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THE DILATON CLASSICAL SOLUTION AND THE SUPERSYMMETRY BREAKING EVOLUTION IN AN EXPANDING UNIVERSE*

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ABSTRACT

The existence of a dilaton field in no scale supergravities implies that the only classical solutions of the dilaton + Einstein equations are those which predict at low temperatures a vast hierarchy between the supersymmetry breaking scale m_{SUSY} (i.e. the typical squark, slepton and/or gaugino mass) and the Planck scale. At relatively low temperatures the dynamical quantities of the matter + dilaton system relax to some critical time trajectories with the property that the $m_{SUSY}(t)$ and $T(t)$ time trajectories stay proportional. The dimensionless ratio m_{SUSY}/T is a fixed point of the theory and it is completely independent of the high temperature condition ($T \simeq M_{Planck}$) of the matter + dilaton system.

1. Introduction

In the framework of $N = 1$ supergravity theories there is an interesting class of models which contain in their spectrum a physical dilaton field ϕ_D. The dilaton couples through gravitational interactions to the trace of the energy momentum tensor of the theory and therefore it interacts non-trivially with the SU(3)×SU(2)×U(1) supermultiplets. Although these interactions are of gravitational strength we show here that the existence of a dilaton field in the effective theory is of major importance and may be necessary to understand the observed scale hierarchies in our universe.

$N = 1$ supergravity models with a dilaton field are the so called no scale models [1-5]. In general any supergravity model which emerges from $D + 4$ spacetime dimensions contains naturally a dilaton after the compactification of the extra D dimensions. For instance the $6 + 4 = 10$ dimensional superstring theories [6] define 4-dimensional no scale type effective theories once the extra 6-dimensions are compactified in a Ricci flat manifold [7,8].

It is crucial that in the no scale supergravity models the spontaneous breakdown of supersymmetry (superhiggs mechanism) [9] is induced by the dilaton supermultiplet [1,3,4] (ϕ_D, ϕ_p, η). The fermionic partner of the dilaton (Majorana fermion) η is the would be Goldstino field, the field which is combined with the two helicity spin 3/2 state and defines the massive spin 3/2 gravitino. The pseudoscalar partner of the dilaton ϕ_p is the Goldstone mode of a global U(1) non-compact symmetry [3,4,10,11]. ϕ_p appears in the no scale supergravity Lagrangian only through its space time derivatives. From these properties of the dilaton supermultiplet it follows that the magnitude of the supersymmetry breaking scale depends crucially on the dilaton vacuum expectation value without any farther dependence on ϕ_p [4,11]:

$$m_{SUSY} = M \exp \alpha\phi_D \tag{1}$$

M is the gravitational scale, $M = (8\pi G_N)^{-1/2} = M_{Planck}/\sqrt{8\pi} = 2.4 \times 10^{18}$

GeV and α is a model-dependent parameter (see later). By m_{SUSY} we denote the typical mass splitting between bosonic and fermionic masses of the SU(3)×SU(2)×U(1) gauge interacting supermultiplets:

$$m^2_{SUSY} = | \, m^2_B - m^2_F \, | \tag{2}$$

From Eq. (1) it is clear that the magnitude of m_{SUSY} in the effective theory is determined by the actual value of the dilaton field $\langle \phi_D \rangle$. The later has to be compatible with the dilaton field equations. Therefore in order to understand why m_{SUSY} is so small today $m_{SUSY} \leq 10^{-16} M$ one has to examine the solutions of the dilaton field equations. However in all no scale models there is an approximate SU(1, 1) symmetry of the vacuum ($\phi_D \rightarrow \phi_D +$ constant) [1,3,4,10,11] and consequently $m_{SUSY}(\langle \phi_D \rangle)$ remains undetermined classically. ϕ_D and ϕ_p correspond to the two non-compact generators of the SU(1,1)/U(1) Kahler manifold [3,4,10,11]. Because of this classical degeneracy, the determination of $m_{SUSY}(\phi_D)$ is achieved at the quantum level of the theory [2-5]. Unfortunately, the quantum gravitational corrections are not controllable at present; therefore an important and self consistent assumption is necessary for the effective low energy theory. We assume therefore that after taking into account the quantum corrections, ϕ_D remains a dilaton field, namely ϕ_D still couples to the trace of the energy momentum tensor of the theory.

This paper is organized as follows:

In Sec. 2 we show that the existence of a dilaton in an effective $N = 1$ no scale supergravity defines unambiguously the dilaton coupling to the matter system (α) as well as the dilaton self coupling (λ). Assuming a Robertson-Walker background and using the Einstein equations we derive the gravity-matter-dilaton classical equations of motion.

In Sec. 3 we give a particular critical solution of the dilaton-matter-gravity system and we discuss its essential properties. This solution implies very precise state equations and keeps the $m_{SUSY}(t)$ and $T(t)$ time trajectories proportional. The fixed ratio $m_{SUSY}/T = \xi$ depends on the number of massless and massive states as well as the couplings α and λ.

In Sec. 4 we discuss the range of the various parameters in a semirealistic situation and we give ξ as a function of those parameters.

In Sec. 5 we demonstrate that for sufficiently low temperatures the only solution of the system is the critical solution in the sense that any other solution relaxes quite rapidly to the critical one.

The Sec. 6 is devoted for conclusions.

2. Classical Equations of Motion

The aim of this work is to derive and solve the classical field equations for the dilaton field in a non-trivial gravitational background. We will restrict ourselves to the class of homogeneous and isotropic solutions with zero space curvature. So the metric is parameterized as usual by the scale factor $R(t)$

$$g_{\mu\nu} = \text{ diag. } (1, \ -R(t), \ -R(t), \ -R(t)) \ . \tag{3}$$

Furthermore we assume that the gauge interacting particles (leptons, sleptons, quarks, squarks, Higgs bosons, higgsinos, gauge bosons and gauginos), are in thermal equilibrium at any time t defining a temperature $T(t)$. We parametrize their energy momentum tensor as usual [12]

$$S^{\mu\nu} = -pg^{\mu\nu} + (\rho + p)u^{\mu}u^{\nu}, \quad u^{\circ} = 1, \quad u_i = 0 \tag{4}$$

p and ρ are the pressure and energy densities of the gauge interacting thermalized system. They are functions of the temperature and the masses of sleptons, squarks, Higgs bosons and gauginos. So, in the absence of any other scale in the effective theory other than $m_{SUSY}(\phi_D)$ (no scale hypothesis), p and ρ are functionals of T and $m_{SUSY}(\phi_D)$. It is important to notice that this later statement is valid only at times where the Higgs vacuum expectation value $\langle H\rangle$ is equal to zero.

$$\left.\begin{aligned}\rho &= \rho(m_{SUSY},T)\\ p &= p(m_{SUSY},T)\end{aligned}\right\}\ \forall\, T > T_c : \langle H\rangle = 0 \tag{5}$$

T_c is the critical temperature of the SU(2)×U(1) $\xrightarrow{T_c}$ U(1)^{em} phase transition.

For the non-gauge interacting particles, such as the pseudoscaler ϕ_p and the gravitino ψ_μ, we assume that their contribution to the total energy momentum tensor is negligible at least for times where $T(t) < M$.

One must keep in mind that the SU(2)×U(1) $\xrightarrow{T_c}$ U(1) phase transition may occur if only and only if $m_{SUSY}(T)$ for $T < T_c$ becomes smaller than the dimensional transmutation scale Q [13,14] of the SU(2)×U(1) radiative breaking mechanism [13-15]

$$\langle H\rangle \neq 0 \iff m_{SUSY}(T) \leq Q \quad \text{for} \quad T < T_c\ . \tag{6}$$

Indeed, the existence of SU(2)×U(1) breaking minima request that a (mass)^2 of a Higgs doublet becomes negative for scale μ smaller than Q [13,14,2-5],

$$\frac{m_H^2}{m_{SUSY}^2} = C(\alpha_i(\mu)) \leq 0 \quad \text{for} \quad \mu \leq Q \tag{7}$$

where the dimensionless function C depends only on the dimensionless parameters of the theory; namely the gauge and Yukawa effective coupling constants. The relation (7) can only occur if the massive supersymmetric particles are not decoupled at scale Q and give non-zero contribution to the mass renormalization of m_H^2 [14]. Detailed analysis of the radiative breaking mechanism [13-15] shows that the transmutation scale Q defined by the equality $C(\alpha_i(Q)) = 0$ is hierarchically smaller than M [14,1-5]: $Q = M\exp[-0(1)/\alpha_2]$ and takes the desired value $Q \simeq 10^{-16}M$ when the gauge coupling constants $\alpha_1,\ \alpha_2$ and α_3 are of order unity at M (gauge unification hypothesis) also when the ratio $\alpha_t(Q)/\alpha_2(Q)$ lies in the range 1/3 to 2 [13-15,1-5] (prediction of the top quark mass).

We are now in a position to derive and solve the dilaton field equations for temperatures $T > T_c\ :\ \langle H\rangle = 0$. First notice that the general dilaton-matter Lagrangian density has the form

$$L_{eff} = \sqrt{-g}\left(-\frac{1}{2}\mathbf{R} + \frac{1}{2}\, g^{\mu\nu}\partial_\mu\phi_D\partial_\nu\phi_D + \mathcal{L}^{matter}(g^{\mu\nu}, m_{SUSY}(\phi_D), \phi_i) - U_D(\phi_D)\right) + \dots \tag{8}$$

where the dots denote non-gauge interacting fields like ψ_μ, ϕ_p etc, and ϕ_i stands for the gauge interacting ones; $\mathcal{L}^{matter}$ is defined so that

$$\mathcal{L}^{matter}\Big|_{\phi_i=0} = 0 \tag{9a}$$

The energy momentum tensor $S^{\mu\nu}$ of Eq. (4) is related with $\mathcal{L}^{matter}$,

$$S^{\mu\nu} = -\frac{2}{\sqrt{-g}}\,\frac{\delta}{\delta g_{\mu\nu}}\,\left(\sqrt{-g}\ \mathcal{L}^{matter}\right) \tag{9b}$$

while the total energy momentum tensor (including dilaton) is

$$T^{\mu\nu} = -\hat{p}g^{\mu\nu} + (\hat{\rho} + \hat{p})u^\mu u^\nu + \partial^\mu\phi_D\partial^\nu\phi_D - \frac{1}{2}\, g^{\mu\nu}(\partial\phi_D)^2 \tag{10a}$$

where we have defined the effective pressure and density of the system as

$$\hat{p} = p - U_D, \quad \hat{\rho} = \rho + U_D \tag{10b}$$

Our main assumption is that the dilaton field keeps its essential property of coupling to the trace of the energy momentum tensor $\hat{S}^\mu_\mu$ of the effective theory, or

$$\Box\phi_D = -a(\hat{\rho} - 3\hat{p}) = -a\hat{S}^{\mu}_{\mu} \tag{11a}$$

with

$$\hat{S}^{\mu\nu} = S^{\mu\nu} + g^{\mu\nu}U_D = -\hat{p}g^{\mu\nu} + (\hat{\rho} + \hat{p})u^{\mu}u^{\nu} \tag{11b}$$

The proportionality constant a in Eq. (11) will be completely determined by consistency requirements as we will see immediately. Indeed, the dilaton Eq. (11a) must not be in contradiction with the fact that $-\hat{p}$ plays the role of an effective potential for the dilaton

$$\Box\phi_D = \frac{\partial\hat{p}}{\partial\phi_D} \tag{12a}$$

So the consistency of our assumption that ϕ_D is a real dilaton field implies (Eqs. (11a,b) and (12a))

$$\frac{\partial\hat{p}}{\partial\phi_D} = -a(\hat{\rho} - 3\hat{p})\ , \quad \forall\, T > T_c \tag{12b}$$

As is stated above, the gauge interacting system defines a thermodynamic system $(\rho,\ p,\ T)$ and consequently it satisfies the integrability condition

$$T\frac{\partial p}{\partial T} = \rho + p \tag{13a}$$

On the other hand, the scale hypothesis (Eq. (5)) implies

$$\left(m_{SUSY}\frac{\partial}{\partial m_{SUSY}} + T\frac{\partial}{\partial T}\right)p = 4p, \quad \forall\, T > T_c \tag{13b}$$

which express the fact that p has scale dimension 4. It then follows (Eq. (13a,b)) that

$$m_{SUSY} \frac{\partial}{\partial m_{SUSY}} p = -(\rho - 3p) \tag{14a}$$

or

$$\frac{\partial}{\partial \phi_D} p = -\alpha(\rho - 3p) \quad . \tag{14b}$$

From Eqs. (12b) and (14b) we find

$$\frac{\partial U_D}{\partial \phi_D} = (a - \alpha)\ (\rho - 3p) + 4aU_D \ , \quad \forall\, T > T_c \tag{14c}$$

Since U_D depends only on ϕ_D while $\rho - 3p$ has an explicit dependence on T as well, the validity of the above equation for every $T > T_c$ implies that the proportionality constant a in Eqs. (11a) and (12b) is not arbitrary but is fixed to $a = \alpha$. Also it implies that U_D has to scale like the fourth power of m_{SUSY}

$$U_D = \lambda m^4_{SUSY} = \lambda M^4 \ \exp\ 4\alpha\phi_D \tag{15}$$

with λ a dimensionless constant. Equation (15) is of main importance; it shows that the existence of a dilaton field in the effective theory request the absence of terms in U_D with different scaling properties than m^4_{SUSY}. For instance, terms like $m^2_{SUSY} M^2$ are forbidden in the effective theory.

Before writing down the system of classical equations which involve ϕ_D, ρ, p and the scale factor R we stress again that Eq. (13b) is valid only if all scales are proportional to m_{SUSY} and when $m_{SUSY}(T) > Q \simeq 10^{-16} M$ so that the Higgs vacuum expectation value is zero, $\langle H \rangle = 0$. In the presence of the renormalization scale μ one must assume that the theory is renormalized using a field dependent renormalization such that $\mu =$ const $\times\, m_{SUSY}(\phi_D)$. Within these assumptions, the classical field equations for ϕ_D, ρ, p and R are

$$\ddot{\phi}_D + 3H\dot{\phi}_D + \alpha\,(\hat{\rho} - 3\hat{p}) = 0 \tag{16a}$$

$$\dot{\hat\rho} + 3H(\hat\rho + \hat p) - \alpha\dot\phi_D(\hat\rho - 3\hat p) = 0 \qquad (16b)$$

$$3H^2 = \hat\rho + \frac{1}{2}\,\dot\phi_D^2 \qquad (16c)$$

$$-\,\mathbf{R}(t) = 6\left(\dot H + 2H^2\right) = \hat\rho - 3\hat p - \dot\phi_D^2 \qquad (16d)$$

In Eqs. (16a-d) and in what follows we work in $M = 2.4 \times 10^{18}$ GeV units for simplicity. By the dot "$\cdot$" we denote derivation with respect to the time t and by $H(t)$ we denote the Hubble function $H = \dot R/R$. Its value at a time t gives the expansion rate of the universe. The first Eq. (16a) is the dilaton equation which is explicitly derived before. Equation (16b) is obtained by the conservation equation of the total energy momentum tensor, $T^{\mu\nu};\ \nu = 0$, and it takes that form once the dilaton Eq. (16a) is used. Finally the two last Eqs. (16c,d) follow from the Einstein equations

$$\mathbf{R}^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}\mathbf{R} = T^{\mu\nu} \qquad (17)$$

Equation (16c) corresponds to $\mu = \nu = 0$ while Eq. (16d) is nothing but the trace of Eq. (17). The trace Eq. (16d) is not an independent one but it can be obtained using Eqs. (16a-c).

3. The Critical Solution

The desired classical solution, $\phi_D(t)$, has to satisfy the system of coupled Eqs (16a-c). In general one may expect that the dilaton trajectory at low temperatures as well as its value at $T \simeq Q = 10^{-16}M$ is sensible and strongly dependent on the initial high temperature dilaton value $\phi_D(T \simeq M)$. If that was the case then the existence of the SU(2) $\times$ U(1) phase transition and the hierarchy generation mechanism in no scale models will be questionable or otherwise ambiguous depending on the initial random value of $\phi_D(T \simeq M)$. However this uncomfortable situation never happens. As we will see below the dilaton time trajectory relaxes at low temperatures to a critical one keeping the dimensionless ratio $m_{SUSY}(\phi_D(t))/T(t)$ constant.

$$\frac{m_{SUSY}(t)}{T(t)} \xrightarrow[t\ \text{large}]{} \xi_c + O\left(\frac{1}{t^{1/4}}\right), \quad T \geq T_c \tag{18}$$

The value of the dimensionless constant ξ_c is of order unity and is completely independent on the initial conditions $\phi_D(T \simeq M)$ or $m_{SUSY}(T \simeq M)$. This low temperature scale identification between m_{SUSY} and T time trajectories is very crucial and guarantees the existence of the SU(2)×U(1) phase transition at a critical temperature $T_c \simeq Q = 10^{-16}M$. The precise value of T_c is completely independent on the initial value of $m_{SUSY}(T \simeq M)$ and depends only on the critical fixed point ξ_c. For temperatures lower than T_c the Higgs vacuum expectation value $\langle H\rangle_T$ and m_{SUSY} will relax to their zero temperature values which are close to the transmutation scale $Q \simeq 10^{-16}$ M [2-5] and the dilaton stops to evolve

$$\langle H\rangle_0 \simeq m_{SUSY}(T \simeq 0) \simeq 10^{-16}M \tag{19}$$

Indeed the appearance of the extra scale Q (quantum generated) implies that for temperatures smaller than Q the values of $\langle H\rangle_T$ and m_{SUSY} are of order Q and follow by the minimization of the zero temperature effective potential; in addition the dilaton acquires a mass of order $Q^2/M \simeq 10^{-5}$ eV. The presence of the scale Q does not affect our previous assumption [Eq. (5)] for $T \gg Q$. We will not examine here this relaxation mechanism but it is obvious that the transition takes place quite rapidly since the value of m_{SUSY} at T_c is already of order Q, so very close to its zero temperature value. The relaxation mechanism becomes more complicated in the case where ϕ_D continues to behave like a dilaton for $T < T_c$; we will examine this situation elsewhere.

During the time interval of m_{SUSY} and T identification the time trajectories of all the dynamical quantities: $\hat{\rho}$, $\hat{p}$, H, ϕ_D, T, and m_{SUSY} are attracted to the following critical ones

$$\hat{\rho}^c = \frac{C_\rho}{t^2} , \quad C_\rho = \frac{6\alpha^2 - 1}{8\alpha^2} \tag{20a}$$

$$\hat{p}^c = \frac{C_p}{t^2} , \quad C_p = \frac{2\alpha^2 - 1}{8\alpha^2} \tag{20b}$$

$$H^c = \frac{1}{2t} \quad \text{or} \quad R^c(t) = ct^{1/2} \tag{20c}$$

$$\phi_D^c = \frac{1}{\alpha} C_D - \frac{1}{2\alpha} \log\, t , \quad \text{or} \quad m^c_{SUSY}(t) = \frac{e^{C_D}}{t^{1/2}} \tag{20d}$$

$$T^c = \frac{C_T}{t^{1/2}} \tag{20e}$$

It is simple to check that the above critical trajectories satisfy the classical field Eqs. (16a-d) with a fixed state equation given by

$$\frac{\hat{\rho}(t)}{\hat{p}(t)} = \frac{6\alpha^2 - 1}{2\alpha^2 - 1} \tag{21}$$

The other important characteristic of the critical trajectories (Eq. (20)) is that the curvature scale vanishes $\mathbf{R} = 0$ even in the presence of coupled massive degrees of freedom ($\rho - 3p \neq 0$). Also the entropy of the thermalized system is conserved with RT = constant even if the thermalized system interacts non-trivially with the non-thermalized dilaton field.

$$\frac{\partial}{\partial t} \left(R^3 S\right) = \frac{\partial}{\partial t} \left[R^3 \left(\frac{\rho + p}{T} \right) \right] = 0 \ . \tag{22}$$

$\mathbf{R} = 0$ and $R^3 S$ = constant are valid because, the non-trivial time evolution of $\phi_D(t)$ keeps the ratio m_{SUSY}/T constant so that ρ/T^4 and p/T^4 are constant quantities.

We show later that the critical solution of Eq. (20) is the only stable one in the sense that any other solution relaxes to the critical one for sufficiently large times.

Before performing that important demonstration we derive some useful relations between the critical point $\xi_c = m^c_{SUSY}/T^c$ and the other dimensionless parameters of the theory; namely the parameter α and λ as well as the number of chiral and vector (N_c and N_V) gauge interacting supermultiplets. In a semirealistic model the range of those parameters is quite restricted as we will see immediately.

The constant coefficients C_D and C_T of the critical solution Eq. (20) as well as ξ_c are completely determined in a specific model once the parameter α and λ are known. Indeed at a given temperature T the density and pressure are functions of T and $\xi = m_{SUSY}/T$:

$$\hat{\rho} = T^4 \left(f_\rho(\xi) + \lambda\xi^4\right) \tag{23a}$$

$$\hat{p} = T^4 \left(f_p(\xi) - \lambda\xi^4\right) \tag{23b}$$

where $f_\rho(\xi)$ and $f_p(\xi)$ are the statistical distributions of the thermodynamical system (ρ, p, T). For instance assuming that all supersymmetric partners of the chiral fermions as well as the gauginos, have masses of order m_{SUSY}, then

$$f_\rho(\xi) = 2N_c \left(I^B_\rho(\xi) + I^F_\rho(0)\right) + 2N_V \left(I^B_\rho(0) + I^F_\rho(\xi)\right) \tag{24a}$$

$$f_p(\xi) = 2N_c \left(I^B_p(\xi) + I^F_p(0)\right) + 2N_V \left(I^B_p(0) + I^F_p(\xi)\right) \tag{24b}$$

where $I^{B,F}_{\rho,p}$ are the statistical integrals for Bose and Fermi distributions.

$$I^{B(F)}_\rho(\xi) = \frac{1}{\pi^2}\int_0^\infty dq\,\frac{q^2\epsilon(\xi)}{e^{\epsilon(\xi)} \mp 1}\,, \quad \epsilon(\xi) = \sqrt{q^2+\xi^2} \tag{25a}$$

$$I^{B(F)}_p(\xi) = \frac{1}{\pi^2}\int_0^\infty dq\,\frac{q^4/\epsilon(\xi)}{e^{\epsilon(\xi)} \mp 1} \tag{25b}$$

N_c, N_V are the number of chiral and vector supermultiplets respectively. In Eq. (24a,b) we assumed for simplicity equal supersymmetry breaking masses. In reality the mass breaking scale of a given supermultiplet takes a value around m_{SUSY}

$$m_i = a_i \, m_{SUSY} \tag{26}$$

where a_i are calculable constants in any specific model [13-15,2-5]. Defining m_{SUSY} such that $\sum a_i/(N_c+N_V) \simeq 1$ Eqs. (24a,b) are good approximate formulas valid in a general model.

The critical constant ξ_c is then determined algebrically by the state Eq. (21) together with (23a,b)

$$f_\rho(\xi) - 3f_p(\xi) - \gamma f_\rho(\xi) + (4-\gamma)\lambda\xi^4 = 0 \tag{27a}$$

where

$$\gamma = \frac{2}{6\alpha^2 - 1} \tag{27b}$$

Once ξ_c is known the coefficients C_T and C_D are also determined using the Eqs. (24a,b)

$$C_T = \left(\frac{6\alpha^2-1}{8\alpha^2}\right)^{1/4} \frac{1}{(f_p(\xi_c)+\lambda\xi_c^4)^{1/4}} \quad \text{and} \quad C_D = \log\left(\xi_c C_T\right) \tag{29}$$

Therefore all critical trajectories are completely fixed once the parameters α, λ, N_c and N_V are known.

4. The Range of the Parameters and ξ_c

In a semirealistic model N_c and N_V are quite well known. For instance in the minimal model with three families and two light Higgs doublets $N_c = 45+4 = 49$, $N_V = 12$ while with four Higgs doublets $N_c = 53$.

The parameter λ is unknown in general and depends on the observable as well as the hidden sector of the theory. However the requirement of vanishing cosmological constant at $T \ll T_c$, fixes this parameter

$$\lambda m_{SUSY}^4 \left|_{T \ll T_c} \simeq \mathcal{O}(\langle H \rangle)^4 \right. \tag{30a}$$

Also, the quantum stability requirement for $\langle H \rangle$ and the validity of the radiative breaking mechanism [13-15] of the SU(2)$\times$U(1) $\rightarrow$ U(1)^{em} implies that [14]

$$m_{SUSY} \left|_{T \ll T_c} \leq g_2 \langle H \rangle \right. \tag{30b}$$

where g_2 is the SU(2) coupling constant. A lower bound for m_{SUSY} comes from the experimentally unobserved charged supersymmetric scalers below 30-40 GeV.

$$\frac{1}{8} \langle H \rangle < m_{SUSY} \left|_{T \ll T_c} \right. \tag{30c}$$

Therefore in all semirealistic supergravity models λ has to lie in the range (see Eqs. (30a,b,c)).

$$10^{-3} \leq \lambda \leq 10 \tag{31}$$

Finally the value of the parameter α depends on the way where m_{SUSY} is related to the gravitino mass $m_{3/2}$. In no scale models there is a strong relation between the normalized dilaton field ϕ_D and the Kähler potential G of the theory [4]

$$\phi_D = \frac{1}{\sqrt{6}} \left. G \right|_{\langle \phi_i \rangle} + \text{ const.} \tag{32}$$

A consequence of this relation is that the dilaton dependence of the gravitino mass is fixed to [4,11]

$$m_{3/2} = Me^{G/2} = M \ \exp\left(\sqrt{3/2}\,\phi_D + \ \text{const.}\right) \ . \tag{33}$$

Therefore, if m_{SUSY} scales like the k^{th} power of $m_{3/2}$ then the parameter $\alpha = k\sqrt{3/2}$.

$$m_{SUSY} = \frac{m_{3/2}^k}{M^{k-1}} \to \alpha = k\sqrt{3/2} \tag{34}$$

For models with a non zero-tree level mass splitting in the gauge interacting sector of the theory, m_{SUSY} is proportional to $m_{3/2}$ and $\alpha = \sqrt{3/2}$. This statement is valid for all models which respect the U(1) non-compact symmetry [3,4,10,11] $\phi_p \to \phi_p +$ const.

The only possible deviation from the $\alpha = \sqrt{3/2}$ rule may arise in the class of no scale models where the tree level $m_{SUSY}^{T.L.}$ is identically zero [4]. For these models the only supersymmetry breaking terms present at the tree level are the gravitino – Goldstino mass terms. This is for instance the case of the SU(N,1)/SU(N)$\times$U(1) model [4] in the absence of tree level gaugino masses. This model was found as the effective four dimensional theory which emerges from the ten dimensional $E_8 \times E_8'$ superstring after compactification of the internal six dimensions on a Ricci flat manifold [7,8]. A non-zero supersymmetry breaking in the graviton-dilaton sector arises under the assumption of the E_8' gaugino condensation [16,17].

Even though $m_{SUSY}^{T.L.} = 0$ (semi)-classically, the non-vanishing mass splitting in the dilaton-graviton supermultiplet is expected to be communicated to the gauge interacting sector through gravitational interactions. Dimensional analysis implies that

$$m_{SUSY} = m_{3/2}\ f\left(\frac{m_{3/2}}{M}, \frac{\Lambda_{co}}{M}\right) \tag{35a}$$

where f is a dimensionless function and Λ_{co} is a loop momentum cutoff scale. If a fundamental theory of gravity exists, then the effective value for Λ_{co} is either

M or $m_{3/2} = M \ \exp \sqrt{3/2}\ \phi_D$, one of the two scales of the theory. (A non-zero vacuum expectation value of a scale $\langle \phi_i \rangle \neq 0$ is also proportional either to M or $m_{3/2}$.) In any case the dimensionless function f depends only on $m_{3/2}/M$. Expanding f in powers of $m_{3/2}/M$ we obtain

$$m_{SUSY} = m_{3/2} \left(C_1 + C_2 \left(\frac{m_{3/2}}{M} \right) + C_3 \left(\frac{m_{3/2}}{M} \right)^2 + \ldots \right) \tag{35b}$$

The first non-vanishing coefficient C_k will determine the parameter $\alpha = k\sqrt{3/2}$ in the effective theory. If for some unknown reason the pertubative expansion of f does not exist, then it is possible that k is not an integer. In any case, the existence of the exact supersymmetry ($m_{3/2} \rightarrow 0$) implies that k is a positive number. The most probable situation is when $k = 1$ or, $k = 2$ in more symmetric cases. In the absence of a strong symmetry reason in the fundamental theory of gravity it is very difficult to understand values of k larger than three.

For any value of the parameter α larger than 1/2 ($\gamma < 4$) there is at least one solution with $\xi_c > 0$ for any N_V, N_c and $\lambda > 0$. In fig. (1) we plot ξ_c as function of λ for different choices of α and with N_c and N_V chosen 49 and 12 respectively. When $\alpha = \sqrt{3/2}$ there is a unique stable critical solution (dashed line). For α larger than 1.6 the situation changes. There are in general three critical solutions $\xi_c^{(1)} < \xi_c^{(2)} < \xi_c^{(3)}$. The smaller one ($\xi_c^{(1)}$) exists for all values of $\lambda > 0$ while $\xi_c^{(2)}$ and $\xi_c^{(3)}$ exist only when $\lambda/(N_c + N_V)$ is sufficiently small. For instance when $\alpha = 2\sqrt{3/2}$ there is one stable critical solution for $\lambda > 2.5 \times 10^{-4}$ while for $\lambda < 2.5 \times 10^{-4}$ there are two stable $\xi_c^{(1)}$, $\xi_c^{(3)}$ and one unstable $\xi_c^{(2)}$ (solid lines). For semirealistic values of λ and α there is only one solution ξ_c which lies in the region

$$0.4 < \xi_c < 6 \tag{36}$$

5. The Stability of the Critical Solutions

In what follows we will show that there always exists at least one stable critical solution for $\alpha > 1/2$ $(\gamma < 4)$ and $\lambda > 0$. Any other type of solution will be unstable in the sense that it relaxes rapidly at a low temperature to a stable critical one. For this purpose it is convenient to choose as independent variables the logarithm of the ratio m/T, the temperature T and the scale factor R,

$$z \equiv \log \frac{m_{SUSY}}{T} = \alpha\phi_D - \log T \; . \tag{37}$$

Also we convert the time derivatives to derivatives with respect to log R using the identity

$$\frac{\partial}{\partial t} A = H \frac{\partial}{\partial \log R} A \equiv H \overset{\circ}{A}(R) \; . \tag{38}$$

Because of the Eq. (16b), it is possible to express $\alpha\overset{\circ}{\phi}_D$ in terms of z and $\overset{\circ}{z}$ without any further dependence on T and R

$$\alpha\overset{\circ}{\phi}_D = \overset{\circ}{z} a(z) - 1 \tag{39a}$$

where $a(z)$ is a positive function of z given in terms of the statistical distributions $f_\rho(z)$, $f_P(z)$ and $f'_\rho(z) = \frac{\partial}{\partial z} f_\rho(z)$

$$a(z) = \frac{4\, f_\rho - f'_\rho}{3(f_\rho + f_P)} \geq 1 \tag{39b}$$

On the other hand Eq. (16c) permits us to express the ratio H^2/T^4 in terms of z and $\overset{\circ}{z}$ only, indeed,

$$\frac{H^2}{T^4} = h(z, \overset{\circ}{z}) > 0; \quad h(z, \overset{\circ}{z}) = \frac{2\alpha^2}{6\alpha^2 - (a(z)\overset{\circ}{z} - 1)^2} \cdot \left(f_\rho(z) + \lambda e^{4z}\right) \tag{40}$$

Finally, using these relations (39) and (40) we may eliminate ϕ_D and T from the dilaton Eq. (16a) and obtain a second order differential equation involving only the function $z(R)$

$$h(z,\overset{\circ}{z})\ \left(a(z)\overset{\circ\circ}{z}+a'(z)\overset{\circ}{z}^{2}\right)+b(z)a(z)\overset{\circ}{z}+V'_{eff}(z)=0 \tag{41a}$$

with

$$b(z)=\frac{1}{2}\ \frac{H^2-\overset{\circ}{H}H}{T^4}=\frac{1}{2}\ \frac{\hat{\rho}-\hat{p}}{T^4}=\frac{1}{2}\ \left(f_\rho(z)-f_p(z)+2\lambda e^{4z}\right) \tag{41b}$$

and

$$V'_{eff}(z)=\frac{1}{3\gamma}\ \left(f_\rho-3f_p-\gamma f_\rho+(4-\gamma)\lambda e^{4z}\right) \tag{41c}$$

Notice that $b(z)$ is a positive function as well as $a(z)$ and $h(z,\overset{\circ}{z})$. Also that $V'_{eff}(z)$ can be interpretated as the derivative of an "effective potential"

$$V_{eff}(z)=\int^{z}dz\ V'_{eff}(z)=\frac{1}{3\gamma}\left(-f_p(z)+\frac{4-\gamma}{4}\lambda e^{4z}-\gamma\int^{z}dz\ f_\rho(z)\right) \tag{42}$$

The critical solutions ($z=$ constant) are those which correspond to the stationary points of $V_{eff}(z)$. Indeed $V'_{eff}(z)=0$ is nothing else but the state Eq. (27) which determines the critical solutions. Because of the positivity of a, h and b the stable solutions are those which correspond to the local minima of V_{eff} while any solution which corresponds to a local maximum is unstable. That there exists at least one stable solution follows from the fact that $V_{eff}(z)$ goes to $+\infty$ when $|z|\to+\infty$ (for $0<\gamma<4,\ \lambda>0$)

$$V_{eff}\longrightarrow\begin{cases}\frac{4-\gamma}{3\gamma}\lambda e^{4z}-\frac{1}{3\gamma}f_\rho(\infty)z, & z>0\\ f_\rho(-\infty)|z|, & z<0\end{cases} \tag{43a}$$

where

$$f_\rho(\infty)=2\frac{\pi^2}{15}\left(N_V+\frac{7}{8}N_c\right) \tag{43b}$$

and

$$f_\rho(-\infty)=2\frac{\pi^2}{8}(N_V+N_c) \tag{43c}$$

If there is only one critical solution then it is always a stable one (see fig. (2a)).

If there are three solutions, ($\alpha \geq 2\sqrt{3/2}$ and $\lambda/N_T \leq 4 \times 10^{-6}$), $z_c^{(1)} < z_c^{(2)} < z_c^{(3)}$, then $z_c^{(1)}$ and $z_c^{(3)}$ are stable while $z_c^{(2)}$ is unstable (see fig. (2b)).

The general solution $z(R)$ of the differential Eq. (41a) describes a dumping oscillation around the local minima of $V_{eff}(z)$. For large R (low temperatures) $z(R)$ relaxes to (one of) the stable critical solution(s) (see fig. 3). The relaxation temperature depends on the initial conditions, while the critical point $z_c = \log\xi_c$ is completely independent of those. For instance starting with an initial z_I close enough to z_c then the trajectory $z(R)$ is given by

$$z(R) = z_c + \left(\frac{R_I}{R}\right)^{1/2} A\cos\left(\omega\,\log\frac{R}{R_I} + \phi\right)\,,\quad R \geq R_I \qquad (44a)$$

where A and ϕ depends on z_I and $\overset{\circ}{z}_I$ while

$$\omega = \sqrt{\delta - \frac{1}{4}}\,,\quad \delta = \frac{V''(z_c)}{a(z_c)h(z_c,0)} \qquad (44b)$$

Eq. (44a) is valid only if $|A| \simeq \mathcal{O}(1)$.

If initially $z(R_I) \gg z_c$ then $z(R)$ has the following asymptotic behavior,

$$z(R) \simeq z_I\; - (3k-1)\,\log\frac{R}{R_I} + \mathcal{O}\left(\frac{1}{R}\right),\;\left(\alpha = k\sqrt{3/2}\right) \qquad (45)$$

while if $z(R_I) \ll z_c$, $z(R)$ behaves as

$$z(R) = z_I +\; \log\frac{R}{R_I} + \mathcal{O}\left(\frac{1}{R}\right) \qquad (46)$$

with $T \cdot R =$ const.

In both cases a relaxation temperature T_R exists where the system enters the dumping oscillation regime described by the Eq. (44a) (see fig. 3).

$$T_R = T_I e^{-z_0} \ , \quad z_0 > 0 \tag{47}$$

with

$$z_0 = \begin{cases} \frac{z_I - z_c}{3k-1} \ , & z_I > z_c \\ -(z_I - z_c) \ , & z_I < z_c \end{cases}$$

For instance, if the magnitude of m_{SUSY} at very high temperature ($T_I \simeq M$) is around the gravitational scale, $10^{-4}T_I < m^I_{SUSY} < 10^4 T_I$, then, the relaxation temperature is bigger than 10^{14} GeV $\simeq 10^{-4}M$ while for m^I_{SUSY} relatively small ($m^I_{SUSY} < 10^{-4}M$), the relaxation temperature is $T_R \simeq m^I_{SUSY}$. Therefore for any random value of m^I_{SUSY} the system relaxes to its critical trajectory ($m_{SUSY}(T) = \xi_c T$) before the SU(2)× U(1) $\xrightarrow{T_c}$ U(1) phase transition. Only if m^I_{SUSY} is hierarchically smaller than M from early beginning $\left(m^I_{SUSY} < 10^{-16}M\right)$ the system does not have enough time to reach its critical trajectory before the SU(2)×U(1) phase transition. In any case only the solutions which predict a vast hierarchy between $m_{SUSY}(T \simeq Q)$ and M are permitted by the dilaton field equations. Consequently the hierarchical ratio $m_{SUSY}/M \leq 10^{-16}$ is perfectly natural when in the effective theory there exists a physical dilaton field.

CONCLUSIONS

In this work we show that the existence of a dilaton in an effective $N = 1$ no scale supergravity implies a very particular time evolution of the gravity-matter-dilaton dynamical system. The cosmological evolution of the universe is qualitatively and quantitatively very different of the standard description where the masses of the particles are taken as constant parameters. Now the masses are dynamical quantities and evolve in a very particular way ($m_i(t) \simeq \xi_c T(t)$). The energy density due to the massive degrees of freedom is never negligible compared to that coming from the massless ones, for all temperatures larger than the dimensional transmutation scale ($T > Q_0$). So the standard notion of decoupling is not applied here.

The ratio m_{SUSY}/T takes a precise fixed value and guarantees that the SU(2)×U(1)→ U(1)^{em} phase transition happens when the universe is sufficiently old with $T_c \simeq Q_0 = 10^{-16}M$. The details of the transition are completely independent of the very high energy condition of the matter + dilaton system; they depend only on the low temperature fixed value of $\xi_c = m_{SUSY}/T$. The fact that ξ_c in any semirealistic model is of order unity, implies that the magnitude of m_{SUSY} around the SU(2)×U(1) phase transition ($T \sim T_c$) is already very close to its zero temperature value, so it ensures that the relaxation of the system to

$$\langle H \rangle_{T\simeq 0} \simeq m_{SUSY}|_{T\simeq 0} \simeq Q_o = 10^{-16}M$$

happens before the recombination period so that the successful results of nucleosynthesis remain undisturbed. It is the first time that one may relate successfully the age of the universe at the SU(2)×U(1) phase transition with the observed scale hierarchies:

$$t(T_c) \leq c_{age}\, \frac{M}{m^2_{SUSY}}$$

where c_{age} is a well determined coefficient given in terms of ξ_c and the other dimensionless parameters of the effective theory.

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FIGURE CAPTIONS

Fig. 1. The dependence of the critical point $\xi_c = m^c_{SUSY}/T^c$ on the parameter λ.

Fig. 2. (a) The effective potential $V_{eff}(z)$ for $\alpha = \sqrt{3/2}$ and $\lambda = 1$ ($N_c = 49$, $N_V = 12$). There is a unique minimum. (b) The effective potential $V_{eff}(z)$ for $\alpha = 2\sqrt{3/2}$ and $\lambda = 10^{-1}$, 2×10^{-4}, 10^{-5} ($N_c = 49$, $N_V = 12$). For $\lambda > 2.5 \times 10^{-4}$ there is one minimum. For $\lambda < 2.5 \times 10^{-4}$ there are two minima and one maximum.

Fig. 3. The m_{SUSY}/T relaxation at low temperatures starting from large (solid line) or small (dashed line) initial values $m_{SUSY}/T > \xi_c$ and $m_{SUSY}/T < \xi_c$ respectively.

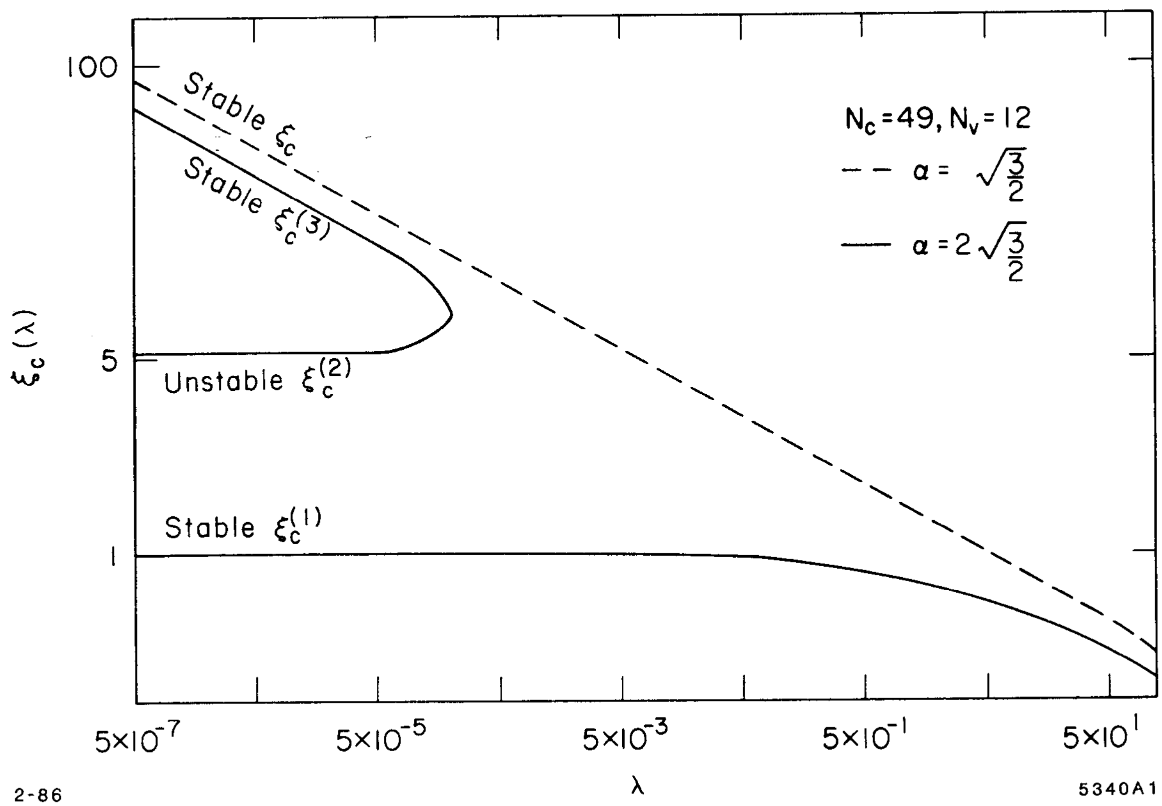
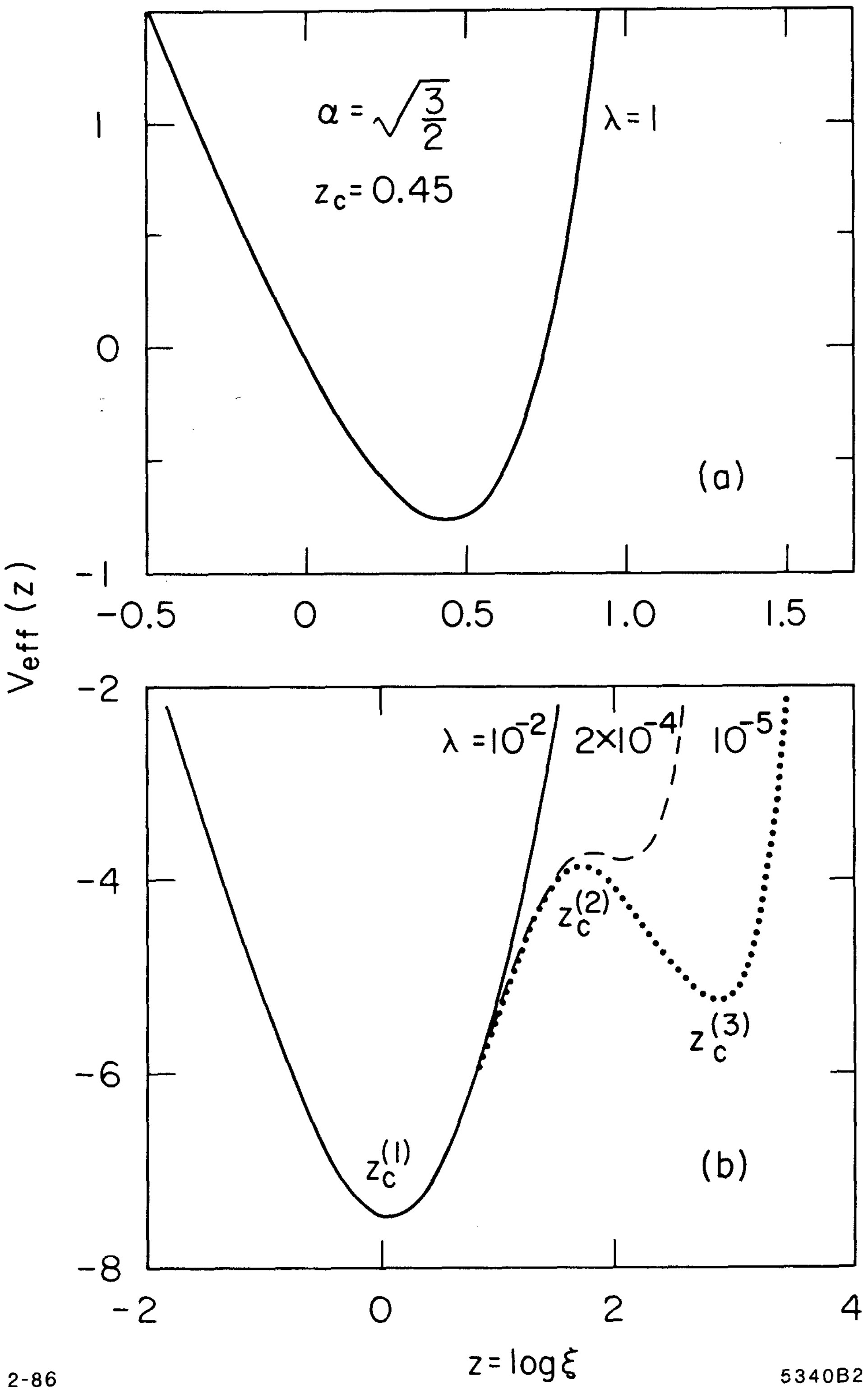

Stable ξc
Stable ξc(3)
Unstable ξc(2)
Stable ξc(1)
Nc=49, Nv=12
α = √(3/2)
α = 2√(3/2)
100
5
1
ξc(λ)
5×10^-7
5×10^-5
5×10^-3
5×10^-1
5×10^1
λ
2-86
5340A1

Fig. 1



$\alpha = \sqrt{\frac{3}{2}}$
$z_c = 0.45$
$\lambda = 1$
(a)
V_{eff} (z)
$\lambda = 10^{-2}$
2×10^{-4}
10^{-5}
$z_c^{(2)}$
$z_c^{(3)}$
$z_c^{(1)}$
(b)
$z = \log \xi$
2-86
5340B2

Fig. 2

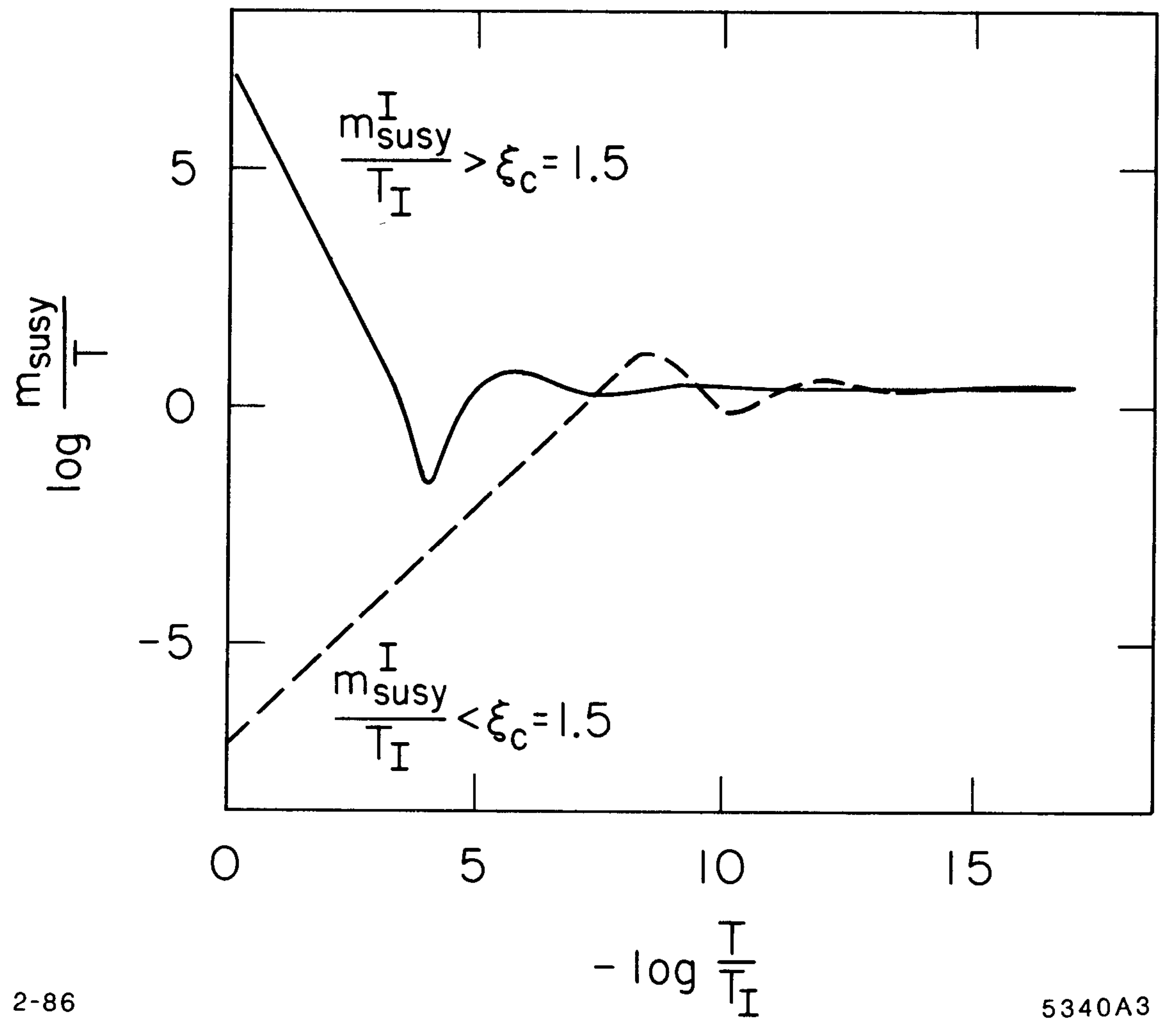


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Fig. 3