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BEAMSTRAHLUNG FROM COLLIDING ELECTRON-POSITRON BEAMS WITH NEGLIGIBLE DISRUPTION*

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ABSTRACT

We present radiative energy loss formulas for beamstrahlung from colliding electron-positron beams which experience negligible disruption, as determined by numerical simulation. Our computer code uses the correct quantum mechanical photon number spectrum for synchrotron radiation emitted by relativistic electrons to simulate with macroparticles the discrete nature of photon emission. For Gaussian beams with small average electron energy loss, we determine energy loss formulas valid for all radiation regimes from classical to extreme quantum mechanical which depend on only two beam parameters, a quantum radiation parameter Υ_0 and a beam energy per unit length, Γ_0.

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1. Introduction

When a high energy electron beam and positron beam from an accelerator collide, the particles in each beam emit synchrotron radiation due to their interaction with the electromagnetic fields generated by the opposite beam (beamstrahlung).[1] Because of the intense fields and relatively short bunch lengths involved, the average number of photons emitted per electron (positron) can be small, while the individual photon energies can be significant fractions of the initial electron energy. This discrete, quantum mechanical nature of photon emission will result in a beam spectrum quite different from that expected from continuous, classical radiation loss. A quantitatively correct description of beam energy loss is of direct interest to the particle accelerator designer and high-energy experimentalist.

The simulation of the complete beam-beam interaction including beamstrahlung and beam disruption (the focussing of one beam by the fields of the opposite beam) is a very involved computational problem. Considerable simplification occurs if disruption of the beams can be neglected. For Gaussian beams, the disruption parameters for the horizontal and vertical directions are[2]

$$D_x = \frac{D_y}{R}, \qquad D_y = \frac{2N_b r_e \sigma_z}{\gamma(1+R)\sigma_y^2} , \tag{1}$$

where $R = \sigma_x/\sigma_y$ is the beam aspect ratio, N_b is the number of beam particles, $\gamma = \mathcal{E}/m_e c^2$, $\mathcal{E}$ is the particle energy, $r_e = e^2/m_e c^2$, and σ_i are the Gaussian beam widths. For high energy beams there can be accelerators of interest in which D is small.

In this paper we consider beamstrahlung in the limit $D \to 0$, so the two colliding beams do not change shape during the collision. Each beam is assumed to be initially monoenergetic with particles radiating according to the local electromagnetic fields they experience as they pass through the opposite beam. The fields generated by an ultra-relativistic beam are essentially transverse to the direction of motion with the electric and magnetic fields locally perpendicular and of equal magnitude. Self-fields of a beam are negligible compared with those generated by the oncoming beam. The fields of a beam are due to the discrete electrons and positrons, but the fields can generally be treated as continuous for the purpose of calculating radiation. A particle with a local radius of curvature ρ in the field of a beam will radiate a quantum over a characteristic distance ρ/γ. Provided that this radiation length is much greater than the inverse longitudinal beam density, σ_z/N_b, the beam field may be treated as locally continuous with scale lengths given by the beam dimensions.[3]

2. Quantum Synchrotron Radiation

With disruption neglected, the analytic expressions for the transverse electromagnetic fields generated by a Gaussian beam can be used throughout the beamstrahlung simulation. In units of the critical field, $F_c = m_e^2 c^3/e\hbar$, the electric field (which equals the magnetic field H in Gaussian units) at a point (x, y, z) relative to the beam center can be written as

$$E(x,y,z)/F_c = (\Upsilon_0/\gamma_0)\ f_R(x,y,z) \tag{2}$$

where

$$\Upsilon_0 = \frac{5}{6}\ \frac{N_b\ r_e\ \lambdabar_e \gamma_0}{(1+R)\sigma_y \sigma_z}\ , \tag{3}$$

$\gamma_0 = \mathcal{E}_0/m_e c^2$, $\mathcal{E}_0$ is the initial beam energy, and $\lambdabar_e = \hbar/m_e c$. The function f_R is given by the following expressions for round ($R = 1$) and flat ($R > 1$) beams respectively,[4]

$$f_{R=1} = \frac{24}{5}\ \frac{e^{-z^2/2\sigma_z^2}}{(2\pi)^{1/2}}\ \frac{1-e^{-r^2/2\sigma_y^2}}{r/\sigma_y}, \qquad r = (x^2+y^2)^{1/2}\ , \tag{4}$$

$$f_{R>1} = \frac{6}{5}\ (1+R)\ \frac{e^{-z^2/2\sigma_z^2}}{(R^2-1)^{1/2}} \cdot \left| w\left(\frac{x+iy}{\sqrt{2(R^2-1)}\ \sigma_y}\right) - e^{-[(x^2/2\sigma_x^2)+(y^2/2\sigma_y^2)]}\ w\left(\frac{x/R+iRy}{\sqrt{2(R^2-1)}\sigma_y}\right)\right|\ , \tag{5}$$

where the complex error function is defined by

$$\omega(\varsigma) = e^{-\varsigma^2}\left[1+\frac{2i}{\sqrt{\pi}}\int_0^{\varsigma} e^{\varsigma'^2} d\varsigma'\right]\ . \tag{6}$$

The utility of defining the beam parameter Υ_0 in Eq. (2) becomes evident when we consider quantum synchrotron radiation from an electron. The quantum mechanical photon number and power spectra for the radiation emitted by an unpolarized ultra-relativistic electron with energy $\mathcal{E} = \gamma m_e c^2$ in a homogenous external electromagnetic field, $F_{\mu\nu}$, can be written as[5]

$$N(\omega) = \frac{P(\omega)}{\hbar\omega} = \frac{\alpha}{\sqrt{3}\pi\gamma^2}\left[\int_{\xi}^{\infty} K_{5/3}\ (\eta)\ d\eta + \left(\frac{\hbar\omega}{\mathcal{E}}\right)^2 \left(1-\frac{\hbar\omega}{\mathcal{E}}\right)^{-1}\ K_{2/3}\ (\xi)\right]\ , \tag{7}$$

where α is the fine structure constant, ω is the photon frequency, K_ν is the modified Bessel function of the second kind, $\xi = 2(\hbar\omega/\mathcal{E})/3\Upsilon(1-\hbar\omega/\mathcal{E})$, $\Upsilon = |\Pi_\mu F^{\mu\nu}\ \Pi^\lambda F_{\lambda\nu}|^{1/2}/m_e c F_c$, and Π_μ is the electron mechanical momentum. The expression (7) is valid when $|\frac{1}{2}F_{\mu\nu}F^{\mu\nu}|^{1/2} \ll F_c$ and $\frac{1}{4}\tilde{F}_{\mu\nu}F^{\mu\nu} = \bar{E}\cdot\bar{H} = 0$. The Lorentz invariant parameter Υ characterizes the quantum mechanical nature of the radiation. Familiar classical radiation corresponds to $\Upsilon \ll 1$ in which the classical synchrotron frequency is $\hbar\omega_c = 3\Upsilon\mathcal{E}/2$, whereas $\Upsilon \gg 1$ corresponds to extreme quantum radiation in which the peak position of the synchrotron power spectrum approaches the electron energy.

The total radiated power from an electron is

$$P = \int_0^{\mathcal{E}/\hbar} P(\omega)\ d\omega = \frac{2}{3}\ \alpha\ \frac{m_e c^3}{\lambda\!\!\!^{-}_e}\ g(\Upsilon) \tag{8}$$

where

$$g(\Upsilon) \simeq \begin{cases} \Upsilon^2, & \Upsilon \ll 1 \\ 0.5564\ \Upsilon^{2/3}, & \Upsilon \gg 1 \end{cases}, \tag{9}$$

and the photon emission rate is

$$N = \int_0^{\mathcal{E}/\hbar} N(\omega)\ d\omega = \frac{5\alpha}{2\sqrt{3}}\ \frac{c}{\lambda\!\!\!^{-}_e}\ \frac{1}{\gamma}\ h(\Upsilon) \tag{10}$$

where

$$h(\Upsilon) \simeq \begin{cases} \Upsilon, & \Upsilon \ll 1 \\ 1.012\Upsilon^{2/3}, & \Upsilon \gg 1 \end{cases}. \tag{11}$$

For intermediate values of Υ, there are no simple analytic forms for the functions $g(\Upsilon)$ and $h(\Upsilon)$. Table I contains representative values of these functions in the range $10^{-3} \le \Upsilon \le 10^3$.

When calculating beamstrahlung in the laboratory frame (i.e. the center of mass frame of the two beams) where $\bar{E}$, $\bar{H}$ and the electron momentum $\bar{\Pi}$ are essentially mutually perpendicular, the above formulas may be used locally for a given beam electron (positron) with $\Upsilon \simeq 2\gamma\ E/F_c$ and E/F_c given by Eq. (2)

for a Gaussian beam.[6] The essential approximation made here is that the local beam field can be treated as homogeneous. The beam field has longitudinal and transverse scale lengths σ_z and $\sigma_\perp = \sigma_{x,y}$, respectively, relative to the electron velocity vector. Provided that for a given electron the radiation length $\rho/\gamma \ll \sigma_z$ or $\Upsilon \gg \lambdabar_e \gamma/\sigma_z$, the beam field can be viewed as homogeneous longitudinally. Similarly if the transverse deflection distance $\rho/\gamma^2 \ll \sigma_\perp$ or $\Upsilon \gg \lambdabar_e/\sigma_\perp$, the field is essentially homogeneous transversely. We restrict our attention in this paper to linear colliders in which these conditions are satisfied for most beam particles.

3. Numerical Simulation

Without disruption the beamstrahlung simulation is reduced conceptually to calculating the relative energy loss $\Delta\mathcal{E}/\mathcal{E}$ of particles in one beam as they pass undeflected through a region of space occupied by the transverse electromagnetic fields generated by the oncoming beam. Dimensionally relative energy loss is related to radiated power by $\Delta\mathcal{E}/\mathcal{E} = P\cdot(\Delta z/c)/\mathcal{E}$ where Δz is the distance the particle travels as it radiates energy $\Delta\mathcal{E}$. Since power is a function of the beam radiation parameter Υ_0 and aspect ratio R (in the absence of transverse beam offsets), only one additional parameter, the beam energy per unit length, must be specified in the simulation. We choose to define this parameter as

$$\Gamma_0 = \frac{5}{9}\ \frac{\gamma_0 \lambdabar_e}{\sigma_z}\ . \tag{12}$$

Consequently only the three physical parameters Υ_0, Γ_0 and R (plus two transverse beam offsets if necessary) are required to characterize beamstrahlung from monoenergetic beams without disruption.[7]

With the beam physics now well defined, it is straightforward to discretize the beamstrahlung problem to a form amenable to numerical simulation. Although it is computationally impossible to track the individual positrons and electrons in each beam, it is also unnecessary. Individual particles move along smooth trajectories determined by the fields generated by the superposition of particles in the oncoming beam. We are not concerned with the stochastic nature of photon emission from individual electrons but only the simulation of the discrete effects of photon emission from many electrons which on average radiate according to Eq. (7). Under these conditions we may simulate the actual beams with macroparticles and mean fields. Macroparticles represent many real particles in a volume element of the beam but have the same charge to mass ratio as an electron. Mean fields replace the electromagnetic fields generated by the oncoming beam in a volume element by a local, constant field.

In the present simulation program, the radiating Gaussian beam is taken as a box of size $2N_\perp\sigma_x \times 2N_\perp\sigma_y \times 2N_\parallel\sigma_z$, with the integers $N_\perp$ and $N_\parallel$ supplied as input parameters. This box is divided into $2N_\perp\, n_\perp \times 2N_\perp n_\perp \times 2N_\parallel n_\parallel$ cubes with the integers $n_\perp$ and $n_\parallel$ also supplied as input. Each cube is a macroparticle with a constant fractional charge (in units of N_b) assigned according to a Gaussian charge distribution. The radiating beam moves undeflected along the z-axis through a region of space occupied by the transverse fields of the oncoming beam. This region is also divided into $2N_\perp n_\perp \times 2N_\perp n_\perp \times 2N_\parallel n_\parallel$ cubes and defines a mean field array. The value of the field in each cube is given by Eq. (2) evaluated at the cube center. The macroparticle and mean field arrays are moved through each other in $4N_\parallel n_\parallel - 1$ equal steps. The distance Δz travelled by these arrays in each step is $\sigma_z/2n_\parallel$. Macroparticles radiate according to the constant field value in the mean field cube that they overlap with at a particular step.

The relative energy loss from a macroparticle at each step is calculated under two different assumptions. The first is the assumption of continuous energy loss in which the energy change is simply $\Delta\gamma/\gamma = P(\Upsilon)\cdot(\Delta z/c)/\gamma m_e c^2$, and the average number of photons emitted per electron is $N_p = N(\Upsilon,\gamma)\cdot\Delta z/c$, where $P(\Upsilon)$ and $N(\Upsilon,\gamma)$ are the total power (8) and emission rate (10) respectively, and $\Upsilon = 2\gamma E/F_c$. The step size Δz is assumed to be sufficiently small that $\Delta\gamma/\gamma$ and N_p are much less than unity. Since the number of macroparticles is assumed to be large, this approach necessarily gives the correct values for all average beam quantities (i.e. first moments of distributions). However, because the number of photons emitted per electron during the collision is typically not a large number, the use of a continuous energy loss algorithm will not give the correct results for beam quantities dependent on the discrete effects of photon emission (e.g. r.m.s. energy spreads). To obtain such information requires the use of a discrete energy loss algorithm.

The determination of the discrete energy loss from a macroparticle consists of two calculations in the simulation. First, if at a given step $p_{r1} \leq N_p$, where p_{r1} comes from a uniform random number generator ($0 \leq p_{r1} \leq 1$), then the macroparticle is allowed to radiate some fraction of its energy. The energy to be radiated is determined by inverting the photon number spectrum (7) using the following standard technique.[8] The cumulative probability of emitting a photon with energy fraction $\hbar\omega'/\mathcal{E}$ in the range $[0,\ \hbar\omega/\mathcal{E}]$ is

$$\Pi_c(\hbar\omega'/\mathcal{E} \in [0,\ \hbar\omega/\mathcal{E}]) = \int_0^{\hbar\omega/\mathcal{E}} N(\omega')\ d(\hbar\omega'/\mathcal{E}) \Big/ \int_0^1 N(\omega')\ d(\hbar\omega'/\mathcal{E})\ , \quad (13)$$

where $0 \leq \Pi_c \leq 1$. The function Π_c can be used as a random number generator with a distribution equivalent to the photon number spectrum. The inverse of this function is defined by $\Omega(\Pi_c) = \hbar\omega/\mathcal{E}$, where $0 \leq \hbar\omega/\mathcal{E} \leq 1$. In the simulation a probability p_{r2} from a uniform random number generator is supplied as the argument of Ω (represented by a two-dimensional table $\Omega(\Pi_c, \Upsilon)$) to obtain the discrete radiated energy loss of a macroparticle, $\Delta\gamma/\gamma = \Omega(p_{r2}) = \hbar\omega/\mathcal{E}$.

4. Simulation Results

Using the code just described, we have determined by numerical simulation the behavior of eight beam related quantities as a function of Υ_0, Γ_0 and R when the beams collide with no transverse offsets and the average electron energy loss during the collision is small (typically less than ten percent). This is the regime of immediate interest in linear colliders, although the simulation code can treat beams with transverse offsets and arbitrarily large energy losses. The first four quantities are averages over the beam distribution after the beams have collided. They are the final average electron energy loss

$$\langle \Delta\mathcal{E}/\mathcal{E}_0 \rangle = \langle (\mathcal{E}_0 - \mathcal{E})/\mathcal{E}_0 \rangle \ , \tag{14}$$

the r.m.s. electron energy spread

$$\sigma_{\mathcal{E}}/\mathcal{E}_0 = \left(\langle (\Delta\mathcal{E}/\mathcal{E}_0)^2 \rangle - \langle \Delta\mathcal{E}/\mathcal{E}_0 \rangle^2 \right)^{1/2} \ , \tag{15}$$

the average photon number per electron $\langle N_p \rangle$ and the average photon energy $\langle \hbar\omega/\mathcal{E}_0 \rangle$.

Four other quantities of interest are the average center of mass energy loss, the average center of mass energy squared loss and the associated r.m.s. energy spreads. Center of mass (CM) averages for two finite size beams require information about both beam distributions. We use a definition for these averages due to Yokoya,[9] the so-called "luminosity-weighted" averages. We define a center of mass coordinate system whose origin is the collision point of the bunch centers, with the positive s-axis along the direction of beam 1 and the x and y axes perpendicular to the s-axis. In addition z_1 (z_2) is a longitudinal coordinate co-moving with beam 1 (beam 2) with origin at the bunch center. The bunch centers collide at time $t = 0$. Since the beams move at the speed of light, a point $(x,\ y,\ s,\ t)$ has longitudinal coordinates $z_1 = s - ct$ and $z_2 = -s - ct$ relative to the two bunch centers, respectively.

Using the coordinate system $(x,\ y,\ s,\ t)$, the luminosity is given by

$$\mathcal{L} = 2f \int n_1(x,\ y,\ s - ct,\ t)\ n_2(x,\ y,\ -s - ct,\ t) dx\ dy\ ds\ cdt\ , \tag{16}$$

where f is the accelerator repetition rate, and n denotes the beam particle density. The CM energy of two particles in beam 1 and beam 2 at some space-time point is $W = S^{1/2} = 2(\mathcal{E}_1 \mathcal{E}_2)^{1/2}$, where S is the CM energy squared. The average CM energy loss is defined to be the luminosity-weighted average of $\Delta W/W_0 = (W_0 - W)/W_0$,

$$\begin{aligned}\langle \Delta W/W_0 \rangle = (2f/\mathcal{L}) \int (\Delta W/W_0) \\ n_1(x,\ y,\ s - ct,\ t)\ n_2(x,\ y,\ -s - ct,\ t)\ dx\ dy\ ds\ cdt\ ,\end{aligned} \tag{17}$$

where $W_0 = 2\mathcal{E}_0$. The r.m.s. CM energy spread is

$$\sigma_W/W_0 = (\langle (\Delta W/W_0)^2 \rangle - \langle \Delta W/W_0 \rangle^2)^{1/2}\ . \tag{18}$$

Similar expressions hold for the average CM energy squared loss $\langle \Delta S/S_0 \rangle$ and r.m.s. spread σ_S/S_0.

For all Υ_0, Γ_0 and R, we have found the following remarkably simple set of radiative energy loss formulas for Gaussian beams without transverse offsets when the average energy loss $\langle \Delta\mathcal{E}/\mathcal{E}_0 \rangle$ is small,

$$\left\langle \frac{\Delta\mathcal{E}}{\mathcal{E}_0} \right\rangle = \frac{2}{3}\,\frac{\alpha}{\Gamma_0}\,g(\Upsilon_0) \tag{19}$$

$$\langle N_p \rangle = \frac{5}{2\sqrt{3}}\,\frac{\alpha}{\Gamma_0}\,h(\Upsilon_0) \tag{20}$$

$$\left\langle \frac{\hbar\omega}{\mathcal{E}_0} \right\rangle = \frac{4}{5\sqrt{3}}\,\frac{g(\Upsilon_0)}{h(\Upsilon_0)} \tag{21}$$

$$\frac{\sigma_{\mathcal{E}}}{\mathcal{E}_0} = a_1 \left\langle \frac{\Delta\mathcal{E}}{\mathcal{E}_0} \right\rangle \left(1 + \frac{a_2}{\langle N_p \rangle}\right)^{1/2} \tag{22}$$

$$\left\langle \frac{\Delta W}{W_0} \right\rangle = b \left\langle \frac{\Delta\mathcal{E}}{\mathcal{E}} \right\rangle \left(1 + \left\langle \frac{\hbar\omega}{\mathcal{E}_0} \right\rangle\right) \tag{23}$$

$$\frac{\sigma_W}{W_0} = d_1 \left\langle \frac{\Delta\mathcal{E}}{\mathcal{E}_0} \right\rangle \left(1 + \frac{d_2}{\langle N_p \rangle}\right)^{1/2} \tag{24}$$

$$\left\langle \frac{\Delta S}{S_0} \right\rangle = 2b \left\langle \frac{\Delta\mathcal{E}}{\mathcal{E}_0} \right\rangle \tag{25}$$

$$\frac{\sigma_S}{S_0} = 2d_1 \left\langle \frac{\Delta\mathcal{E}}{\mathcal{E}_0} \right\rangle \left(1 + \frac{d_3}{\langle N_p \rangle}\right)^{1/2} , \tag{26}$$

where the functions g and h are defined by Eqs. (8) and (10), and the energy loss coefficients a_i, b and d_i are given in Table II. The dependence of these expressions on the aspect ratio R is essentially contained in Υ_0 with residual variations of only a few percent for $1 \le R < \infty$. The expressions (19)–(26) when used with the coefficients in Table II can be considered accurate to the level of a few percent.

Analytic expressions exist for some of these formulas in the classical and extreme quantum radiation regimes. Equation (19) agrees well with a classical result of Bassetti and Gygi-Hanney[10] for $\langle \Delta\mathcal{E}/\mathcal{E}_0 \rangle$ in which their R-dependent form factor is well approximated by the factor $1 + R$ in Υ_0.[11] Our expressions (19)–(22) also agree with analytic results obtained by Yokoya for round beams ($R = 1$) in the classical and quantum regimes.[9] Our simulations indicate that these expressions are valid for all aspect ratios with Υ_0 defined by Eq. (3). The formulas for $\langle \Delta W/W_0 \rangle$ and σ_W/W_0 in the quantum regime differ from the analytic expressions given by Yokoya because we use the exact form $W = S^{1/2} = 2(\mathcal{E}_1\mathcal{E}_2)^{1/2}$ for the CM energy of two particles rather than the approximation $W \simeq W_0(1 - (\Delta\mathcal{E}_1 + \Delta\mathcal{E}_2)/2\mathcal{E}_0)$ of Yokoya when $\langle \Delta\mathcal{E}/\mathcal{E}_0 \rangle \ll 1$. This approximation for W is not correct in the quantum regime where particles can radiate substantial fractions of their initial energy through one photon.

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References

1. B. Richter, IEEE Trans. Nucl. Sci. NS-32, 3828 (1985).

2. R. Hollebeek, Nucl. Instrum. Methods 184, 333 (1981).

3. The discreteness of the beam can affect the synchrotron power spectrum under special conditions, namely when the individual particle fields (opening angle $1/\gamma$) within the beam do not overlap enough longitudinally for the resulting beam field to be spatially continuous, i.e. $\sigma_\perp/\gamma \ll \sigma_z/N_b$ where $\sigma_\perp = \sigma_{x,y}$. The field can in principle act like a wiggler resulting in a broad second peak near $\omega_d \sim \gamma^2 \; N_b c/\sigma_z \gg \omega_c \sim \gamma^3 c/\rho$ (the classical synchrotron frequency). We restrict our attention to the normal situation in linear colliders where ω_d exceeds the particle energy so the second peak cannot occur kinematically.

4. M. Bassetti, *et al.*, IEEE Trans. Nucl. Sci. NS-30, 2182 (1983).

5. See A. A. Sokolov and I. M. Ternov, *Synchrotron Radiation* (Pergamon, Berlin, 1968), and references therein. See also, W.–Y. Tsai and A. Yildiz, Phys. Rev. **D8**, 3446 (1973), and P. Chen and R. J. Noble, SLAC–PUB–4050 (August 1986).

6. We assume that electrons radiate independently and not coherently since the normal situation in linear colliders is that the synchrotron radiation wavelength is much less than the bunch length.

7. The numerical coefficients in Υ_0 and Γ_0 as well as the factor $1 + R$ in the denominator of Υ_0 have been chosen in hindsight to yield simple energy loss formulas for all aspect ratios in all radiation regimes without extraneous numerical factors.

8. J. M. Dawson, Rev. Mod. Phys. 55, 403 (1983).

9. K. Yokoya, KEK Report 85-9, October 1985; KEK Report 85-53, October 1985, submitted to Nucl. Instrum. Methods.

10. M. Bassetti and M. Gygi-Hanney, LEP Note 221, CERN, 1980.

11. P. Wilson, in Laser Acceleration of Particles (Malibu, California, 1985), (AIP, New York, 1985), p. 560.

TABLE I

Representative values of the functions $g(\Upsilon)$ and $h(\Upsilon)$ in the range $10^{-3} \leq \Upsilon \leq 10^3$.

Υ	$g(\Upsilon)$	$h(\Upsilon)$
10^{-3}	9.94×10^{-7}	9.99×10^{-4}
10^{-2}	9.45×10^{-5}	9.91×10^{-3}
10^{-1}	6.55×10^{-3}	9.30×10^{-2}
1	1.82×10^{-1}	7.16×10^{-1}
10	1.84	4.24
10^2	1.11×10^1	2.13×10^1
10^3	5.56×10^1	1.01×10^2

TABLE II

Behavior of the energy loss coefficients a_i, b and d_i as a function of the beam radiation parameter Υ_0 when $\langle \Delta\mathcal{E}/\mathcal{E}_0 \rangle \lesssim 0.1$.

Υ_0	a_1	a_2	b	d_1	d_2	d_3
$\lesssim 10^{-2}$	0.41	30	0.42	0.32	10	10
10^{-1}	0.38	30	0.43	0.31	10	10
1	0.31	33	0.44	0.27	14	10
10	0.25	43	0.45	0.24	18	11
10^2	0.22	53	0.46	0.22	22	12
$\gtrsim 10^3$	0.20	63	0.47	0.21	26	13