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VERY HIGH ENERGY COLLIDERS*

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1. Introduction

The first particle accelerators were built roughly fifty years ago. These first machines had energies of the order of MeVs and were used to study a world that looked relatively simple. Matter was composed of four basic constituents: protons, neutrons, electrons, and neutrinos. These constituents interacted via four forces: the weak (to account for radioactivity); the electromagnetic (to account for the interaction between charges and currents); the strong (to bind the nucleus together); and the gravitational (to account for the interaction of masses at large distances.) All our attempts at understanding matter were guided by two dynamical principles – relativity and quantum mechanics.

In the intervening years, the energy of our accelerators has grown by six orders of magnitude to reach the TeV level. Our old view of what were the elementary constituents of matter has turned out to be wrong. The simple picture of four constituents became ever more complicated as machines of higher energy were built and more and more mesons and isobars of the nucleon were discovered. In the early 1960s there were more than one hundred of the "elementary particles." All of this was swept away in the 1960s to be replaced with the quark model, wherein the proton, the neutron, all of those mesons and other particles became composites of combinations of quarks and antiquarks.

In these last fifty years we seem to have lost one force, for our present picture is that the weak and the electromagnetic forces are but different manifestations of the same basic force. Our theoretical colleagues are struggling (so far unsuccessfully) with models that try to combine the strong force and perhaps even gravity into a unified picture.

Our dynamical principles remain the same. Relativity and quantum mechanics are still our guide and space is still thought to be continuous although some are questioning that, too.

Experiments and theory of the last fifty years have given rise to our present generation of models that allow us to calculate what happens at the fundamental level down to distances as short as 10^{-17} centimeters. The key to this great advance in our understanding of the physical world has been the accelerators that have allowed experiments that probe matter to ever smaller distances. We have gone from Cockcroft–Walton generators to Van de Graaffs to cyclotrons to synchrotrons to strong focusing to linacs to colliding beams to superconductivity. The energy of our machines has gone up by six orders of magnitude while the cost per unit energy has gone down by nearly five orders of magnitude in the same period of time. To continue our study of the fundamental nature of matter we will need more powerful and cost-effective accelerators that will probe distances where we already know our present theoretical models to be inadequate.

The Tevatron project at the Fermi National Accelerator Laboratory has demonstrated the practicality of superconducting magnet technology for large accelerators. The SSC project now in the R&D phase in the United States builds on this technology in the design of a machine to reach 40 TeV in the center of mass at what appears to be a cost per unit energy that continues the trend to reduced unit costs.

The SLC project at the Stanford Linear Accelerator Center is nearing completion and is intended to demonstrate a new colliding beam technique, the linear collider, that reduces the cost per unit energy of electron–positron colliders. This technique can in principle make practical electron colliders of very high energy.

Eventually, we will want to build accelerators of much higher energy than those we talk about now, and in this paper I will do some large extrapolation to see what kind of machines those might be. It is not enough to consider energy, for it is also necessary that the intensity (luminosity) be sufficient to study the physical processes of interest. In looking at intensity issues I base my analysis on the physics that we know and some very general scaling laws. In looking at both electron and proton machines, the electron machines will turn out to be more promising and I will review the basic design principles of very high energy linear colliders.

2. Luminosity and Energy Requirements

A. PROTON MACHINES

Protons are composite particles. Their constituents are three valence quarks (u, u, d); gluons that are exchanged between the quarks to bind the system together; and the so-called "Sea" quarks which are virtual quark–antiquark pairs generated by the interaction of the gluons and the valence quarks. This multitude of constituents (partons) within the proton share the proton's energy.

A proton–proton collision is like two bags, each containing many constituents, hurtling at each other. The hard collisions, the ones that lead to the production of large mass phenomena, are collisions of one of the constituents in one of the bags with a constituent in the other bag. These hard collisions are relatively improbable, and when they occur tend to produce final state particles with large transverse momentum and leave behind a collection of excited debris in the bags. The individual partons tend to have low energy fractions and so the center of mass energy in the parton–parton collision is, on the average, much smaller than the center of mass energy of the proton–proton system.

Figure 1 shows the momentum distribution within the proton of the valence quarks, the Sea quarks and the gluons.[1] The quantity x is the fraction of the proton momentum carried by a given constituent. The momentum distribution is itself a function of the momentum transfer in the hard collision of the constituents. For example, the valence quark momentum distribution is shown schematically at several momentum transfers in Fig. 2. The higher the momentum transfer the smaller is the average fraction of the momentum of the proton carried by a particular constituent.

What all of this means is that while the total cross section for a proton–proton collision is very large, the partial cross section for the interesting hard collisions is very small and depends strongly on the mass of the final state produced. The cross section for the production of some final state with a mass M plus the excited

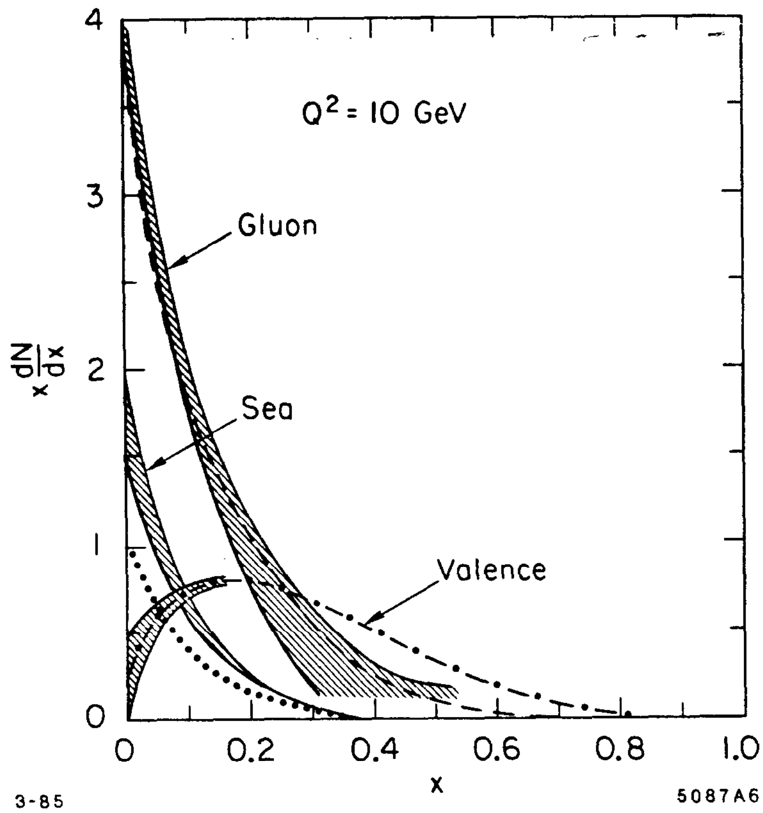

Fig. 1. The gluon, valence quark, and Sea quark distributions at a momentum transfer of 10 GeV^2 from Ref. 1.

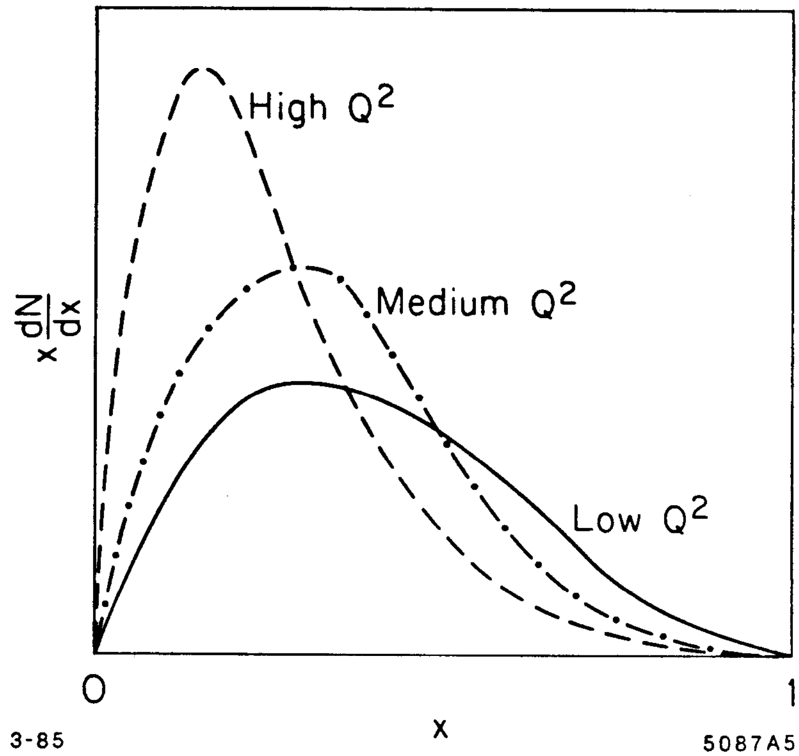

Fig. 2. The evolution of the valence quark distribution as Q^2 increases. At higher Q^2 the distribution becomes more peaked and is shifted to lower x.

proton fragments X has an energy and mass dependence given by

$$\sigma(M+X) \propto \frac{1}{M^2} f\left(\frac{M^2}{E^{*2}}\right) \tag{1}$$

where E^* is the center of mass energy of the proton–proton system. An example of the energy and mass dependence of the cross section is given in Fig. 3. It shows the cross section for the production of a Higgs boson as a function of Higgs mass for various proton–proton center of mass energies. This cross section decreases rapidly with increasing mass at a fixed center of mass energy and decreases rapidly with decreasing center of mass energy at a fixed boson mass.

One can do this kind of analysis for any process one cares to study, and from this kind of analysis can define the "discovery limit" of a given machine. This discovery limit relates the mass of the phenomena that can be studied to both the center of mass energy and the luminosity of a proton–proton collider. The SSC, for example, now in the preliminary design phase, has a design center of mass energy of 40 TeV and a luminosity of 10^{33} cm^{-2} s^{-1}. I list in the table below the upper limit on the mass detectable in various kinds of phenomenon where this upper limit is set at that mass that results in a handful of events in a running year.

Final State	Mass Limit (TeV)
Jet pairs	8.0
lepton pairs	0.4
W'	3.6
Z'	1.6
η_T	3.2
$\tilde{g}$	4.8
$Q\bar{Q}$	4.8
H	1.0
Mean Limit	3.0

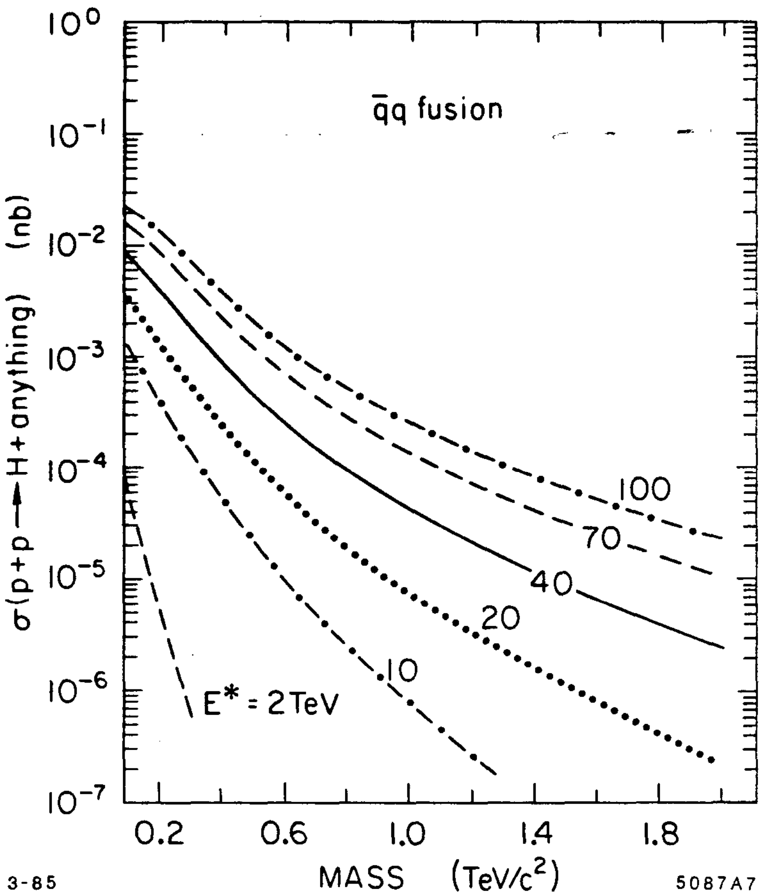

Fig. 3. The total cross section for Higgs boson production by quark–antiquark fusion in proton-proton collisions as a function of Higgs boson mass for various center of mass energies from Ref. 1.

The SSC thus has a discovery limit that depends on the process studied and ranges from 0.4 TeV for new lepton pairs, to 8 TeV for jet pair formation. A crude mean for the mass reach of the SSC is about 3 TeV. However, it should be noted that because of the energy and mass dependence of the cross section for a given process (Eq. (1)), the SSC is a "discovery machine" at this TeV mass region, and is a precision machine giving very high event rates at a few hundred GeV mass.

We now have to look at the requirements for a proton machine going beyond the SSC. Suppose we want to move up a decade in mass. To move the "discovery" limit up by a factor of ten we have to increase the energy or the luminosity or both. Equation (1) shows that raising the center of mass proton–proton collision energy by a factor of ten and the luminosity by a factor of a hundred over those of the SSC moves this discovery limit up by the required factor of ten. Can one build such a machine using storage ring technology, and could one use such a machine if one could build it? I think the answer is no in both cases.

An obvious problem with the machine will be the luminosity lifetime. Particles will be lost from the circulating beams by proton–proton interactions at the collision point. This is already a significant problem at the SSC, where the luminosity lifetime for the presently favored design is about 20 hours. In our super SSC with an energy ten times higher than the SSC, we would probably get our luminosity up by a factor of a hundred by increasing the number of bunches circulating in the machine by a factor of ten, and getting the other factor of ten from the decreased size of the colliding bunches resulting from the adiabatic damping that occurs in acceleration to the higher energy. If one gets the luminosity up in this fashion and adds in the increase of the total cross section expected from the increase in the center of mass energy, the luminosity lifetime goes down by a factor of fifteen from the SSC value to roughly 1.5 hours. This is probably too short a lifetime to allow for injection and ramping up to energy in a storage ring design.

As far as the experimental detectors are concerned, the problems are probably overwhelming. There are approximately 100 proton–proton interactions per beam–beam collision, and I don't believe that you can make detection apparatus to stand that kind of rate. There are some who argue now that the 10^{33} luminosity of the SSC will be very hard to use with "real world" detectors; and I doubt that anyone can demonstrate a usable detector technology at 10^{35}.

B. Electron–Positron Machines

In contrast to protons, from what we know now electrons and positrons are elementary particles. There are no "partons" to share the momentum of the primary electron and proton and thus reduce the effective collision energy. *The energy you build is what you get.* However, cross sections for particular processes are small and thus large luminosities are required. The cross section for a given process is given by

$$\sigma_i \approx 10^{-37} E^{*^{-2}} R_i \quad (\mathrm{cm}^2) \tag{2}$$

where E^* is in TeV and R_i is the ratio of the cross section for process i divided by the cross section for mu pair production through the electromagnetic interaction only. Some typical values of R_i are listed in the table below.

Final State	R
$\mu^+\mu^-$	1.2
$Q\bar{Q}$ (charge $\frac{2}{3}$)	2.0
$Q\bar{Q}$ (charge $\frac{1}{3}$)	1.2
W^+W^-	25
Z^0Z^0	25
$Z^0\gamma$	25
Z^0H	0.2
Z'	1000
ρ_T	7
$\tilde{\nu}\,\tilde{\bar{\nu}}$	0.6

We can define "discovery" limits for the electron–positron machines, too. I will set the required yield as 100 events per 10^7 seconds. The table below gives the center of mass energy at which one would get 100 events in an integrated luminosity of 10^{40} cm^{-2} s^{-1}.

Channel	E^* (TeV) at $L = 10^{33}$
$Q\bar{Q}$ (charge $\frac{2}{3}$)	4.5
Jet–Jet (old quarks)	10.0
$Z^0 H$	1.4
$\widetilde{W}^+\widetilde{W}^-$	4.5
$\tilde{\nu}\,\tilde{\bar{\nu}}$	2.5

As in the case of the proton machines, one spans quite a range of masses as one looks at different processes. Here an integrated luminosity of 10^{40} is enough to study jet–jet phenomena up to 10 TeV mass or to study Z^0 plus Higgs production to 1.4 TeV mass. I will interpret this table as implying that *very roughly* a machine with 3 TeV in the center of mass requires a luminosity of 10^{33}. The luminosity required for machine of other energies is given by

$$\mathcal{L} = 10^{33} \left(\frac{E^*}{3}\right)^2 \text{ cm}^{-2}\text{ s}^{-1} \tag{3}$$

where the center of mass energy E^* is in units of TeV.

There are background processes in electron–positron collisions which will eventually give multiple events per beam crossing for sufficiently high luminosity. The dominant background is the so-called two photon process. However, the total cross section for this process is much smaller than the background generating cross section in proton–proton collisions and there is no problem with the two photon process until luminosities are much higher than 10^{35} cm^{-2} s^{-1}.

C. A Quick Summary of Proton and Electron Colliders

For proton colliders:

1. The effective center of mass energy is much lower than the proton–proton center of mass energy.
2. Cross sections are proportional to $M^{-2}f(M^2/E^{*2})$
3. The SSC has an effective discovery limit of 3 TeV if its luminosity is 10^{33} cm^{-2} s^{-1}. To go to higher energy, the energy, the luminosity or both have to be increased.
4. If the luminosity is held fixed, the machine energy must be scaled roughly as the square of the mass limit.

For electron–positron colliders:

1. The energy built is what you get.
2. The cross section is proportional to ${E^*}^{-2}$.
3. The luminosity required is proportional to the square of the cm energy and is roughly given by

$$\mathcal{L} = 10^{33}\left(\frac{E^*(\mathrm{TeV})}{3}\right)^2 \qquad (4)$$

4. Background is not a problem until the luminosities are much larger than 10^{35} cm^{-2} s^{-1}.

3. The Basic Design of High Energy Linear Electron Colliders

The technique in use up to now for electron–positron colliders is that of the colliding beam storage ring. This technology is well understood and is being used to construct the 27 km. circumference LEP storage ring at CERN. However, the cost of storage rings at fixed luminosity scales as the square of the center of mass energy and so runs into "fiscal feasibility" problems at energies much higher than LEP's. A technique with different scaling laws is required and I believe that that technique is the linear collider.

The basic design of high energy linear colliders is much more complicated than that of high energy electron storage rings. In colliding beam storage rings the technology is well known and the limits on performance are well understood. It is possible to write a few simple equations that define the parameters of an optimized storage ring and determine its costs for any choice of energy and luminosity. However, linear electron colliders are new and we are still learning to understand them. In this section I will summarize some of the basic design equations and constraints and give a few examples of parameters for very high energy machines. My aim is to introduce some realism into the discussion of new technologies for acceleration.

The beam–beam interaction can be much stronger in a linear collider than in a storage ring. In an electron–positron collider the collective fields of one beam will focus a single particle in the other beam, as illustrated in Fig. 4. The strength of the interaction is measured by a dimensionless parameter D (the disruption parameter) which is the ratio of the bunch length to the focal length of an equivalent lens. For round trigaussian beams D is given by

$$D = \frac{\sigma_z}{F} = \frac{r_e \sigma_z N}{\gamma \sigma_{r_o}^2} \tag{5}$$

where the bunch has a longitudinal standard deviation σ_z, a radial standard deviation σ_{r_o}, a number of particles N and an energy γ in rest mass units; r_e

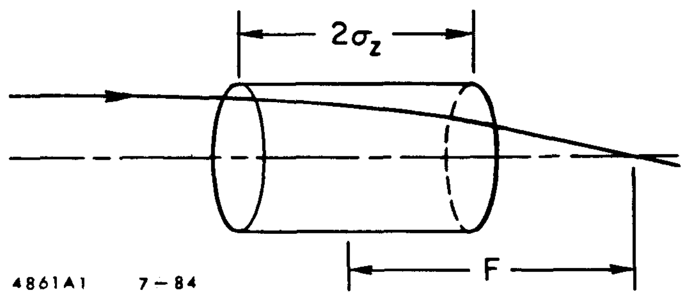

Fig. 4. The effect on a particle in one beam of the macroscopic fields from all of the particles in the other beam in a linear collider.

is the classical electron radius; and F is the small amplitude focal length of an equivalent thin lens. The effective fields in a linear collider tend to be very large and the focal lengths tend to be small. For example, in the SLC project now under construction at SLAC, the fields are on the order of megagauss, F is on the order of millimeters, and D is about one.

The luminosity equation of a linear collider is given by

$$\mathcal{L} = \frac{N^2 f}{4\pi} \left\langle \frac{1}{\sigma_r^2} \right\rangle \equiv \frac{N^2 f}{4\pi\sigma_{r_o}^2} H \tag{6}$$

where the charge in the two bunches is assumed equal, f is the collision frequency, σ_{r_o} is the radial standard deviation of the charge distribution before the collision, and H is an enhancement factor which measures the effect of the beam–beam interaction on the transverse dimension of the beams during the collision. The beam–beam interaction in linear colliders can be so strong that a kind of mutual pinch occurs, reducing the radius of both beams during the collision period and hence enhancing the luminosity. H has been calculated by means of a computer simulation by Hollebeek,[2] and his results for a round gaussian beam are shown in Fig. 5. H is by definition 1 at small values of the disruption parameter and rises to an asymptotic value of around six for disruption parameters greater than two.

The large effective fields in the collision region can generate very intense synchrotron radiation. At high luminosity the synchrotron radiation, called "beamstrahlung", dominates the energy spread in the beams. Classically, the synchrotron radiation spectrum is a universal function of the photon energy divided by a parameter E_c called the critical energy.

$$E_c = 3\hbar c \frac{\gamma^3}{2\rho} \tag{7}$$

In this equation h is Planck's constant, c is the velocity of light, γ is the energy in rest mass units, and ρ is the bending radius of the particle in the field of

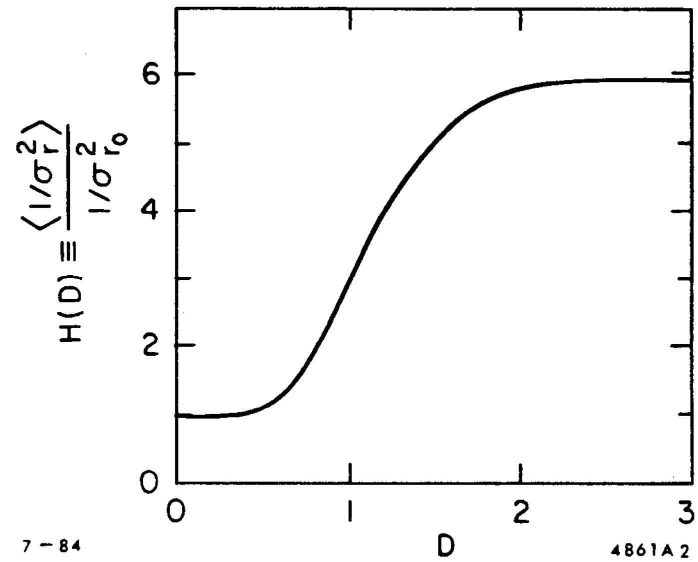

Fig. 5. The luminosity enhancement factor, H, as is a function of the disuption parameter, D.

the other beam. Classically, if the beamstrahlung photon energy is measured in units of the critical energy, the spectrum is like that shown by the heavy line in Fig. 6, rising to a maximum at $x = 1$ and decreasing exponentially for $x > 1$. This classical spectrum is good as long as the beam energy divided by the critical energy is much greater than one.

What happens in the case where the beam energy divided by the critical energy is less than 1? Clearly we can't have the classical spectrum, for energy conservation would be violated. R. Noble,[3] and T. Himel and J. Siegrist[4] have worked out this problem and the results are shown by the dashed line in Fig. 6. If effect, the beamstrahlung spectrum follows the classical spectrum up to $x = E_b/E_c$ and then drops rapidly to zero. In this case, less beamstrahlung is emitted than the classical equations imply.

The ratio of beam energy to critical energy for a uniform cylindrical beam is given by

$$\frac{E_b}{E_c} = \frac{Df^{\frac{1}{2}}P}{2\sqrt{3}\hbar c\gamma r_e^2(\pi\mathcal{L})^{\frac{3}{2}}} \tag{8}$$

where P is the power in one beam and $\mathcal{L}$ is the luminosity. A useful rough approximation for a gaussian beam is

$$\frac{E_b}{E_C} \approx 3 \times 10^{-3}\frac{f^{\frac{1}{2}}DP}{EL^{3/2}} \tag{9}$$

where P is measured in megawatts, E is in TeV, and L is in units of 10^{33} cm^{-2} s^{-1}.

It turns out that all low energy machines like the SLC are in the classical regime and all interesting very high energy machines are in the quantum mechanical regime. For the SLC, E_b/E_c is about 100 and safely classical. A high energy machine which might be of interest could have a beam energy of 1.5 TeV, a luminosity of 10^{33}, a frequency of 1,000 hertz, a disruption parameter of one, and a beam power of 1 megawatt. Such a machine would have E_b/E_c of 0.06 and would be very definitely in the quantum mechanical regime.

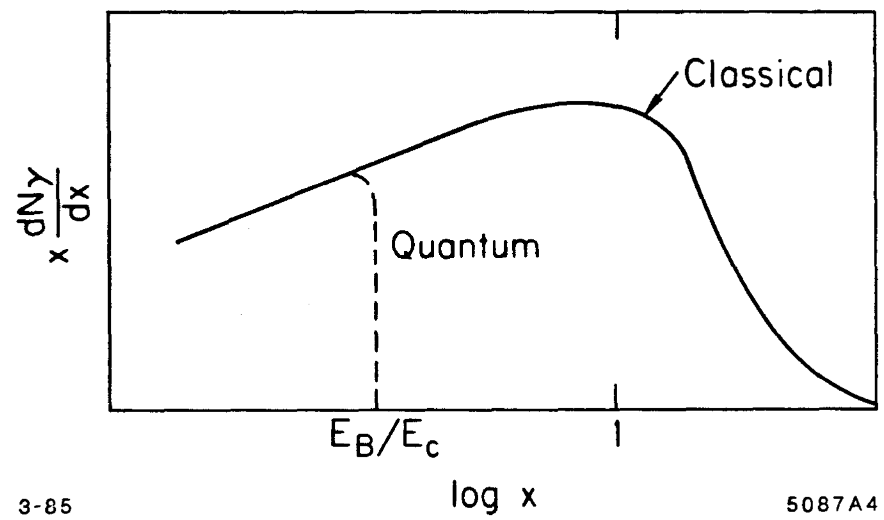

Fig. 6. A schematic of the synchrotron radiation spectrum in the classical and quantum mechanical limits.

The fractional energy loss δ of a particle in one beam in passing through the other beam is given by

$$\begin{aligned}\delta_{QM} &\approx \delta_{\text{classical}} \times \left(\frac{E}{E_c}\right)^{4/3} \\ &\approx 16\left(\frac{DP\ (\text{MW})}{fE\ (\text{TeV})}\right)^{1/3}\end{aligned} \tag{10}$$

A parameter of importance for the high energy physics experiments to be done with the machine is the center of mass energy spread which is given by

$$\sigma_{E/E} \approx \frac{\delta_{QM}}{2\sqrt{3}} \tag{11}$$

For most experiments, it is desireable that δ be less than about 0.35, or $\sigma_{E/E}$— less than about 10%. This is all very new, and hence I would not be surprised if I had lost a factor of 2 here or there in the above equations. I hope I have not lost too many of them.

In specifying what happens at the collision point in a linear collider, there are nine parameters to be related by four equations. The parameters are energy, luminosity, energy spread at the collision point, disruption parameters, beam power, beam radius at the collision point, bunch length, collision frequency, and number of particles per bunch. Three of the equations (luminosity, disruption parameter and energy spread) are given above. The fourth is an almost trivial relation between beam power, number of particles per bunch and repetition frequency.

What does all this mean for very high energy machines? I cannot claim to have fully digested the implications of the quantum mechanical beamstrahlung regime on machine design. Rather than trying to develop an optimized set of parameters, I will give several sets of consistent parameters for a machine of sufficiently high energy and luminosity to be interesting. I will take a center of mass energy of 10 TeV; a luminosity of 10^{34} cm^{-2} s^{-1}; an interaction region β function of 1 millimeter (though I have no idea if the magnets can be made strong enough to realize such a small beta); a disruption parameter of 0.1, which implies a H of 1; and a center of mass energy spread of about 10%. Three sets of consistent parameters are given in the table below. In the table ϵ_N is the invariant emittance defined as $\gamma\sigma_x\sigma_{x'}$.

Consistent Non–Optimized Sets of Parameters

P (MW)	1	3	10
f(HZ)	3000	9000	30,000
$N(e^+$ or $e^-)$	4.1×10^8	4.1×10^8	4.1×10^8
σ_z(mm)	3.4×10^{-4}	1×10^{-3}	3.4×10^{-3}
$\epsilon_N(M)$	4×10^{-9}	1.2×10^{-8}	4×10^{-8}
σ_{r_o}(microns)	6.4×10^{-4}	1.1×10^{-3}	2.0×10^{-3}

In all of the cases, the energy delivered to the collision region per bunch of electrons or positrons is constant. As the total power in the beam increases, the invariant emittance, and hence the radius at the collision point also increases. In all of these cases the invariant emittance is considerably smaller than that of the SLC and the beam radii are tiny indeed.

I emphasize again that these parameter sets are not meant to be taken as optimized sets—they are only consistent sets. It will take much more work to arrive at an optimized set of realizable parameters and that work will probably have to include development of advanced technology to make possible working with extremely small beams.

4. Conclusions

My conclusions are relatively simple, but represent a considerable challenge to the machine builder.

High luminosity is essential. We may in the future discover some new kind of high cross section physics, but all we know now indicates that the luminosity has to increase as the square of the center of mass energy. A reasonable luminosity to scale from for electron machines would be 10^{33} cm^{-2} s^{-1} at a center of mass energy of 3 TeV.

The required emittances in very high energy machines are small. It will be a real challenge to produce these small emittances and to maintain them during acceleration. The small emittances probably make acceleration by laser techniques easier, if such techniques will be practical at all.

The beam spot sizes are very small indeed. It will be a challenge to design beam transport systems with the necessary freedom from aberration required for these small spot sizes. It would of course help if the beta functions at the collision points could be reduced.

Beam power will be large—to paraphrase the old saying, "power is money"—and efficient acceleration systems will be required.

References

1. This figure as well as Fig. 3 is reprinted with the permission of the authors from "Supercollider Physics," E. Eichten *et al.*, Rev. Mod. Phys. 56, 579 (1984), and Fermilab Pub-84/17-T, February, 1984.
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