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ON THE REALIZATION OF CONTINUOUS FLAVOR SYMMETRIES IN LARGE N LATTICE QCD WITH SUSSKIND FERMIONS*

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Over the past ten years there have been many discussion of chiral symmetry breaking (χSB) in strongly coupled lattice gauge theories.[1] A certain percentage of these studies[2] work with a nearest neighbor fermion *Hamiltonian*, using either the naive discretization of the Dirac equation or Susskind's staggered fermions.[3] Many authors claim to find spontaneous breakdown of continuous symmetries in these models. I have always been puzzled by this since the models in question are always equivalent to N_F flavors of staggered fermions and the symmetries in question are simply the isospinlike transformations between the different flavors. Of course, one can choose a basis for the continuum fermion fields such that these symmetry generators carry γ_5's (and one[2] usually does), but the result is still puzzling.

Several years ago I found a resolution of this puzzle, but, since interest in these matters seemed to have waned, I neglected to write it up. The confusion of several colleagues over the past two years has convinced me that there may still be some interest in presenting it.

We will work within the context of the $1/N$ approximation and discuss the effective potential for local bilinears $(-1)^r \psi_i^+(r)\psi_j(r)$ whose expectation values break the flavor symmetry. We prove the following things.

1. At $N = \infty$ QCD has several inequivalent degenerate vacua, some of which break the flavor symmetry, but one of which does not. These vacua are separated from each other in field space—they are isolated minima of the potential.

2. The degeneracy of the $N = \infty$ vacuum is accidental; a consequence of the fact that at $N = \infty$ different flavors contribute additively to the vacuum energy. At next order in $1/N$ the degeneracy is split and the symmetric vacuum is preferred.

Note that, as a consequence of the isolation of the $N = \infty$ minima, the $1/N$ expansions around the symmetry breaking vacua will show no signs of disease (until their large order behavior is studied). The tunneling amplitude from these false vacua to the symmetric vacuum is of order e^{-N}. This reconciles the results of previous authors with the naive prejudice that the flavor symmetries are unbroken.

A more general derivation of the results that flavor symmetries are unbroken may be obtainable by use of the methods of Witten and Vafa.[4] I have not investigated this avenue of approach since it gives no insight into the apparently consistent picture of symmetry breaking found in previous work.

The Hamiltonian that we have to study is

$$H = H_{\text{gauge}} + i \sum_{m} \chi_i^+(x)\eta_m(x) \times \left[U(x, x+\hat{m})\chi_i(x-\hat{m}) - U^+(x, x-\hat{m})\chi_i(x-\hat{m})\right] \tag{1}$$

i is the flavor index running from 1 to N_F. H_{gauge} is the usual lattice gauge Hamiltonian for an $SU(N)$ gauge group.

The effective potential is defined as the Legendre transform of the vacuum energy in the presence of a source term

$$\delta H = \sum (-1)^x \chi_i^+ m_{ij} \chi_j \qquad m^+ = m \ . \tag{2}$$

The $(-1)^x$ in (2) is necessary for charge conjugation invariance. Without it we would be studying the theory in the presence of chemical potentials for fermion number densities, rather than the $\bar{\psi}\psi$ effective potential. δH breaks the single unit translation symmetries which are the true residues of chirality on the lattice.[3] It also breaks $SU(N)$ if m is not proportional to the unit matrix.

By an $SU(N)$ rotation, we can choose a basis for the ψ's for which m is diagonal

$$\delta H - \sum_i (-1)^x m_i \chi_i^+ \chi_i \ . \tag{3}$$

It is then easy to derive a path integral formula for the vacuum energy

$$e^{-W(\{m_i\})} = \int dU \; e^{-S(U)} \, e^{\sum_{i=1}^{N_f} L(m_i)} \; . \tag{4}$$

Here S is the continuous time Euclidean action for the pure gauge theory and $e^{L(m)}$ is the determinant of the lattice Dirac operator

$$e^L = \det\left(\sum_m \eta_m(x) \left(U(x,y)\delta(y - x + \hat{m}) - U^+(x,y)\delta(y - x - \hat{m}) \right) \right) \; . \tag{5}$$

In the large N limit the average of the product of two gauge invariant functions factorizes and

$$W(\{m_i\}) = -\sum_{i=1}^{N_f} \langle L(m_i) \rangle \equiv -\sum_{i=1}^{N_f} \frac{\int dU \, e^{-S(U)} L(m_i)}{\int dU \, e^{-S(U)}} \; . \tag{6}$$

This is the exact analog of the Coleman-Witten[5] argument for continuum QCD.[6] The effective potential $V(\{\phi_i\})$ is also a sum of independent terms for each quark

$$V(\{\Phi_i\}) = \sum_i U(\phi_i) \; . \tag{7}$$

In the continuum large N theory we can always choose the m_i (and thus ϕ_i) to be positive by a continuous chiral rotation. A non-trivial minimum of V has each ϕ_i at a non-trivial minimum of U. Since there is no reason for U to have more than one minimum, all the ϕ_i must be equal. On the lattice, however, the transformation that takes m to $-m$ is discrete and changes the gauge field configuration. it is essentially single unit lattice translation. Thus $L(m,U) \neq L(-m,U)$ although $\langle L^n(m) \rangle = \langle L^n(-m) \rangle$. Consequently, $U(\phi)$ will have two non-trivial minima at $\pm\, \phi_0$, if it has any at all.

If $\phi_0 \neq 0$, large N lattice QCD (with Susskind fermions) has $2^{N_F - 1}$ inequivalent vacua where each ϕ_i is $\pm\, \phi_0$. (Note that two vacua with *all* the ϕ_i reversed are equivalent. They are related by a discrete lattice chiral transformation—a one unit translation.) All but one of these minima spontaneously break $SU(N_f)$.

This degeneracy is clearly due to the "additive quark" nature of the $N = \infty$ limit. We next examine the $1/N$ corrections to the vacuum energy. The value of the effective potential at any of its minima is given by a value of $W(\{m_i\})$ at $m_i = 0$. W is multivalued at $m = 0$. The branch of W corresponding to a particular pattern of signs for the ϕ_i is picked out by choosing the opposite pattern of signs for the m_i (to lower the energy) and then taking $m \to 0$.

The $1/N$ correction to the vacuum energy is the dispersion of $\sum_i L(m_i)$:

$$\delta W(\{m_i\}) = -\left\langle \left(\sum_i L(m_i)\right)^2 - \left\langle \sum_i L(m_i) \right\rangle^2 \right\rangle . \tag{8}$$

Since we are interested in configuration in which the m_i are equal up to a sign, the term which breaks the degeneracy is

$$-\sum_{i \neq j} \langle L(m_i) L(m_j) \rangle . \tag{9}$$

Each term in the sum is either $\langle L^2(m) \rangle$ or $\langle L(m)L(-m) \rangle$. Since $L(m) \neq L(-m)$ for fixed gauge field configuration

$$\langle (L(m) - L(-m))^2 \rangle > 0 . \tag{10}$$

(Note that this is a strict inequality; $>$ *is not* $\gtrsim$) and

$$\langle L^2(m) \rangle > \langle L(m)L(-m) \rangle . \tag{11}$$

The vacuum energy is lowered by having all the ϕ_i equal. Thus, as advertised, $1/N$ corrections to the effective potential choose the symmetric vacuum.

It should be emphasized again that the consequences of this result cannot be seen in finite orders of the $1/N$ expansion around a particular vacuum. The zeroth order degenerate vacua are isolated and tunneling between the false vacua and the true one is an e^{-N} effect.

CONCLUSIONS

The results of this investigation lead one to question again the conventional interpretation of the even-odd "continuous chiral symmetry" of Euclidean Susskind fermions. This symmetry can be realized in the continuum either as a vector current or an "axial isospin" current $\phi^{+}\gamma_5 T_3 \phi$ and can be either spontaneously broken or conserved in the continuum limit.

The two vacua (in which the symmetry is broken respectively conserved) are related by a continuous chiral transformation and are degenerate and have identical physics. This is not so on the lattice and the question of which vacuum is preferred in weakly coupled near-continuum QCD has never been answered. The issue is not really resolved by Monte-Carlo calculations (which prefer the vacuum with spontaneous breaking) because they work with a mass term which orients the vacuum in the direction which spontaneously breaks the "chiral" symmetry and "extrapolate" their results to zero mass.*

The conventional picture of spontaneous breakdown of the even-odd symmetry is based on strong coupling large N expansions. We have seen that such considerations can be misleading in the Hamiltonian formalism unless all vacua are taken into account and $1/N$ corrections to the effective potential computed. Indeed, in the time continuum limit, the even-odd symmetry becomes one of the flavor symmetries we have studied here. So the conventional calculation certainly chooses the wrong vacuum in the time continuum limit. Unfortunately, on a Euclidean lattice the symmetric vacuum corresponds to the expectation value for a one link operator and it is not possible to extend the simple arguments we have made here.

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* This probably means that the resolution of the question will have little practical effect on Monte-Carlo calculations if one believes that reasonable results can be obtained at fairly large values of the mass term. Even if the weakly coupled lattice theory prefers the vacuum in which the "chiral" lattice symmetry is unbroken, the mass term may be seen as a trick for forcing the lattice system into a spontaneously broken vacuum which becomes physically equivalent in the continuum limit.

However it seems a bit unreasonable for the asymmetric vacuum to be energetically preferred for all finite values of the time lattice spacing and to be disfavored in the time continuum limit.† I suspect that there is at least a finite range of time lattice spacings and gauge couplings for which the conventional calculations are finding a meta stable false vacuum. Perhaps this is even true for the Euclidean symmetric version of the theory.

Even in this case there is a possible escape for someone who would like to believe that the $1/N$-string coupling expansion around a spontaneously broken vacuum can eventually be made into a tool for calculating the properties of physical hadrons. (Similar remarks reply to the Hamiltonian calculations of Ref. 2.) The meta stable symmetry breaking vacua of the lattice theory share many properties with *some* of the degenerate vacua of the continuum. Even if the lattice vacuum preserves the even-odd continuous symmetry, some states which break this symmetry become degenerate with the vacuum in the continuum limit. It is not implausible to expect that these are meta stable states which are smooth extrapolations of the vacua found in many strong coupling, large N calculations. Then the conventional calculational schemes would work (in principle), although their theoretical justification would be somewhat more complicated than one had thought.

† Actually, there is a scenario where this is reasonable. Imagine plotting the difference in energy between the symmetric and asymmetric vacua in the time lattice spacing $(a), -1/N$ plane. We now that if $a = 0$ it is negative for a range of N near $N = \infty$ and vanishes at $N = \infty$. There might be a line of zeroes in the $a - \frac{1}{N}$ plane coming in to the origin.(see fig.1) Then the symmetric vacuum would be preferred on the small N side of this line. The finite "a" theory would have a first order phase transition at a finite value of N.At strong coupling, where one link operators probably are suppressed, this scenario is particularly plausible.

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