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PLASMA ACCELERATORS*

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ABSTRACT

In this paper we discuss plasma accelerators which might provide high gradient accelerating fields suitable for TeV linear colliders. In particular we discuss two types of plasma accelerators which have been proposed, the Plasma Beat Wave Accelerator and the Plasma Wake Field Accelerator. We show that the electric fields in the plasma for both schemes are very similar, and thus the dynamics of the driven beams are very similar. The differences appear in the parameters associated with the driving beams. In particular to obtain a given accelerating gradient, the Plasma Wake Field Accelerator has a higher efficiency and a lower total energy for the driving beam. Finally, we show for the Plasma Wake Field Accelerator that one can accelerate high quality low emittance beams and, in principle, obtain efficiencies and energy spreads comparable to those obtained with conventional techniques.

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1. INTRODUCTION

Recently there have been two similar types of plasma acclerator schemes proposed. The Plasma Beat Wave Accelerator (PBWA)[1,2] employs two laser beams beating at the plasma frequency to drive the plasma while the Plasma Wake Field Accelerator (PWFA)[3–5] replaces the laser beams by a bunched relativistic electron beam. Since the two schemes make use of different sources, the corresponding mechanisms that drive the plasma waves are different. In the PBWA, it is the ponderomotive force which comes from the beating lasers that drives the plasma, whereas in the PWFA the driving bunch is decelerated by the plasma and thus transfers energy to the plasma wave. Other than this difference, however, the two schemes are very similar. In both cases large longitudinal electric fields are generated in the plasma which oscillates at the fundamental plasma frequency ω_p. These fields are then used to accelerate an electron beam.

In this paper we study the PBWA and the PWFA in parallel to point out both the similarities and the differences in the two schemes. In Section 2 we begin with a calculation of the plasma wave induced by two beating lasers for the PBWA and by a relativistic electron bunch for the PWFA. From this we calculate the longitudinal and transverse electric fields due to the plasma wave.

Next in Section 3 we use the fields calculated to treat several accelerator physics issues. Since in both cases the plasma is driven by bunches (lasers) of finite cross section, there is both a transverse electric field and a transverse *variation* of the longitudinal electric field. The transverse field is used to calculate focusing (and defocusing) effects while the radial variation of the electric field is used to calculate induced energy spread. Since the phase velocity of the plasma wave is not c, there is phase slippage along the wave. This is quite large for the PBWA and must be included in the design considerations. For the PWFA phase slippage restricts the energy of the driving electron bunch. To complete this section, the energy requirement and the efficiency are estimated.

In Section 4 we compare the PWFA with the PBWA using four numerical examples which serve to illustrate possibilities in design. We choose parameters which would yield either an interesting experiment or the first stage of an actual accelerator. Since the PBWA is somewhat more restrictive in design, we first fix a design with fields ranging from 1 to 10 GeV/m using two different types of lasers. The design for the PWFA is then chosen to match the critical parameters of the PBWA. We conclude this section with a discussion comparing the two schemes.

In Sections 3 and 4 we find that the efficiency of energy transfer from the plasma to an accelerated electron bunch is rather low. This, however, is due to the particular model and parameters chosen for the calculation. In Section 5 we show two alternative methods for improving the efficiency. In addition, we show that it is possible to have a matched emittance which is consistent with TeV collider needs.

In the following sections we follow Refs. 2, 5 and in particular Ref. 6 in most of the calculations. Sections 2, 3 and 4 are quite similar to Ref. 6 although Section 2 is somewhat more general here. Section 5 is new work which explores briefly the possibilities of improving efficiency.

In the next section we will treat the plasma oscillations in the linear approximation since this is completely adequate for our purpose. Discussions of nonlinear plasma oscillations due to a driving electron beam can be found in Refs. 7 - 10 and in Ref. 5.

2. FIELDS IN A PLASMA WAVE OF FINITE EXTENT

We consider a uniform cold plasma of density n_0 with stationary ions. The linear plasma oscillations will be driven resonantly by two beating laser frequencies for the PBWA, while for the PWFA they will be shock excited by a thin disk of relativistic electrons. To find the electric field in the plasma wave for both schemes, we start with the linearized, nonrelativistic fluid equations,

$$\begin{aligned} &\frac{\partial n_1}{\partial t} + n_0(\nabla \cdot \vec{v}_1) = 0 \\ &\frac{\partial \vec{v}_1}{\partial t} = \frac{e\vec{\mathcal{E}}_1}{m} + \frac{\vec{F}_{ext}}{m} = \frac{\vec{F}}{m} , \end{aligned} \tag{2.1}$$

and solve for the perturbed plasma density n_1. In Eq. (2.1) $\vec{\mathcal{E}}_1$ is the electric field due to n_1, and $\vec{F}_{ext}$ is the external force due to either a driving beam or a beating laser.

2.1 The Plasma Beat Wave Accelerator

In the case of the PBWA the force is most easily calculated from a Hamiltonian which has been averaged over the fast oscillation of the laser frequency. This leaves only the beating effect of the two laser frequencies.

For a model of the laser we consider two plane waves which are modulated radially to obtain the desired transverse profile. The nonrelativistic Hamiltonian which governs motion of the electrons in the plasma is

$$H = \frac{(\vec{p} - e\vec{A}/c)^2}{2m} \quad , \tag{2.2}$$

and the vector potential for the incoming laser is

$$\vec{A} = \frac{c\vec{E}_0(r)}{\sqrt{2}} \left(\frac{\cos(k_1 z - \omega_1 t)}{\omega_1} + \frac{\cos(k_2 z - \omega_2 t)}{\omega_2} \right) \quad , \tag{2.3}$$

where ω_i and k_i are the frequency and wave vector of the two laser lines, and $\vec{E}_0(r)$ describes the transverse profile and polarization of the beam. The frequency and wave vector are related by the dispersion relation of electromagnetic waves in a plasma,

$$\omega^2 = k^2 c^2 + \omega_p^2 \quad , \tag{2.4}$$

where ω_p is the plasma frequency

$$\omega_p = \left[\frac{4\pi e^2 n_0}{m} \right]^{1/2} \quad . \tag{2.5}$$

A simple way to calculate the ponderomotive potential due to the beating lasers is to average the Hamiltonian over the fast oscillation at the laser frequency. Here we assume that the difference in frequency between the two laser lines is much smaller than the laser frequency. Averaging over a time of $2\pi/\omega$ yields the Hamiltonian

$$\langle H\rangle = \frac{\vec{p}^2}{2m} + \frac{e^2 E_0^2(r)}{4m\omega^2}[1+\cos(k_\delta z - \omega_\delta t)] \quad , \tag{2.6}$$

where $\omega_\delta = \omega_1 - \omega_2$ and $k_\delta = k_1 - k_2$, and we have dropped the index on ω. The last term is simply the ponderomotive potential due to a beating laser with a finite cross section.

In order to provide useful acceleration over a significant distance, it is necessary that the plasma wave phase velocity be close to c, the speed of a high energy injected electron beam. As we shall see, the phase velocity of the plasma wave is matched to the 'phase velocity' of the beat pattern. This is the group velocity of electromagnetic waves in a plasma,

$$v_g = \frac{\omega_\delta}{k_\delta} \simeq \frac{d\omega}{dk} = c\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} \quad . \tag{2.7}$$

Due to Eq. (2.7) above it is sometimes useful to define $\gamma_p = \omega/\omega_p$ since this would be the relativistic energy factor of a particle travelling at that speed.

For the sake of a comparison with the PWFA later in this paper, we will select a radial dependence of the ponderomotive potential given by

$$E_0^2(r) = 2E_0^2 \begin{cases} K_2(k_p a)\ I_0(k_p r) + \frac{1}{2} - \frac{2}{(k_p a)^2} - \frac{r^2}{2a^2} & r < a \\ I_2(k_p a)\ K_0(k_p r) & r > a \end{cases} \quad , \tag{2.8}$$

where K_n and I_n are modified Bessel functions. This radial profile is parabolic near the origin but falls off exponentially for $r > a$. It was chosen to yield a simple parabolic dependence in the equations below.

To obtain a good coupling to the response of the plasma we set the difference of the laser frequencies equal to the plasma frequency

$$\begin{aligned} \omega_\delta &= \omega_p \\ k_\delta &= k_p, \end{aligned} \tag{2.9}$$

where we have noted that the plasma wave number k_p is equal to k_δ.

To use the above results we need the divergence of the force due to the Hamiltonian in Eq. (2.6). This is given by

$$\nabla \cdot \vec{F} = 4\pi e^2 n_1 + e^2 B_0(r) + e^2 B_1(r) \cos(k_p z - \omega_p t) \quad , \tag{2.10}$$

where Poisson's equation has been used to substitute for $\nabla^2 \phi_1$ and

$$B_1(r) = \begin{cases} \frac{E_0^2 k_p^2}{4m\omega^2}(1 - r^2/a^2) & r < a \\ 0 & r > a \end{cases} \quad , \tag{2.11}$$

$$B_0(r) = \begin{cases} \frac{E_0^2 k_p^2}{2m\omega^2}\left(K_2(k_p a) I_0(k_p r) - \frac{2}{k_p^2 a^2}\right) & r < a \\ \frac{E_0^2 k_p^2}{2m\omega^2} I_2(k_p a) K_0(k_p r) & r > a \end{cases} \quad . \tag{2.12}$$

Substituting into Eq. (2.1) yields

$$\frac{\partial^2 n_1}{\partial t^2} + \omega_p^2 \, n_1 = -\frac{\omega_p^2}{4\pi}\left[B_0(r) + B_1(r) \cos(k_p z - \omega_p t)\right] \quad . \tag{2.13}$$

To calculate the solution to (2.13), let the laser pulse begin at $k_p z - \omega_p t = 0$. If the plasma is undisturbed ahead of the laser pulse, the solution is

$$n_1(r, z, t) = -\frac{B_0(r)}{4\pi}[1 - \cos(k_p z - \omega_p t)] - \frac{B_1(r)}{8\pi}(k_p z - \omega_p t) \sin(k_p z - \omega_p t). \tag{2.14}$$

The solution above has two distinct terms. The first term is due to the shock excitation of the plasma by the front of the laser pulse, while the second term is due to the resonant driving of the plasma by the beating lasers. Since we would like to let the second term build up over many cycles, the second term will be much larger than the first term. In addition, in an actual device the laser pulse would turn on more gradually thus reducing the shock excitation. For these reasons we will neglect the first term in Eq. (2.14) in the following analysis.

With $n_1(r,z,t)$ in hand, we now must find the electric field $\vec{\mathcal{E}_1}$ due to the plasma oscillation. Since the magnetic field due to a linear plasma wave vanishes, we can simply use Poisson's equation,

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}\phi_1)+\frac{\partial^2\phi_1}{\partial z^2}=-4\pi e n_1 \quad . \tag{2.15}$$

If we have a laser pulse of length τ, the amplitude of the plasma density wave will reach its peak value at the end of the pulse. From Eq. (2.14) this is given by

$$n_1^{max}(r)=\frac{\omega_p\tau E_0^2k_p^2}{32\pi m\omega^2}(1-r^2/a^2) \qquad r<a, \tag{2.16}$$

and the potential can be shown to be

$$\phi_1=R(r)\sin(k_pz-\omega_pt) \tag{2.17}$$

with

$$R(r)=\frac{\omega_p\tau eE_0^2}{4\omega^2m}\begin{cases}K_2(k_pa)\ I_0(k_pr)+\frac{1}{2}\left(1-\frac{r^2}{a^2}\right)-\frac{2}{(k_pa)^2} & ,\quad r<a\\ I_2(k_pa)\ K_0(k_pr) & ,\quad r>a\end{cases} \quad . \tag{2.18}$$

The longitudinal and transverse electric fields for $r<a$ for the PBWA are thus given by

$$\mathcal{E}_z=-\frac{\omega_p\tau k_peE_0^2}{4\omega^2m}\left\{K_2(k_pa)\ I_0(k_pr)+\frac{1}{2}\left(1-\frac{r^2}{a^2}\right)-\frac{2}{(k_pa)^2}\right\}\cos(k_pz-\omega_pt)\ ,$$

$$\mathcal{E}_r=-\frac{\omega_p\tau k_peE_0^2}{4\omega^2m}\left\{K_2(k_pa)\ I_1(k_pr)-\frac{r}{k_pa^2}\right\}\ \sin(k_pz-\omega_pt)\ . \tag{2.19}$$

2.2 THE PLASMA WAKE FIELD ACCELERATOR

For the case of the PWFA the situation is very similar. We only need to change the laser source term in Eq. (2.13). For the case of a driving beam of density n_b, the divergence of the force is given by

$$\nabla \cdot \vec{F} = 4\pi e^2 (n_1 + n_b) \ . \tag{2.20}$$

Following Ref. 5, consider a driving beam with density profile

$$n_b = \sigma(r)\delta(z - v_b t) \ . \tag{2.21}$$

Then the solution for the perturbed density is given by

$$n_1(r) = \begin{cases} k_p \sigma(r) \sin(k_p z - \omega_p t) & k_p z - \omega_p t < 0 \\ 0 & k_p z - \omega_p t > 0 \ . \end{cases} \tag{2.22}$$

To compare with the PBWA we use a parabolic distribution given by

$$\sigma(r) = \begin{cases} \frac{2N}{\pi a^2}(1 - r^2/a^2) & r < a \\ 0 & r > a \end{cases} \ , \tag{2.23}$$

where N is the total number of particles in the driving bunch. Once again it is possible to calculate the longitudinal and transverse electric fields due to the plasma wave.[5] These are given by

$$\begin{aligned} \mathcal{E}_z &= \frac{-16eN}{a^2} \left\{ K_2(k_p a)\ I_0(k_p r) + \frac{1}{2}\left(1 - \frac{r^2}{a^2}\right) - \frac{2}{(k_p a)^2} \right\} \cos(k_p z - \omega_p t) \ , \quad r < a \\ \mathcal{E}_r &= \frac{-16eN}{a^2} \left\{ K_2(k_p a)\ I_1(k_p r) - \frac{r}{k_p a^2} \right\} \sin(k_p z - \omega_p t) \ , \quad r < a \ . \end{aligned} \tag{2.24}$$

Thus the electric fields for the two schemes turn out to be remarkably similar.

For reasons which we will discuss later, the transverse size of the driven beam must be somewhat smaller than the transverse size of the laser beams or the driving electron beam. In addition if $k_p a \gg 1$, then the electric fields for both schemes are of the following form:

$$\begin{aligned} \mathcal{E}_z &\simeq -A\left(1-\frac{r^2}{a^2}\right)\cos(k_p z-\omega_p t) \\ \mathcal{E}_r &\simeq 2A\frac{r}{k_p a^2}\sin(k_p z-\omega_p t) \end{aligned} \qquad r \ll a \qquad (2.25)$$

where

$$A = \begin{cases} \dfrac{\omega_p \tau k_p e E_0^2}{8\omega^2 m} & PBWA \\ \dfrac{8eN}{a^2} & PWFA \end{cases} \qquad (2.26)$$

Other than different coefficients, the forces that the driven electrons experience share the same physical characteristics in both schemes. To be specific there is a longitudinal force $e\mathcal{E}_z$ that either accelerates or decelerates the driven bunch of electrons, and there is a transverse force $e\mathcal{E}_r$ shifted in phase which either will focus or defocus the driven bunch (see Fig. 1). From Fig. 1 it is clear that we have both acceleration and focusing over 1/4 of the plasma wavelength.

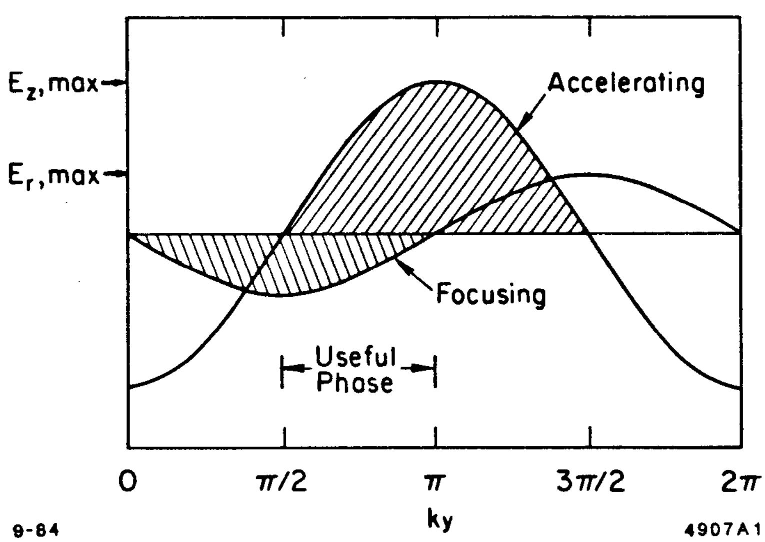

Fig. 1.

3. ACCELERATOR PHYSICS ISSUES

In this section we discuss some accelerator physics issues which are relevant to both schemes of plasma accelerators. To begin we concentrate on the quality and intensity of a driven electron bunch with finite transverse extent. In particular we treat the transverse oscillations and the energy spread due to the transverse variation of the accelerating field. We then discuss other issues such as phase slippage, spot size and driving beam energy for the PBWA and PWFA. The details in the discussion of these issues are different for the two schemes since we choose to fix different parameters in the two cases. Finally, in order to address the question of intensity, we discuss the efficiencies of both schemes.

3.1 THE BETA FUNCTION

In this paper the beta function is defined to be the wavelength/2π of the transverse oscillation at some instantaneous phase ϕ along the plasma wave. In the last section we saw that, except for a difference in coefficients, the PBWA and PWFA have the same electric fields. We also pointed out that there is a useful phase between $\pi/2$ and π along the plasma wave. In general there will be some phase slippage between the plasma wave and the driven beam. If this phase slippage is slow, then we can calculate the transverse focusing effects as if the beam were at a fixed phase on the wave.

The differential equation governing the transverse oscillations of a highly relativistic particle is

$$\frac{d^2x}{dz^2} = e\frac{\mathcal{E}_x}{\gamma mc^2} , \tag{3.1}$$

where γmc^2 is the particle's instantaneous relativistic mass. Thus, for small radius from Eq. (2.25), we have

$$\frac{d^2x}{dz^2} = \left[\frac{eA\sin\phi}{k_p a^2 \gamma mc^2}\right] x . \tag{3.2}$$

Identifying the coefficient of x above with β^{-2} yields the beta function,

$$\beta = \left[\frac{k_p a^2 \gamma mc^2}{eA\sin\phi}\right]^{1/2} . \tag{3.3}$$

3.2 ENERGY SPREAD

From Eq. (2.25) it is evident that for a driving beam with finite transverse size, the longitudinal field varies transversely. Consider a driven bunch with transverse radius b which moves along the axis of the plasma wave. Since the field varies parabolically in the transverse direction, the average energy gain is reduced slightly and an energy spread is induced. If we assume that the beam is already very relativistic, then the average change in energy for one stage is

$$\Delta E_{ave} = \Delta E \left(1 - \frac{2}{3}\left(\frac{b}{a}\right)^2\right), \tag{3.4}$$

where ΔE is the energy gain for a particle on the axis of the plasma wave. The corresponding energy spread induced in one stage for the model we have chosen is

$$\left[\frac{\delta(\Delta E)}{\Delta E}\right]_{rms} = \frac{\sqrt{2}}{3}\left(\frac{b}{a}\right)^2 . \tag{3.5}$$

3.3 THE TRAPPING PARAMETER

The trapping parameter is defined to be the ratio of the plasma density perturbation n_1 to the unperturbed density n_0. Physically, this parameter indicates the linearity of the plasma oscillation. Since we work in the linear approximation for the plasma wave in both schemes, α should be kept reasonably small. For the case of the PBWA, we assume that the plasma oscillation saturates at the end of the laser, which corresponds to[11,12]

$$\alpha \equiv \frac{n_1}{n_0} \simeq \frac{1}{4} \quad PBWA. \tag{3.6}$$

For the case of the PWFA we take L and $\mathcal{E}_z$ as chosen parameters. In addition, to scale the transverse effects, we fix the ratio between the transverse size of the driving bunch and the plasma wavelength, a/λ_p. This in turn determines the plasma wavelength and the plasma density. In order to check that the plasma wave so generated is indeed a linear wave, we must calculate α, which in this case is given by

$$\alpha = \frac{e\mathcal{E}_z}{mc\omega_p} \qquad PWFA. \tag{3.7}$$

3.4 Phase Slippage

For both accelerator schemes the phase velocity of the plasma wave is not equal to the velocity of the driven bunch. This means that the driven bunch will slip in phase along the plasma wave as it is accelerated. For the PBWA we maximize $\mathcal{E}_z$ for a given L by optimizing the phase shift δ. If we choose a laser frequency ω, an acceleration length L, and a phase slippage δ for speed of light particles; then the plasma frequency is given by[2]

$$\omega_p = \left(\frac{2\delta c\omega^2}{L}\right)^{1/3} . \tag{3.8}$$

On the other hand, the acceleration gradient that the driven bunch sees varies along L due to the phase slippage. If the total phase slippage over the entire acceleration length is δ, then the average acceleration gradient is related to the ideal gradient by a phase slip form factor $\sin\delta/\delta$, that is

$$e\mathcal{E}_z^{ave} = \alpha mc\omega_p \frac{\sin\delta}{\delta} . \tag{3.9}$$

Here the phase has been allowed to slip from the top of the cosine down one side so that the bunch is always in a focusing region. The average acceleration gradient can be maximized for a given L if

$$\delta \simeq \frac{5\pi}{16} \quad \text{and} \quad \frac{\sin\delta}{\delta} \simeq 0.85 \quad PBWA. \tag{3.10}$$

For the PWFA we consider only relativistic driving and driven bunches. In addition we require that the final energy of the driving bunch after the distance L is still relativistic. In this case we can calculate the phase slippage along the plasma wave since the plasma wave phase velocity is equal to the velocity of the driving bunch. Following Ref. 5 we integrate the relative velocity along the length L to obtain

$$\delta \simeq \frac{\pi L}{\lambda_p}\left[(\gamma_{1i}\gamma_{1f})^{-1} - (\gamma_{2i}\gamma_{2f})^{-1}\right] \quad PWFA. \tag{3.11}$$

Since in an actual high energy accelerator the second term would be quite small, we will neglect it when using Eq. (3.11).

3.5 THE TRANSVERSE SIZE

We need the transverse size to calculate the transverse dynamics of the driven bunch. For the PBWA to make the optimum use of the laser beam it is necessary to match the Rayleigh length R to the acceleration section. Following Ref. 2 we choose the section to be twice the Rayleigh length. This in turn determines the diffraction limited spot size,

$$a^2 = \frac{R\lambda}{\pi} = \frac{L\lambda}{2\pi} = \frac{2\delta c^2\omega}{\omega_p^3} \qquad PBWA, \tag{3.12}$$

where Eq. (3.8) has been used to eliminate L. For the PWFA since we would like to fix the number of particles in the driving bunch, the transverse size is determined by the desired accelerating field,

$$a = \left[\frac{8r_e N_1 mc^2}{e\mathcal{E}_z}\right]^{1/2} \qquad PWFA, \tag{3.13}$$

where r_e is the classical electron radius.

3.6 THE ENERGY REQUIREMENT

In the PBWA the laser beam power for the beam profile given in Eq. (2.8) is

$$W = \frac{\pi a^2}{2} \frac{E_0^2 c}{8\pi} . \tag{3.14}$$

If we assume that we have a laser pulse length τ, the energy necessary to drive the plasma wave density to αn_0 is[2]

$$W\tau = \frac{\alpha \delta m^2 c^5}{e^2 \omega_p} \left(\frac{\omega}{\omega_p} \right)^3 \qquad PBWA . \tag{3.15}$$

where Eq. (3.12) has been used to eliminate a^2. On the other hand, the energy in the driving bunch for the PWFA is simply given by

$$W\tau = N_1 E_1 \qquad PWFA . \tag{3.16}$$

3.7 THE EFFICIENCY

The overall efficiency of the accelerators here can be divided into three parts. The first part is the efficiency of conversion of 'wall plug' energy to either laser energy or electron beam energy. These two efficiencies may be quite different, however, we will not discuss them here. The second efficiency is the conversion of either laser or electron beam energy to plasma energy. The third efficiency is that for conversion of the plasma energy to the driven electron beam. The efficiency of the transfer of energy from the laser to the plasma has been calculated for the PBWA model we have chosen.[2] For a general phase shift δ the ratio of the plasma energy to the laser energy is given by

$$\eta_1 = \frac{P.E.}{W\tau} = \frac{\alpha\delta}{4} . \tag{3.17}$$

If laser depletion is included in the analysis, this number will be reduced slightly.

The efficiency of the transfer of energy from an electron beam to the plasma is quite different. In this case one must consider the beam loading effects. If we could treat the bunch as a macro-particle, then for a very relativistic driving bunch we could extract nearly all of its energy before it's velocity changed enough to yield a phase slip. However, due to beam loading this is not possible since the leading edge of the driving bunch loses essentially no energy to the plasma while the trailing edge loses twice as much as that calculated for a point-like particle. Thus, for very short bunches, we can only extract about 1/2 of the energy

$$\eta_1 = \frac{1}{2} \quad PWFA \, . \tag{3.18}$$

For longer bunches of electrons, one can improve this factor and also improve the *transformer ratio*[13,14] at the expense of the peak field. This technique might be difficult to realize in the PWFA with the model we consider here since the strong transverse fields due to the head of the driving bunch would focus the tail. Therefore, we will not consider it in this section; however, it is an important possiblity if the transverse fields can be reduced.

The final efficiency to calculate is that from the plasma to the driving bunch. This efficiency is the same for both cases provided that the characteristics of the plasma wave are the same. The total acceleration gradient experienced by a bunch with N_2 particles in a plasma wave is

$$G \equiv \frac{dE_2}{dz} = e\mathcal{E}_z f - 4e^2 \frac{N_2}{b^2} \quad , \tag{3.19}$$

where $e\mathcal{E}_z$ is the peak longitudinal electric field, and f is a factor less than unity which takes into account phase slippage or shifts in phase from the peak accelerating field. The second 'beam loading' term is due to the plasma wake induced by the trailing bunch. The efficiency is given by the total energy gained by the bunch divided by the plasma energy,

$$\eta_2 = N_2 G L \left(\frac{\mathcal{E}_z^2}{8\pi} \frac{\pi a^2}{2} L \right)^{-1} \quad . \tag{3.20}$$

This efficiency has a maximum when

$$N_2 = \frac{f\mathcal{E}_z b^2}{8e} \ , \tag{3.21}$$

and the value is given by

$$\eta_2^{max} = f^2 \frac{b^2}{a^2} \ . \tag{3.22}$$

For the PWFA f can be taken to be essentially unity while for the PBWA f is given by Eq. (3.10). This yields

$$\begin{aligned} \eta_2^{max} &\simeq .72\frac{b^2}{a^2} \qquad PBWA \\ \eta_2^{max} &\simeq \frac{b^2}{a^2} \qquad PWFA \end{aligned} \ . \tag{3.23}$$

4. COMPARISONS AND DISCUSSION

Now we come to a detailed comparison between the PBWA and the PWFA. As mentioned earlier, our guide will be the self consistency among all relevant accelerator parameters within each scheme. Our approach is to choose a set of parameters in each scheme that we fix from the beginning. The remaining parameters in each scheme can then be calculated in terms of those chosen parameters. The scaling to different sets of chosen parameters is straight forward using the results of the previous section. To make a fair comparison we will study two sets of sample accelerators with the same acceleration gradient and the same length L. In addition, to make the comparison meaningful to real experiments, we employ only those laser and electron beams that are presently available. Under these considerations, the parameters that should be fixed in the two schemes are quite different. In particular, for the PBWA we need to fix the laser frequency ω by choosing a particular laser source. If we then fix the length L of the acceleration section, the phase slippage determines the plasma frequency ω_p. This means that the longitudinal electric field $\mathcal{E}_z$ is a derivable quantity. On the other hand, the energy gradient in the PWFA is chosen so that the intensity and dimensions are not far from realizable values. As we shall see, in spite of this difference it is possible to match the acceleration gradients.

4.1 Numerical comparisons

To keep the dimensions to a laboratory scale, we select the acceleration lengths to be 10 cm and 100 cm. These two lengths are then combined with two different laser frequencies, the *Nd*: Glass laser and the CO_2 laser, to form four sets of sample calculations. For the PBWA the parameter α is chosen to be 0.25, which is approximately the saturation value,[11] and the phase slippage is taken to be the optimum value given in the previous section. Finally, we assume that the laser pulse length and the growth time for the plasma wave τ is about 159 cycles ($\omega_p\tau = 1000$).

Since the PWFA is ~~not~~ so restrictive in its design, we can now set the parameters to match some of those for the PBWA. In particular we use the same acceleration gradient and the same a/λ_p. The number of particles in the driving bunch is taken from the present number in the SLC and the bunch length is assumed to be somewhat less than the plasma wavelength. The initial and final energies of the driving bunch are selected so that the final energy of the bunch tail is 90% of its initial energy. As we can see from Tables 1 and 2, the phase slippage for the PWFA is much smaller than that for the PBWA. All parameters except the efficiency and the energy in the driving beam turn out to be quite comparable. In particular note that the focusing for both schemes is quite strong. The energy required for the driving bunch is consistently higher for the PBWA; however, because it is less efficient in these examples, the number of particles which can be driven is comparable to the PWFA.

4.2 Discussion

The examples above seem to favor the Plasma Wake Field Accelerator especially for the longer accelerator sections. This is due to the divergence of the laser. For longer Rayleigh lengths it is necessary to have a larger spot and thus more peak power to obtain the same intensity at the spot. On the other hand the particle beam is assumed not to diverge. This is true because the emittance of the beam is typically much smaller than the corresponding wavelength/π for

Table 1. Plasma Beat Wave Accelerator

Chosen Parameters	Values			
ω [sec^{-1}]	Nd: Glass	1.78×10^{15}	CO_2	1.78×10^{14}
L [cm]	10	100	10	100
α	0.25	0.25	0.25	0.25
δ [rad]	$5\pi/16$	$5\pi/16$	$5\pi/16$	$5\pi/16$
$\sin\delta/\delta$	0.85	0.85	0.85	0.85
$\omega_p\tau$	1000	1000	1000	1000
Derived Parameters				
ω_p [10^{13} sec^{-1}]	2.65	1.23	.571	.265
n_0 [10^{16} cm^{-3}]	21.7	4.67	1.00	0.22
$e\mathcal{E}_z$ [GeV/m]	9.38	4.36	2.00	0.94
a [mm]	0.13	0.41	0.41	1.30
a/λ_p	1.82	2.70	1.25	1.82
β [$\sqrt{\gamma/\sin\phi}$ mm]	0.18	0.57	0.57	1.80
N [10^{10}]	$1.95\eta_2$	$9.04\eta_2$	$4.19\eta_2$	$1.95\eta_2$
W_T [J]	23.9	515.4	11.1	239.2

the laser. In addition it is possible to use magnetic focusing elements to define the size of a charged particle beam. The problem of the divergence of the laser beam might be solved by using lasers sufficiently intense to self focus in the plasma; however, this possibility was not considered since it lies outside the scope of the simple models given here. In addition, for the PBWA parameters chosen, the laser power is somewhat below the critical value for relativistic self focusing.[15,16]

Unfortunately, for both schemes the efficiency η_2 and the energy spread induced are directly related. Thus, if a small energy spread is necessary, then η_2 will necessarily be small for both schemes. The efficiency η_1 of the PWFA was better in all cases because the energy transfer from the laser to the plasma is limited by Eq. (3.17) to quite a small value. There is a possible solution to this problem. Since the laser is not depleted very much, it might be possible to reuse the beam after a suitable amplification. This would yield a very high

Table 2. Plasma Wake Field Accelerator

Chosen Parameters	Values			
L [cm]	10	100	10	100
$e\mathcal{E}_z$ [GeV/m]	9.38	4.36	2.00	0.94
N_1	5×10^{10}	5×10^{10}	5×10^{10}	5×10^{10}
E_1 [GeV]	1.04	4.84	0.22	1.04
a/λ_p	1.82	2.70	1.25	1.82
Derived Parameters				
a [mm]	0.25	0.36	0.54	0.78
$\delta[10^{-3} rad]$	5.5	2.5	42	18
ω_p $[10^{13}\ \mathrm{sec}^{-1}]$	1.37	1.41	.439	.438
n_0 $[10^{16}\ \mathrm{cm}^{-3}]$	5.90	6.18	.606	.604
α	0.38	0.17	0.25	0.11
β $[\sqrt{\gamma/\sin\phi}\ \mathrm{mm}]$	0.28	0.59	0.73	1.52
N_2 $[10^{10}]$	$2.25\eta_2$	$2.25\eta_2$	$2.25\eta_2$	$2.25\eta_2$
$W_T = N_1 E_1$ [J]	8.33	38.8	1.76	8.33

repetition rate and looks quite attractive; however, this possibility needs much more study.

There is one final problem for the PBWA. We have assumed that the plasma wave would grow over $1000/2\pi$ cycles. If there are density fluctuations greater than about .2%, then the wave would saturate much sooner. This case would require a much larger laser energy in order to drive the plasma to the desired field in a shorter time.

5. THE EFFICIENCY AND THE TRANSVERSE EMITTANCE

One primary problem in the preceding discussion is that the plasma accelerators discussed have very low efficiency, η_2. If we require that the induced energy spread due to the *transverse* variation of the acceleration field is say 1 %, then the maximum efficiency of transfer of plasma energy to the electron beam is

$$\eta_2^{max} \simeq 0.02 \; . \tag{5.1}$$

Even the η_2^{max} above cannot be realized since it would require full beam loading and thus would yield 100 % energy spread (since at the tail of the bunch the accelerating field is zero).

Typically in an RF structure η_2^{max} is limited by the fact that the wake field of a bunch contains modes other than the fundamental accelerating mode. Thus, if the bunch current is increased until all energy is extracted from the fundamental, there is still energy radiated into all higher modes. This of course depends upon the longitudinal bunch distribution as well. For example, for the SLAC accelerating structure with a 1 mm Gaussian bunch[17]

$$\eta_2^{max} \simeq 0.3 \qquad \text{SLAC} \; . \tag{5.2}$$

However, operating at this efficiency would yield a beam with 100 % energy spread. Therefore, the efficiency is sacrificed for an acceptable energy spread. For SLC operation the full energy spread can be kept to about 1 % provided that

$$\eta_2 \simeq 0.03 \qquad \text{SLC} \; . \tag{5.3}$$

This is achieved by balancing the beam loading effects against the curvature of the RF to achieve optimum energy spread.

In the case of plasma accelerators one can expect a similar reduction factor from η_2^{max} to η_2. The problem is that η_2^{max} is too small.

The problem of small efficiency is related to transverse focusing and the transverse emittance. As mentioned earlier, the small efficiency is basically proportional to the energy spread. Since the longitudinal electric field varies with radius, the trailing beam size b must be kept small compared to the leading beam size a. Therefore, the efficiency $\eta_2 = (b/a)^2$ is small. However, the variation of the longitudinal field is related to the strength of the strong radial electric fields which focus the trailing beam. Thus, both the beta function and the beam size are coupled to the efficiency. In addition, for self consistency, one must require the emittance of the accelerated beam to be

$$\epsilon = \frac{b^2}{\beta} \ . \tag{5.4}$$

Using Table 2 we can calculate the emittance necessary for the example in column 1. First let us assume that a transverse variation of acceleration gradient of 1% is acceptable. In this case the trailing beam must have a size

$$b = .036 \text{ mm} \ . \tag{5.5}$$

To calculate the beta function consider an injected 10 GeV electron bunch which resides at a reasonable phase $\phi = 0.1$ on the plasma wave. Then the beta function for transverse focusing is

$$\beta = 12.4 \text{ cm} \ . \tag{5.6}$$

Therefore, using Eq. (5.4) above and assuming perfect matching, one finds an emittance

$$\epsilon \simeq 1 \times 10^{-8} \text{ m} \tag{5.7}$$

or an invariant emittance of

$$\gamma\epsilon \simeq 2 \times 10^{-4} \text{ m} \ . \tag{5.8}$$

This value is much larger than the nominal value for the SLC,

$$[\gamma\epsilon]_{SLC} = 3 \times 10^{-5} \text{ m} \ . \tag{5.9}$$

For Tev linear colliders we would like emittances much smaller than the present SLC emittance. In this sense the previous example does not match the desired characteristics of the trailing beam. If a very low emittance beam were injected, the resulting beam size would be reduced and the efficiency would suffer. Therefore, it is useful to attempt a solution to both the problem of efficiency and the problem of large emittance. In the next two sections we discuss two possible solutions to the problems of emittance and efficiency.

5.1 A Modification of the transverse profile

All of the results in Tables 1 and 2 were calculated with particular models of the driving beam. In this section we show how changing the transverse profile of the driving bunch can ~~dramatically~~ affect both efficiency and emittance.

In this section we consider the PWFA with a driving bunch profile

$$\sigma(r) = \begin{cases} \dfrac{N}{\pi a^2} & r < a \\ 0 & r > a\ . \end{cases} \tag{5.10}$$

There is a corresponding example for the PBWA, but we will restrict the discussion to the PWFA in this section.

The solution for the perturbed density n_1 is again given by Eq. (2.22), and the fields behind the bunch are

$$\begin{aligned} \mathcal{E}_z &= \frac{-4eN}{a^2}\{1 - k_p a K_1(k_p a)\, I_0(k_p r)\ \}\cos(k_p z - \omega_p t)\ , \quad r < a \\ \mathcal{E}_r &= \frac{4eNk_p}{a} K_1(k_p a)\, I_1(k_p r)\ \sin(k_p z - \omega_p t)\ , \quad r < a. \end{aligned} \tag{5.11}$$

Comparing this with Eq. (2.24) we see that for large $k_p a$, the longitudinal field is quite constant and the radial field is quite small. Explicitly, for $k_p a$ large but $k_p r$ small, we have

$$\begin{aligned} \mathcal{E}_z &\simeq \frac{-4eN}{a^2}\cos(k_p z - \omega_p t)\ , \quad r \ll a \\ \mathcal{E}_r &\simeq (2\pi)^{1/2}(k_p a)^{3/2} e^{-k_p a}\frac{eN}{a^3} r\ \sin(k_p z - \omega_p t)\ , \quad r \ll a. \end{aligned} \tag{5.12}$$

Following Section 3.1 we can calculate the beta function for small transverse oscillations,

$$\beta = \left[\frac{\gamma}{\sin\phi} \frac{a^3 e^{k_p a}}{(2\pi)^{1/2}(k_p a)^{3/2} N r_e} \right]^{1/2} . \tag{5.13}$$

Due to the exponential factor in Eq. (5.13) above, we can increase the beta function quite easily in this model.

To calculate the efficiency with limited energy spread, it is useful to approximate the longitudinal electric field for both $k_p a$ and $k_p r$ large. In this case

$$\mathcal{E}_z \simeq \frac{-4eN}{a^2} \left(1 - \frac{1}{2} \left(\frac{a}{r} \right)^{1/2} e^{-k_p(a-r)} \right) \cos(k_p z - \omega_p t) . \tag{5.14}$$

From Eq. (5.14) above we see that the field is quite constant for $r < a$ and drops *exponentially* to 1/2 its value at $r = a$. Therefore, the field is essentially constant until $r \simeq a$.

If we consider a trailing beam with full width b, the full energy spread induced by the spread in $\mathcal{E}_z$ is

$$\left[\frac{\delta(\Delta E)}{\Delta E} \right]_{\text{full}} = \frac{1}{2} \left(\frac{a}{b} \right)^{1/2} e^{-k_p(a-b)} . \tag{5.15}$$

In this case the efficiency η_2 is again given by

$$\eta_2^{max} = \left(\frac{b}{a} \right)^2 . \tag{5.16}$$

It is useful to calculate an example which yields high efficiency and low energy spread. Consider the example shown in Table 2 column 1. To maintain the acceleration field of 9.38 GeV/m, we must reduce a by $\sqrt{2}$. To maintain the ratio a/λ_p, we increase the plasma frequency by the same factor:

$$\begin{aligned} a &= .175\,\text{mm} \\ \omega_p &= 1.94 \times 10^{13}\,\text{sec}^{-1} \end{aligned} \tag{5.17}$$

which yields

$$k_p a = 11.3 . \tag{5.18}$$

This is quite large and is fine for the approximations made in Eq. (5.14).

Using Eq. (5.15) if we restrict the full energy spread to about 1 %, the driving beam radius b can be increased to

$$\frac{b}{a} \simeq .63 \tag{5.19}$$

which yields an efficiency

$$\eta_2^{max} \simeq .40 \quad . \tag{5.20}$$

To check the transverse focusing we use Eq. (5.13) to find

$$\beta \simeq 5.7\sqrt{\gamma/\sin\phi}\,\mathrm{mm} \tag{5.21}$$

which, for a 10 GeV bunch with $\sin\phi = .1$, yields

$$\beta \simeq 2.5\,\mathrm{m} \ . \tag{5.22}$$

If we assume perfect matching, the emittance at 10 GeV is

$$\epsilon = b^2/\beta \simeq 4.9 \times 10^{-9} \ , \tag{5.23}$$

which yields an invariant emittance

$$\gamma\epsilon \simeq 1 \times 10^{-4} \ . \tag{5.24}$$

The emittance above is *not* the rms emittance since it includes essentially all of the beam. To compare with rms emittances one might divide by a factor of 5. Even so, it is still quite large; however, it can be dramatically reduced by increasing k_p by a factor of 2 without affecting the efficiency or peak field. Keeping a and b fixed this yields

$$\begin{aligned} k_p &\to 2k_p \\ \beta &\to 420\,\mathrm{m} \\ \epsilon &\to 2.9 \times 10^{-11} \\ \gamma\epsilon &\to 5.7 \times 10^{-7} \ . \end{aligned} \tag{5.25}$$

The emittance above is much closer to interesting values for large linear colliders. The example above should be considered an illustration, but it is by no means optimum. The primary purpose is to illustrate that neither the efficiency nor the transverse emittance are *fundamental* problems. In the next section we show yet another possible way to increase the efficiency.

5.2 VAN DER MEER'S SUGGESTION FOR ENHANCING THE EFFICIENCY[18,19]

S. van der Meer suggests in Ref. 18 that one way of solving the problem of small η_2 might be to *decrease* the radius of the beam relative to the plasma wavelength so that $k_p a \ll 1$. This causes the resultant electric field to be much more constant over the dimension of the driving bunch even in the case of a parabolic transverse profile. More importantly, the wake field is not so strongly dependent on the transverse size of the bunch which allows one to obtain a large efficiency even for small b/a.

To calculate the efficiency in this case let us return to Eq. (2.24). We can write the efficiency as

$$\eta = \frac{N_2 \Delta E_2}{N_1 \Delta E_1} , \tag{5.26}$$

where ΔE_1 is the energy loss of a particle in the first bunch over a length L, and ΔE_2 is the net energy gain, including beam loading, of a particle in the second bunch. As usual, we assume that the trailing bunch is placed at a point of maximum acceleration. Since the field varys little over the bunches in this case , we estimate η_2 using the field at the center. From Eq. (2.24) we have

$$\begin{aligned} \Delta E_1 &\simeq \frac{8e^2 N_1 L}{a^2} f_1 \\ \Delta E_2 &\simeq \frac{16e^2 N_1 L}{a^2} f_1 - \frac{8e^2 N_2 L}{b^2} f_2 , \end{aligned} \tag{5.27}$$

where

$$\begin{aligned} f_1 &= K_2(k_p a) + \frac{1}{2} - \frac{2}{(k_p a)^2} \\ f_2 &= K_2(k_p b) + \frac{1}{2} - \frac{2}{(k_p b)^2} . \end{aligned} \tag{5.28}$$

The efficiency above is maximum when

$$N_2 = N_1 \left(\frac{b}{a}\right)^2 \frac{f_1}{f_2} \tag{5.29}$$

and has the value

$$\eta_2^{max} = \left(\frac{b}{a}\right)^2 \frac{f_1}{f_2} \ . \tag{5.30}$$

For the cases considered previously ($k_p a \gg 1$, $k_p b \gg 1$) we find

$$f_{1,2} \to \frac{1}{2} \ . \tag{5.31}$$

However, for the opposite case, if we expand the Bessel function for small argument, we find

$$\begin{aligned} f_1 &\simeq \frac{(k_p a)^2}{8} \left(\frac{3}{4} - C + \ln\frac{2}{k_p a}\right) , \qquad k_p a \ll 1 , \\ f_2 &\simeq \frac{(k_p b)^2}{8} \left(\frac{3}{4} - C + \ln\frac{2}{k_p b}\right) , \qquad k_p b \ll 1 , \end{aligned} \tag{5.32}$$

where $C = .577\cdots$ is Euler's constant. This yields an efficiency

$$\eta_2^{max} \simeq \frac{\frac{3}{4} - C + \ln\frac{2}{k_p a}}{\frac{3}{4} - C + \ln\frac{2}{k_p b}} , \qquad k_p a \ , \ k_p b \ll 1 \ . \tag{5.33}$$

To complete the calculation we need the energy spread induced by the accelerating field. From Eq. (2.24) for $k_p a \ll 1$ and $k_p r \ll 1$, the longitudinal electric field is

$$\mathcal{E}_z \simeq -2eNk^2 \left(\frac{3}{4} - C + \ln\frac{2}{k_p a} - \left(\frac{r}{a}\right)^2 + \frac{1}{4}\left(\frac{r}{a}\right)^4\right) \ . \tag{5.34}$$

Therefore, for a trailing beam of radius b, the full energy spread is

$$\left[\frac{\delta(\Delta E)}{\Delta E}\right]_{\text{full}} \simeq \frac{(b/a)^2}{\frac{3}{4} - C + \ln\frac{2}{k_p a}} , \qquad k_p a \ll 1 \ . \tag{5.35}$$

Finally it is also useful to calculate the beta function in this case. Returning to Eq. (2.24) for small $k_p a$ and small $k_p r$, the transverse electric field is

$$\mathcal{E}_r \simeq \frac{4eNk_p}{a^2} r \ . \tag{5.36}$$

Following Section 3.1, this yields the beta function

$$\beta = a \left[\frac{1}{4N_1 r_e k_p} \frac{\gamma}{\sin\phi} \right]^{1/2} \ . \tag{5.37}$$

To illustrate this technique it is again useful to show an example. However, in this case it is necessary to modify the design somewhat more. Consider again the example in Table 2 column 1. In order to keep the accelerating field at 9.38 GeV/m, we will reduce a and N while keeping k_p fixed. (If we were to decrease k_p instead, the plasma wake would become nonlinear.) If we fix $k_p a = 0.1$, this yields

$$\begin{aligned} a &\simeq 2.2 \times 10^{-6}\,\mathrm{m} \\ N_1 &\simeq 5 \times 10^{8} \ . \end{aligned} \tag{5.38}$$

Using Eq. (5.35) and keeping the full spread to about 1 % yields the transverse size of the trailing beam

$$\left(\frac{b}{a}\right)^2 \simeq .032 \ . \tag{5.39}$$

The maximum efficiency can now be calculated using Eq. (5.33)

$$\eta_2^{max} \simeq .65 \ . \tag{5.40}$$

Finally, we can calculate the beta function from Eq. (5.37),

$$\beta \simeq 4.3 \times 10^{-6} [\gamma / \sin\phi]^{1/2}\,\mathrm{m} \ , \tag{5.41}$$

which, for a 10 GeV bunch and $\sin\phi = .1$, yields

$$\beta = 1.9\,\mathrm{mm} \ . \tag{5.42}$$

The emittance necessary for matching to the beam size is therefore given by

$$\epsilon \simeq 7.6 \times 10^{-11}\,\mathrm{m}\ , \tag{5.43}$$

which yields an invariant emittance

$$\gamma\epsilon \simeq 1.5 \times 10^{-6}\,\mathrm{m}\ . \tag{5.44}$$

The example above illustrates again that it is possible to obtain both small matched emittances and good efficiency for the trailing bunch. However, the two methods are quite different; the beta functions differ by 5 orders of magnitude and the beam sizes differ by 3 orders of magnitude. This leads to additional problems for the second scheme.

One problem is that when we decrease $k_p a$ we increase the relative magnitude of the transverse electric field. This leads to a small beta function. But the driving beam must be focused by external magnets. If we assume an emittance for the *driving bunch* equal to that of the trailing bunch, then to obtain the required beam size ($a = 2.2\,\mu$m), we would need a beta function due to external focusing of

$$\beta_{ext} \simeq 1.5\,\mathrm{m}\ . \tag{5.45}$$

To obtain the beta function above would require quite strong focusing even with the extremely low emittance driving beam assumed above. Such a low emittance for the driving beam could probably be obtained with the damping rings used for the accelerated bunch. However, since we have not optimized the design, there may be other solutions to this problem as well.

A second problem with the strong focusing is that the particles which oscillate in the focusing fields emit synchrotron radiation. The average loss in a smooth focusing system is given by[5]

$$\frac{dE}{dz} = \frac{2}{3} r_e mc^2 \frac{\gamma^4 b^2}{\beta^4}\ . \tag{5.46}$$

Using the results obtained above, we find that a trailing 10 GeV particle with a transverse amplitude b loses energy at the rate

$$\frac{dE}{dz} \simeq 1.7 \text{ MeV/m} . \tag{5.47}$$

This is quite large but is still small compared to the acceleration rate. But, as the beam is accelerated the synchrotron radiation increases rapidly. Once again, with more careful design, we may be able to solve this problem since it is very sensitive to β. Therefore, in spite of these difficulties, we believe that the second method for enhancing the efficiency is also quite promising.

6. CONCLUSION

In this paper we have discussed plasma accelerators which have possible applications to TeV linear colliders. The two schemes, the Plasma Beat Wave Accelerator and the Plasma Wake Field Accelerator, are very similar in that the field oscillation is supported by simple linear plasma oscillations which have phase velocity close to the speed of light. They differ in that the plasma is driven in the first case by high power beating lasers and in the second case by an intense, high-energy electron bunch.

In the preceding comparisons we showed that in most essentials we get comparable results in the two schemes. However, the PWFA is somewhat more efficient. It has another advantage in that it may be easier to manipulate high energy electron bunches rather than high power laser beams.

In the last section we focused on two important issues, the efficiency and the required emittance to obtain that efficiency. We showed two solutions which increase the maximum efficiency to levels exceeding those for conventional structures. Therefore, we do not believe that efficiency is a fundamental limitation.

There are many questions which we have not addressed. We have said nothing about many of the more practical questions of how we obtain the driving bunches or lasers and how efficient that process is, and we have not treated the problem of how to stage the devices.

We have instead attempted to focus on rather idealized problems to gain insight into the basic physics of both schemes. In doing so we have not addressed questions of transformer ratio for the PWFA. This important subject is treated in another paper in these proceedings.[20] We have limited our treatment to driving bunches which are short compared to the plasma wavelength. In the case of longer bunches one must fold the longitudinal charge distribution with the wake field to obtain the field both within and behind the bunch. In this way for asymmetric triangular bunches one can enhance the transformer ratio at the expense of peak field for a given intensity driving bunch.[13,14] However, in treating this problem it is necessary to include the action of the transverse wake of the driving bunch on itself in a self consistent way. This may cause difficulties for long bunches in that the tail of the bunch will be focused by the head. However, if the transverse fields are small (as in Section 5.1), they have little effect, and the triangular bunch idea looks much more promising.

To conclude, we would like to emphasize that this paper has attempted to explore the feasibility of both the PBWA and PWFA on fundamental grounds. We have certainly not explored the parameter space completely, and thus the designs sketched here are simply examples. They were chosen to illustrate that very high accelerating fields (1 - 10 GeV/m) can be obtained in both the PBWA and the PWFA, and more importantly, that other limitations (such as the efficiency) do indeed have solutions. Therefore, we conclude that both the PBWA and the PWFA continue to be interesting possibilities for new acceleration mechanisms for TeV linear colliders.

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