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ON THE HADRONIZATION OF TOP QUARKS*

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ABSTRACT

Gluon bremsstrahlung off heavy quarks like top is expected to be strongly damped. Top production on the Z^0 resonance will then lead to quasi-exclusive final states consisting of two top hadrons plus at most a few soft pions. We point out that quark-hadron duality can be realized as an average over two-body decay modes of Z^0 into top. The states involved are not just the stable top hadrons, but necessarily have to include higher excitations, namely *p*-, *d*-, etc. wave configurations of $(t\bar{q})$. Their strong decays in turn lead to soft pions. Some consequences of this picture are discussed.

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1. The Problem

Very general considerations lead to the expectation that the fragmentation function for quarks becomes harder and harder as the mass of the quark increases, asymptotically going to a delta-like function.[1] Such a behavior is implemented by a fragmentation function like the following:[2]

$$
\begin{aligned}
D_Q(z) &\simeq \frac{4}{\pi}\,\frac{\sqrt{\epsilon_Q}}{z\left[1-\frac{1}{z}-\frac{\epsilon_Q}{1-z}\right]^2} \\
\epsilon &\simeq 0.15\left(\frac{m_c}{m_Q}\right)^2
\end{aligned}
\qquad (1)
$$

for a heavy quark Q with mass M_Q.

This expectation has been tested quite successfully for charm and bottom quarks. For top quarks with $m_t \simeq 40$ GeV one gets $\epsilon_t \simeq 2\times 10^{-4}$ and an average energy loss in e^+e^- annihilation of $\sim \sqrt{\epsilon_t}\,E_{\text{beam}}$. On the Z^0 this amounts to ~ 600 MeV. Thus the top hadrons carry away almost all the energy and roughly one GeV is all that is left in energy for additional pions. This scenario even allows for a sizeable fraction of all top events to be made up by just two top mesons.[8]

There are then two ways to calculate the cross section for top production:

(A) For $e^+e^- \to Z^0 \to t\bar{t}$, t denoting top quarks, one obtains[3]

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} &= \frac{3\beta}{64\pi^2}\left(\frac{G_F\,m_Z^2}{2\sqrt{2}}\right)^2 \frac{Q^2}{(Q^2-m_Z^2)^2+(m_Z\Gamma_Z)^2}\,(v_e^2+a_e^2)\cdot N\ , \\
N &= (v_t^2+a_t^2)\beta^2(1+\cos^2\theta)+2v_t^2(1-\beta^2)+4\tau_e v_t a_t\,\beta\cos\theta\quad , \qquad (2) \\
\tau_e &= \frac{2a_e\,v_e}{a_e^2+v_e^2}
\end{aligned}
$$

with

$$a_e = -1 \ , \qquad v_e = -1 + 4\sin^2\theta_W$$

$$a_t = 2I_3 = 1 \ , \qquad v_t = 2I_3 - 4q_t \sin^2\theta_W = 1 - \frac{8}{3}\ \sin^2\theta_W \quad . \tag{3}$$

The generalization of (2) and (3) to other quarks is rather obvious.

Furthermore the t quarks are produced with a high degree of polarization on the Z^0; for its longitudinal component one finds[3]

$$P_L = -\frac{2v_t a_t\,\beta(1+\cos^2\theta) + 2\tau_e(a_t^2\beta^2 + v_t^2)\cos\theta}{(v_t^2 + a_t^2)\beta^2(1+\cos^2\theta) + 2v_t^2(1-\beta^2) + 4\tau_e v_t a_t\,\beta\cos\theta} \quad . \tag{4}$$

(B) For calculating $e^+e^- \to Z^0 \to H_t\overline{H}_t' - H_t$ and H_t' being two top hadrons that are not necessarily identical — one has to specify their quantum numbers. The difference in mass of pseudoscalar mesons — T — and vector mesons — T^* — is due to hyperfine splitting. Thus it will decrease as m_t increases:

$$M(T^*) - M(T) \ \sim\ 5 \text{ MeV} \left(\frac{40 \text{ GeV}}{m_t}\right) \quad , \tag{5}$$

i.e. for all practical purposes T and T^* will be mass degenerate. This implies also that T^* will decay weakly almost all the time:[4]

$$\Gamma\,(T^* \to T\gamma) \ \lesssim\ 10^{-3}\,\Gamma(T^*) \quad . \tag{6}$$

Accordingly one has to compute (at least) the following three transition rates

$$e^+e^- \to Z^0 \to T\,\overline{T},\ T^*\,\overline{T} + T\,\overline{T}^*,\ T^*\,\overline{T}^* \quad .$$

One finds generalizing the analysis of Ref. [10]:

$$\frac{d\sigma}{d\Omega}\ \left(e^+e^- \to Z^0 \to T\,\overline{T}\right) = N_{PP}\,\beta^3 \sin^2\theta\, v_t^2\, |F_{PP}|^2 \tag{7}$$

$$\frac{d\sigma}{d\Omega}\left(e^+e^- \to Z^0 \to T^*\,\overline{T} + T\,\overline{T}^*\right) = N_{PV}\,\beta \left\{ \frac{\beta^2}{2}(1+\cos^2\theta)v_t^2\,|EF_{PV}|^2 \right.$$

$$+ \left[1 + \frac{\beta^2}{2}\left(\frac{f^2\beta^2 + 1 - 2f}{1-\beta^2}\right)\sin^2\theta\right] a_t^2\,|EF_{PV}^5|^2$$

$$\left. + 4\beta\cos\theta\, v_t a_t \,\mathrm{Re}\; E^2\, F_{PV}^5\, F_{PV}^* \right\} \tag{8}$$

$$\frac{d\sigma}{d\Omega}\left(e^+e^- \to Z^0 \to T^*\,\overline{T}^*\right) = N_{VV}\,\beta^3 \left\{ \frac{\beta^2}{4}\ \sin^2\theta\, a_t^2\,|4E^2F_{VV}^5|^2 \right.$$

$$+ \left[2(1+\tilde{f})^2\gamma^2 + \frac{1}{2}\sin^2\theta \right. \tag{9}$$

$$\left.\left. \times \left(2 + (1+2\tilde{f}\gamma^2)^2 - 2\gamma^2(1+\tilde{f})^2\right)\right] v_t^2\,|F_{VV}|^2 \right\}$$

with $\gamma^2 = \frac{1}{1-\beta^2}$ and N_{PP}, N_{PV}, N_{VV} being normalization factors (a discussion which differs from this one in spirit and in detail can be found in Ref. [9]).

Constraints imposed by conservation of angular momentum produce these expressions; the dynamics is contained in the form factors F_{PP}, $F_{PV}^{(5)}$, f, $F_{VV}^{(5)}$ and $\tilde{f}$ which are defined as follows:

$$\langle T(p)\,\overline{T}(\bar{p})|j_\mu|0\rangle = (p-\bar{p})_\mu\, F_{PP}\, v_t \tag{10}$$

$$\left\langle T(p)\,\overline{T}^*(\bar{p},\epsilon)|j_\mu|0\right\rangle = i\epsilon_{\mu\alpha\beta\gamma}\, q_\alpha\, p_\beta\, \epsilon_\gamma\, v_t\, F_{PV} \tag{11}$$

$$\left\langle T(p)\,\overline{T}^*(\bar{p},\epsilon)|j_\mu^5|0\right\rangle = \left(p\cdot q\,\epsilon_\mu - f\epsilon\cdot q\,p_\mu - \frac{1-f}{2}\ \epsilon\cdot q\,q_\mu\right) a_t\, F_{PV}^5 \tag{12}$$

$$\left\langle T^*(p,\epsilon)\,\overline{T}^*(\bar{p},\bar{\epsilon})|j_\mu|0\right\rangle = \left[\epsilon\cdot\bar{\epsilon}(q-2p)_\mu + (1+\tilde{f})(q\cdot\bar{\epsilon}\,\epsilon_\mu - q\cdot\epsilon\,\bar{\epsilon}_\mu)\right] v_t\, F_{VV} \tag{13}$$

$$\left\langle T^*(p,\epsilon)\,\overline{T}^*(\bar{p},\bar{\epsilon})|j_\mu^5|0\right\rangle = (q-2p)_\mu\, i\epsilon_{\alpha\beta\gamma\delta}\, q_\alpha\, p_\beta\, \epsilon_\gamma\, \bar{\epsilon}_\delta\, a_t\, F_{VV}^5 \tag{14}$$

where $q = p + \bar{p}$.

The phase space and angular dependence of these exclusive processes is quite distinct from that of the quark reaction as described by eq. (2). To express these differences in real quantitative terms one needs a specific ansatz for calculating the various form factors. Yet even independent of that one can make the following crucial observation: the processes $e^+e^- \to Z^0 \to T\,\overline{T}, T^*\,\overline{T}^*$ do *not* show a linear dependence on $\cos\theta$ and thus will *not* exhibit a forward-backward asymmetry A_{FB}, only $e^+e^- \to Z^0 \to T^*\,\overline{T} + T\,\overline{T}^*$ will! This feature is actually easily understood since T and $\overline{T}$ [T^* and $\overline{T}^*$] are mesons belonging to the same isospin multiplet (and not just to equivalent ones as in the case of $t, \bar{t}$ or $T, \overline{T}^*$ + h.c.). Therefore — apart from isospin breaking — no forward-backward asymmetry can be established in $Z^0 \to H_t\overline{H}_t$: for $d\sigma(Z^0 \to H_t(\vec{p})\,\overline{H}_t(-\vec{p})) = d\sigma(Z^0 \to \overline{H}_t(\vec{p})\,H_t(-\vec{p}))$ is shown to hold by employing isospin rotations. Then summing over the three channels $Z^0 \to T\,\overline{T},\ T^*\,\overline{T} + T\,\overline{T}^*,\ T^*\,\overline{T}^*$ will yield a significantly "diluted", *i.e.* reduced A_{FB} relative to the one appearing in $Z^0 \to t\bar{t}$.

This observation by itself is not very surprising since one cannot trust a perturbative treatment using quarks (and gluons) just above threshold. The problem arises when one connects this with the expectation sketched in the beginning that gluon bremsstrahlung is so damped that $Z^0 \to T\,\overline{T},\ T^*\,\overline{T}$+h.c., $T^*\,\overline{T}^*$ represents a large fraction of all top events for $m_t = 40$ GeV or even lower.

Such a statement cannot be correct for two reasons: firstly these exclusive channels become quickly insignificant due to a form factor suppression $F(q^2) \sim \frac{4m_t^2}{q^2}\,\theta(q^2 - 4m_t^2)$. Secondly, due to the very general concept of quark-hadron duality it hardly makes sense that at the Z^0 resonance, which for $m_t = 40$ GeV is ~ 13 GeV above threshold, the inclusive process $Z^0 \to H_t + X$ should not be well described by $Z^0 \to t\bar{t}$. In the following we explore this concept of duality in more detail for the problem at hand.

2. Realization of Duality

Cross sections calculated in terms of quarks and gluons should represent a good description of hadronic cross sections when the inclusive rate under study is built up from a number N of exclusive channels. The relevant question then is how large this number N has to be and at which energy interval Δ above production threshold this requirement is met.

We have stated above as part of the problem that gluon bremsstrahlung off top quarks appears as a very inefficient way to generate additional final states. However it is important to note that duality can also be realized by summing over two-body modes

$$d\sigma\left(e^+e^- \to Z^0 \to t\bar{t}\right) \simeq \sum_{i,j=1}^{N} d\sigma\left(e^+e^- \to Z^0 \to H_t^i\, \overline{H}_t^j\right) \tag{15}$$

where H_t^i denotes the various allowed top hadrons: in addition to the s-wave $(t\bar{q})$ states T, T^* (and the baryons Λ_t) there appear the p-wave states $(t\bar{q})_{\ell=1}$ with quantum numbers $J^P = 0^+,\ 1^+,\ 2^+$, the d-wave states etc.

The explicit examples given in eqs. (7)-(9) show that $N > 2$ to avoid a contradiction in (15). If the p wave states are included one has $N = 6$ (even ignoring baryons) and (15) should be realized at least in an approximate way. This general expectation is not based on an actual calculation of (15) for $N = 6$: for $N > 2$ there enter so many a priori unknown form factors that such a calculation of exclusive modes becomes impractical. Instead we base our expectation on experience gained in analogous cases like charm and bottom production.

As already stated the mass difference $M(T^*) - M(T)$ is due to hyperfine splitting which shows a $1/m_t$ dependence leading to $M(T^*) - M(T) \ll M(B^*) - M(B) < M(D^*) - M(D)$. However the mass splittings between the s- and p- etc. wave $(Q\bar{q})$ states do not exhibit such a dependence on the heavy quark mass.

One actually predicts[5]

$$\overline{M}(T_{\ell=1}) - M(T) \sim \overline{M}(B_{\ell=1}) - M(B) \sim \overline{M}(D_{\ell=1}) - M(D) \sim 500 \text{ MeV} \quad (16)$$

where $\overline{M}$ denotes the average p-wave mass. It is interesting to note in this context that ARGUS has recently presented evidence[6] for a $D^*(2420)$ meson with $J^P = 2^+$ or 1^+ decaying to $D^*\pi$.

From this discussion we conclude that starting at around 1 GeV above threshold 6 different top hadrons will be produced with $J^P = 0^-, 1^-, 0^+, 1^+, 1^{+\prime}, 2^+$; around 2 GeV above threshold d-wave $(t\bar{q})$ states will appear. Therefore we expect that at $\sim$ 2-3 GeV above top production threshold duality should start to operate both for the total rate and for angular distributions.

If — contrary to this expectation — the data showed non-monotonic features when taken at different energies (*i.e.* also off the Z^0 resonance) then one would have to rely on a dispersion relation ansatz to extend the applicability of duality:[7] one calculates

$$d\bar{\sigma} = \frac{\Delta}{\pi} \int_{(2m_t)^2}^{\infty} ds' \, \frac{d\sigma(s')}{(s'-s)^2 + \Delta^2} \quad (17)$$

and compares it with its experimentally determined value using $\Delta \sim$ 2-3 GeV.

To conclude: even when gluon bremsstrahlung is strongly damped, as expected for very heavy quarks, duality will set in to operate just a few GeV above threshold. This happens because a large number of quasi-two-body channels $H_t^i \overline{H}_t^j$ $(i, j = 1, \ldots, N)$ contributes.

3. Further Consequences

(i) While T, T^* and Λ_t decay weakly, the higher excitations $T_{\ell=1,2,...}$ will undergo strong decays to $T + \pi's$, $T^* + \pi's$. Therefore one does indeed expect extra pions unconnected to T or T^* decays carrying an energy of a few hundred MeV.

(ii) Accordingly there is no reliable correlation between the electric charge of the produced top hadron H_t and the weakly decaying T or T^*.

(iii) As stated in the beginning the top quarks will be produced with a high degree of longitudinal polarization on the Z^0. Since also the T^* vector mesons decay weakly part of this t polarization can in principle be measured via semileptonic decay asymmetries. The size of this effect obviously depends on the relative production rate of T^* versus T. This in turn will be affected by the ratio of $T_{\ell=1,...} \to T^*\pi's$ versus $T_{\ell=1,...} \to T\pi's$.

(iv) If indeed $t\bar{t}$ production predominately materializes into two-body modes $H_t^i \overline{H}_t^j$ then it might not be a hopeless enterprise to study the spectroscopy of top mesons. Again, polarization effects would represent highly valued tools for disentangling the information.

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