SLAC – PUB – 3857
December 1985
(M)

Exclusive Hadronic and Nuclear Processes in QCD*

Stanley J. Brodsky

Stanford Linear Accelerator Center
Stanford University
Stanford, CA 94305
and
Institute for Theoretical Physics
University of California
Santa Barbara, CA 93106

1. INTRODUCTION

With the advent of quantum chromodynamics as a complete theory of the strong interactions, one can now anticipate understanding the underlying dynamics and composition of both hadrons and nuclei at a fundamental level. In the last few years considerable progress has been made in calculating short distance hadronic scattering and production *amplitudes* in terms of quark and gluon subprocesses.[1–6] This in turn has led to a basic understanding of exclusive hadron and nuclear scattering processes at large momentum transfer as well as progress in describing the structure of hadronic and nuclear wavefunctions in terms of their fundamental quark and gluon degrees of freedom.[7]

QCD has two essential properties which make calculations of processes at short distances or high momentum transfer tractable and systematic. The critical feature is asymptotic freedom: the effective coupling constant $\alpha_s(Q^2)$ which controls the interactions of quarks and gluons at momentum transfer Q^2 vanishes logarithmically at large Q^2:

$$\alpha_s(Q^2) = \frac{4\pi}{\beta \log(Q^2/\Lambda^2_{\rm QCD})} \qquad (Q^2 \gg \Lambda^2) \tag{1.1}$$

[Here $\beta = 11 - \frac{2}{3}n_f$ is derived from the gluonic and quark loop corrections to the effective coupling constant; n_f is the number of quark contributions to the vacuum polarizations with $4m_f^2 \lesssim Q^2$.] The parameter $\Lambda_{\rm QCD}$ normalizes the value of $\alpha_s(Q_0^2)$ at a given momentum transfer $Q_0^2 \gg \Lambda^2$, within a specific renormalization or

⋆ Work supported in part by the Department of Energy, contract DE-AC03-76SF00515, by the National Science Foundation, Grant PHY82-17853, and by the National Aeronautics and Space Administration, at the University of California at Santa Barbara.

Invited talk presented at the Nuclear Chromodynamics: Quarks and Gluons in Particles and Nuclei, Santa Barbara, California, August 12-23, 1985.

cutoff scheme. Recently α_s has been determined fairly unambiguously using the measured branching ratio for upsilon radiative decay $\Upsilon(b\bar{b}) \to \gamma X$:[8]

$$\alpha_s(0.157\ M_\Upsilon) = \alpha_s(1.5\ \text{GeV}) = 0.23 \pm 0.03 \tag{1.2}$$

Taking the standard $\overline{MS}$ dimensional regularization scheme, this gives $\Lambda_{\overline{MS}} = 119^{+52}_{-34}$ MeV. In more physical terms, the effective potential between infinitely heavy quarks has the form $[C_F = 4/3$ for $n_c = 3]$,[9]

$$\begin{aligned} V(Q^2) &= -C_F \frac{4\pi\alpha_V(Q^2)}{Q^2} \\ \alpha_V(Q^2) &= \frac{4\pi}{\beta \log(Q^2/\Lambda_V^2)} \qquad (Q^2 \gg \Lambda_V^2) \end{aligned} \tag{1.3}$$

where $\Lambda_V = e^{5/6}\Lambda_{\overline{MS}} \simeq 270 \pm 100$ MeV. Thus the effective physical scale of QCD is $\sim 1\ f_m^{-1}$. At momentum transfers larger than this scale, α_s becomes small, QCD perturbation theory becomes applicable, and a microscopic description of short-distance hadronic and nuclear phenomena in terms of quark and gluon subprocesses becomes viable.

Complementary to asymptotic freedom is the existence of factorization theorems for both exclusive and inclusive processes at large momentum transfer. In the case of exclusive processes (in which the kinematics of all the final state hadrons are fixed at large relative mass) the hadronic amplitude can be represented as the product of a hard-scattering amplitude for the constituent quarks convoluted with a distribution amplitude for each incoming or outgoing hadron.[1-3] (See Sect. 2.) The distribution amplitude contains all of the bound-state dynamics and specifies the momentum distribution of the quarks in the hadron. The hard scattering amplitude can be calculated perturbatively as a function of $\alpha_s(Q^2)$. The predictions can be applied to form factors, exclusive photon-photon reactions, photoproduction, fixed-angle scattering, *etc.* In the case of the simplest processes, $\gamma\gamma \to M\bar{M}$[10] and the meson form factors, rigorous all-order proofs can be given.

The central unknown in the QCD predictions is the composition of the hadrons in terms of their quark and gluon quanta.[1] Recently, several important tools have been developed which allow specific predictions for the hadronic wavefunctions directly from the theory. A primary tool is the use of light-cone quantization to construct a consistent relativistic Fock state basis for the hadrons and their observables in terms of quark and gluon quanta. The distribution amplitude and the structure functions are defined directly in terms of these light-cone wavefunctions. The form factor of a hadron can be computed exactly in terms of a convolution of initial and final light-cone Fock state wavefunctions.[11]

A second important tool is the use of QCD sum rules to provide constraints on the moments of hadron distribution amplitudes.[3] This method has already yielded important information on the momentum space structure of hadrons which we review in Sect. 4. A particularly important advance is the construction of nucleon distribution amplitudes, which together with the QCD factorization formulae, predict the correct sign and magnitude as well as scaling behavior of the proton and neutron form factors.[3]

Another recent advance has been the development of a formalism to calculate the moments of distribution amplitudes using lattice gauge theory.[12] The initial results are extremely interesting — suggesting a highly structured oscillating momentum-space valence wavefunction for the meson. The results from both the lattice calculations and QCD sum rules also demonstrate that the light quarks are highly relativistic in the bound state wavefunctions. This gives further indication that while non-relativistic potential models are useful for ennumerating the spectrum of hadrons (because they express the relevant degrees of freedom), they are not reliable in predicting wavefunction structure.

2. EXCLUSIVE REACTIONS

We will be interested in hadronic and nuclear processes in which all final particles are measured at large invariant masses compared with each other, *i.e.*, large momentum transfer exclusive reactions. This includes form factors of hadrons and nuclei at large momentum transfer Q and large angle scattering reactions such as photoproduction $\gamma p \to \pi^+ n$, nucleon-nucleon scattering at large momentum transfer, photodisintegration $\gamma d \to np$ at large angles and energies, *etc.* A crucial result is that such amplitudes factorize[1–3] at large momentum transfer in the form of a convolution of a hard scattering amplitude T_H which can be computed perturbatively from quark-gluon subprocesses multiplied by process-independent "distribution amplitudes" $\phi(x, Q)$ which contain all of the bound-state non-perturbative dynamics of each of the interacting hadrons. To leading order in $1/Q$ the scattering amplitude has the form

$$\mathcal{M} = \int_0^1 T_H(x_j, Q) \prod_{H_i} \phi_{H_i}(x_j, Q)[dx]. \tag{2.1}$$

Here T_H is the probability amplitude to scatter quarks with fractional momentum $0 < x_j < 1$ from the incident to final hadron directions, and ϕ_{H_i} is the probability amplitude to find quarks in the wavefunction of hadron H_i collinear up to the scale Q, and

$$[dx] = \prod_{j=1}^{n_i} dx_j \delta\left(1 - \sum_k^{n_i} x_k\right). \tag{2.2}$$

The key to the derivation of this factorization of perturbative and non-perturbative dynamics is the use of the Fock basis $\{\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i)\}$ defined at equal $\tau = t + z/c$ on the light-cone to represent relativistic color singlet bound states. The λ_i specify the helicities; $x_i \equiv (k_i^0 + k_i^z)/(p^0 + p^z)$, $(\sum_{i=1}^n x_i = 1)$, and $\mathbf{k}_{\perp i}$, $(\sum_{i=1}^n \mathbf{k}_{\perp i=0})$, are the relative momentum coordinates. Thus the proton is represented as a column vector state ψ_{qqq}, ψ_{qqqg}, $\psi_{qqq\bar{q}q}$ ... In the light-cone gauge, $A^+ = A^0 + A^3 = 0$, there are no ghosts, and only the minimal "valence" Fock state needs to be considered at large momentum transfer; any additional quark or gluon forced to absorb large momentum transfer yields a power-law suppressed contribution to the hadronic amplitude. For example, at large Q^2, the baryon form factor can be systematically computed by iterating the valence Fock state wavefunction equation of motion wherever large relative momentum occurs. To leading order the kernel is effectively one-gluon exchange. The sum of the hard gluon exchange contributions is the gauge invariant amplitude T_H. The residual factor from the wavefunction is the distribution amplitude ϕ_B which plays the role of the wavefunction at the origin in the analogous non-relativistic calculation. Thus we obtain the form: [See Fig. 1(a)]

$$F_B(Q^2) = \int_0^1 [dy] \int_0^1 [dx] \phi_B^\dagger(y_j, Q) T_H(x_i, y_j, Q) \phi_B(x_i, Q), \tag{2.3}$$

where to leading order in $\alpha_s(Q^2)$, T_H is computed from $3q + \gamma^* \to 3q$ tree graph amplitudes: [Fig. 1(b)]

$$T_H = \left[\frac{\alpha_s(Q^2)}{Q^2}\right]^2 f(x_i, y_j) \tag{2.4}$$

and

$$\phi_B(x_i, Q) = \int [d^2k_\perp] \psi_V(x_i, \mathbf{k}_{\perp i}) \theta(Q^2 - k_{\perp i}) \tag{2.5}$$

is the valence three-quark wavefunction [Fig. 1(c)] evaluated at quark impact separation $b_\perp \sim \mathcal{O}(Q^{-1})$. Since ϕ_B only depends logarithmically on Q^2 in QCD, the main dynamical dependence of $F_B(Q^2)$ is the power behavior $(Q^2)^{-2}$ derived from scaling of the elementary propagators in T_H. Thus, modulo logarithmic factors,

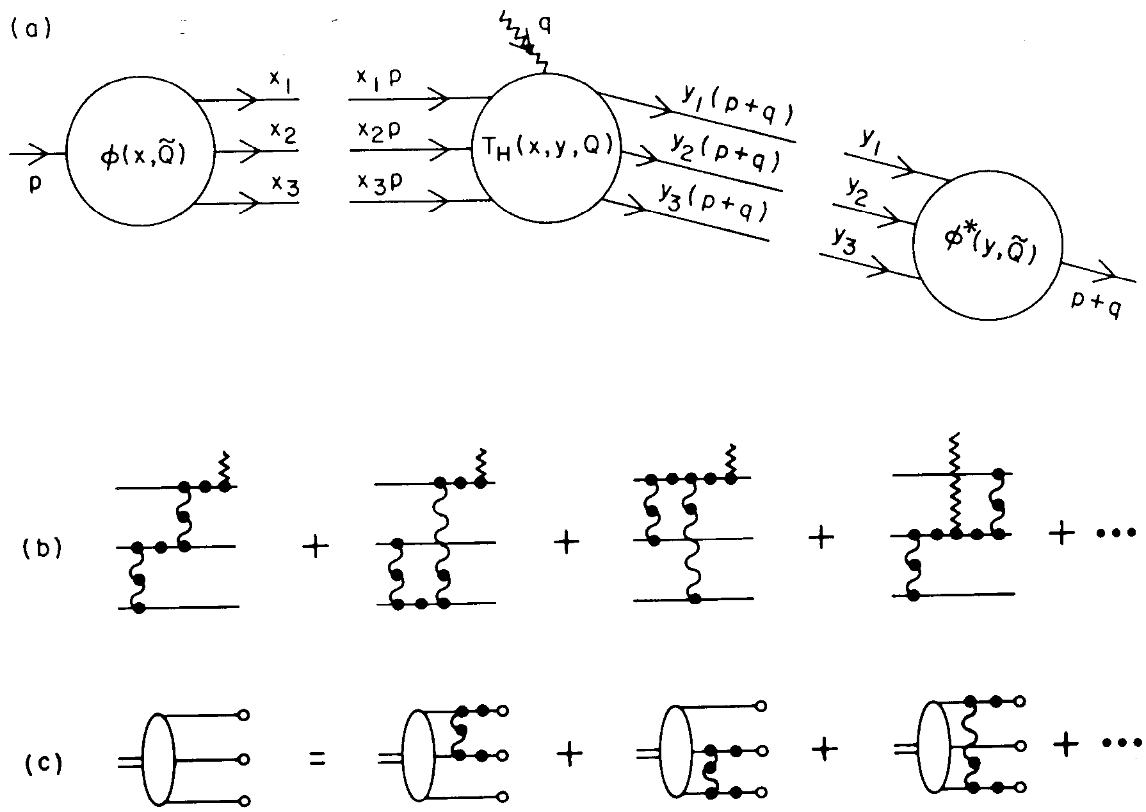

Figure 1: (a) Factorization of the nucleon form factor at large Q^2 in QCD. The optimal scale $\tilde{Q}$ for the distribution amplitude $\phi(x, \tilde{Q})$ is discussed in Ref. 1. (b) The leading order diagrams for the hard scattering amplitude T_H. The dots indicate insertions which enter the renormalization of the coupling constant. (c) The leading order diagrams which determine the Q^2 dependence of $\phi_B(x, Q)$.

one obtains a dimensional counting rule for any hadronic or nuclear form factor at large Q^2(helicity $\lambda = \lambda' = 0$ or $\frac{1}{2}$) [13]

$$F(Q^2) \sim \left(\frac{1}{Q^2}\right)^{n-1} . \tag{2.6}$$

$$F_1^N \sim \frac{1}{Q^4}, \quad F_\pi \sim \frac{1}{Q^2}, \quad F_d \sim \frac{1}{Q^{10}}, \tag{2.7}$$

where n is the minimum number of fields in the hadron. Since quark helicity is conserved in T_H and $\phi(x_i, Q)$ is the $L_z = 0$ projection of the wavefunction, total hadronic helicity is conserved at large momentum transfer for any QCD exclusive reaction.[13] The dominant nucleon form factor thus corresponds to $F_1(Q^2)$ or $G_M(Q^2)$; the Pauli form factor $F_2(Q^2)$ is suppressed by an extra power of Q^2. In the case of the deuteron, the dominant form factor has helicity $\lambda = \lambda' = 0$. The general form of the logarithmic dependence of $F(Q^2)$ can be derived from the operator product expansion at short distance or by solving an evolution equation for the distribution amplitude computed from gluon exchange [Fig. 1(c)], the only QCD contribution which falls sufficiently slowly at large transverse momentum to effect the large Q^2 dependence.

The distribution amplitude for a baryon is determined by an evolution equation which can be derived for the Bethe-Salpeter equation at large transverse momentum projected on the light-cone:

$$\left(Q^2 \frac{\partial}{\partial Q^2} + \frac{3C_F}{2\beta}\right) \phi(x_i, Q) = \frac{C_B}{\beta} \int [dy] V(x_i, y_i) \phi(y_i, Q), \tag{2.8}$$

where $C_F = (n_c^2 - 1)/2n_c = 4/3$, $C_B = (n_c + 1)/2n_c = 2/3, \beta = 11 - (2/3)n_f$, and $V(x_i, y_i)$ is computed to leading order in α_s from the single-gluon-exchange kernel. The evolution equation automatically sums to leading order in $\alpha_s(Q^2)$ all of the contributions from multiple gluon exchange which determine the tail of the valence wavefunction and thus the Q^2-dependence of the distribution amplitude. The general solution of this equation is

$$\phi(x_i, Q) = x_1 x_2 x_3 \sum_{n=0}^{\infty} a_n \left(\ell n \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n} \tilde{\phi}_n(x_i), \tag{2.9}$$

where the anomalous dimensions γ_n and the eigenfunctions $\tilde{\phi}_n(x_i)$ satisfy the characteristic equation:

$$x_1 x_2 x_3 \left(-\gamma_n + \frac{3C_F}{2\beta}\right) \tilde{\phi}_n(x_i) = \frac{C_B}{\beta} \int_0^1 [dy] V(x_i, y_i) \tilde{\phi}_n(y_i). \tag{2.10}$$

In the large Q^2 limit, only the leading anomalous dimension γ_0 contributes to the form factor.

A useful technique for solving the evolution equations is to construct completely antisymmetric representations as a polynomial orthonormal basis for the distribution amplitude of multiquark bound states. In this way one obtains a distinctive classification on nucleon (N) and delta (Δ) wave functions and the corresponding Q^2 dependence which discriminates N and Δ form factors. The antisymmetrization technique is presented in detail in ref. 14 for nuclear systems.

The result for the large Q^2 behavior of the baryon form factor in QCD is then[1-3]

$$G_M(Q^2) = \frac{\alpha_s^2(Q^2)}{Q^4} \sum_{n,m} d_{nm} \left(\ell n \frac{Q^2}{\Lambda^2} \right)^{-\gamma_m - \gamma_n} \tag{2.11}$$

where the γ_n are computable anomalous dimensions of the báryon three-quark wave function at short distance and the d_{mn} are determined from the value of the baryon distribution amplitude $\phi_B(x, Q_0^2)$ at a given point Q_0^2, and the normalization of T_H. The dominant part of the form factor comes from the region of the x integration where each quark has a finite fraction of the light cone momentum; the end point region where the struck quark has $x \simeq 1$ and spectator quarks have $x \sim 0$ is suppressed by quark (Sudakov) form factor gluon radiative corrections.

In Table I we give a summary of the main scaling laws and properties of large momentum transfer exclusive and inclusive cross sections which are derivable starting from the light-cone Fock space basis and the perturbative expansion for QCD.

Table I Comparison of Exclusive and Inclusive Cross Sections

Exclusive Amplitudes	Inclusive Cross Sections
$\mathcal{M} \sim \prod \phi(x_i, Q) \otimes T_H(x_i, Q)$	$d\sigma \sim \prod G(x_a, Q) \otimes d\hat{\sigma}(x_a, Q)$
$\phi(x, Q) = \int^Q [d^2k_\perp] \psi_{\rm val}^Q(x, k_\perp)$	$G(x, Q) = \sum_n \int^Q [d^2k_\perp][dx]' \lvert\psi_n^Q(x, k_\perp)\rvert^2$
Measure ϕ in $\gamma\gamma \to M\bar{M}$	Measure G in $\ell p \to \ell X$
$\sum_{i \in H} \lambda_i = \lambda_H$	$\sum_{i \in H} \lambda_i \neq \lambda_H$
Evolution	
$\frac{\partial \phi(x,Q)}{\partial \log Q^2} = \alpha_s \int [dy] V(x, y) \phi(y)$	$\frac{\partial G(x,Q)}{\partial \log Q^2} = \alpha_s \int dy\ P(x/y) G(y)$
$\lim_{Q\to\infty} \phi(x, Q) = \prod_i\ x_i \cdot C_{\rm flavor}$	$\lim_{Q\to\infty} G(x, Q) = \delta(x) C$

Power Law Behavior

$\frac{d\sigma}{dx}(A+B\to C+D)\cong\frac{1}{s^{n-2}}f(\theta_{\rm c.m.})$ $\frac{d\sigma}{d^2p/E}(AB\to CX)\cong\sum\frac{(1-x_T)^{2n-1}}{(Q^2)^{n_{\rm act}-2}}f(\theta_{\rm c.m.})$

$n=n_A+n_B+n_C+n_D$ $n_{\rm act}=n_a+n_b+b_c+n_d$

T_H: expansion in $\alpha_s(Q^2)$ $d\hat{\sigma}$: expansion in $\alpha_s(Q^2)$

Complications

End point singularities	Multiple scales
Pinch singularities	Phase-space limits on evolution
High Fock states	Heavy quark thresholds
	Higher twist multiparticle processes
	Initial and final state interactions

As shown in Fig. 2 the power laws predicted by perturbative QCD are consistent with experiment.[15]

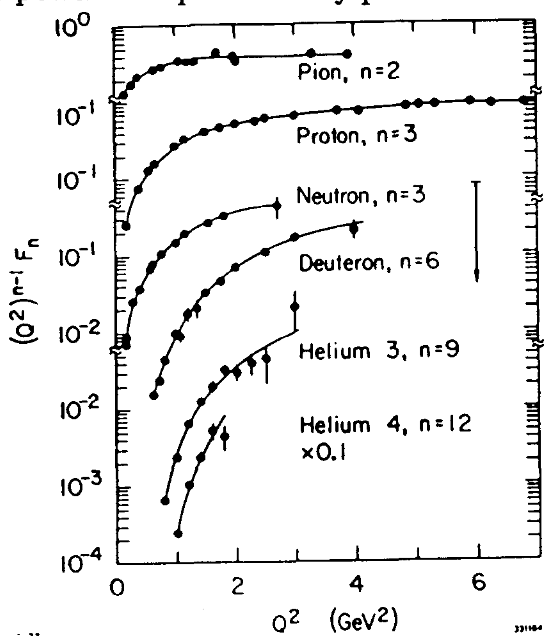

Figure 2: Comparison of experiment with the QCD dimensional counting rule $(Q^2)^{n-1}F(Q^2)\sim$ constant for form factors.

The near constant behavior of $Q^4G_M(Q^2)$ at large Q^2 [see Fig. 3] provides a direct check that the minimal Fock state in the nucleon contains three quarks and that the quark propagator and the $qq\to qq$ scattering amplitudes are approximately scale-independent. More generally, the nominal power law predicted for large momentum transfer exclusive reactions is given by the dimensional counting rule $\mathcal{M} \sim Q^{4-n_{\mathrm{TOT}}} F(\theta_{cm})$ where n_{TOT} is the total number of elementary fields which scatter in the reaction. The predictions are apparently compatible with experiment. In addition, for some scattering reactions there are

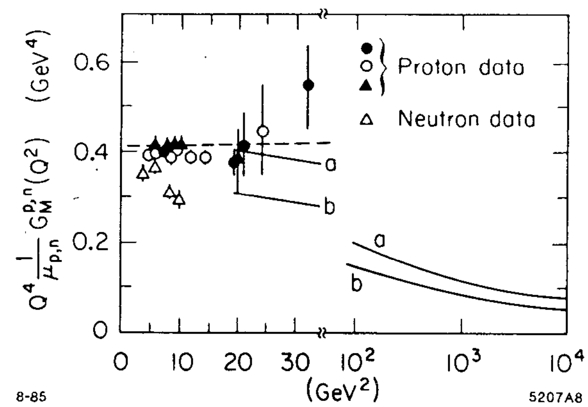

Figure 3: Perturbative QCD predictions for the proton (curve a) and the neutron (curve b) form factors given by ref. 3. The data are from ref. 15

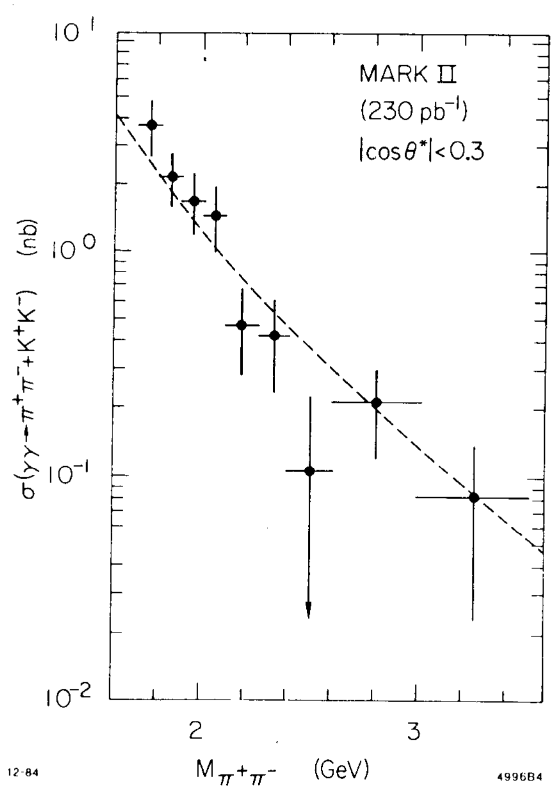

Figure 4: Measured cross section[16] for $\gamma\gamma \to \pi^+\pi^-$ plus $\gamma\gamma \to K^+K^-$

integrated over the angular region $|\cos\theta^*| < 0.3$. The errors contain systematic as well as statistical contributions. The curve is the perturbative QCD prediction.

contributions from multiple scattering diagrams (Landshoff contributions) which together with Sudakov effects can lead to small power-law corrections, as well as a complicated spin and amplitude phase phenomenology.[2,17] As shown in Fig. 4, recent measurements of $\gamma\gamma \to \pi^+\pi^-$, K^+K^- at large invariant pair mass are beautifully consistent with the QCD predictions[13] which are essentially independent of the shape of the distribution amplitude. In principle it should be possible to use measurements of the scaling and angular dependence of the $\gamma\gamma \to M^0\bar{M}^0$ reactions to measure the shape of the distribution amplitude $\phi_M(x,Q)$.[10] Thus far experiment has not been sufficiently precise to measure the modifications of dimensional counting rules predicted by QCD.

The actual calculation of $\phi(x,Q)$ from QCD requires non-perturbative methods such as lattice gauge theory, or more directly, the solution of the light-cone equation of motion[1]

$$\left[M^2 - \sum_{i=1}^{n}\left(\frac{k_\perp^2 + m^2}{x}\right)_i\right]\Psi = V_{\rm QCD}\Psi \qquad (2.$$

The explicit form for the matrix representation of $V_{\rm QCD}$ and a discussion of the infrared and ultraviolet regulation required to interpret this result is given in ref. 1.

Checks of the normalization of $(Q^2)^{n-1}F(Q^2)$ require independent determinations of the valence wavefunction, as has been obtained through QCD sum rules. [See Sect. 4] It has also been suggested that the relatively large normalization of $Q^4G_M^p(Q^2)$ at large Q^2 can be understood if the valence three-quark state has small transverse size, *i.e.*, is large at the origin.[18] The physical radius of the proton measured from $F_1(Q^2)$ at low momentum transfer then reflects the contributions of the higher Fock states $qqqg$, $qqq\bar{q}q$ (or meson cloud), *etc.* A small size for the proton valence wavefunction (*e.g.*, $R^p_{qqq} \sim 0.2$ to 0.3 fm) can also explain the large magnitude of $\langle k_\perp^2\rangle$ of the intrinsic quark momentum distribution needed to understand hard-scattering inclusive reactions. The necessity for small valence state Fock components can be demonstrated explicitly for the pion wavefunction, since $\psi_{q\bar{q}/\pi}$ is constrained by sum rules derived from $\pi^+ \to \ell^+\nu$ and $\pi^0 \to \gamma\gamma$. One finds a valence state radius $R^\pi_{q\bar{q}} \sim 0.4$ fm, corresponding to a probability $P^\pi_{q\bar{q}} \sim \frac{1}{4}$.

3. HADRONIC WAVEFUNCTIONS IN QCD

In order to make further contact between QCD and experiment, we require knowledge of the hadronic wavefunctions. The most convenient representation of the wavefunction is in terms of its Fock components $\{\psi_n\}$ defined at equal τ on the light-cone. Given the ψ_n, we can calculate current matrix elements (form factors) in terms of overlap integrals, structure functions from $|\psi_n|^2$ integrated over the $\mathbf{k}_{\perp i}$, and distribution amplitudes from $\psi_{q\bar{q}}$ or ψ_{qqq} integrated over the $\mathbf{k}_{\perp i}$.

Is it conceivable that the light cone equation of motion 2.12 for QCD could be solved? Recently, H.C. Pauli and I have begun a program to see whether a numerical evaluation is possible. The basic step is to impose periodic boundary conditions in $z^- = z - t$. The light-cone momenta $k^+ = k^\circ + k^-$ of each constituent take on discrete values

$$k^+ = \frac{2\pi}{L} n, \quad n = 1, 2...$$

with $k^- = \frac{k_\perp^2 + m^2}{k^+}$. The total charge Q and total light-cone momenta $P^+ = \Sigma k_i^+$ commute with H_{LC} and thus can be simultaneously diagonalized. In the case of field theories, 1 space and 1 time dimension, there are only a finite number of Fock states that can have a given P^+, since the k_i^+ are positive and the sum is conserved. Thus H_{LC} has a block diagonal form and can be readily diagonalized by analytic or numerical methods. We have applied this procedure to the Yukawa $\bar{\psi}\phi\psi$ and Schwinger models (QED with massless and massive fermions in $1+1$ dimensions) with very encouraging results. In the case of massless fermions it is necessary to include zero mode $k_i^+ = 0$ Fock components. The renormalization procedure and other technical aspects are discussed in ref. 19. The spectrum obtained from the Yukawa model agrees with that obtained using much more arduous methods. In the case of the Schwinger model one immediately finds that the $Q = 0$ spectrum is equivalent to that of non-interacting bosons of mass squared e^2/π.

In the case of QCD in $3+1$ dimensions one can introduce the $k_\perp$ degrees of freedom with a discretized Cartesian or cylindrical basis. By choosing light-cone gauge $A^+ = 0$ all the gluon degrees of freedom are physical. Unlike lattice gauge theory there are no difficulties with doubling of the fermion spectrum. We expect that increasingly accurate results for the spectrum and wavefunctions will be obtained as the "harmonic resolution" $K = LP^+/2\pi$ is increased since this allows a finer sampling of the x_i dependence of the wavefunctions. Further discussion on the role of K is given in ref. 19.

Until complete solutions are obtained, we need to be content with constraints derived indirectly from theory. As emphasized in Sect. 2, there are indications that the *valence* wavefunction is more compact and relativistic than the overall properties of the hadron. The first indications from both lattice gauge theory[12] and QCD sum rules[3] (see the next section) suggest that the pion distribution amplitude is highly structured in momentum space.

4. QCD SUM RULE CONSTRAINTS ON HADRON WAVEFUNCTIONS

Useful constraints[3] on the lowest moments of the distribution amplitude can be obtained using the QCD sum rule approach of the ITEP Group or by resonance saturation of vertex functions.[20] Although the numerical accuracy of these complementary methods is not known the general agreement between their predictions and overall consistency with other hadron phenomenology lends credence to their validity.

Let us first illustrate the QCD sum rule method for the case of the pion distribution amplitude. The moments $\langle x^n \rangle$ are expressible as matrix elements of gauge invariant local operators:

$$(z \cdot P)^{n+1} f_\pi \langle x^n \rangle = \langle \Omega | O_n(x) | \pi(p) \rangle \equiv \langle \Omega | \bar{d} \gamma \cdot z \gamma_5 (iz \cdot \overleftrightarrow{D})^n u | \pi(p) \rangle$$

$$\text{where} \qquad \langle x^n \rangle = \int_{-1}^{1} dx \; x^n \phi_\pi(x)$$

Here $x = x_1 - x_2$, $\langle x^\circ \rangle = 1$, $f_\pi \cong 133$ MeV, p^μ is the pion four momentum, z^μ is a light-like vector: $z^2 = 0, z \cdot p = p^+$, and $\overleftrightarrow{D}_\mu = \overrightarrow{D}_\mu - \overleftarrow{D}_\mu$, where $\overrightarrow{D} = \overrightarrow{\partial}_\mu - igA^a_\mu \cdot \frac{\lambda^a}{2}$. This relation is simplest in the gauge $z \cdot A^+ = 0$. The state $|\Omega\rangle$ is the true QCD vacuum.

In order to obtain constraints on the $\langle x^n \rangle$ one considers the correlation function between two of the O_n:

$$\begin{aligned} I_{no}(z,q) &= i \int d^4y \; e^{iq \cdot y} \langle \Omega | T \; O_n(y) O_o(0) | \Omega \rangle \\ &= (z \cdot q)^{n+2} I_{no}(q^2). \end{aligned}$$

The "signal" between $O_0^{(0)}$ and $O_n(y)$ is carried by the pion, higher meson resonances, and the continuum. At high $q^2 \to -\infty, y^2 \sim \mathcal{O}(1/Q^2)$ and the operator product expansion allows one to calculate I_{no} as an expansion in powers of

$1/q^2$ involving perturbative and $\langle G^2\rangle$ and $\langle\bar{\psi}\psi\rangle$ "vacuum condensate" contributions. On the other hand, $I_{no}(q^2)$ can be computed from a dispersion integral over hadron intermediate states. The dual indentification of the power law and resonance contribution (expressed via a Borel transformation) then leads to numerical constraints on the lowest moment: The best fit obtained in ref. 5 is

$$\langle x^2\rangle_\pi = 0.40, \qquad \langle x^2\rangle_{A_1} = 0.04 - 0.07$$
$$\langle x^4\rangle_\pi = 0.24.$$

($\langle x^4\rangle_{A_1}$ is small but not determined accurately.) The value of the renormalization scale μ^2 is of the order 1.5 to 2.5 GeV^2.

The relatively large values for the second and fourth moments imply that the pion distribution is quite broad. An additional constraint on the distribution amplitude is that ϕ vanishes at least as fast as ϕ_π^{asympt} at the endpoints $x \to \pm 1$. Together these constraints imply a *double-humped* distribution; the model proposed in ref. 5 is

$$\phi_\pi(x,\mu) = \frac{15}{4}x^2(1-x^2).$$

There are a number of approximations which make it difficult to assess the numerical accuracy of the results. Nevertheless the distribution amplitudes derived by Chernyak and Zhitnitsky[3] serve as useful forms for making QCD predictions for exclusive processes.

One of the consequences of the QCD sum rule approach is a striking dependence of the shape of the ρ-meson distribution amplitude on its helicity. This can be traced to the fact that the $\langle\bar{\psi}\psi\bar{\psi}\psi\rangle$ contribution changes sign because of the helicity dependence of the gluon-exchange interaction. A simple model for the ρ distribution amplitude which satisfies the moment constraints is:

$$\phi^\rho(x,\mu) = \phi_{\text{asympt}}(x)\begin{cases} \dfrac{15}{16}x_1x_2 & \lambda = \pm 1 \\ 1 + \dfrac{3}{2}\left((x_1-x_2)^2 - \dfrac{1}{5}\right) & \lambda = 0. \end{cases}$$

In each case the evolution from $\mu = 500$ MeV can be computed by expanding in terms of two lowest order Gegenbauer polynominal eigensolutions. The strong helicity dependence of the ρ distribution amplitude has interesting consequences for the angular dependence of $\gamma\gamma \to \rho\rho$ cross sections.

The requirement that the nucleon is the $I = \frac{1}{2}$, $S = \frac{1}{2}$ color singlet representation of three quark fields in QCD uniquely specifies the x_i permutation symmetry of the proton distribution amplitude:

$$\phi_{\uparrow}^{p}(x_i, \mu) \propto \frac{1}{\sqrt{6}} \left[d_{\uparrow} u_{\downarrow} u_{\uparrow} + u_{\uparrow} u_{\downarrow} d_{\uparrow} - 2u_{\uparrow} d_{\downarrow} u_{\uparrow}\right] \frac{1}{8} f_N \left[\phi_N(x_1 x_2 x_3) + \phi_N(x_3 x_2 x_1)\right]$$
$$+ \frac{1}{\sqrt{2}} \left[d_{\uparrow} u_{\downarrow} u_{\uparrow} - u_{\uparrow} u_{\downarrow} d_{\uparrow}\right] \cdot \frac{1}{8\sqrt{3}} f_N \left[\phi_N(x_3 x_2 x_1) - \phi_N(x_1 x_2 x_3)\right]$$
$$+ (1 \rightarrow 2) + (2 \rightarrow 3)$$

The neutron distribution amplitude is determined by the substitution $\phi_n = -\phi_p$ $(u \rightarrow d)$. Moments of the nucleon distribution amplitude can be computed from the correlation function of the appropriate local quark field operators that carry the nucleon quantum numbers.

The model wavefunction proposed in ref. 3, consistent with the derived moments, is

$$\phi_N(x_1 x_2 x_3) = \phi_{\text{asympt}} \cdot \left[11.35(x_1^2 + y_2^2) + 8.82 x_3^2 - 1.68 x_3 - 2.94 - 6.72(x_2^2 - x_1^2)\right]$$

where $\phi_{\text{asympt}} = 120\ x_1 x_2 x_3$. The renormalization scale is $\mu \cong 1$ GeV. The normalization of the nucleon valence wavefunction is also determined:

$$f_N(\mu = 1 \text{ GeV}) = (5.2 \pm 0.3) \times 10^{-3} \text{ GeV}.$$

A striking feature of the QCD sum rule prediction is the strong asymmetry implied by the first moment: 65% of the proton momentum (at $P_z \Rightarrow \infty$) is carried by the u quark with helicity parallel to that of the proton. [See Fig. 5.] The two remaining quarks each carry $\sim$ 15 to 20% of the total momentum.

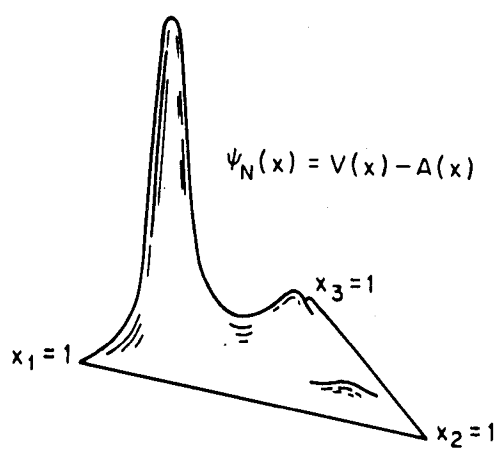

Figure 5: QCD sum rule prediction for the proton distribution amplitude.

From ref. 3.

The striking shape of the CZ wavefunction is due to the fact that only the first few eigensolutions to the nucleon evolution equation are used as a basis. Since one is so far from full evolution, there is no compelling reason why this should be correct. The essential feature of the sum rule predictions is the strong asymmetry, together with the value of f_N which give perturbative predictions for the proton and neutron form factors consistent both in *sign* and *magnitude* with experiment. [See Fig. 2].

5. APPLICABILITY OF PERTURBATIVE QCD EXCLUSIVE PROCESSES

An important question in QCD phenomenology is the range of applicability of the factorization formula and perturbation theory for exclusive processes. Recently Isgur and Llewellyn Smith[21] have argued that the non-perturbative contributions could be dominant even at very large momentum transfer, obscuring any possibility of empirically testing the perturbative predictions.

The Isgur-Llewellyn Smith discussion is based on several assumptions:

(1) They note that the asymptotic form of the distribution amplitude gives contributions to Q^2F_π and Q^4G_M which are $\approx \frac{1}{2}$ and 10^{-2} of the observed pion and nucleon form factors, respectively, at the largest Q^2 measured. However, $\phi_{\rm asympt}$ is not expected to be applicable to physical hadrons until enormous Q^2 where the non-singlet structure functions are fully evolved to delta-functions at x=0. Furthermore, we note that those perturbative predictions that are independent of the shape of distribution amplitudes such as the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to K^+K^-$ cross sections are in excellent agreement with experiment in form and magnitude. (See Sect. 2.) As noted in Sect. 4, the wavefunctions derived by Chernyak and Zhitnitsky[3] consistently give form factor predictions in agreement with the experimental sign and magnitude. (See Fig. 2)

(2) It has been claimed that nonperturbative contributions to electromagnetic form factors calculated from the overlap of non-relativistic quark-model wavefunctions can be numerically large even though such contributions are asymptotically power-law suppressed compared to the hard-scattering perturbative contributions. Such calculations are highly sensitive to the boosted form of hadron wavefunctions. If $\psi_{NR} \sim exp[-a\mathbf{k}^2]$, then the boosted wavefunction has the form $\psi_{LC} \sim [exp - ak_\perp^2/x(1-x)]$ in light-cone variables. Using the Drell-Yan convo-

lution formula this form gives a strongly (Gaussian) suppressed contribution at large q^2 as shown at this workshop by Jacob and Kisslinger.[22] The calculations of ref. 21 are based on wavefunctions which are only power- law suppressed at $x = 1$ and exponentially in $k_\perp^2$, apparently violating rotational invariance.

(3) Isgur[21] has argued that higher-twist contributions $(1/Q^2)^{n+1}$ can dominate leading twist $(\alpha_s/Q^2)^n$ contributions until very large Q^2 since the latter are numerically suppressed by the small value of $\alpha_s(Q^2)$. However we note that in explicit calculations of the higher twist terms one finds at least as many powers of $\alpha_s(Q^2)$ as occur in the leading twist result.

(4) In some cases, perturbative calculations are sensitive to endpoint regions of integration in x_i, and are thus numerically sensitive to non-perturbative effects. This criticism is particularly valid for the Chernyak and Zhitnitsky wavefunction in which a spectator quark in T_H carries only $\sim \frac{1}{6}$ of the light-cone momentum. If the quark propagators have an intrinsic mass-scale μ^2 then the proton form factor has denominators of the form,

$$\sim Q^2 + M/\langle x\rangle\langle y\rangle \sim Q^2 + 36\mu^2.$$

This would not be inconsistent with the mass corrections of the phenomenological form factors if $\mu \lesssim 200$ MeV.

Clearly the QCD sum rule wavefunctions have potential difficulties with endpoint singularities unless this region is strongly suppressed in T_H — *e.g.,* by the Sudakov quark form factors. A more compelling reason to be suspicious of the applicability of the QCD hard scattering formula to exclusive reactions is the striking behavior of the spin asymmetry A_N and spin correlations observed at $p_T \gtrsim 1$ GeV in large angle $pp \to pp$ scattering.[23,24] However, here the theory is much more complicated than the form factor predictions, because of Landshoff pinch singularities. The strong spin dependence of baryon wavefunctions as indicated by the QCD sum rule approach may also be very relevant to the eventual understanding of the anomalous spin results.

6. RECENT DEVELOPMENTS IN THE THEORY OF EXCLUSIVE PROCESSES

In this section I will outline some areas of recent progress in applying QCD perturbation theory to high momentum transfer exclusive process.

(1) The complete calculation of the tree graph structure of both $\gamma\gamma \to M\bar{M}$[13] and $\gamma\gamma \to B\bar{B}$[25] amplitudes has now been completed. The CZ proton distribution amplitudes give predictions for $\gamma\gamma \to p\bar{p}$ in rough agreement with the experimental normalization, although the production energy is too low for a clear test. The $\gamma^*\gamma^* \to M\bar{M}$ amplitudes for off-shell photons have now been calculated by Gunion *et al.* .[26] The results show important sensitivity to the form of the meson distribution amplitudes. The consequences of $|gg\rangle$ mixing in singlet mesons in $\gamma\gamma$ processes is discussed in ref. 27.

(2) Mass corrections to QCD hard scattering amplitudes for a number of heavy quark production amplitudes have been computed. Exclusive pair production of heavy hadrons $|Q_1\bar{Q}_2\rangle, |Q_1Q_2Q_3\rangle$ consisting of higher generation quarks ($Q_i = t, b, c$ and possibly s) can be reliably predicted[28] within the framework of perturbative QCD, since the required wavefunction input is essentially determined from nonrelativistic considerations. The results can be applied to e^+e^- annihilation, $\gamma\gamma$ annihilation, and W and Z decay into higher generation pairs. The normalization, angular dependence, and helicity structure can be predicted away from threshold, allowing a detailed study of the basic elements of heavy quark hadronization. A particularly striking feature of the QCD predictions is the existence of a zero in the form factor and e^+e^- annihilation cross section for zero-helicity hadron pair production close to a specific timelike value $q^2/4M_H^2 = m_h/2m_\ell$ where m_h and m_ℓ are the heavier and lighter quark masses, respectively. (See Fig. 6)

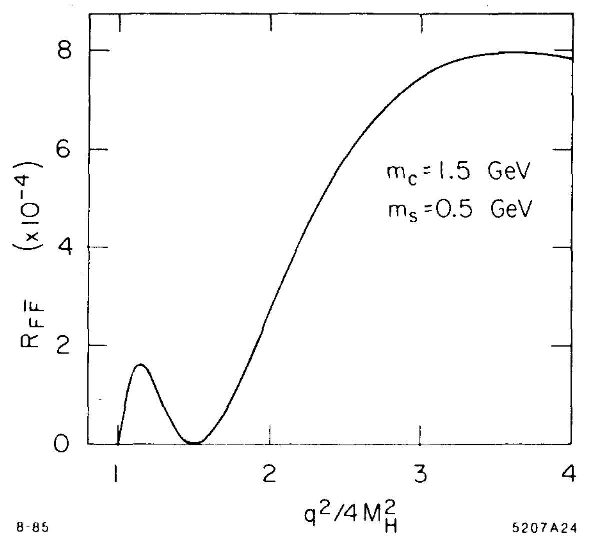

Figure 6: Perturbative QCD prediction[28] for $R_{F\bar{F}} = \frac{\sigma(e^+e^- \to F\bar{F})}{\sigma(e^+e^- \to \mu^+\mu^-}$.

This zero reflects the destructive interference between the spin-dependent and spin-independent (Coulomb exchange) couplings of the gluon in QCD. In fact, all pseudoscalar meson form factors are predicted in QCD to reverse sign from spacelike to timelike asymptotic momentum transfer because of their essentially monopole form. For $m_h > 2m_\ell$ the form factor zero occurs in the physical region.

(3) The formal properties of distribution amplitudes, their relation to the Bethe-Salpeter amplitudes, operator product expansion, and the use of conformal symmetry is discussed in refs. 5 and 29. The complete analysis of meson form factors though next to leading order is discussed in ref. 30. It has been conjectured[31] that the eigensolutions of the evolution equations for distribution amplitudes can be specified from conformal symmetry and the values of the anomalous dimensions for QCD if $\beta = 0$ (which can be effected by modifying the number of fermions). This has been verified to two-loop order in ref. 31 for $\phi^3_{(6)}$ theory using dimensional regularization. However, the conformal predictions are not consistent with the explicit calculations of the order α_s^2 kernel in QCD. The reason for this breakdown is apparently related to the infrared sensitivity of the ladder contribution to the evolution equation kernel.

(4) The methods developed in ref. 1 can be used to calculate other types of exclusive amplitudes such as weak and electromagnetic hadron decays. In addition, higher twist contributions, such as the leading $O(1/Q^2)$ longitudinal contribution to the pion structure function at $x \to 1$, have been confirmed experimentally in $\pi N \to \pi^+\pi^- X$ experiments.[33] In the case of inclusive jet experiments one calculate the contribution of "direct" higher twist amplitudes such as $\pi g \to q\bar{q}$ which lead to dijet events in πN collisions with no beam fragments.[34] A beautiful confirmation of these QCD predictions has been reported in ref. 35.

6. APPLICATIONS TO NUCLEAR PHYSICS

There are a number of interesting consequences of quark and gluon degrees of freedom in nuclei which are outside the usual domain of traditional nuclear physics.

(1) The nuclear force at very short distances can be calculated by perturbative methods.[36] A treatment of this type from the standpoint of evolution equations of the six-quark system and a derivation of the short-distance repulsion of the

nucleon-nucleon configuration is discussed in ref. 37.

(2) The observed light-cone 6-quark wavefunction has five-independent color singlet components[38] and can be systematically evaluated at short distances.[37,38] This leads to exact analytic results for the deuteron form factor at large momentum transfer and an understanding of the role of hidden color. This is discussed in detail in Ji's talk at this workshop. QCD predicts extra, hidden color degrees of freedom in all nuclei. Such exotic states should be excitable in Compton scattering, $\gamma d \to \gamma d$, *etc.*

(3) The QCD prediction for $f_d(Q^2) \equiv F_d(Q^2)/F_N^2(Q^2/4)$ for the leading (helicity-zero to helicity-zero) deuteron form factor is remarkably consistent with experiment for $Q^2 > 1$ GeV^2 when expressed in terms of reduced nuclear amplitudes, a formalism which covariantly removes the fall-off due to nucleon substructure. (See Fig. 7) Scaling laws for other high momentum transfer nuclear exclusive processes such as $\gamma d \to pn$ are discussed in ref. 40. The possibility of zeros in the non-leading helicity nuclear form factors analogous to the zeros that occur in heavy quark hadron form factors should be investigated.

(4) The fact that the nucleon is a composite system whether considered as Skyrmion soliton or as quark-gluon bound state, implies that it does not obey a *local* Dirac equation in an external potential.

(5) There are a number of novel QCD effects which arise because of coherent effects in nuclear targets. These include effects which occur during the propagation[42] of quarks and gluons through nuclear matter such as the Landau-Pomeranchuk formation zone,[43] the breakdown of factorization at incident parton energies below a scale set by the nuclear size,[42] shadowing phenomena, color transparency in high momentum transfer quasi-exclusive reactions,[44] *etc.*

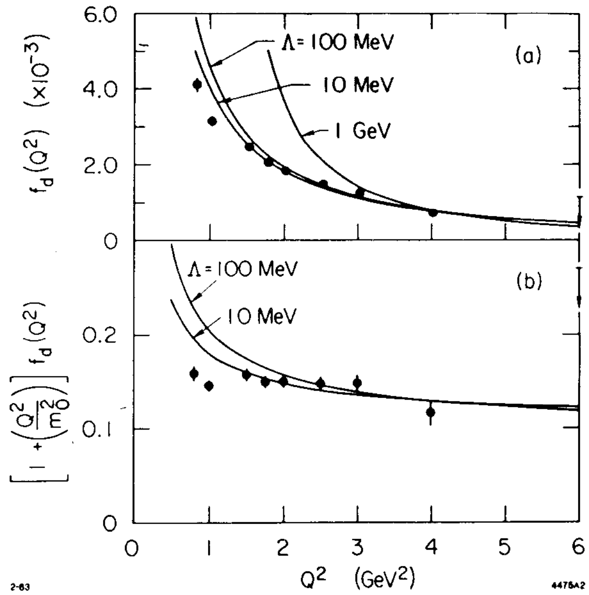

Figure 7: (a) Comparison of the asymptotic QCD prediction[40] for $f_d(Q^2)$ with experiment using $F_N(Q^2) = [1 + (Q^2/0.71\text{GeV}^2)]^{-2}$. The normalization is fit at $Q^2 = 4$ GeV^2. (b) Comparison of the prediction $[1 + (Q^2/m_0^2)]f_d(Q^2) \propto (\ell n Q^2)^{-1-(2/5)(C_F/\beta)}$ with data. The value $m_0^2 =$ 0.28 GeV^2 is used.

ACKNOWLEDGEMENTS

I wish to thank C. Carlson, J. Gunion, K. Hornbostel, Tao Huang, C.R. Ji, G.P. Lepage, H.C. Pauli, and M. Sawicki for discussions. This research was supported in part by the National Science Foundation under Grant No. PHY82-17853, supplemented by funds from the National Aeronautics and Space Administration, at the University of California at Santa Barbara.Work also supported by the Department of Energy, contract DE-AC03-76SF0015.

REFERENCES

1. G.P. Lepage and S.J. Brodsky, Phys. Rev. D**22**, 2157 (1980).

2. A. Mueller, Phys. Rept. **73**, 237 (1981).

3. V.L. Chernyak and I.R. Zhitnitskii, Phys. Rept. **112**, 173 (1984), Nucl. Phys. B**246**, 52 (1984). Xiao-Duang Xiang, Wang Xin-Nian, and Huang Tao, BIHEP-TH-84 **23** and **29** (1984).

4. G.P. Lepage, S.J. Brodsky, T. Huang and P.B. Mackenzie, CLNS-82/522, published in Banff Summer Institute 81 (1981).

5. S.J. Brodsky, Y. Frishman, G.P. Lepage and C. Sachrajda, Phys. Lett. **91**B, 239 (1980).

6. A.V. Efremov and A.V. Radyushkin, Phys. Lett. **94**B, 245 (1980).

7. For additional discussion of applications of QCD to nuclear physics and references, see S.J. Brodsky, published in the proceedings of the conference "New Horizons in Electromagnetic Physics," University of Virginia, April 1982; S.J. Brodsky, T. Huang and G.P. Lepage, SLAC-PUB-2868 (1982) published in Springer Tracts in Modern Physics, Vol. **100**, "Quarks and Nuclear Forces," ed. D. Fries and B. Zeitnitz (1982); S.J. Brodsky and G.P. Lepage, in the proceedings of the Eugene Few Body Conference 1980: 247C (Nucl. Phys. **A363**, 1981) S.J. Brodsky, in the proceedings of the NATO Pacific Summer Institute Progress in Nuclear Dynamics, Vancouver Island (1982), S.J. Brodsky and C.R. Ji, SLAC-PUB-3747 (1985).

8. C. Klopfenstein *et al.* CUSB 83-07 (1983).

9. S.J. Brodsky, G.P. Lepage, P.B. Mackenzie, Phys. Rev. D**28**, 228 (1983). The problem of setting the scale of the argument of the running coupling constant is discussed in this paper.

10. S.J. Brodsky and G.P. Lepage, Phys. Rev. D**24**, 1808 (1981). The calculation of $\gamma\gamma \to B\bar{B}$ is given by G.R. Farrar, E. Maina, and F. Neri, RU-85-08 (1985).

11. S.D. Drell and T.M. Yan, Phys. Rev. Lett. **24**, 181 (1970).

12. S. Gottlieb and A.S. Kronfeld, CLNS-85/646 (June 1985). A.S. Kronfeld and D.M. Photiadis, Phys. Rev. D**31**, 2939 (1985), and S. Gottlieb (these proceedings).

13. S.J. Brodsky and G.R. Farrar, Phys. Rev. Lett. **31**, 1153 (1973), and Phys. Rev. D**11**, 1309 (1975). S.J. Brodsky and G.P. Lepage, Phys. Rev. D**24**, 1808 (1981).

14. S.J. Brodsky and C.-R. Ji, SLAC-PUB-3706, -3707 and -3148 (1985).

15. M.D. Mestayer, SLAC-Report 214 (1978) F. Martin *et al.* , Phys. Rev. Lett. **38**, 1320 (1977); W.P. Schultz *et al.* Phys. Rev. Lett. **38**, 259 (1977); R.G. Arnold *et al.* Phys. Rev. Lett. **40**, 1429 (1978); SLAC-PUB-2373 (1979); B.T. Chertok, Phys. Lett. **41**, 1155 (1978); D. Day *et al.* Phys. Rev. Lett. **43**, 1143 (1979). Summaries of the data for nucleon and nuclear form factors at large Q^2 are given in B.T. Chertok, in Progress in Particle and Nuclear Physics, Proceedings of the

International School of Nuclear Physics, 5th Course, Erice (1978), and Proceedings of the XVI Rencontre de Moriond, Les Arcs, Savoie, France, 1981.

16. G. Gidal *et al.* LBL-19992, June 1985.

17. S.S. Kanwal, Phys. Lett. **142**B, 294 (1984).

18. S.J. Brodsky, T. Huang, G.P. Lepage, in Particles and Fields 2, edited by A.Z. Capri, A. N. Kamal, Plenum (1983), and T. Huang, SLAC-PUB-2580 (1980), published in the Proceedings of the XXth International Conference on High Energy Physics, Madison, Wisconsin (1980).

19. H.C. Pauli and S.J. Brodsky, Phys. Rev. D, 1985.

20. M.J. Lavelle, Imperial College preprint, TP/84-84/58 (1984). S.J. Brodsky and C.-R. Ji, to be published (1985).

21. N. Isgur and C.H. Llewellyn Smith, Phys. Rev. Lett. **52**, 1080 (1984). N. Isgur, Progress in Particle and Nuclear Physics, Vol. 13, "Nucleon Physics with Chromodynamics: From High Q^2 to Baryon Spectroscopy to Nuclear Physics," ed. A. Faessler.

22. O.C. Jacob and L. Kisslinger, Carnegie-Mellon preprint 1985.

23. A.D. Krisch, UM-He-85-19 (1985).

24. J. Szwed, Phys. Lett. **93**B, 485 (1980); Acta Polon. B**14**, 239 (1983).

25. G.R. Farrar and F. Neri, Phys. Lett. **130**B, 109 (1983).

26. J. Gunion, D. Millers, and K. Sparks, UC-Davis 85-02 (1985).

27. G.W. Atkinson, N. Tsokos and J. Sucher, Proc. of 6th International Symposium on Two-Photon Physics, 1984.

28. S.J. Brodsky and C.R. Ji, Phys. Rev. Lett. (1985). SLAC-PUB-3856.

29. S.J. Brodsky, C.R. Ji, and M. Sawicki, Phys. Rev. D**32**, 1530 (1985).

30. M.H. Sarmadi, Phys. Lett. **143**B, 471 (1984); F.-M. Dittes and A.V. Radyushkin, Phys. Lett. **134**B, 359 (1984); G.R. Katz, preprint CLNS 84/623 (1984).

31. S.J. Brodsky, Y. Frishman, and G.P. Lepage, SLAC-PUB-3755 (1985).

32. E. Berger and S.J. Brodsky, Phys. Rev. Lett. **42**, 940 (1979).

33. H.B. Greenlee *et al.* Phys. Rev. Lett. **55**, 1555 (1985).

34. E. Berger and S.J. Brodsky, Phys. Rev. D**24**, 2428 (1981).

35. M.W. Arenton *et al.* Phys. Rev. D**31**, 984 (1985).

36. For a recent discussion of progress in the derivation of nuclear forces from QCD-based models, see K. Maltman and N. Isgur, Phys. Rev. Lett. **50**, 1827 (1983), E.L. Lomon, MIT preprint CTP No. 1116 (1983); and references therein. The quark interchange mechanism for $N-N$ scattering is discussed in J.F. Gunion, S. Brodsky, and R. Blankenbecler, Phys. Rev. D**8**, 287 (1973). Qualitative QCD-based arguments for the repulsive $N-N$ potential at short distances are given in C. Detar, HU-TFT-82-6 (1982); M. Harvey, Nucl. Phys. **A352**, 301 (1981); **A352**, 326 (1981); R.L. Jaffe, Phys. Rev. Lett. **24**, 228 (1983); and G.E. Brown, in Erice 1981, Proceedings, Quarks and the Nucleus. The possibility that the deuteron form factor is dominated at large momentum transfer by hidden color components is discussed in V.A. Matveev and P. Sorba, Nuovo Cimento **45**A, 257 (1978).

37. The six-quark evolution formalism is developed and the leading anomalous dimensions for the deuteron are given in C.-R. Ji and S.J. Brodsky, SLAC-PUB-3148 and to be published.

38. M. Harvey, Nucl. Phys. **A352**, 301 (1981); **A352**, 326 (1981). V. Matveev and P. Sorba, Nuovo Cimento Lett. **20**, 435 (1977). Similar considerations for nonrelativistic system are given in A. Faessler *et al.* Nucl. Phys. **A402**, 555 (1983); S. Furui and A. Faessler Nucl. Phys. **A397**, 413 (1983).

39. S.J. Brodsky, C.-R Ji and G.P. Lepage, Phys. Rev. Lett. **51**, 83 (1983).

40. S.J. Brodsky and J.R. Hiller, Phys. Rev. C**28**, 475 (1983). S.J. Brodsky and B.T. Chertok, Phys. Rev. Lett. **37**, 269 (1976); Phys. Rev. D**114**, 3003 (1976). S.J. Brodsky, in Proceedings of the International Conference on Few Body Problems in Nuclear and Particle Physics, Laval University, Quebec (1974).

42. The eikonal scattering of quarks in hard scattering processes such as massive lepton pair production, including $k_\perp$ smearing and induced radiation is discussed by G. Bodwin, S.J. Brodsky, and G.P. Lepage, SLAC-PUB-2927 (1982) and S.J. Brodsky, SLAC-PUB-3263 (1984), published in Acta Physica Polonica. Other uses of the Drell-Yan process for studying quark and gluon distributions and shadowing

processes in nuclei are discussed in L.L. Frankfurt and M.I. Strikman, Leningrad preprint 838 (1983).

43. L. Landau and I. Pomeranchuk, Dok. Akademii Nauk SSSR **92**, 535 (1953), and **92**, 735 (1953); L. Stodolsky, MPI-PAE/TH 23/75 (1981); I.M. Dremin, Lebedev preprint 250 (1981). The Landau-Pomeranchuk effect, which is an essential component of factorization theorems for inclusive reactions, predicts that there is no induced hard radiation within a nucleus for incident constituents of energy large compared to the nucleon length. This leads to new conditions for the validity of QCD factorization for the Drell-Yan process, *etc.*

44. Because of color cancellations, QCD predicts no initial or final state corrections to quasi-elastic high momentum transfer reactions such as $\pi A \to \pi N(A-1)$. See A.H. Mueller, to be published in Proceedings of the Moriond Conference (1982). Applications to elastic hadron-nucleus amplitudes are given in S.J. Brodsky and B.T. Chertok, Phys. Rev. Lett. **37**, 269 (1976). Color singlet cancellations for valence states interacting in nuclei are discussed in G. Bertsch, S.J. Brodsky, A.S. Goldhaber and J.F. Gunion, Phys. Rev. Lett. **47**, 297 (1981). Further discussion may be found in S.J. Brodsky, SLAC-PUB-2970 (1982), published in the Proceedings of the XIIIth International Symposium on Multiparticle Dynamics, Volendam, The Netherlands (1982).