On $D^0 - \overline{D}^0$ mixing and CP violation

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ABSTRACT

Observable $D^0 - \overline{D}^0$ mixing would strongly indicate New Physics. The MARK III group has found intriguing, though so far marginal evidence for $Prob.(D^0 \to \overline{D}^0) \sim 1\%$. Effects due to quantum statistics are highly important in properly interpreting the signal and treating the background. We analyze these effects in general. $D^0 - \overline{D}^0$ mixing will also lead to like-sign dileptons in deep inelastic neutrino nucleon scattering. We point out that if $Prob.(D^0 \to \overline{D}^0) \sim 1\%$ held then certain decays like $D^0 \to K_S \pi^0$ could exhibit CP asymmetries on the percent level. Extended Higgs (or Technicolour) models with flavour changing neutral currents would be the only good candidates. Therefore we urge further experimental study along this direction.

I. Introduction

$D^0 - \overline{D}^0$ mixing is being searched for by two different methods:

(i) Like-sign di-leptons from semileptonic $D\overline{D}$ decays; various groups have obtained upper limits on Prob.$(D^0 \to \overline{D}^0)$ of at most a few percent [1].

(ii) Searching for D^0 decays into K^+ mesons. The MARK III collaboration has recently presented data[2] which can be interpreted as suggesting $D^0 - \overline{D}^0$ mixing on the 1% level. Such an analysis in general suffers from the drawback that there exists another physics process leading to the same final state, namely doubly Cabibbo supressed decays (hereafter referred to as DCSD). Unfortunately the strength of such more exotic decays has neither been measured before nor treated theoretically in a detailled way. This gap has to be filled, in particular since observable $D^0 - \overline{D}^0$ mixing would signal new physics as explained later on.

This note will be organized as follows: first we analyze the highly important consequences quantum statistics has on the strength of the signal sought - $D^0 - \overline{D}^0$ mixing - and on the size of the background - doubly Cabibbo suppressed D decays; then we apply a detailled theoretical analysis to DCSD; next we point out on which level $D^0 - \overline{D}^0$ mixing will produce like-sign dileptons in neutrino scattering; then the phenomenology of CP violation in D^0 decays is discussed; finally we estimate the size of $D^0 - \overline{D}^0$ mixing expected in the Standard Model and address the issue of how physics beyond the Standard Model would affect $D^0 - \overline{D}^0$ mixing.

II. Quantum Statistics and $D^0 - \overline{D}^0$ Mixing

The strength of $D^0 - \overline{D}^0$ mixing can conveniently be characterized in the following way:

$$Prob.(D^0 \to \overline{D}^0) \equiv \frac{BR(D^0 \to l^- + X)}{BR(D^0 \to l^+ + X)} = \frac{x^2 + y^2}{2}$$

with $x = \frac{\Delta m}{\Gamma}, \Delta m = |m_2 - m_1|, y = \frac{\Delta \Gamma}{2\Gamma}, \Delta \Gamma = |\Gamma_2 - \Gamma_1|$. However in most cases the D mesons are produced pairwise. It was first pointed out in ref.3 and later

treated in a more general situation in ref.4 that because of quantum statistics one has to consider whether the $D\overline{D}$ pair is produced with relative orbital angular momentum even or odd.

The initial $D^0\overline{D}^0$ state with momentum k_1, k_2 and time of decay t_1, t_2 is given by

$$|i> = |D^0(k_1, t_1)\overline{D}^0(k_2, t_2) > +(-1)^l|D^0(k_2, t_2)\overline{D}^0(k_1, t_1) > \qquad (1)$$

with l=relative orbital angular momentum and

$$|D^0(t) > = g_+(t)|D^0 > +\frac{q}{p}g_-(t)|\overline{D}^0 > \qquad (2)$$

$$|\overline{D}^0(t) > = \frac{p}{q}g_-(t)|D^0 > +g_+(t)|\overline{D}^0 > \qquad (3)$$

$$\frac{p}{q} = \frac{1+\epsilon}{1-\epsilon} \qquad (4)$$

$$g_\pm = \frac{1}{2}(e^{im_1 t - \frac{1}{2}\Gamma_1 t} \pm e^{im_2 t - \frac{1}{2}\Gamma_2 t}) \qquad (5)$$

Furthermore be f_1, f_2 two possible final states in neutral D decays with the corresponding amplitudes defined as follows:

$$A(D^0 \longrightarrow f_i) = A_i \qquad (6)$$

$$A(\overline{D}^0 \longrightarrow f_i) = \overline{A}_i, \quad i = 1, 2 \qquad (7)$$

In the limit of $x, y \ll 1$, which is quite appropriate for $D^0 - \overline{D}^0$ mixing, one then obtains

$$rate(|D^0\overline{D}^0|_l) \propto \begin{cases} \frac{1}{2\Gamma^2}\{(x^2 + y^2)(A_1 A_2 - \overline{A}_1\overline{A}_2)^2 + \\ (2 - x^2 + y^2)(A_1\overline{A}_2 - \overline{A}_1 A_2)^2\} & \text{l=odd} \\ \frac{1}{2\Gamma^2}\{3(x^2 + y^2)(A_1 A_2 + \overline{A}_1\overline{A}_2)^2 + \\ (2 - 3x^2 + 3y^2)(A_1\overline{A}_2 + A_2\overline{A}_1)^2 + \\ 8y(A_1 A_2 + \overline{A}_1\overline{A}_2)(A_1\overline{A}_2 + A_2\overline{A}_1)\} & \text{l=even} \end{cases} \qquad (8)$$

For simplicity we have assumed CP invariance in deriving eq.(8).

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Searching for $D^0 - \overline{D}^0$ mixing via like-sign dileptons is described by a special case of eq.(8):

$$A_1 = A_2 = A; \quad \overline{A}_1 = \overline{A}_2 = 0 \quad for \quad f_1 = f_2 = l^{\pm} + ... \quad in \quad D^0 \overline{D}^0 \longrightarrow l^{\pm} l^{\pm}$$

$$(9)$$

$$A_1 = \overline{A}_2 = A; A_2 = \overline{A}_1 = 0 \quad for \quad f_1 = l^+ + ..., f_2 = l^- + ... \quad in \quad D^0 \overline{D}^0 \longrightarrow l^+ l^-$$

$$(10)$$

Therefore (for $x, y \ll 1$)

$$\left| \frac{N(l^{\pm} l^{\pm})}{N(l^+ l^-)} \right|_{D^0 \overline{D}^0} = \begin{cases} \frac{x^2 + y^2}{2} & \text{l=odd} \\ \frac{3(x^2 + y^2)}{2} & \text{l=even} \end{cases} \qquad (11)$$

The other method employs, as mentioned in the Introduction, non-leptonic decays with strangeness $S = \pm 2$ in the final state. A simple example is provided by

$$f_1 = f_2 = K^- \pi^+ \quad or \quad K^- \rho^+; \quad thus \quad A_1 = A_2 = A; \quad \overline{A}_1 = \overline{A}_2 = \overline{A} \quad (12)$$

$$\left| \frac{N(K^- \pi^+, K^- \pi^+)}{N(K^- \pi^+, K^+ \pi^-)} \right|_{D^0 \overline{D}^0} = \begin{cases} \frac{x^2 + y^2}{2} & \text{l=odd} \\ \frac{3(x^2 + y^2)}{2} + 4|\rho|^2 + 8y|\rho| & \text{l=even} \end{cases} \qquad (13)$$

where in the last line we have used that $\overline{A}$ being doubly Cabibbo suppressed is tiny compared to A; $|\rho|^2 \equiv \frac{|\overline{A}|^2}{|A|^2}$ naively equals $tg^4 \theta_C$. We will come back to this point in the next section.

From eq.(13) one reads off an important result: for l=odd only mixing, i.e. x and/or $y \neq 0$, can produce $S = \pm 2$ final states of the type $f_1 = f_2 = 2 - body$ $final$ $state$ $f = K\pi, K\rho, K^* \pi$, but DCSD cannot[5]. For $f_1 \neq f_2$ this result does not hold in general anymore. E.g. for $f_1 = K^- \pi^+, f_2 = K^- \rho^+$

one obtains from eq.(8) for x=y=0:

$$\left|\frac{N(K^{\pm}\pi^{\mp}, K^{\pm}\rho^{\mp})}{N(K^{\pm}\pi^{\mp}, K^{\mp}\rho^{\pm})}\right|_{l=odd} = |\rho_{PV} - \rho_{PP}|^2 \tag{14}$$

where we have defined

$$\rho_{PP} = \frac{A(\overline{D}^0 \longrightarrow K^-\pi^+)}{A(D^0 \longrightarrow K^-\pi^+)}, \quad \rho_{PV} = \frac{A(\overline{D}^0 \longrightarrow K^-\rho^+)}{A(D^0 \longrightarrow K^-\rho^+)} \tag{15}$$

If doubly Cabibbo suppressed decays possessed a universal suppression factor relative to Cabibbo favoured modes, i.e. $\rho_{PP} = \rho_{PV} = \rho$ then DCSD still could not contribute to $S = \pm 2$ final states for l=odd even when $f_1 \neq f_2$. However as we will discuss in some detail in the next sect. such a universal suppression factor very likely does not exist.

If f is a genuine 3-body final state like $K\pi\pi$ with the $K\rho$ and $K^*\pi$ combinations *taken out*, then one has to refine the analysis given above: for now the final state is not characterized anymore by just one momentum k. Fortunately the MARK III data show[2]that almost all $K\pi\pi$ final states originate from PV modes.

A last example for illustration:

$$\left|\frac{N(l^+\nu K^-, K^-\rho^+)}{N(l^+\nu K^-, K^+\rho^-)}\right|_{D^0\overline{D}^0} = \begin{cases} \rho_{PV}^2 + \frac{1}{2}(x^2 + y^2) & \text{l=odd} \\ \rho_{PV}^2 + \frac{3}{2}(x^2 + y^2) & \text{l=even} \end{cases} \tag{16}$$

To summarize this chapter's discussion: if the $D^0\overline{D}^0$ pair is produced with an *odd* relative angular momentum, then the contribution of DCSD to $S = \pm 2$ final states is strongly suppressed; it actually vanishes for f+f final states with f=2-body final state. For an *even* relative orbital angular momentum the mixing signal is enhanced (see eq.(13))- however the physical background process, namely DCSD, now contributes also with increased strength.

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Thus it is very relevant to note that $D^0\overline{D}^0$ occur in a p wave in the reactions

$$e^+e^- \longrightarrow 1\gamma \longrightarrow D\overline{D}, \quad e^+e^- \longrightarrow D^*\overline{D} + D\overline{D}^* \longrightarrow D\overline{D}\pi \qquad (17)$$

and in an s wave in

$$e^+e^- \longrightarrow D^*\overline{D} + D\overline{D}^* \longrightarrow D\overline{D}\gamma, e^+e^- \longrightarrow D^*\overline{D}^* \longrightarrow D\overline{D} + \pi\pi, \pi\gamma, \gamma\gamma \qquad (18)$$

III. Doubly Cabibbo Suppressed D Decays (=DCSD)

On the quark level there is a simple relationship between Cabibbo favoured and doubly Cabibbo suppressed transitions:

$$\mathcal{L}(\Delta C = -1, \Delta S = 1) = -tg^2\theta_c\mathcal{L}(\Delta C = -1, \Delta S = -1) \qquad (19)$$

Invoking duality ideas one can proceed to calculate the *inclusive* hadronic decays using these quark operators. The two types of D decays proceed via quark diagrams with the same topology (namely both via spectator diagrams as well as W exchange diagrams); the phase space corrections are identical (at least as long as the masses of the up and down quarks can be ignored); also the strong final state interactions are identical. Thus one finds for the total rate of DCSD:

$$\Gamma(D^0 \longrightarrow S = 1) = tg^4\theta_c\Gamma(D^0 \longrightarrow S = -1) \qquad (20)$$

However it is unlikely that just one universal suppression factor, be it $tg^4\theta_c$ or some other number, will affect the individual DCSD. We will consider two relevant examples to explain this point, namely the rates for $D^0 \longrightarrow K^+\pi^-$ vs. $D^0 \longrightarrow K^-\pi^+$ and for $D^0 \longrightarrow K^+\rho^-$ vs. $D^0 \longrightarrow K^-\rho^+$.

To compute these rates we follow the general procedure of Bauer and Stech[6]: in the effective Hamiltonian one replaces the product of quark currents by the

product of the corresponding hadron currents. This amounts to employing a factorization approximation, i.e.

$$A(D^0 \longrightarrow K^- \pi^+) \simeq < K^- |\bar{s}\gamma_\mu c|D^0 >< \pi^+|\bar{u}\gamma_\mu d|0 > \tag{21}$$

$$A(D^0 \longrightarrow K^+ \pi^-) \simeq < \pi^- |\bar{d}\gamma_\mu c|D^0 >< K^+|\bar{u}\gamma_\mu s|0) > \tag{22}$$

The matrixelements are then determined to be

$$< \pi^+|\bar{u}\gamma_\mu d|0 > = f_\pi p_\mu^\pi \tag{23}$$

$$< K|\bar{s}\gamma_\mu c|D^0 > = (p^D + p^K - \frac{m_D^2 - m_K^2}{q^2}q)_\mu \frac{m_{F^*}^2}{m_{F^*}^2 - q^2} + \frac{m_D^2 - m_K^2}{q^2}\frac{m_F^2}{m_F^2 - q^2}q_\mu \tag{24}$$

with $q = p^D - p^K$. The other matrixelelments are given by the analogous expressions. One then calculates

$$|\rho_{PP}|^2 = |\frac{A(D^0 \to K^+ \pi^-)}{A(D^0 \to K^- \pi^+)}|^2 \simeq |\frac{f_K}{f_\pi}|^2 \frac{m_D^4}{(m_D^2 - m_K^2)^2}\frac{m_F^4}{(m_F^2 - m_K^2)^2}tg^4\theta_c \sim 2.1 tg^4\theta_c \tag{25}$$

for $f_K = 166 \quad MeV, \quad f_\pi = 133 \quad MeV$. Thus SU(3) breaking introduces a significant enhancement for DCSD leading to two pseudoscalar mesons.

For decays leading to a pseudoscalar plus a vector meson one follows the analogous procedure:

$$A(D^0 \to K^- \rho^+) \simeq < K^- |\bar{s}\gamma_\mu c|D^0 >< \rho^+|\bar{u}\gamma_\mu d|0 > \tag{26}$$

$$A(D^0 \to K^+ \rho^-) \simeq < \rho^- |\bar{d}\gamma_\mu c|D^0 >< K^+|\bar{u}\gamma_\mu s|0 > \tag{27}$$

To compute the matrixelements that involve vectormesons we again use the pre-

scription of Stech[6]:

$$< \rho^+ |\bar{u}\gamma_\mu d|0 > = f_\rho m_\rho^2 \epsilon_\mu^\rho \tag{28}$$

$$< \rho^- |\bar{d}\gamma_\mu c|D^0 > \simeq 2m_\rho i \frac{\epsilon^\rho \cdot p^D}{q^2} \frac{m_F^2}{m_F^2 - q^2} q_\mu + ... \tag{29}$$

where in eq.(29) we have listed only those terms that contribute when the proper two matrixelements are contracted. Putting everything together we find

$$|\rho_{PV}|^2 = |\frac{A(D^0 \to K^+ \rho^-)}{A(D^0 \to K^- \rho^+)}|^2 \sim 0.8 |\frac{f_K}{f_\rho m_\rho}|^2 tg^4 \theta_c \simeq 0.45 \quad tg^4 \theta_c \tag{30}$$

i.e. a sizeable suppression relative to naive expectation, this time due to SU(6) breaking.

Comparing eq.(25) and (30) leads to the conclusion that at least in the factorization approximation there is no universal suppression factor for DCSD. At the same time it is interesting to note that there is no clear contradiction to the ratio of the inclusive rates as expressed in eq.(20): an enhancement in the PP decay is largely offset by a suppression in the PV mode as expected from duality ideas.

IV. $D^0 - \overline{D}^0$ mixing and like-sign dileptons

The CDHS collaboration has recently published data[7] on prompt like-sign dimuons in neutrino scattering:

$$\frac{N(\mu^\mp \mu^\mp)}{N(\mu^- \mu^+)} \sim 0.02 - 0.04 \tag{31}$$

It should be kept in mind that the signal barely reaches the 3σ level; nevertheless it is worthwhile to consider the possibility that the signal is real. The "wrong sign" muons are said to be quite consistent with coming from D decays as far as their kinematical distributions are concerned. Invoking $c\bar{c}$ production from gluons would reproduce this pattern; however perturbative calculations yield a rate roughly 30 times smaller than the observed signal.

We want to point out that also $D^0 - \overline{D}^0$ mixing would produce like-sign dimuons. For $Prob.(D^0 \to \overline{D}^0) = 0.02$ one obtains

$$\frac{N(\mu^{\mp}\mu^{\mp})}{N(\mu^{-}\mu^{+})} \sim 0.015 \tag{32}$$

assuming $D^* : D$ production of 3:1 which leads to $N(D^0) \sim 3N(D^+)$. One should also note that $D - \overline{D}$ mixing will lead to a ratio of like-sign to opposite-sign dimuons that is to a very good approximation independant of the beam energy - in clear contrast to the $c\overline{c}$ hypothesis

V. CP Violation in D^0 Decays

At first sight it might look pretentious to suggest searching for CP violation in D decays when the strength of $D^0 - \overline{D}^0$ mixing does not exceed 1 % . Yet one should keep in mind that the usual mixing observables (like like-sign dileptons etc.) depend on x^2, y^2 (see eq.(11,13)). Mixing observed at strength ~ 1 % thus corresponds to $x \sim \frac{1}{10}$. There are CP asymmetries that are only linear in x and therefore less suppressed. The main ingredient in this method (which previously has been discussed in a detailled way for B decays[8]) is to find final states that can be reached both in D^0 and $\overline{D}^0$ decays; examples are

$$f = K_S\pi^0, K_S\eta, K_S\omega, K_S\phi, K^+K^- \tag{33}$$

The first four modes are Cabibbo allowed while the last one is Cabibbo suppressed; their branching ratios range from 1/2 % to a few %. Then one finds

$$\frac{\sigma(e^+e^- \to l^+/K^- + Xf) - \sigma(e^+e^- \to l^-/K^+ + Xf)}{\sigma(e^+e^- \to l^+/K^- + Xf) + \sigma(e^+e^- \to l^-/K^+ + Xf)} = \begin{cases} 2xsin2\phi & \text{if l=even} \\ 0 & \text{if l=odd} \end{cases} \tag{34}$$

where $sin2\phi$ contains the complex phase responsible for CP violation.We also want to emphasize that now it isonly the s wave configuration for $D^0 - \overline{D}^0$ that can exhibit the interestingeffect, namely CP violation. Eq.(34) shows what was

mentioned above, namely that D^0 decays to certain final states can exhibit CP asymmetries of order $0.1 \cdot phase$, when $D^0 - \overline{D}^0$ are produced in an l=even configuration. Later we will give some estimates on the possible size of such effects.

VI. $D^0 - \overline{D}^0$ mixing and New Physics

The authors of two recent papers [9] have emphasized that

(i) long-distance dynamics presumably dominate $D - \overline{D}$ mixing and that

(ii) $Prob.(D^0 \to \overline{D}^0)$ could reach values of at most a few times 10^{-3}. Thus mixing observed at the 1% level or more would be evidence for New Physics.

We agree with these findings in general and will present the details of our calculation elsewhere. Calculations based on dispersion relations and observed decay rates implicitely include all of long distance dynamics. Therefore New Physics would contribute to $D - \overline{D}$ mixing only via short distance dynamics.

Here we will give only the main findings of our analysis on possible New Physics; the detailled results will be presented in a subsequent paper. It is very unlikely that $D^0 - \overline{D}^0$ mixing would be significantly enhanced by the presence of a fourth family, righthanded currents or supersymmetry states. The best candidate model would be based on a non-minimal Higgs sector.

Actually it has been pointed out before [10] that such models could give

$$\Delta m_D \sim (1-2) \cdot 10^{-13} GeV, \quad x_D \sim 0.1 \tag{35}$$

The ratio of quark masses squared $(\frac{m_t}{m_b})^2 \sim 100$ is the main reason in these models for $\Delta m_D \gg \Delta m_K$. Furthermore there exists no clear reason in these models why CP violating phases like $sin(2\phi)$, see eq.(34), should be particularly small.

VII. Summary

Proper consideration of quantum interference effects is highly important when searching for $D^0 - \overline{D}^0$ mixing at the percent level. Observing such a signal would be evidence for physics beyond the Standard Model. In that case it would make sense to perform a dedicated search for CP violation in D decays.

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