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(2,0) supersymmetry and space-time supersymmetry in the heterotic string theory*

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ABSTRACT

Criteria for unbroken $N = 1$ space-time supersymmetry in the heterotic string theory in the presence of background fields are discussed. We make use of the construction of the fermion vertex operator in the Neveu-Schwarz-Ramond model. (2,0) world-sheet supersymmetry is shown to be one of the necessary conditions for space-time supersymmetry in most cases. Constraints on the various background fields implied by (2,0) world-sheet supersymmetry are derived, taking into account the effect of σ-model loop corrections. Special care is taken to study the effect of local Lorentz and gauge anomaly on these constraints. Our analysis determines the constraints unambiguously up to field redefinitions.

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1. Introduction

It has become clear through recent studies that non-linear σ-models provide an adequate description of strings moving in arbitrary background fields[1–12]. One of the main successes of the non-linear σ-models so far has been to provide us information about equations of motion of the massless fields in the string theory through the β-functions[4–9]. At present σ-model provides us with a major tool for finding non-trivial solutions of the classical string field equations. General arguments have been presented to show that σ-models formulated on a Calabi-Yau manifold has vanishing β-function to all orders in the perturbation theory[13], and provide a solution of the classical string equations of motion, although recent studies indicate that this may not be the case[14–16].

In this paper we shall not pursue the question of obtaining the equations of motion of the string theory from the σ-model. Instead we shall use the σ-model to analyze another important question in the heterotic string theory[17], namely, under what condition on the background fields does the theory admit an unbroken space-time supersymmetry? This question has been discussed by various authors by studying the field theoretic limit of the string theory[4], and also by analyzing the σ-model[18–20] describing the Green-Schwarz superstring[21] in arbitrary background fields in the light-cone gauge. The advantage of using the Green-Schwarz version of the σ-model is that space-time supersymmetry is manifest in this formulation. But in this model it is very difficult to carry out the analysis beyond the σ-model tree level, i.e. beyond the lowest order in the inverse string tension. The reason is that in a general background field the tree level action for this model does not contain the most general renormalizable Lagrangian. This may be illustrated by looking at two particular terms in the σ-model lagrangian:

$$G_{ij}(X)\partial_- X^i \partial_+ X^j + iS^\alpha(\delta_{\alpha\beta}\partial_- + \tfrac{1}{2}\omega_i^{ab}[\Sigma^{ab}]_{\alpha\beta})S^\beta \tag{1.1}$$

where X^i and S^α are two dimensional bosonic and fermionic fields, transforming in the vector and spinor representations of the $SO(8)$ Lorentz group respectively.

G_{ij} is the background metric, ω_i^{ab} is the spin connection constructed from G_{ij} and Σ^{ab} denotes the generators of the SO(8) group in the spinor representation. The point to note is that the coupling constant ω, instead of being taken as completely arbitrary, is taken to be related to G, although there is no obvious symmetry that tells us to do so. As a result, when we start doing quantum corrections, the description of the theory becomes ambiguous, and depends on the procedure of subtracting ultraviolet divergences, since there is no constraint which prevents us from adding a finite local counterterm involving the $S^\alpha S^\beta$ operator to the action. Requirement of $N = 1$ space-time supersymmetry may provide a way out, but since our purpose is to determine the conditions on the background fields required by $N = 1$ supersymmetry, we must first start from a general background field, calculate the effective action, and then impose the constraint of $N = 1$ supersymmetry on the effective action.

The problem does not arise in the Neveu-Schwarz-Ramond formulation of the theory[22] , since for an arbitrary background field it has an $N = \frac{1}{2}$ (or (1,0)) supersymmetry[4–6,23–30] , and the requirement of this symmetry prevents us from adding any local counterterm to the action besides those which corresponds to redefinitions of various background fields $G_{ij}(x)$, $B_{ij}(x)$ and $A_i^M(x)$[30] . The disadvantage of this formulation is that it is difficult to understand space-time supersymmetry in this model. Recently, however, the space-time supersymmetry charge has been constructed by Friedan, Martinec and Shenker[31,32] in the N-S-R model in flat background. Hence a viable approach would be to try to generalize their construction for the heterotic string in arbitrary background field, and try to see under what restriction on the background fields we can construct a conserved supersymmetry charge.

This is the approach we shall pursue in this paper. We shall show that one of the necessary conditions for getting unbroken space-time supersymmetry in the heterotic string theory is that the σ-model must have an extended (2,0) world sheet supersymmetry.* We then analyze the quantum corrections in the Neveu-Schwarz-Ramond model and study how they modify the constraint equations on the background fields in order to have (2,0) supersymmetry. Special care is taken to analyze the effect of local Lorentz and gauge anomalies in this model. The reason is that the equations which give us the criteria for (2,0) supersymmetry at the σ-model tree level involve a three form,

$$S_{ijk} = \tfrac{1}{2}(B_{ij,k} + B_{jk,i} + B_{ki,j}) \tag{1.2}$$

where B_{ij} is the background antisymmetric tensor field. Since the effect of local Lorentz and gauge anomalies forces us to modify the transformation laws of B under these transformations, S_{ijk} does not transform covariantly under these transformations any more. We may construct a covariant tensor H_{ijk} by adding gauge and Lorentz Chern-Simons terms to S_{ijk} with appropriate coefficients. But the definition of H_{ijk} is ambiguous, since we may add any covariant three form to H, without affecting its gauge and Lorentz covariance properties. Only a careful analysis of the quantum corrections in the σ-model determines what tensor should replace S in the equations determining the criteria for (2,0) supersymmetry in this model. This question is of more than academic interest, since we must know the precise form of the constraint equations for generating space-time supersymmetric field configurations with non-vanishing torsion[19,20].

In our analysis we determine the three form H which appears in the constraint equations for (2,0) supersymmetry. We also find that by making a field redefinition of the background fields G_{ij}, B_{ij} and A_i^M we may always bring the constraint equations in the form of tree level constraint equations. Of course the new fields have complicated transformation properties under local gauge and Lorentz transformations, that is how the constraint equations become invariant

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* This relation was first discussed by Hull and Witten, who analyzed the classical action for a (1,0) supersymmetric model, and found that the constraints on the background fields for this model to have a (2,0) supersymmetry form a subset of the conditions derived in Ref.[24] for unbroken $N = 1$ space-time supersymmetry.

under these transformations despite the presence of an apparently non-covariant term S_{ijk} in this equation.

Sec.2 of this paper is devoted to a general discussion of (1,0) and (2,0) supersymmetric σ-models. Sec.3 discusses the relationship between space-time supersymmetry and world-sheet (2,0) supersymmetry. In Sec.4 we review the role of local Lorentz and gauge anomalies in the σ-model, and discuss how they are expected to affect the constraint equations for (2,0) supersymmetry. Sec.5 contains the analysis of the effect of σ-model loop corrections on (2,0) supersymmetry. We also compare our result with recent proposals by Strominger[19] and by Hull[20]. We conclude in Sec.6 by stating the main results of this paper and future prospects.

2. (1,0) and (2,0) supersymmetry in the heterotic string theory

In this section we shall study the two dimensional σ-model describing the heterotic string in arbitrary background fields and derive the constraints on the background fields under which the σ-model has an extended supersymmetry. In the covariant formulation the dynamical variables of the two dimensional theory are the ten bosonic coordinates X^μ, ten right-handed Majorana-Weyl coordinates λ^μ and thirty two left handed Majorana-Weyl coordinates ψ^s.* The coupling constants of the theory are the various background ten dimensional fields. We shall consider a field configuration where six of the ten dimensions (denoted by the index i) are compactified, whereas the other four dimensions remain flat. Let $G_{ij}(x)$, $B_{ij}(x)$ and $A_i^M(x)$ be the vacuum expectation values of the graviton, the anti-symmetric tensor, and the gauge fields respectively along the compact dimensions. The σ-model describing the heterotic string in such a background is most conveniently written down in terms of the superfields[33], which are defined as,

$$\begin{aligned} \Phi^i &= X^i + \theta\lambda^i, \quad i = 1,...6 \\ \Lambda^s &= \psi^s + \theta F^s, \quad s = 1,...32 \end{aligned} \tag{2.1}$$

where θ is an anti-commuting parameter and F^s is an auxiliary field, required for the superspace formulation of the theory. The action for the heterotic string is then given by[4–6,24–30] ,†

* In previous papers[5,26,30] we had taken λ^μ's to be left-handed and the ψ's to be right-handed. We change our convention here in order to comply with the more generally accepted notation.

† Since we are formulating the theory in a flat world sheet, the background vacuum expectation value of the dilaton field does not couple to the σ-model lagrangian. Instead, it corresponds to adding a term proportional to $\partial_\alpha\partial_\beta\Phi - \delta_{\alpha\beta}\partial^2\Phi$ to the energy-momentum tensor[8,31,34] .

$$S = \frac{1}{2\pi\alpha'}\int d\sigma d\tau d\theta\Big[- i\big(G_{ij}(\Phi) + B_{ij}(\Phi)\big) D\Phi^i \partial_- \Phi^j$$
$$- \Lambda^s\big(\delta_{st} D - i A_i^M (T^M)_{st} D\Phi^i\big)\Lambda^t\Big] \tag{2.2}$$

plus the free field action involving the fields X^A, λ^A (A=7,...10). Here T^M ia a generator of the gauge group, σ, τ are the variables labelling the string world sheet, and

$$\partial_\pm = \frac{1}{\sqrt{2}}(\partial_\tau \pm \partial_\sigma)$$
$$D = \partial_\theta + i\theta\partial_+ \tag{2.3}$$

We shall also need the component form of the action for our analysis. The most convenient form of this action is given in terms of the tangent space coordinates. We define the vielbein fields $e_i^a(x)$ through the relation,

$$G_{ij}(x) = e_i^a(x) e_j^a(x) \tag{2.4}$$

and the fields λ^a as,

$$\lambda^a = e_i^a(X)\lambda^i \tag{2.5}$$

After eliminating the auxiliary fields F^s by their equations of motion the action (2.2) may be written in terms of the component fields as,

$$\frac{1}{4\pi\alpha'}\int d\tau d\sigma\Big[G_{ij}(X)\partial_\alpha X^i \partial^\alpha X^j + \epsilon^{\alpha\beta} B_{ij}(X)\partial_\alpha X^i \partial_\beta X^j + 2i\big(\lambda^a \partial_- \lambda^a$$
$$+ \lambda^a(\omega_k^{ab}(X) - S_k^{ab}(X))\lambda^b \partial_- X^k\big) + 2\big(i\psi^s\partial_+\psi^s + A_i^M(X)(T^M)_{st}\partial_+ X^i \psi^s \psi^t\big)$$
$$+ iF_{ab}^M(X)\psi^s(T^M)_{st}\psi^t\lambda^a\lambda^b\Big] \tag{2.6}$$

where,

$$S_{ijk} \equiv S_k^{ab} e_i^a e_j^b = \tfrac{1}{2}(\partial_i B_{jk} + \partial_j B_{ki} + \partial_k B_{ij}) \tag{2.7}$$

and ω_i^{ab} is the torsion free spin connection constructed from the vielbein fields

$e_i^a(x)$. F_{ij}^M is the field strength constructed from the gauge field $A_i^M(x)$.

The supersymmetry transformation laws of the various component fields are given by,

$$\begin{aligned}&\delta X^i = \epsilon\lambda^i; \quad \delta\lambda^i = i\partial_+ X^i \epsilon;\\ &\delta\psi^s = i\epsilon\lambda^i A_i^M (T^M)_{st}\psi^t\end{aligned} \tag{2.8}$$

where ϵ is a right-handed Majorana-Weyl spinor. Following Ref.24 we call it a (1,0) supersymmetry.

If it so happens that the action (2.6) is invariant under a rigid chiral symmetry,

$$\lambda^i \to J_j^i(X)\lambda^j \tag{2.9}$$

then the action (2.6) is also invariant under a second supersymmetry obtained by replacing λ^i by $J_j^i(X)\lambda^j$ everywhere in (2.8). In order that the second supersymmetry anticommutes with the first one, J_j^i must satiafy,[23,24]

$$\begin{aligned}&J_j^i J_k^j = -\delta_k^i\\ &N_{ij}^k \equiv J_i^\ell J_{[j,\ell]}^k - J_j^\ell J_{[i,\ell]}^k = 0\end{aligned} \tag{2.10}$$

i.e. J must be a complex structure. In terms of superfields the new supersymmetry transformation may be expressed as,

$$\begin{aligned}&\delta\Phi^i = \epsilon J_j^i(\Phi) D\Phi^j\\ &\delta\Lambda^s = i\epsilon J_j^i(\Phi) A_i^M(\Phi) D\Phi^j (T^M)_{st}\Lambda^t\end{aligned} \tag{2.11}$$

Following Ref.24 we shall say that the model now has a (2,0) supersymmetry. Using Eq.(2.11) we may calculate the variation of the classical action under the second supersymmetry transformation. The result is,

$$\delta S = \frac{i}{2\pi\alpha'}\Bigg[\int d\tau d\sigma d\theta \epsilon (G_{ij}J^i_\ell + G_{\ell i}J^i_j) D\Phi^\ell D\partial_+\Phi^j + \tilde{\nabla}_k J^j_\ell D\Phi^i D\Phi^\ell \partial_+\Phi^k G_{ij}$$

$$+ D\Phi^k D\Phi^j (T^M)_{st}\Lambda^s\Lambda^t J^i_j F^M_{ki}\Bigg] \tag{2.12}$$

where,

$$\tilde{\nabla}_i J^j_k = J^j_{k,i} + (\Gamma_i{}^j{}_\ell - S_i{}^j{}_\ell)J^\ell_k - (\Gamma_i{}^\ell{}_k - S_i{}^\ell{}_k)J^j_\ell \tag{2.13}$$

Γ being the Christoffel symbol constructed out of the metric $G_{ij}(x)$. Thus in order for (2.6) to be invariant under the second supersymmetry transformation, we must have,[23,24]

$$\begin{aligned} &G_{k\ell}J^k_i J^\ell_j = G_{ij} \\ &J^j_{k,i} + (\Gamma_i{}^j{}_\ell - S_i{}^j{}_\ell)J^\ell_k - (\Gamma_i{}^\ell{}_k - S_i{}^\ell{}_k)J^j_\ell = 0 \\ &J^k_i F^M_{jk} - J^k_j F^M_{ik} = 0 \end{aligned} \tag{2.14}$$

Since J^i_j is a complex structure, we may introduce complex coordinates z^α, $z^{\bar\alpha}$ such that in this coordinate system,

$$J^\alpha_\beta = i\delta^\alpha_\beta; \quad J^{\bar\alpha}_{\bar\beta} = -i\delta^{\bar\alpha}_{\bar\beta}; \quad J^\alpha_{\bar\beta} = J^{\bar\alpha}_\beta = 0. \tag{2.15}$$

Eqs.(2.14) may then be written as,

$$\begin{aligned} &G_{\alpha\beta} = G_{\bar\alpha\bar\beta} = 0 \\ &\Gamma_{\bar\alpha\beta\gamma} - S_{\bar\alpha\beta\gamma} = \Gamma_{\alpha\bar\beta\bar\gamma} - S_{\alpha\bar\beta\bar\gamma} = S_{\alpha\beta\gamma} = S_{\bar\alpha\bar\beta\bar\gamma} = 0 \\ &F^M_{\alpha\beta} = F^M_{\bar\alpha\bar\beta} = 0 \end{aligned} \tag{2.16}$$

The second set of equations tells us that it is always possible to choose a gauge where $B_{\alpha\beta} = B_{\bar\alpha\bar\beta} = o$.

3. Space-time supersymmetry

In this section we shall discuss the relationship between the (2,0) world sheet supersymmetry and the space-time supersymmetry in the heterotic string theory. We shall follow the treatment of Friedan, Shenker and Martinec[31] for the construction of the space-time supersymmetry generator in the Neveu-Schwarz-Ramond model. Hence we shall first very briefly recall the construction of the supersymmetry generator for string theory in flat background.

The main ingredient for constructing the supersymmetry generator is the spin operator S^α. [We shall use the index α for denoting the spinor coordinates as well as the complex coordinates of the internal manifold, but this will not cause any confusion.] A simple way to construct these operators is to first bosonize the 10 Majorana-weyl fermions λ^μ into five right moving scalar fields H^i as,

$$\begin{aligned} \lambda^1 + i\lambda^2 &\sim : e^{iH^1} : \\ \lambda^3 + i\lambda^4 &\sim : e^{iH^2} : \\ &\;\;. \\ &\;\;. \\ \lambda^9 + i\lambda^{10} &\sim : e^{iH^5} : \end{aligned} \tag{3.1}$$

The 32 spin operators are then defined as,

$$S^\alpha \sim : e^{\frac{i}{2}(\pm H^1 \pm H^2 \ldots \pm H^5)} : \tag{3.2}$$

and the supersymmetry charge Q^α is then given by,

$$Q^\alpha \sim e^{-\frac{1}{2}\phi} S^\alpha \tag{3.3}$$

where ϕ is a bosonic field obtained by bosonizing the ghost fields. Since both ϕ

and S^α are right moving fields, Q^α satisfies,

$$(\partial_\tau - \partial_\sigma) Q^\alpha = 0 \tag{3.4}$$

and hence $\int d\sigma Q^\alpha$ is a conserved charge.

For an interacting theory bosonization may be carried out in the interaction picture, where the time dependence of various fields are given by free field equations of motion. The interaction hamiltonian for the fermionic fields is mapped into the interaction hamiltonian for the bosonic fields. The equations of motion of various operators in the Heisenberg picture are then obtained by calculating their commutators with the interaction hamiltonian. Since Q^α, in general, does not commute with the interaction hamiltonian, it will no longer satisfy (3.4) and no longer give a conserved supersymmetry charge. The situation can change, however, if some components of Q^α commute with the interaction hamiltonian. Since the ghost action remains (almost) free[34] in the presence of background fields, it is enough to ensure that some components of S^α commute with the interaction hamiltonian. We shall give a specific construction which gives rise to such a situation, although we do not prove that this is the only way to obtain a conserved supercharge.*

Our construction requires the manifold to have a complex structure† J_i^j, and a rigid chiral symmetry of the theory under the transformation,

$$\lambda^i \to J_j^i \lambda^j \tag{3.5}$$

As was shown in Sec.2, this is enough for the theory to have (2,0) supersymmetry. We shall now show how to construct a conserved space-time supersymmetry charge in such a theory. The existence of the complex structure allows us to

* A similar construction for Calabi-Yau manifolds was indicated in Ref.31.

† It has recently been suggested by Hull that it may be enough to have an almost complex structure on the manifold for unbroken supersymmetry. We do not consider this possibility in this paper.

choose complex coordinates on the manifold, which we shall denote by z^{α}, $z^{\bar{\alpha}}$. We may also choose a complex basis for the fermionic fields, the transformation (3.5) has a simple form in this basis,

$$\lambda^{\alpha} \to i\lambda^{\alpha}; \quad \lambda^{\bar{\alpha}} \to -i\lambda^{\bar{\alpha}}. \tag{3.6}$$

As a result of this symmetry $G_{\alpha\beta}$ and $G_{\bar{\alpha}\bar{\beta}}$ vanishes, and $G_{\alpha\bar{\beta}}$ may be expressed as,

$$G_{\alpha\bar{\beta}} = e^{a}_{\alpha} e^{\bar{a}}_{\bar{\beta}} \tag{3.7}$$

where e's are the vielbein fields. Let us define,

$$\lambda^{a} = e^{a}_{\alpha}\lambda^{\alpha}; \quad \lambda^{\bar{a}} = e^{\bar{a}}_{\bar{\alpha}}\lambda^{\bar{\alpha}} \tag{3.8}$$

We may now bosonize the λ^{a}, $\lambda^{\bar{a}}$ fields in the interaction picture as,

$$\lambda^{a} \sim : e^{iH^{a}} :, \quad \lambda^{\bar{a}} \sim : e^{-iH^{a}} :, \quad a = 1,2,3 \tag{3.9}$$

(The fields H^4 and H^5 of course still remains free fields since the fields λ^7,...λ^{10} are non-interacting). The existence of the symmetry (3.5) guarantees that the terms in the σ-model action couples λ^{a} to $\lambda^{\bar{b}}$, but there is no coupling of the form $\lambda^{a}\lambda^{b}$ or $\lambda^{\bar{a}}\lambda^{\bar{b}}$. In other words the total connection (including the $\psi\psi\lambda\lambda$ coupling) that couples to λ must have $U(3)$ holonomy.

It turns out that the existence of unbroken space-time supersymmetry requires a stronger condition, namely, that this connection should have $SU(3)$ holonomy.* This gives some further constraints on the background fields besides those obtained by demanding (2,0) supersymmetry. At the tree level of the

* We could start directly by requiring SU(3) holonomy, and then derive the existence of an almost complex structure and the rigid chiral symmetry (3.5). But we want to separately state the two conditions since the σ-model loop corrections will be discussed only for the constraint implied by Eq.(3.5).

sigma model, these constraints are,

$$J^{ij} F^M_{ij} = 0 \tag{3.10}$$

and that the connection $\Gamma - S$ has $SU(3)$ holonomy. As a result of these constraints the composite operator $\epsilon^{abc}\lambda^a\lambda^b\lambda^c$ satisfies the free field equation,

$$(\partial_\tau - \partial_\sigma)(\epsilon^{abc}\lambda^a\lambda^b\lambda^c) = 0 \tag{3.11}$$

Hence this operator commutes with the interaction hamiltonian. In the bosonized picture this means that the operator

$$: e^{i(H^1+H^2+H^3)} : \tag{3.12}$$

commutes with the interaction hamiltonian and hence obeys the equation of motion of a free right moving field. As a result,

$$(\partial_\tau - \partial_\sigma)\left[e^{-\frac{\phi}{2}} : e^{\frac{i}{2}[H^1+H^2+H^3 \pm H^4 \pm H^5]} :\right] = 0 \tag{3.13}$$

since ϕ, H^4 and H^5 are free fields. This gives us a conserved space-time supercharge,

$$\int d\sigma \left[e^{-\frac{\phi}{2}} : e^{\frac{i}{2}[H^1+H^2+H^3 \pm H^4 \pm H^5]} :\right] \tag{3.14}$$

whose four components are generated by four choices of the signs in front of H^4 and H^5.

Thus we have shown that in this particular way of constructing the space-time supercharge the world sheet (2,0) supersymmetry is a necessary condition for the existence of unbroken space-time supersymmetry. The rigid chiral symmetry given in (3.5) is needed to ensure the absence of certain operators (e.g. $\lambda^{\bar{a}}\lambda^b\lambda^c$) on the right hand side of Eq.(3.11). Since this consideration is not limited to the σ-model tree level, the presence of the rigid chiral symmetry (3.5) and hence also a (2,0) supersymmetry remains an integral part of the criterion for unbroken space-time supersymmetry even when the loop effects in the σ-model are taken into account.

Although the constraints for unbroken space-time supersymmetry have been derived by other means before, the advantage of using the Neveu-Schwarz-Ramond model is that all the quantum corrections are well defined in this model, and so we may actually compute corrections to these conditions using perturbation theory. In the next two sections we shall study how the condition for (2,0) supersymmetry undergoes modification under quantum corrections in the σ-model.

Finally we should mention that in our construction of the supersymmetry charge we have not encountered any constraint on the vacuum expectation value of the dilaton field. This is not surprising, since we have only used equations of motion of various fields in our analysis, which are independent of the vev of the dilaton field. In order that the supersymmetry charge (3.14) converts physical vertex operators into physical vertex operators, we must ensure that it commutes with the BRST operator[31,34–37] . Since the expression for the BRST charge involves the dilaton vacuum expectation value, we expect that the requirement of BRST invariance of the supersymmetry charge will give further constraints on the dilaton field as well.

4. Local Lorentz and gauge anomaly in the σ-model

In Sec.2 we derived restrictions on various background fields in order to have (2,0) supersymmetry in the σ-model. These equations are summarized in Eqs.(2.14). These equations, however, are not invariant under local Lorentz and gauge transformations[24,26,30,39] once we take into acount the effect of one loop chiral anomaly[38] . To see this let us define local Lorentz and gauge transformations on the background fields as,

$$\begin{aligned} \delta e_i^a(x) &= \Theta^{ab}(x) e_i^b(x) \\ \delta A_i^M(x) &= \partial_i \Theta^M(x) + f^{MNP} A_i^N(x) \Theta^P(x) \end{aligned} \tag{4.1}$$

together with appropriate transformations on the two dimensional fields. Here Θ^{ab} and Θ^M are infinitesimal parameters for local Lorentz and gauge transformations respectively. Under these transformations the effective action changes as[30] ,*

$$\begin{aligned} \delta S^{(1-loop)} = -\frac{i}{8\pi} \int d\tau d\sigma d\theta [& D\Theta^M(\Phi)\partial_- \Phi^i A_i^M(\Phi) - A_i^M(\Phi) D\Phi^i \partial_- \Theta^M(\Phi) \\ & - D\Theta^{ab}(\Phi)\omega_i^{ab}(\Phi)\partial_-\Phi^i + \omega_i^{ab}(\Phi) D\Phi^i \partial_- \Theta^{ab}(\Phi)] \end{aligned} \tag{4.2}$$

which may be cancelled by redefining the transformation laws of the antisymmetric tensor field B_{ij} as,

$$\delta B_{ij} = -\frac{1}{4}\alpha'(\partial_{[i}\Theta^M A_{j]}^M - \partial_{[i}\Theta^{ab}\omega_{j]}^{ab}) \tag{4.3}$$

Under these modified transformation laws, S_{ijk}, as defined in Eq.(2.7), is no longer invariant under local Lorentz and gauge transformations. If, however, we

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* We must interchange all the ∂_+'s with ∂_-'s in order to compare the results of this section with those in Ref.[5,26,30].

define,

$$H_{ijk} = S_{ijk} + \frac{\alpha'}{8}[\Omega_3(A)_{ijk} - \Omega_3(\omega)_{ijk}] \tag{4.4}$$

where

$$\Omega_3(A)_{ijk} = \tfrac{1}{2}[A^M_{[i}F^M_{jk]} + \tfrac{1}{3}f^{MNP}A^M_{[i}A^N_jA^P_{k]}] \tag{4.5}$$

is the Chern-Simons three form, then H is invariant under local Lorentz and gauge transformations. Hence we may expect S to be replaced by H in Eq.(2.14).

There is, however, an ambiguity in defining H, since we may add any local Lorentz and gauge invariant antisymmetric rank three tensor, constructed out of Yang-Mills field strength, the curvature tensor and H itself, to the right hand side of Eq.(4.4), without spoiling the gauge and local Lorentz invariance of H. This ambiguity may also be seen in a slightly different form as follows. Let T^M_i and T^{ab}_i be two gauge and Lorentz covariant tensors. If we now define,

$$\hat{H}_{ijk} = \tfrac{1}{2}(\hat{B}_{ij,k} + \hat{B}_{jk,i} + \hat{B}_{ki,j}) + \frac{\alpha'}{8}[\Omega_3(\hat{A}^M)_{ijk} - \Omega_3(\hat{\omega}^{ab})_{ijk}] \tag{4.6}$$

where,

$$\hat{A}^M_i = A^M_i + T^M_i, \quad \hat{\omega}^{ab}_i = \omega^{ab}_i + T^{ab}_i$$

$$\hat{B}_{ij} = B_{ij} - \frac{\alpha'}{4}(A^M_{[i}T^M_{j]} - \omega^{ab}_{[i}T^{ab}_{j]}) \tag{4.7}$$

then it can be easily shown that $\hat{H}$ is invariant under local Lorentz and gauge transformations. We may then expect that when we take into account the effect of one loop Lorentz and gauge anomaly, the requirement of (2,0) supersymmetry should give constraints on the background fields of the form,

$$G_{k\ell}J^k_iJ^\ell_j = G_{ij}$$

$$J^j_{k,i} + (\Gamma_i{}^j{}_\ell - \hat{H}_i{}^j{}_\ell)J^\ell_k - (\Gamma_i{}^\ell{}_k - \hat{H}_i{}^\ell{}_k)J^j_\ell = 0 \tag{4.8}$$

$$J^k_iF^M_{jk} - J^k_jF^M_{ik} = 0$$

where $\hat{H}$ is calculated from (4.6) with some specific choice of the connections

$\hat{A}^M = A^M + T^M$ and $\hat{\omega}^{ab} = \omega^{ab} + T^{ab}$. We shall show in the next section that this is indeed the case and derive expressions for $\hat{A}$ and $\hat{\omega}$.

5. Effects of radiative corrections on (2,0) supersymmetry

We shall begin the discussion in this section with a simple calculation. We shall calculate the contribution to the one loop effective action involving the Φ^i fields from the Λ^s loop, and study the variation of this effective action under the second supersymmetry transformation given in Eq.(2.11). Since the Λ^s loop is the only source of gauge anomaly, we expect that by studying the variation of this effective action, we shall know how the requirement for (2,0) supersymmetry is affected by gauge anomaly. What makes this calculation simple is that, as we shall show, we may directly calculate the variation of the effective action under (2,0) supersymmetry, without calculating the effective action itself.

Now, as can be seen from Eq.(2.2), the gauge field A_i^M appears in the tree level action only in the combination $A_i^M D\Phi^i$. Thus naively one would expect that the the one loop effective action may be expressible as a (non-local) function of the composite superfield $A_i^M D\Phi^i$. Also, since,

$$\delta_{gauge}(A_i^M D\Phi^i) = D\Theta^M + f^{MNP}\Theta^P(A_i^N D\Phi^i) \tag{5.1}$$

even in the gauge variation of the effective action A_i^M should appear in the same combination. By examining expression (4.2) for the gauge anomaly we find, however, that it contains a term proportional to $A_i^M\partial_-\Phi^i$ as well. The solution to this puzzle lies in the fact that in defining the effective action we are allowed to add arbitrary local counterterms to the action* and in this case, in order to reproduce Eq.(4.2) we must add a local term involving $A_i^M\partial_-\Phi^i$ to the effective action. The effective action whose gauge variation has the form (4.2) may be expressed as,

$$S^{(\Lambda)} = f(A_i^M D\Phi^i) - \frac{i}{4\pi}\int d\tau d\sigma d\theta A_i^M\partial_-\Phi^i A_j^M D\Phi^j \tag{5.2}$$

where f is a non-local function which depends only on the combination $A_i^M D\Phi^i$,

* This corresponds to a redefinition of various background fields

and which transforms under a gauge transformation as,

$$\delta_{gauge} f = \frac{i}{8\pi} \int d\tau d\sigma d\theta A_i^M(\Phi) D\Phi^i \partial_- \Theta^M \tag{5.3}$$

so that $\delta_{gauge} f$ depends on A_i^M only through the combination $A_i^M D\Phi^i$. The advantage of representing the effective action in the form (5.2) lies in the fact that under the second supersymmetry transformation given in Eq.(2.11),

$$\delta_{susy}(A_i^M D\Phi^i) = D(\epsilon A_i^M J_j^i D\Phi^j) + f^{MNP} A_k^N D\Phi^k (\epsilon A_i^P J_j^i D\Phi^j) \tag{5.4}$$

i.e. $A_i^M D\Phi^i$ transforms as in a gauge transformation with the gauge parameter $\epsilon A_i^M J_j^i D\Phi^j$. Thus the variation of f under the second supersymmetry transformation may be obtained by replacing Θ^M by $\epsilon A_i^M J_j^i D\Phi^j$ in (5.3). On the other hand the supersymmetry variation of the second term in (5.2) may be calculated explicitly. The final result is,

$$\delta_{susy} S^{(A)} = -\frac{i}{8\pi} \int d\tau d\sigma d\theta \epsilon D\Phi^i D\Phi^\ell \partial_- \Phi^k J_\ell^j \Omega_3(A)_{ijk} \tag{5.5}$$

Combining this with (2.12) we see that in order that the full effective action is invariant under the second supersymmetry transformation, S_{ijk} must be replaced by $S_{ijk} + \frac{\alpha'}{8}\Omega_3(A)_{ijk}$ in Eq.(2.14). This analysis thus tells us that the effect of gauge anomaly on (2,0) supersymmetry is indeed to change the constraint equations to the form given in Eq.(4.8). $\hat{H}$ is calculated from Eq.(4.6) by setting $T^M = 0$.†

† Of course at one loop order we could argue on general dimensional grounds that T^M must vanish, since there is no gauge and Lorentz covariant tensor that could be constructed to this order. A tensor like $F_{jk}^M H^{jk}{}_i$ can be added to A_i^M with an explicit power of α' and hence can contribute to the right hand side of Eq.(4.6) at two loop order. But this explicit calculation demonstrates that there is no subtle effect, and that the gauge invariance of the theory is indeed consistent with (2,0) supersymmetry.

A similar analysis for graphs involving the Φ-loops becomes much more complicated, since the variation of the non-local part of the effective action cannot be expressed as the variation of the action under a local Lorentz transformation. Hence we must use some indirect method for analyzing this contribution. But before doing that, let us first point out an apparent contradiction in the results we have obtained so far. According to our analysis the criteria for having an extended supersymmetry requires modification when we include the one loop contribution to the effective action. However, there is a superfield formulation for theories with (2,0) supersymmetry once the tree level equations (2.14) are satisfied. In this formulation we may calculate the effective action using extended supergraphs, and hence never spoil the (2,0) supersymmetry. How then is our analysis consistent with this result?

It turns out that the solution to this puzzle is also the key to the understanding of how the equations (2.14) are modified by Lorentz anomaly. The point is that different methods of computation of the effective action may differ by finite local terms. It was shown in Ref.30 that the addition of any local term in the lagrangian which respects the (1,0) supersymmetry may always be absorbed into a redefinition of the coupling constants G_{ij}, B_{ij} and A_i^M. Hence our result can be consistent with the results of (2,0) superfield calculation only if the new equations (4.8) can be brought into the form of Eq.(2.14) after a field redefinition. To check this, consider the second of Eqs.(4.8), written in complex coordinates,

$$\Gamma_{\bar\alpha\beta\gamma} - \hat H_{\bar\alpha\beta\gamma} = \Gamma_{\alpha\bar\beta\bar\gamma} - \hat H_{\alpha\bar\beta\bar\gamma} = \hat H_{\alpha\beta\gamma} = \hat H_{\bar\alpha\bar\beta\bar\gamma} = 0 \tag{5.6}$$

According to our argument ,it must be expressible as,

$$\Gamma'_{\bar\alpha\beta\gamma} - S'_{\bar\alpha\beta\gamma} = \Gamma'_{\alpha\bar\beta\bar\gamma} - S'_{\alpha\bar\beta\bar\gamma} = S'_{\alpha\beta\gamma} = S'_{\bar\alpha\bar\beta\bar\gamma} = 0 \tag{5.7}$$

where Γ' and S' are Christoffel symbol and torsion constructed out of some new metric $G'_{\alpha\bar\beta}$ and new antisymmetric tensor $B'_{\alpha\bar\beta}$. In order to verify that this is indeed the case, we express Eq.(5.6) in terms of G_{ij} and B_{ij},

$$\begin{aligned}
&\tfrac{1}{2}(G_{\bar{\alpha}\beta,\gamma} - G_{\bar{\alpha}\gamma,\beta}) - \tfrac{1}{2}(B_{\bar{\alpha}\beta,\gamma} - B_{\bar{\alpha}\gamma,\beta} + B_{\beta\gamma,\bar{\alpha}}) - \frac{\alpha'}{8}\Omega_3(A)_{\bar{\alpha}\beta\gamma} = 0 \\
&\tfrac{1}{2}(B_{\alpha\beta,\gamma} + B_{\beta\gamma,\alpha} + B_{\gamma\alpha,\beta}) + \frac{\alpha'}{8}\Omega_3(A)_{\alpha\beta\gamma} = 0
\end{aligned} \tag{5.8}$$

The other equations in (5.6) are obtained by complex conjugating (5.8).

Since $d\Omega_3(A)$ is proportional to $Tr(F \wedge F)$, and since $F_{\alpha\beta} = F_{\bar{\alpha}\bar{\beta}} = 0$, $d\Omega_3(A)$ has two holomorphic and two antiholomorphic indices. (In other words it is a (2,2) form.) Hence,

$$\partial_{[\delta}(\Omega_3(A))_{\alpha\beta\gamma]} = \partial_{[\bar{\delta}}(\Omega_3(A))_{\alpha\beta\gamma]} = 0 \tag{5.9}$$

showing that $\Omega_3(A)_{\alpha\beta\gamma}$ and $\Omega_3(A)_{\bar{\alpha}\beta\gamma}$ may be expressed as,

$$(\Omega_3(A))_{\alpha\beta\gamma} = D_{\alpha\beta,\gamma} + D_{\beta\gamma,\alpha} + D_{\gamma\alpha,\beta} \tag{5.10}$$

$$(\Omega_3(A))_{\bar{\alpha}\beta\gamma} = C_{\bar{\alpha}\beta,\gamma} - C_{\bar{\alpha}\gamma,\beta} + D_{\beta\gamma,\bar{\alpha}} \tag{5.11}$$

for some tensors $C_{\bar{\alpha}\beta}$ and $D_{\alpha\beta}(= -D_{\beta\alpha})$. Similarly $\Omega_3(A)_{\alpha\bar{\beta}\bar{\gamma}}$ may be written as,

$$(\Omega_3(A))_{\alpha\bar{\beta}\bar{\gamma}} = C_{\alpha\bar{\beta},\bar{\gamma}} - C_{\alpha\bar{\gamma},\bar{\beta}} + D_{\bar{\beta}\bar{\gamma},\alpha} \tag{5.12}$$

Note that we are not assuming any symmetry properties of C, so that $C_{\bar{\alpha}\beta}$ and $C_{\beta\bar{\alpha}}$ are completely independent of each other. Substituting (5.10) and (5.11) in (5.8) we see that we may express Eqs.(5.8) as,

$$\tfrac{1}{2}(G'_{\bar{\alpha}\beta,\gamma} - G'_{\bar{\alpha}\gamma,\beta}) - \tfrac{1}{2}(B'_{\bar{\alpha}\beta,\gamma} - B'_{\bar{\alpha}\gamma,\beta} + B'_{\beta\gamma,\bar{\alpha}'}) = \tfrac{1}{2}(B'_{\alpha\beta,\gamma} + B'_{\beta\gamma,\alpha} + B'_{\gamma\alpha,\beta}) = 0 \tag{5.13}$$

where,

$$G'_{\bar{\alpha}\beta} = G_{\bar{\alpha}\beta} - \frac{\alpha'}{8}(C_{\bar{\alpha}\beta} + C_{\beta\bar{\alpha}})$$

$$B'_{\bar{\alpha}\beta} = B_{\bar{\alpha}\beta} + \frac{\alpha'}{8}(C_{\bar{\alpha}\beta} - C_{\beta\bar{\alpha}}) \tag{5.14}$$

$$B'_{\alpha\beta} = B_{\alpha\beta} + \frac{\alpha'}{4}D_{\alpha\beta}$$

Because of the last two equations of (5.6) we can no longer choose $B_{\alpha\beta}$ to be zero, but we may choose $B'_{\alpha\beta}$ to be zero.

With the lesson that we learned we may proceed to determine how Eqs.(2.14) gets affected by Lorentz anomaly. The key point is that it must be affected in such a way that the resulting equations may be expressed as (5.7) after a field redefinition. Furthermore, any set of Lorentz and gauge covariant constraint equations which reduce to Eq.(5.7) after a field redefinition is a valid set of constraints, since if there is more than one set of such constraints satisfying the above criteria, they may be transformed into each other by a field redefinition. We shall seek for a constraint equation of the form (4.8). We take $\hat{H}$ of the form given in Eq.(4.6) with T^M set to zero. As before, we may convert the new constraint equations to the old ones by a field redefinition if $d\Omega_3(\hat{\omega}) = Tr[R(\hat{\omega}) \wedge R(\hat{\omega})]$ is a (2,2) form, i.e. if $R(\hat{\Omega})$ is a (1,1) form.* A convenient choice is,

$$\hat{\omega}_i^{ab} = \omega_i^{ab} + \hat{H}_i^{ab} \tag{5.15}$$

With this choice $\hat{H}$ appears on both sides of the equation (4.6), but it can be solved iteratively for $\hat{H}$. If we calculate the corresponding connection in the

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* It has been argued by Strominger[19] that this is the only case where we can obtain a perturbative solution to the constraint equations, whose zeroth order solution is a Calabi-Yau manifold.

coordinate basis,

$$\hat{\Gamma}_{i\ k}^{\ j} = G^{j\ell}(e_\ell^a \partial_i e_k^a + \hat{\omega}_i^{ab} e_\ell^a e_k^b) = \Gamma_{i\ k}^{\ j} + \hat{H}_{i\ k}^{\ j} \tag{5.16}$$

where Γ denotes the Christoffel symbol, then, after using the constraint equations (4.8), the various components of $\hat{\Gamma}$ in complex coordinate system are given by

$$\begin{aligned} \hat{\Gamma}_{\alpha\ \gamma}^{\ \beta} &= G^{\beta\bar{\beta}} G_{\bar{\beta}\gamma,\alpha} \\ \hat{\Gamma}_{\alpha\ \bar{\gamma}}^{\ \bar{\beta}} &= \hat{\Gamma}_{\alpha\ \gamma}^{\ \bar{\beta}} = o \\ \hat{\Gamma}_{\alpha\ \bar{\gamma}}^{\ \beta} &= G^{\beta\bar{\beta}}(G_{\bar{\beta}\alpha,\bar{\gamma}} - G_{\alpha\bar{\gamma},\bar{\beta}}) \end{aligned} \tag{5.17}$$

The curvature tensor constructed from it may be shown to be a (1,1) form with correction terms of order α' by using the equation

$$G_{\alpha[\bar{\beta},\bar{\gamma}]\delta} - G_{\delta[\bar{\beta},\bar{\gamma}]\alpha]} = O(\alpha') \tag{5.18}$$

which again follows from the constraint equations (4.8). Removal of the extra terms of order α' will need addition of new terms of order α' to the right hand side of Eq.(5.15), i.e. of order α'^2 to the definition of $\hat{H}$. Note that in $\hat{\omega}$, $\hat{H}$ is added to the spin connection, whereas in Eq.(4.8) the covariant derivative is taken with respect to a connection where $\hat{H}$ is subtracted from the Christoffel symbol.

A different choice of the connection $\hat{\omega}'$ has been proposed by Strominger. Expressed in terms of $\hat{\Gamma}'$ defined by Eq.(5.16) with $\hat{H}$ replaced by $\hat{H}'$, the various components of this connection take the form,

$$\begin{aligned} \hat{\Gamma}'^{\ \beta}_{\alpha\ \gamma} &= G^{\beta\bar{\beta}} G_{\bar{\beta}\gamma,\alpha} \\ \hat{\Gamma}'^{\ \bar{\beta}}_{\alpha\ \bar{\gamma}} &= \hat{\Gamma}'^{\ \bar{\beta}}_{\alpha\ \gamma} = \hat{\Gamma}'^{\ \beta}_{\alpha\ \bar{\gamma}} = 0 \end{aligned} \tag{5.19}$$

The other components of $\hat{\Gamma}'$ are obtained by complex conjugating the indices. The curvature tensor constructed from this connection is a (1,1) form, and hence

(5.19) also gives a valid choice of $\hat{\omega}$ to be used in Eq.(4.8). Expressed in terms of real coordinates, this corresponds to choosing a different $\hat{\omega}'$, given by,

$$
\begin{aligned}
(\hat{\omega}')_i^{ab} = \omega_i^{ab} + (\hat{H}')_i^{ab} - \tfrac{1}{2} E^{ja} E^{kb} (\hat{H}')_{i'j'k'} \\
(\delta_i^{i'} \delta_j^{j'} \delta_k^{k'} - \delta_i^{i'} J_j^{j'} J_k^{k'} + J_i^{i'} \delta_j^{j'} J_k^{k'} + J_i^{i'} J_j^{j'} \delta_k^{k'})
\end{aligned}
\tag{5.20}
$$

where E^{ak} is the inverse of the vielbein e_i^a. Again $\hat{H}'$ may be obtained by solving Eq.(4.6) iteratively. The values of G_{ij} and B_{ij} obtained by solving Eq.(4.8) with the connection $\hat{\omega}'$ will be related to those obtained by solving Eq.(4.8) with the connection $\hat{\omega}$ by a field redefinition which, in general, will involve the complex structure J as well.

In fact, since by a field redefinition we may bring Eq.(4.8) in the form of Eq.(2.14), we could just solve Eq.(2.14) without worrying about any quantum corrections at all. The fields G_{ij} and B_{ij} obtained by solving these equations will again be related to those obtained by solving Eq.(4.8) by a field redefinition given in Eqs.(5.14), although these fields will have complicated Lorentz transformation properties under local Lorentz and gauge transformations.

We conclude this section by pointing out that our general considerations do not tell us what choice of $\hat{\omega}$ will express the constraint equations in terms of the physical fields, where the definition of a physical field is such that the β-functions (i.e. the equations of motion) expressed in terms of the physical fields will not involve the complex structure. Since the field redefinition which takes us from one choice of $\hat{\omega}$ to another involves the complex structure in general, certainly not all choices of $\hat{\omega}$ will express the constraint equations in terms of the physical fields. In order to settle this question one has to carry out a computation of the effective action in a scheme which does not depend on the complex structure, study the variation of the effective action under (2,0) supersymmetry, and determine the constraints that must be satisfied in order that the above variation vanishes.

6. Conclusion

In this paper we have analyzed the criteria for unbroken space-time supersymmetry in the heterotic string theory, starting from the σ-model describing the propagation of the heterotic string in arbitrary background fields. We use the Neveu-Schwarz-Ramond formulation of the string theory and try to construct a conserved supercharge following the methods of Ref.31. The construction requires the background fields to satisfy certain restrictions, one of which is the existence of unbroken (2,0) world sheet supersymmetry in the σ-model.

We then discuss the restriction imposed on the background fields by the requirement of (2,0) supersymmetry. At the tree level the equations involve the curl of the antisymmetric tensor field B_{ij}. Since this field has anomalous transformation laws under local Lorentz and gauge transformations, the tree level equations are not invariant under these transformations and must be modified by quantum corrections. The requirement of local gauge and Lorentz invariance however does not uniquely specify what the corrections to the equations should be, since we may add any Lorentz and gauge covariant terms to these equations without destroying their invariance properties. With the help of explicit calculation and some general arguments we have been able to determine the correct equations describing the requirement of (2,0) supersymmetry in this model. The main result is that the modified equations can always be transformed into the original tree level equations by a field redefinition. The redefined metric G'_{ij} and the antisymmetric tensor field B'_{ij} have complicated transformation properties under local Lorentz and gauge transformations which compensate for the non-trivial transformation properties of dB' appearing in these equations. We have also expressed the criteria for (2,0) supersymmetry in terms of the fields G_{ij} and B_{ij}, which transforms in the standard way under local Lorentz and gauge transformations. Our result says that the effect of quantum corrections is to replace dB in the tree level equation by a gauge and Lorentz covariant three form $\hat{H}$ with the property that $d\hat{H}$ is a (2,2) form when expressed in complex coordinates.

[The existence of (2,0) supersymmetry requires the manifold to have a complex structure, and hence ensures the existence of a complex coordinate system]. Furthermore, if there are two or more three forms satisfying the above requirements we can use any one of them in our equations. Since we have shown that each set of equations using a particular three form $\hat{H}$ can be deformed into the form of tree level equations by field redefinition, two sets of equations using different three forms can also be deformed into each other by a field redefinition.* We have constructed specific examples of $\hat{H}$ satisfying these requirements. These forms of $\hat{H}$ have previously been advocated by Strominger[19] and by Hull[20] from different considerations.

It is tempting to conjecture that the complete set of equations obtained by requiring unbroken $N = 1$ supersymmetry may be transformed into the corresponding equations at the σ-model tree level after a field redefinition. Then if we obtain a solution to these equations at the tree level we shall obtain a solution to all orders in the perturbation theory. This result will also be consistent with the general arguments presented by Witten[18], showing that in the effective four dimensional theory the F term in the superpotential does not receive any contribution from the σ-model loop corrections. As a result the position of a supersymmetric minimum is unaffected by the σ-model loop corrections. (Ref.[15,16], however, seems to provide a counterexample to this result.) In any case it will be interesting to see how the radiative corrections in the σ-model affect the various other constraint equations for unbroken space-time supersymmetry. One should also be able to compare these results with the higher order corrections calculated by various authors[40] by trying to supersymmetrize the Green-Schwarz action. We hope to return to these questions in the near future.

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* Of course we must stay away from pathological choices for which the field redefinition is singular. The safest rule is to take the O(1) contribution to H to be dB and use the freedom of choice on the $O(\alpha')$ contribution.

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