare mass, and thus the only counter-term of the theory is the wave function renormalization counterterm Z.[♮12]

In the vector-like theory, with the Landau guage there are no finite wave function corrections. Thus the Ansatz (6) for the fermionic propagator is the proper one and $\Gamma(S)$ is finite. However, this is not the case in the presence of the Yukawa-type interactions (2). The scalar exchange *does* contribute to the wave function renormalization and thus Γ as a function of the bare-unrenormalized propagator S should be properly rewritten in terms of the renormalized propagator S_R. The relation between these two propagators is the following:[♮13]

$$iS_R^{-1} \equiv \left\{ \left[1 + f(p^2) \right] \not{p} + \Sigma \right\} = Z^{-1} i S^{-1} \tag{19}$$

Consistent with the truncation we used in evaluating $\Gamma(S)$, the finite wave function renormalization term $f(p^2)$ and the wave function counterterm Z should

♮12 I am grateful to Helen Quinn for this observation.

♮13 For the sake of simplified notation we suppress the flavor indices.

be evaluated up to $\mathcal{O}(h^2)$. To this order iS_R^{-1} is diagrammatically presented in Fig. 5. The subtraction point $p = \mu$ is chosen so that $f(\mu^2) = 0$. This then determines Z and $f(p^2)$ as:

$$Z = 1 - h^2 \mathcal{F}(\mu^2) \tag{20}$$

$$f(p^2) = h^2 \Big[\mathcal{F}(p^2) - \mathcal{F}(\mu^2) \Big] \tag{21}$$

with $\mathcal{F}(p^2)$ being diagrammatically presented in Fig. 5. Since $f(p^2) \propto h^2$, Γ as a function of the bare-unrenormalized propagator S shall be reexpressed in terms of the renormalized propagator S_R^0:

$$iS_R^{o-1} \equiv \not{p} + \Sigma \tag{22}$$

and corrections to Γ due to $f(p^2)$ is included only to $\mathcal{O}(h^2)$. Thus, to the order we are working with $\Gamma(S)$ is *finite* and has the following form:

$$\Gamma(S) = \Gamma_0(S_R^0) + \Gamma_{1-GB}(S_R^0) + \Delta\Gamma(S_R^0) \tag{23}$$

with

$$\Delta\Gamma(S_R^0) = \frac{\delta\Gamma_0}{\delta S}\Big|_{S=S_R^0} S_R^0 \Big[S_R^{0-1} - S^{-1} \Big] S_R^0 + \Gamma_{1-SC}\ (S_R^0) \tag{24}$$

The operator $\frac{\delta}{\delta S}$ refers to the functional derivative and in Eq. (24) integration over the momentum is implied. Γ_0 and Γ_{1-GB} are given by Eqs. (15) and (16), respectively, while $\Delta\Gamma$ is of the following explicit form:

$$\Delta\Gamma = -\frac{h^2}{8\pi^2}\int_0^\infty dpp^3 \left\{ Tr_f \left[\frac{\Sigma^2(p^2)[p^2 - \Sigma^2(p^2)]}{[p^2 + \Sigma^2(p^2)]^2}[\mathcal{F}(p^2) - \mathcal{F}(\mu^2)]\right] + N_f \mathcal{F}(p^2)\right\} \tag{25}$$

with

$$\begin{aligned} \mathcal{F}(p^2) = \frac{1}{16\pi^2}\Bigg\{ & \left[\ell n\frac{\Lambda^2}{m_\phi^2} - 1 - \sum_{\ell=1}^{\infty}\frac{1}{(\ell+2)}\left(\frac{p^2}{p^2+m_\phi^2}\right)^\ell\right] N_f \\ & - \sum_{\ell=0}^{\infty}\frac{(2\ell+1)!!2^{\ell+1}p^{2\ell}}{(\ell+1)!(\ell+2)} Tr_f \int_0^\infty \frac{d(k^2)(k^2)^{\ell+1}\Sigma^2(k^2)}{[k^2+\Sigma^2(k^2)][k^2+p^2+m_\phi^2]^{2\ell+2}}\Bigg\} \end{aligned} \tag{26}$$

Obviously, the part of $\Delta\Gamma$ which depends on Σ' is finite as long as $\Sigma(p^2)$ falls-off faster than $1/p^2$.

The natural choice for the subtraction point is the choice of the on-shell renormalization:

$$\mu_{aa} = \Sigma_{aa}(\mu^2) \qquad a = 1,\ 2,\ \ldots\ N_f \tag{27}$$

which ensures that $\Sigma(p^2)$ corresponds to the pole of the renormalized propagator defined in (19).

As expected one sees that the properly renormalized finite free energy Γ has the similar symmetry structure. All the conclusions regarding Γ_0 and Γ_{1-GB} remain the same. On the other hand $\Delta\Gamma$ has a different functional form from the one of Γ_{1-SC}. However, one can again see that the expansion in terms of Σ^2 has *the same sign* in front of each term. Namely, the term proportional to $Tr_f\Sigma^2$ is positive, while the term proportional $(Tr_f\Sigma^2)^2$ is again negative and again damped by factors Λ_{HC}^4/m_ϕ^4 and $1/N_f$ compared to the term $Tr_f\Sigma^4$.

Therefore, from the form of Γ one has an indication that the flavor-symmetric vacuum remains stable in the renormalizable theory.

There is also another new feature: while in an effective theory with the cut-off Λ parameter the leading contribution of Γ_{1-SC} was $\mathcal{O}(\frac{h^2}{8\pi^2})$ compared to Γ_0, the leading contribution of $\Delta\Gamma$ is $\mathcal{O}\left(\frac{h^2}{8\pi^2}\,\frac{\Lambda_{HC}^2}{m_\phi^2}\right)$ compared to Γ_0. Here Λ_{HC} is again the scale at which the value of Σ drops off sharply (see Fig. 3). Therefore, as $m_\phi \to \infty$, the contribution from the 1 - scalar exchange becomes negligible, i.e., the decoupling takes place. This observation therefore implies that any change in the symmetry of the vacuum due to the Yukawa-type interaction (2) is not likely, because the contribution to Γ from this interaction is damped by a factor Λ_{HC}^2/m_ϕ^2.

In order to make the above analysis quantitative we shall now explicitly evaluate Γ with a particular Ansatz for Σ presented on Fig. 3 with the dotted line.

4.2 Explicit Evaluation of Γ

Exact solution for $\Sigma(p^2)$ as arises by solving the gap equation (8) is difficult to obtain. However, one can be satisfied with a less ambitious task by assuming a certain shape for $\Sigma(p^2)$ with free variational parameters and minimize Γ with respect to these parameters (Rayleigh-Ritz variational principle). This can enable one to extract a nature of the symmetry pattern for Σ.

We shall use the Ansatz for $\Sigma(p^2)$ presented on Fig. 3 with the dotted line. It is of the following form:

$$\Sigma(p^2) = \begin{cases} \sigma\Lambda_{HC}\,, & p \le \Lambda_{HC} \\ 0\,, & p > \Lambda_{HC} \end{cases} \tag{28}$$

Here Λ_{HC} is intuitively a scale at which g, the gauge coupling constant and correspondingly the Yukawa coupling h become large,[♯14] i.e., of order 1. We choose σ, the constant matrix in the flavor space, to be a variational parameter. This simplified Ansatz agrees with our intuition (see Fig. 3, solid line) for what the momentum dependence of Σ should be. On the other hand the essential symmetry structure in the flavor space is still encompassed in the variational parameter σ.

We shall also assume:[♯15]

$$\sigma_{aa} \leq 1 \qquad a = 1,\ 2,\ \dots\ ,\ N_f \tag{29}$$

The subtraction point is then chosen to be:

$$\mu_{aa} = \Lambda_{HC}\sigma_{aa} \qquad a = 1,\ 2,\ \dots\ ,\ N_f \tag{30}$$

In agreement with discussion of Sect. 3, that the dynamical breakdown of the symmetry is driven by formation of fermionic condensates and that FCNC could be suppressed we shall also expand Γ in terms of the small parameter Λ^2_{HC}/m^2_ϕ. Finally, Γ is of the following form:

$$\Gamma_0 = \frac{\Lambda^4_{HC}}{16\pi^2}\, Tr_f \left\{ -\ell n\left(1+\sigma^2\right) + 3\left[\sigma^2 - \sigma^4 \ell n\left(1+\frac{1}{\sigma^2}\right)\right]\right\} \tag{31}$$

♯14 It can be shown [14, 15] that for this theory h evolves proportionally to the gauge coupling g at low momenta, i.e., h reaches very soon the infrared fixed point proportional to g.

♯15 One can show that for $\sigma_{aa} > 1$ the effective potential Γ can be cast in the following form

$$\Gamma = ATr_f \ell n(1+\sigma^2) + BTr_f \frac{1}{\sigma^2} + \mathcal{O}\left(\frac{1}{\sigma^4}\right)$$

with A being negative. The effective potential is *not* bound from below and it *does not* have a minimum for $\sigma_{aa} > 1$.

$$\Gamma_{1-GB} = \frac{\Lambda_{HC}^4}{16\pi^2}\frac{3(N_c^2-1)g^2}{N_c 8\pi^2}$$

$$\times Tr_f\left\{-\sigma^2 + \sigma^4 \ell n\left(1+\frac{1}{\sigma^2}\right)\left[1+\frac{1}{2}\,\ell n\left(1+\frac{1}{\sigma^2}\right)\right]\right\} \tag{32}$$

$$\Delta\Gamma = \frac{\Lambda_{HC}^4}{16\pi^2}\frac{h^2}{8\pi^2}\frac{\Lambda_{HC}^2}{12m_\phi^2}\left\{Tr_f\left[\sigma^2 - 8\sigma^4 + 16\sigma^6\right.\right.$$

$$\left.\times\left[\ell n\left(1+\frac{1}{\sigma^2}\right) - \frac{1}{2(1+\sigma^2)}\right] + \mathcal{O}\left(\frac{\Lambda_{HC}^2}{m^2\phi}\right)\right] N_f - 6\left(\frac{\Lambda_{HC}^2}{m_\phi^2}\right)$$

$$\times Tr_f\left[\sigma^2 - 8\sigma^4 + 16\sigma^6\left[\ell n\left(1+\frac{1}{\sigma^2}\right) - \frac{1}{2(1+\sigma^2)}\right]\right] \tag{33}$$

$$\left.\times Tr_f\left[\sigma^2 - \sigma^4\ell n\left(1+\frac{1}{\sigma^2}\right)\right]\right\}$$

with $\Gamma = \Gamma_0 + \Gamma_{1-GB} + \Delta\Gamma$.

Expansion in powers of $\sigma < 1$ simplifies $\Delta\Gamma$ to the following form:

$$\Delta\Gamma = \frac{\Lambda_{HC}^4}{16\pi^2}\frac{h^2}{8\pi^2}\frac{\Lambda_{HC}^2}{12m_\phi^2}\left\{Tr_f(\sigma^2 - 8\sigma^4)N_f\right.$$

$$\left. - 6\left(\frac{\Lambda_{HC}^2}{m_\phi^2}\right)^2 \left(Tr_f\sigma^2\right)^2 + \mathcal{O}(\sigma^6)\right\} \tag{34}$$

The explicit form for Γ, as presented in Eqs. (32 - 34) again substantiates all the claims we made in Sec. 4.1. It clearly shows, that the flavor symmetric vacuum is stable.

4.3 LARGE N_f LIMIT

Observation that the terms which have a chance of breaking the flavor symmetry, are suppressed by a factor $1/N_f$ relatively to other terms of the form $Tr_f F(\Sigma^2)$, is crucial when studying a theory with $N_f \to \infty$ and $h^2 N_f = \text{const.}$ In this case one sees that to *all orders* in h the only contribution to Γ which survives is the one of Fig. 5 where the empty fermionic line denotes the free propagator of the massless fermion. Note, that these terms with one fermionic line being empty arise as the leading expansion in terms of Λ_{HC}^2/m_ϕ^2. This contribution is obviously of the form $h^2 N_f \; Tr_f \; F(\Sigma^2)$ and has not a chance of breaking flavor.

5. Conclusions

In this paper we studied the structure of the effective potential of the two-body fermionic condensates in a vector-like theory with chiral fermions *and* the flavor invariant Yukawa-type interaction between fermions and scalars. We concentrate on the dynamical breakdown of the flavor symmetry with the formation of the two-body fermionic condensates.[♯16]

Although the contribution from the 1-scalar exchange to the effective potential gives a new structure, different from the one of the gauge-boson exchange, the *sign* in front of this new term is wrong: flavor symmetric vacuum remains stable. The conclusion is the same, whether one treats the theory as a renormalizable theory of elementary fermions and scalars or as an effective theory with

♯16 In Ref. [12], J. C. Pati and myself persued the problem of dynamically generated fermionic masses and mixing angles within the theory where Yukawa couplings and scalar masses already break the flavor symmetry. Such a theory may arise as an effective theory of composite fields based on local super-gravity interactions. We show that the realistic dynamically generated fermionic mass matrix <u>can</u> arise.

the cut-off parameter Λ, with Λ being the compositness scale of fermions and scalar.

Also, as the scalar mass $m_\phi \to \infty$ the contribution to the effective potential from the 1-scalar exchange becomes negligible in a renormalizable theory, thus decoupling takes place.

In the limit when the number of flavors $N_f \to \infty$, one concludes (to all orders in the Yukawa coupling expansion) that dynamical breakdown of the flavor is also *not* possible.

Thus, our approach indicates that in the presented theory the world with dynamically broken flavor may not be favored.

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Figure Captions

Fig. 1 Diagrams contributing to the free energy Γ. The blobs on the solid lies denote the fermionic propagator with dynamically induced mass. The wiggly line denotes the gauge boson propagator.

Fig. 2 Diagrams appearing in the gap-equation. The blobs on the solid lines again denote the fermionic propagators with dynamically induced mass Σ and the wriggly line denotes the gauge boson propagator.

Fig. 3 The momentum dependence of the dynamically induced mass (solid line). The scale Λ_{HC} denotes (intuitively) the scale at which the coupling constants become strong. The dotted line represents an approximate Ansatz for Σ, with σ being a variational parameter.

Fig. 4 The diagram of order h^2 which contributes to Γ.

Fig. 5 Diagrammatically presented form of the renormalized propagator exact up to $\mathcal{O}(h^2)$ and $\mathcal{O}(g^2)$.

Fig. 6 The only remaining diagrams in $N_f \to \infty$ limit.

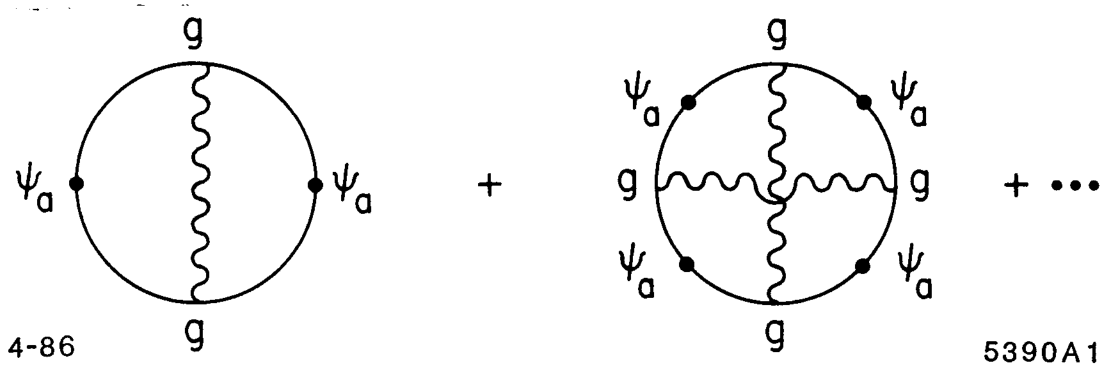


Fig. 1

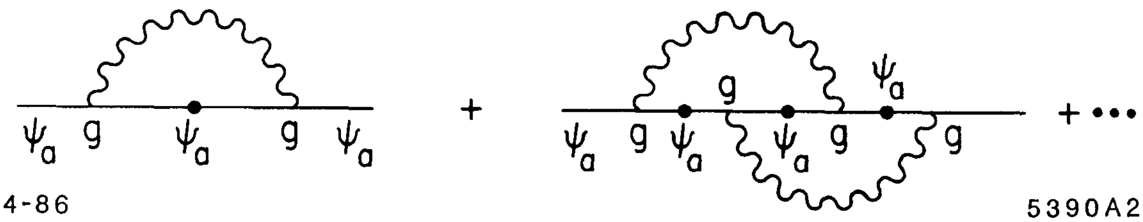
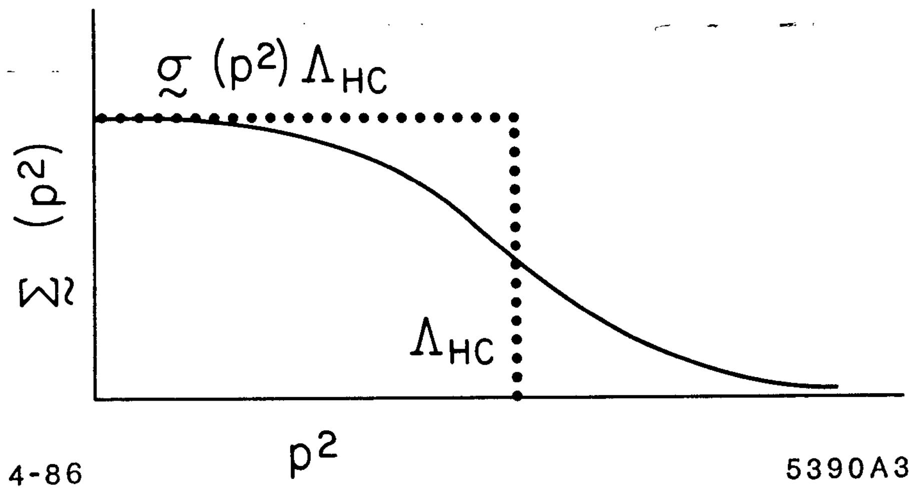
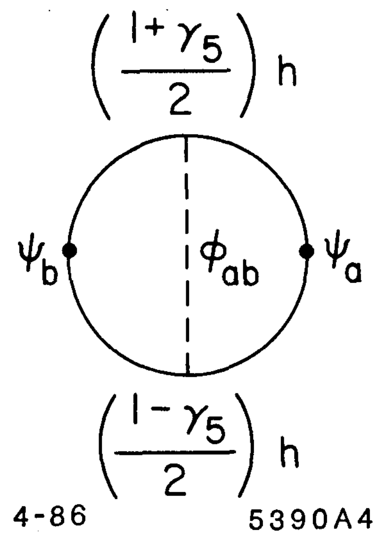


Fig. 2



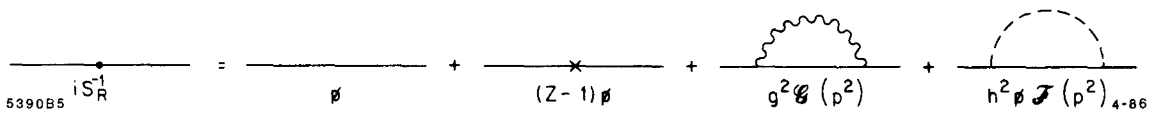
$\underset{\sim}{\sigma}$ (p^2) Λ_{HC}
$\underset{\sim}{\Sigma}$ (p^2)
Λ_{HC}
p^2
4-86
5390A3

Fig. 3



$\left(\frac{1+\gamma_5}{2}\right) h$
ψ_b
ϕ_{ab}
ψ_a
$\left(\frac{1-\gamma_5}{2}\right) h$
4-86
5390A4

Fig. 4



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iS_R^{-1}
=
$\not{p}$
+
$(Z-1)\not{p}$
+
$g^2 \mathcal{G}(p^2)$
+
$h^2 \not{p}\, \mathcal{F}(p^2)$
4-86

Fig. 5

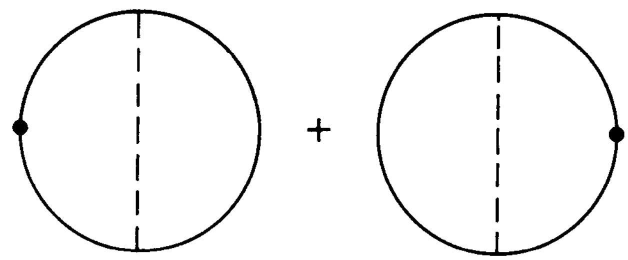

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Fig. 6