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Comments on Axion Strings*

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ABSTRACT

An interesting problem involving axion strings and monopoles is clarified. Also discussed are the gravitational Hall effect (*iii*), gravitational dyons (*iv*), anomalous superconductivity (*v*) and classical anomalies (*vi*).

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In the literature, there have recently appeared articles illuminating the properties of Higgs vortices, or cosmic strings, and the fermion zero modes that live on them. [1,2] Yet, several questions remain unanswered.

I will begin this letter by summarizing earlier results, principally those of ref.[2]. Following this I will discuss certain puzzles that have arisen concerning axion strings. In section (ii) I describe the unusual physics of axion string monopole interactions. Section (iii) resolves a puzzle concerning gravitational fields and anomalies. Section (iv) discusses the Kaluza-Klein monopole, a special case in which the results of (ii) and (iii) overlap. In section (v) I deal with the nature of the fermion currents, and in (vi) the relationship between topological and quantum mechanical concepts. We will see that the axion string provides a *gedanken* laboratory, bringing together several quite interesting concepts in field theory and gravity.

(i) Background

The axion is the Goldstone boson of a spontaneously broken $U(1)$ symmetry, introduced to solve the CP problem of QCD. It has couplings to fermions with ordinary electric charge, as many as desired, though here we consider only a single Dirac fermion with coupling

$$\bar{\psi} e^{i\alpha\gamma_5} \psi.$$

One fermion-loop quantum effects induce a photon-axion interaction term

$$L = \frac{e^2}{32\pi^2} \int \theta(x) F^{\mu\nu}(x) \widetilde{F}_{\mu\nu}(x) dx, \tag{1}$$

where $\theta(x)$ is the dimensionless, real, scalar axion field. This term is not a total derivative because θ is not a constant.

As occurs whenever there is a spontaneously broken $U(1)$, this model has string-like solutions which are topologically stable.† The string configuration for θ is

$$\theta = \theta_0 + n\varphi \qquad\qquad 0 < \varphi < 2\pi \quad n = \text{integer},$$

in which φ is the azimuthal angle and we choose for simplicity n=1. In a beautiful paper, Callan and Harvey discussed the strange physics of the axion string interactions with the electromagnetic field[2]. I will summarize their findings here.

The problem is that for strings the Lagrangian (1) is ill-defined, because of the discontinuity in θ as it crosses 2π[2]. We can replace (1) with

$$L = \frac{e^2}{64\pi^2} \int \partial^\mu \theta A^\nu \widetilde{F}_{\mu\nu} dx \tag{2}$$

which is a meaningful expression for θ, however now we are afflicted with a Lagrangian which is not gauge invariant. The gauge variation of (2) is

$$\frac{\delta}{\delta\omega} L = \frac{e^2}{16\pi^2} \int d^4x [(\partial^\nu \partial^\mu \theta) \widetilde{F}_{\mu\nu} - \partial^\nu (\partial^\mu \theta \widetilde{F}_{\mu\nu})],$$

and as long as $[\partial_\mu, \partial_\nu]\theta = 0$, the gauge variance is a surface term which vanishes at infinity and is not important to local physics. But the string solutions are just those in which the commutator is *not* zero, in fact, for a simple string

$$[\partial_x, \partial_y]\varphi = 2\pi\delta(x)\delta(y).$$

Thus

$$\frac{\delta}{\delta\omega} L = \frac{e^2}{8\pi} \int d^4x \delta(x)\delta(y) \widetilde{F}_{\mu\nu} = -\frac{e^2}{4\pi} \int d^2x \epsilon^{ab} F_{ab} \tag{3}$$

and we are forced to conclude that gauge invariance breaks down on the string. One might think that this gauge non-invariance poses a problem for the quan-

† The instanton effects, which alter the picture at low energies by giving the axion a mass and the string at least one domain wall, are at present neglected. All the results considered here apply to the physics around the string out to a distance $\sim$ 10 m. The string thickness is at most 10^{-12} fermi[3].

tum theory, but it doesn't, because we have neglected an important property of strings.

In the fermion sector the string solution has the property that there exist fermion zero-modes. [4] Zero modes are the solutions to the Dirac equation when the string is treated as a classical background. These solutions are localized around the string, and a fermionic excitation may carry arbitrarily small energy as long as it propagates along the string. For the purposes of this paper all we need to know about zero-modes is that for each 3+1 dimensional Dirac fermion that obtains its entire mass from string coupling $\bar{\psi}e^{i\gamma_5\theta}\psi$(as in the axion model under discussion), there corresponds an effectively 1+1 dimensional massless chiral fermion moving in the +z direction along the string, and for each Dirac fermion coupled by $\bar{\psi}e^{-i\gamma_5\theta}\psi$ there is a 1+1 dimensional chiral fermion moving in the -z direction. To excite fermion modes that propagate off the string would require energies greater than the mass of a pair fermions, which, because of the Higg's phenomenon, is the symmetry breaking scale of the model. As far as the fermions are concerned, at energies below the symmetry breaking scale the theory is a dimensionally reduced one of massless 1+1 dimensional fermions.

For our model of a single Dirac fermion the zero mode is chiral and moves in only one direction along the string. A 1+1 dimensional gauge theory with a single chiral fermion is known to be sick; the gauge symmetry is broken by quantum effects. [5,6] This quantum anomaly implies that in the dimensionally-reduced string world of the fermion, the electromagnetic current is not conserved:

$$\partial^a J_a = \frac{1}{2\pi}\epsilon^{ab}\partial_a A_b \qquad\qquad a, b = 0, 1. \tag{4}$$

Equivalently, the zero-mode contribution to the fermion determinant is not gauge invariant. So

$$\frac{\delta}{\delta\omega}\int \{d\psi d\bar{\psi}\} e^{iL[\psi,\bar{\psi}]} = \frac{e^2}{4\pi}\int d^2x\epsilon^{ab}\partial_a A_b.$$

and since this term appears with opposite sign to (3), the two will cancel and gauge invariance is preserved in the full theory.

The current non-conservation in the 1+1 dimensional fermion world can be understood in a very intuitive fashion[2]. From (2) we can write a current that must exist in the surrounding space whenever $\partial_\mu \theta \neq 0$ in the vacuum and there is an applied external field. This is

$$J_\mu = \frac{\delta}{\delta A^\mu} L_\theta = \frac{e^2}{32\pi^2} \partial^\nu \theta \widetilde{F}_{\nu\mu}. \tag{5}$$

What is the nature of this current? Note that it has nothing to do with the motion of any of the charged particles of the theory, all of which are massive off the string. It is a Hall current which moves perpendicular to the direction of an applied field. For a string along the z-axis with an applied parallel electric field the current per unit length is

$$J_\rho = -\frac{e}{4\pi^2 \rho} E,$$

and the amount of charge being driven onto the string is

$$\frac{e}{2\pi} E.$$

This charge appears in the dimensionally reduced fermion world according to (4), and the cancellation of anomalies is explained by an exchange of charge between the string and higher dimensions.

(ii) Monopoles and Axion Strings

In a famous paper [7] Witten showed that a magnetic monopole acquires an electric charge of $\frac{e\theta}{2\pi}$ in a vacuum in which the CP parameter θ is non-zero. The presence of the axion complicates the electric field of the monopole at short ranges, but it has been shown that the distant field is determined by the Witten charge with θ given by its asymptotic value. [8] Both these results may be easily obtained by being a little more careful in deriving the vacuum current (5).

Actually,

$$J_\mu = \frac{e^2}{32\pi^2}\partial^\nu[\theta \widetilde{F}_{\mu\nu}]$$

and the electric charge of a monopole is

$$Q = \int d\vec{x} J_0 = \frac{e^2}{32\pi^2}\int d\vec{x}\nabla\cdot[\theta\vec{B}] = \frac{e\theta_\infty}{2\pi}$$

which holds true whether θ is a dynamical variable or not.

A problem appears when we take into account the topologically stable string solutions of the axion field. It seems that moving a monopole azimuthally around a string will change the electric charge; moving it around by 2π would seem to increment the charge by one unit. It thus appears that electric charge is not conserved.

This problem may be easily understood using the Lagrangian (2), and considering what currents are induced in the vacuum in the process of moving the monopole around. A monopole circling the string will have an electric dipole field with its moment along the string. This has two effects. It generates an anomaly current in the string and a Hall current in the vacuum. Net charge will flow away from the monopole. In this example of a single orbiting monopole some of the charge will flow radially outward to infinity and some radially inward to the string, where it is carried off to infinity in an anomalous current. Whether the monopole charge is increased or decreased depends of course on the direction of the string, the direction of the orbit and the sign of the magnetic charge.

We can make this more quantitative by imagining a solenoid of magnetic current with an axion string running along its axis[Fig 1]. The electric field inside the inductor is

$$E = 2\pi\Omega\frac{Q_m}{L},$$

where $\frac{Q_m}{L}$ is the magnetic charge per unit length, which is rotating with frequency Ω. There will be a Hall current exchanging charge between the string and the solenoid. The string current is determined by the anomaly,

$$\partial_a J^a_{string} = \frac{e^2}{2\pi} E = \frac{Q_m}{L} \Omega$$

and if Ω is constant the current is

$$J_{string} = e^2 \frac{Q_m}{L} \Omega z + constant.$$

In a time T the amount of charge entering, along the string, a section of the solenoid of length L is

$$Q_e = e^2 Q_m \Delta\theta.$$

Finally, using the Dirac quantization condition, $Q_m = \frac{2\pi}{e^2}$, the change in the electric charge per monopole is

$$\frac{e\Delta\theta}{2\pi}.$$

Since the dyon charge must support an electric field, we can expect that a monopole at rest near a string will feel a tangential force in the direction that will bring its charge closer to zero. If let go it will spin away, its classical trajectory determined by energy conservation, $E = \frac{1}{2}mv^2 + k|\theta|$, where k is a constant and θ is measured from the sheet and angle where the electric charge is zero.

A related problem for axionic domain walls has already been discussed. [9,10] Because of the anomaly, the Peccei-Quinn symmetry is not exact,and the axion is not really a massless Goldstone boson. In the semi-realistic invisible axion models $m_a \sim 10^{-5}$ev. The statement above that $\langle\theta\rangle = \theta_0 + \phi$ is really true only for radial distances less than the Compton wavelength of the axion, which can be on the order of meters. Beyond this $\langle\theta\rangle$ varies only in planar regions of thickness λ_a, the domain walls, which begin on the string and stretch to infinity. To each string there are N domain walls attached, where N is a model dependent number. Theta is constant between the domain walls, smoothly interpolating from one value to another across the wall. The sum of $\Delta\langle\theta\rangle$ for all of the walls attached to a given string must add up to 2π. Because $\langle\theta\rangle$ is different on either side of a wall, the charge of the monopole will change by one unit as it passes through a minimal domain wall. This is consistent with the above discussion of strings.

(iii) The Gravitational Hall Effect

An axion string is an energy-momentum diode because energy-momentum can flow in only one direction along the string. It turns out that the results of part (*i*) are closely paralleled by the gravitational contribution to the action[2]. Anomalies require a CP violating term

$$L = \frac{1}{768\pi^2}\int \theta\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\tau}R^{\lambda\tau}{}_{\alpha\beta}d^4x.$$

In the presence of an axion string we can remove the 2π ambiguity in θ at the cost of having a gauge variant Lagrangian,

$$L = -\frac{1}{192\pi^2}\int \epsilon^{\mu\nu\alpha\beta}\partial_\mu\theta(\Gamma^\lambda_{\nu\rho}\partial_\alpha\Gamma^\rho_{\beta\lambda} - \frac{2}{3}\Gamma^\lambda_{\nu\rho}\Gamma^\tau_{\alpha\lambda}\Gamma^\rho_{\beta\tau})d^4x. \tag{6}$$

Standard manipulations may be used to show that under a coordinate reparameterization[2]

$$\delta L = \frac{1}{48\pi}\int \epsilon^{ab}\partial_\sigma\eta^\rho\partial_a\Gamma^\sigma_{\rho b}d^2x \qquad a,b = t,z. \tag{7}$$

Apparently, general coordinate invariance is broken and energy momentum is not conserved in the theory of equation (6).

Just as in the electromagnetic case, the dimensionally reduced theory of massless fermions on the string has an anomaly, but this time it is the energy-momentum of the fermion zero modes which is not conserved, because of a purely gravitational anomaly [11]

$$\Theta_{a\ ;b}^{\ b} = \frac{1}{48\pi}\partial_c\partial_d\Gamma^c_{ae}\epsilon^{de}. \tag{8}$$

When confined to a reparameterization in the z,t plane, the gauge non-invariance of equation (7) is exactly canceled by the effect the anomaly in the 1+1 dimensional fermion world of the string.

Once again paralleling the electromagnetic example, (6) may be varied with respect to the metric to obtain, when θ is given its string vacuum expectation value, the energy-momentum in the string vacuum induced by an applied gravitational field,

$$\frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} = \Theta^{\gamma\nu} = \frac{1}{348\pi^2}\frac{\epsilon^{\mu\nu\alpha\beta}}{\sqrt{-g}}(\theta_{,\mu}R_{\alpha\beta}^{\ \ \gamma\delta})_{;\delta} + (\nu \leftrightarrow \gamma). \tag{9}$$

It was suggested that the interaction of axion strings with gravitational fields would be rather odd[2]. If the picture were the same as for purely electromagnetic fields, the presence of a parallel gravitational field would induce an energy momentum Hall current feeding into an anomalous 1+1 dimensional energy momentum current along the string. A radial Hall current would be depositing energy, but no momentum, on the string, but since the zero mode is massless its energy must equal its momentum. The discrepancy must somehow be accounted for.

This problem is best elucidated by going to the linearized version of Einstein's field equations. This permits us to deal with strictly external sources, neglecting the gravitational field of the string itself, or of any currents that may be induced. As usual, we take

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad |h_{\mu\nu}| \ll 1.$$

A convenient definition, the "Lorentz" gauge, and the resulting form of Einstein's equations are [12]

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$$

$$h_{\mu\ ,\alpha}^{\ \alpha} = 0$$

$$\partial^2 \bar{h}^{\mu\nu} = 8\pi G T^{\mu\nu}.$$

Now we can use (11) to find the energy-momentum Hall current in the linearized approximation

$$\Theta^{\nu\gamma} = \frac{1}{192\pi^2}\epsilon^{\mu\nu\alpha\beta}[\theta_{,\mu}(h_{\alpha\ ,\ \beta}^{\ \gamma\ \delta} + h_{\beta\ ,\alpha}^{\ \delta\ \ \gamma})]_{,\delta} + (\nu \leftrightarrow \gamma).$$

Using $\theta = \varphi$ everywhere off the string, this becomes

$$\Theta^{\nu\gamma} = \frac{1}{192\pi^2}\frac{1}{\rho}\epsilon^{\hat{\rho}\nu\alpha\beta}\partial_\alpha \partial^2 h_{\beta}^{\ \gamma} + (\nu \leftrightarrow \gamma). \tag{10}$$

But this is zero! The linearized form of Einstein's equations require that $\partial^2 h_{\beta}^{\ \gamma} = 0$. Hence there is no Hall current in this case. The problem stated above is just simply not a problem at all. Of course there will be complicated interactions between the string and gravity, but it is not properly called a Hall effect because the analogy with electromagnetism breaks down in the weak field limit.

We can understand this better by thinking more about the 1+1 gravitational anomaly for the embedded string. To make the analogy with the electromagnetic case we must have the external gravitational field strictly parallel to the string, so that the tidal forces acting on the zero modes do not bend the string itself. In the linearized theory, this implies that the potential for the external sources has a saddle point along the string. Let us take the potential ϕ to depend only on z in the vicinity of the string for a section of any length. In this case the 1+1 dimensional anomaly in string world is proportional to $\partial_z^3\phi(z)$. But since in this region is $\partial_z^2\phi(z) = 0$, by the Einstein equation, we find that the 1+1 anomaly must be zero.

(iv) Kaluza-Klein Dyons and Axion Strings

One may derive the gravitational analog to the Witten dyon charge quite easily with a calculation similar to that in *(ii)*. A more careful derivation of the energy-momentum tensor (11) gives

$$\Theta^{\gamma\nu} = \frac{1}{348\pi^2} \frac{\epsilon^{\mu\nu\alpha\beta}}{\sqrt{-g}} (\theta R_{\alpha\beta}{}^{\gamma\delta})_{;\mu;\delta} + (\nu \leftrightarrow \gamma).$$

The 'electric' type mass is

$$M_\theta = \int d\vec{x}\Theta^{00} = \frac{1}{72\pi^2} \oint ds_i \epsilon^{ijk} [\theta R^0{}_j]_{;k}. \tag{11}$$

This result may be obtained in a slightly more tortuous fashion involving certain anomalous commutators of the linearized metric. [13]

Now the essential question is, do there exist metric configurations which are asymptotically flat, but with non-zero flux integral M_θ? The answer is, no. The closest we can come to this kind of configuration is the NUT metric, which has serious problems. A standard form of the metric is

$$ds^2 = \left[1 - 2\frac{Mr + N^2}{r^2 + N^2}\right] [dt + 2N(1 - \cos\bar{\theta})d\phi]^2$$

$$- \left[1 - 2\frac{Mr + N^2}{r^2 + N^2}\right]^{-1} dr^2 - (r^2 + N^2)(d\bar{\theta}^2 + \sin^2\bar{\theta}d\phi^2),$$

where $\bar{\theta}$, the angular coordinate, is to be distinguished from θ. This metric, when put into (13) and θ is held constant, yields

$$M_\theta = \frac{\theta}{18\pi} N.$$

However, this metric has the bizarre property that the curvature goes to zero at infinity though the metric does not become Minkowski: for $r \to \infty \quad \bar{\theta} \to 0$,

$$ds^2 = dt^2 - dr^2 - r^2 d\Omega^2,$$

but for $r \to \infty \quad \bar{\theta} \to \pi$,

$$ds^2 = (dt + 4N d\phi)^2 - dr^2 - r^2 d\Omega^2.$$

There are two ways to interpret this. One is to redefine the time coordinate $d\tau = dt + 4N d\phi$, in which case τ must be periodic. The causality problems with having such closed-time-like curves are enough to make us discard this possibility immediately. The other way is to keep the coordinates as they are. The $8N dt d\phi$ cross term on the half-space $\bar{\theta} = \pi$ corresponds to a line source of angular momentum per unit length N, with zero rest mass, stretching from infinity to the origin. This interpretation too has fundamental problems, in that the string prevents it from being a localized solution, and a line source of zero rest-mass violates the energy conditions of general relativity. We see that the gravitational analog of the magnetic monopole, and hence of the Witten dyon charge is very problematic. This should actually be reassuring, because if sense could be made of it we would face a contradiction with the conclusion of the previous section.

The Kaluza-Klein monopole is obtained by generalizing the discussion to a 5-dimensional space-time. There, the line singularity can be eliminated from the NUT metric by compactifying the fifth coordinate, rather than that of ordinary time. This is desirable for reasons that have nothing to do with NUT-monopoles: we cannot see another dimension. It then happens that the configuration appears, on scales larger than the compactification radius $\sim N$, like a 4 dimensional magnetic monopole. From the preceeding discussion it should be clear that this monopole will be a dyon if the θ parameter is non-zero, and that its interactions with an axion string will be the same as in (ii).

(v) Anomalous Superconductivity

Witten showed that for strings from broken gauge symmetries, which are not anomalous, superconductivity ocurrs if the fermion zero modes carry ordinary electric charge[1]. That a different type of superconductivity ocurrs in the case axion strings the following very simple argument will show. Superconductivity for a normal 1+1 dimensional theory, with no gauge anomaly, comes from the equation of motion for the current

$$\textit{anomaly free}: \qquad \partial_z \rho - \partial_t j = \frac{e^2 E}{2\pi}. \tag{12}$$

The first term on the left is the screening term and the second is the superconductivity term. This is to be compared to a similar equation for axion strings , determined solely by the anomaly

$$\textit{anomalous}: \qquad \partial_t \rho - \partial_z j = \frac{e^2}{2\pi} E. \tag{13}$$

At first encounter it appears that the axion string does not superconduct. In fact, immediately after an electric field is turned on, equation (12) exhibits superconductivity,

$$\textit{anomaly free}: \qquad \partial_t j = \frac{e^2 E}{2\pi}, \tag{12a}$$

while (12) gives

$$\textit{anomalous}: \qquad \partial_t \rho = \frac{e^2 E}{2\pi}. \tag{13a}$$

From (i) we know that (12a) comes from the sudden appearance of a Hall current which deposits charge along the string[Fig. 2]. Were we wait long enough the currents and charges should reach a steady state. In this case (12) exhibits total screening,

$$anomaly\ free: \qquad \partial_z \rho = \frac{e^2 E}{2\pi}, \tag{12b}$$

while (13) gives

$$anomalous: \qquad \partial_z j = \frac{e^2 E}{2\pi} \tag{13b}.$$

(13b) has this form because the current at any point along the string consists of charge deposited not just at that point, but at all points upstream[Fig. 2]. Recall that chiral fermions can travel in only one direction along the string.

In spite of the clear difference in the form of (12) and (13), as well as their solutions for the above limiting cases, we can safely say that the axion strings superconduct because of the simple observation that the chiral current on the axion string must satisfy the condition

$$\rho = j$$

and therefore (13a) really is a superconducting solution like (12a), and (13b) really does involve screening like (12b). A distinguishing feature of anomalous superconducting currents is that they must be charged currents.

[14]

(vi) Classical Anomalies

A final remark is called for about this apparently classical explanation of the 1+1 dimensional gauge anomaly. A classical anomaly ocurrs where there is charge non-conservation in a classical field theory due to a topologically imposed singularity. For example, we can take a single real scalar field which is periodic. Just as a regular field theory can be thought of as a system of coupled harmonic oscillators at every point in space-time, the periodic theory may be viewed as a system of coupled pendula. The condition of periodicity requires a choice of scale v, and to make this into a sensible quantum theory would require us to treat this theory as the low-energy limit of a spontaneously broken theory, but at the classical level the theory is well defined. There are two gauge invariant ways to couple this classical scalar field to electromagnetism. One is

$$\mathcal{L} = v^2 \frac{1}{2}(\partial_\mu \phi - eA_\mu)^2 - \frac{1}{4}F^2,$$

where a gauge transformation consists of

$$A_\mu \to A_\mu + \partial_\mu \lambda \qquad \phi \to \phi - \lambda.$$

And the other is

$$\mathcal{L} = v^2 \frac{1}{2}(\partial_\mu \phi)^2 + c\phi F\widetilde{F} - \frac{1}{4}F^2,$$

in which ϕ does not change under a guage transformation. The reader will recognize this second case as the effective lagrangian of the axion model discussed in this paper, which we know to exhibit classical charge-nonconservation along a line singularity in space [or a 2- dimensional surface singularity in space-time]. This charge non-conservation may be accounted for by putting a chiral fermion on the singularity and *quantizing* it.* How is it that the 1+1 dimensional anomaly, which is a purely quantum effect, is explainable in terms of a classical anomaly from higher dimensions? Evidently, the one-loop quantum effect in the dimensionally reduced fermion theory is equivalent to a periodicity condition on a purely classical scalar field in two higher dimensions. Efforts to understand this mystery are at present being undertaken by the author.

⋆ This is reminiscent of an effect involving skyrmions, where the soliton of the bosonized QCD Lagrangian seems to know all about quark confinement.

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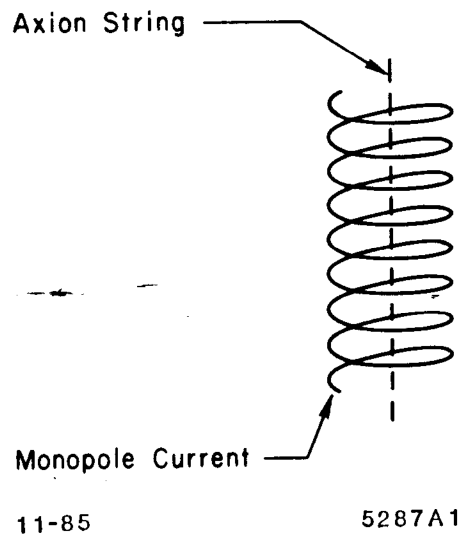

FIG. 1

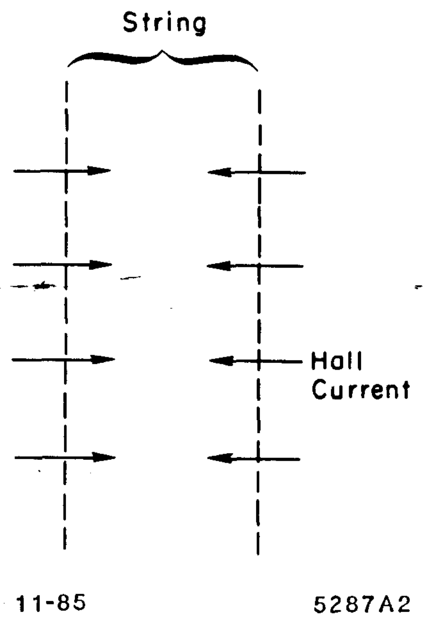

FIG. 2

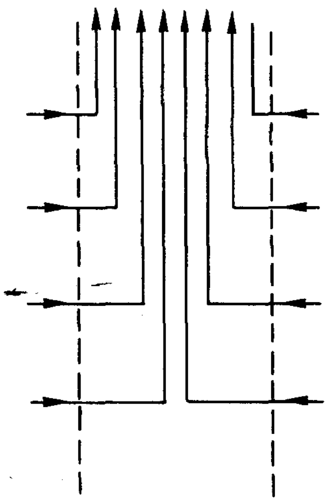

FIG. 3