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Collider Scaling and Cost Estimation*

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1. Introduction

The primary motivation for high energy physicists to study new acceleration mechanisms is to find a way to build colliders at energies above the SSC at costs less than the SSC. Cost considerations are, unfortunately, crucial. It is simply not useful to know how to build an accelerator that would cost 100 billion dollars. Although it is difficult to make cost estimates without knowing the technology, I believe the attempt is useful.

In comparing a linear collider, assumed to be electron positron, with the SSC, a circular proton-proton machine, we need to know the parameters of each that will attain "equivalent" physics. Strictly there is no such equivalence. The two machines have different strengths and weaknesses and are in many ways complimentary. Nevertheless we can establish equivalent parameters for the production of particular final states and make some kind of average over different such states.[1] For the purposes of this lecture I will assume the following parameters to be equivalent:

	SSC	e^+e^- Collider
Beam energy	20 + 20 TeV	1.5 + 1.5 TeV
Luminosity	10^{33} cm^{-2} sec^{-1}	10^{33} cm^{-2} sec^{-1}

In discussing the cost scaling I will often refer to cost estimates for this "SSC equivalent" e^+e^- collider. These costs should be compared with a value of about 2 billion dollars for the SSC. This is the SSC cost without detectors, site, contingency or escalation. As in the SSC case the real cost would be about a factor of two higher than the values given.

2. Scaling Laws and Cost Estimation

It would be technically feasible to construct two SLAC-like linear accelerators, producing beams of electrons and positrons, respectively, up to 1.5 TeV. The problem is that if one bases cost estimates on a simple scaling of the existing SLAC linear accelerator parameters, then costs are excessive. Optimization of parameters for a linear collider is very likely to lead to numbers different from those pertaining to SLAC. Specifically, gradient, wavelength, mechanical tolerances, structural parameters, focusing systems, to name but a few would have to be quite different. Whether research and development based on such an optimization of parameters would lead to a practical machine whose cost is lower than that of the SSC is far from certain, but is not excluded. I try in this lecture to perform such an optimization despite the relative lack of detailed costs.

Costs for existing linear accelerators are associated with physical length, average power, peak power, and energy storage per pulse, and the scaling laws associated with each of these parameters as a function of wavelength and accelerating gradient can be determined. If we rather arbitrarily assume linear relations between costs and each of these parameters then one can obtain cost estimates as a function of the parameters and look for those values that would minimize the cost. It must be remembered that the exercise leads us to parameters and technology very far from existing linear colliders and cannot, therefore, be treated as a realistic estimate of actual cost. It is nevertheless an interesting exercise and may indicate where efforts should be directed.

We are considering only technologies that employ radio frequency power sources, driving near field accelerating structures. I will examine, in turn, the requirements on accelerating gradients, total stored RF energy, peak RF power, and finally average power consumption.

2.1 Accelerating Gradient Requirements

There is a rather obvious relationship between length and the cost of a linear collider and most early efforts at developing new technology were aimed at increasing the accelerating gradients in order to reduce these costs. The length proportional costs might lie somewhere in the range between $10,000 and $100,000 per meter (civil construction alone would be near the lower figure; the cost of SLAC in current dollars is somewhere in the middle of the range). For the purposes of cost optimization I will assume $30,000 per meter.

A gradient of 20 MeV per meter (as at the SLC), would imply, for our SSC equivalent, a total length of the order of 150 kilometers and a total linear cost of the order of 5 billion dollars. Gradients as high as 150 MeV per meter have been achieved in a SLAC structure. With this the length would be reduced to 20 kilometers, but the cost remains still relatively high: of the order of .6 billion. If we are aiming for costs substantially less than the SSC, it would seem prudent to aim for an accelerating gradient somewhat, but not greatly, larger than this.

The limit set by breakdown is believed to rise as the inverse wavelength to the 7/8 power (see Fig. 1). Another limit is set when heating in the accelerating structure would cause momentary melting of its surface. This limit has been discussed by Norman Kroll[2] and Perry Wilson[3] and occurs somewhere between 300 and 1000 MeV per meter at 10 cm and scales as the inverse wavelength to the 1/8th power (Fig. 1 is taken from Perry Wilson's paper[3]). This scaling law assumes that the cavity is filled for a time scaled from that used in SLAC; shorter fill times raise this limit. When the wavelength falls below 100 microns,

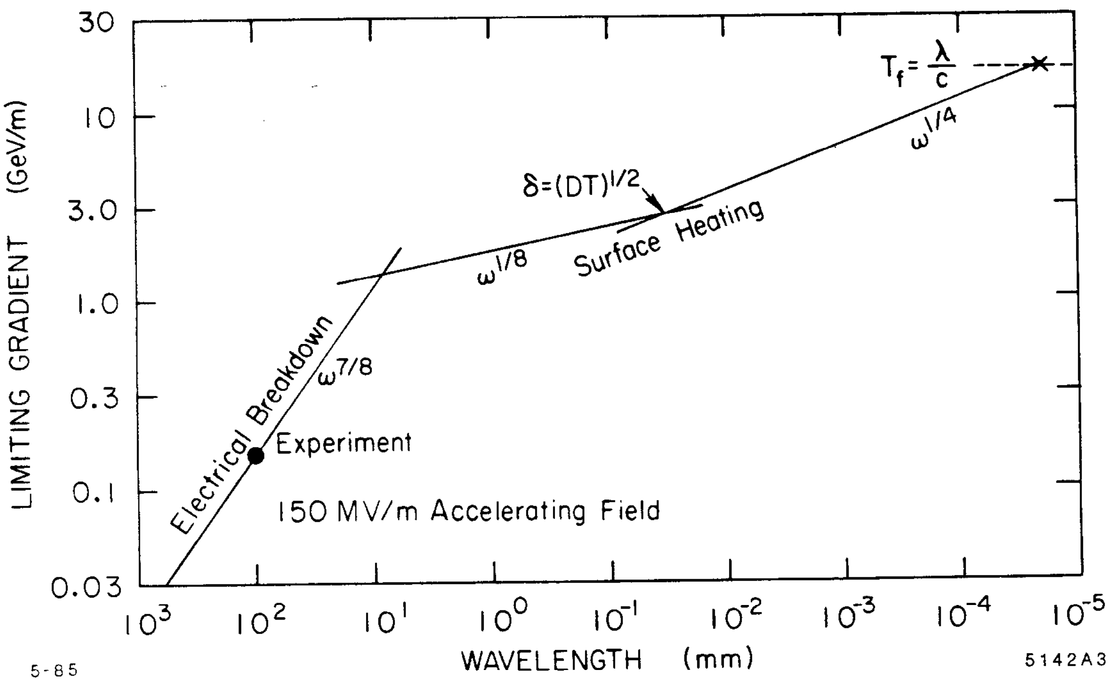

Fig. 1. Limitations on gradient as a function of wavelength due to electric field breakdown and surface heating in a SLAC-type disk-loaded structure.

the scaling law changes and rises as the inverse 1/4 power; the change arising because the temperature becomes limited by the specific heat of the surface material instead of its conductivity. At a wavelength of 10 microns, fields over a plane mirror as high as 4 GeV per meter have been recorded without damage to the surface.

As an example, if we assume a wavelength of less than 1 cm, we can hope for gradients of the order of 500 MeV per meter, and in that case, the linear costs of our SSC equivalent would be of the order of 180 million dollars; substantially less than the SSC.

2.2 Total Stored RF Energy Considerations

If, as in the SLC, the wavelength is 10 cm, but the accelerating gradient 500 MeV per meter, then the total RF energy required would be approximately 40 million Joules. If the costs associated with this stored energy are similar to those at SLAC, i.e., about \$1000 per Joule,[4] then this would cost 40 billion dollars, clearly unreasonable.

The stored energy J is reduced by lowering the gradient E_a but such reduction will increase the linear costs. The stored energy is also reduced by reducing the wavelength (by the square):

$$J \text{ (both beams)} = \sqrt{S}\,\frac{E_a\lambda^2}{4c_1} \quad \left(\text{mks if} \sqrt{S} \text{ and } E_a \text{ are in volts}\right) \ . \tag{2.1}$$

The c_1 depends on the structure geometry, for SLAC:

$$c_1 = k_1\lambda^2 \approx .2 \times 10^{12} \text{ Vm/coulomb} \ . \tag{2.2}$$

The only limit to how small the wavelength can be, is set by wake field effects. These, as we shall see below, set a limit on the number of particles that can be accelerated without having their emittance excessively increased. This number of particles per bunch, determines both the average power requirement and the beamstrahlung. All these will be discussed later and will require the wavelength to be of the order of a millimeter. If I arbitrarily take $\lambda = 1.4$ mm as an example then for $E_a = .5$ GeV/m and a SLAC-like structure

$$J = 4000 \text{ Joules} \quad .$$

If I assume 50% filling efficiency then the RF source must supply a total of 8000 Joules. This to be compared with 40 million Joules for λ =10 cm.

In estimating the costs of providing this RF energy at 1 mm, we cannot use the price associated with the 10 cm wavelengths. Estimates of the cost of, for instance, a Free Electron Laser, which could provide this wavelength are harder to come by. The recent Livermore experiments[5] suggest that it will be of the order of \$10,000 per Joule ($\sim$ 10 times that of a klystron), and the resulting cost would then be around 80 million dollars, far less than for the conventional wavelength even at the higher cost per Joule. Despite the extreme uncertainty in such cost estimates, the above illustrates the need to research the use of short wavelengths.

2.3 Peak Power Consideration

If an acceleration gradient of 500 MeV per meter is required and a 10 cm wavelength employed, then the stored RF energy is of the order of 40 million Joules. If the power is supplied over a fill time of .8 microseconds (as at SLAC), then the peak power requirement of the RF source is 50 terawatts. The cost to provide this peak power with conventional klystrons would be of the order of 40 billion dollars.[4] The peak power, as well as the stored energy cost is excessive.

If the wavelength is reduced, then the stored energy goes down as the square of the wavelength, but the fill time also goes down as the 3/2th power, resulting in the peak power going down as the root:

$$\tau \propto \lambda^{2/3} \; ; \qquad P(\text{peak}) = \frac{J}{\tau} \propto \frac{\sqrt{S} E_a \lambda^{1/2}}{c_2} \tag{2.3}$$

for a SLAC type structure made of copper

$$c_2 = .35 \times 10^6 \ \text{V}^2 \ \text{m}^{-1/2} \ \text{W}^{-1} \ . \tag{2.4}$$

In the case described above, the wavelength is 1.4 mm, the fill time scales to 1.3 nanoseconds and the peak power requirement is 6 terawatts. The cost per peak power for klystrons and an induction linac driven free electron laser (FEL) are similar[5] ($\sim 7 \times 10^{-4}$ \$/watt), and thus, the cost would still be of the order of 4 billion dollars. Unless the cost per unit of peak power can be significantly reduced from this value, the total cost, even after optimization at a lower gradient, remains considerably higher than that of the current SSC. As a result, there is great interest in power sources which have much lower cost per unit of peak power.

Possible new technologies that would work at short wavelengths include lasers, bunch compression prior to a free electron laser, laser-driven photo-diode amplifiers, laser-driven solid state amplifiers; and finally, almost any power source followed by pulse compression schemes.

The cost per unit of peak power for a short pulse laser is already far lower (of the order of 1/1000)[6] than that for conventional RF power sources, and it is this low cost per peak power that has made lasers attractive sources for the generation of high accelerating gradients. Unfortunately, their very short wavelength (of the order of 10 microns) makes it difficult to accelerate bunches of the required magnitude in conventional cavities, although the use of plasma beatwaves and other unconventional structures might overcome this problem. The second difficulty, as noted in Section 2.2, is that the cost[6] per unit of average power for such lasers is very much higher than for more conventional RF sources.

Perhaps the most promising technique, as of this review, is the use of an induction linac driven free electron laser, together with binary pulse compression[7] by a factor of say 32 (five stages). If the capital cost per unit of peak power could be reduced by a factor of twenty, then the peak power cost in our example would be reduced to of the order of 200 million dollars.

Another promising solution is to use a laser driven photo-diode amplifier. The "Lasertron" is such a device operating at normal microwave frequencies. It is similar to a conventional klystron but employs a gun employing a photocathode illuminated by RF modulated laser light. Scaling the lasertron, as now conceived, down to millimeter wavelengths seems impractical, but work on a microlasertron that could be so scaled has been inspired by a proposal of W. Willis and is being worked on at BNL and SLAC.[8]

2.4 Average Power Consumption

Simple extrapolation of current technology would need thousands of terawatts of average beam power. Besides the inevitable operating cost of such energy, there are clearly related capital costs in the conversion of electrical power first to RF fields and then to the beam. Somehow the average beam power must be reduced.

The beam power $P = NfE$ is given by the number of particles per bunch (N), the frequency (f) and the energy (E). The requirements for N and f are set by the need for a high luminosity (10^{33} cm^{-2} sec^{-1} for an SSC equivalent). The luminosity for round beams is given by $L = N^2 f/A$, where A is the cross-sectional area of the two beams at the final focus. Combining these two relationships, and noting that the energy times the cross-sectional area of the beams is given by the invariant beam emittance (ϵ_n) times the focus parameter (β), we obtain for the power per beam

$$P_B \approx 1 \times 10^{-12} \, \frac{L\epsilon_n\beta}{N} \quad \text{(mks)} \ . \tag{2.5}$$

This relationship implies that a large number of particles per bunch (N) is desirable. But a limit to this is set by two considerations: (1) beamstrahlung and (2) wakefields. I will take the approach of leaving discussion of beamstrahlung for the moment, assuming that it is not a problem, and consider only the wakefield constraints.

If N is too large then for a given structure and wavelength, wakefields will cause (1) excessive energy spread of the beam (longitudinal wake) and (2) blow up of the transverse beam size if the initial beam is slightly off axis. I consider them in turn:

Longitudinal Wake

The longitudinal wakefields generate an energy spread in the accelerated beam given[9] by

$$\frac{\Delta E}{E} = e\,\frac{B_0(\sigma_z/\lambda)\cdot C_0 N}{E_a\,\lambda^2} \tag{2.6}$$

where B_0 depends on the geometry of the structure and on the bunch length σ_z. As σ_z/λ goes to zero, B_0 goes to a constant of the order of 4. The C_0 is the wavelength independent loss parameter ($k_0 = C_0/\lambda^2$), which is dependent only on the structure geometry.

It is instructive to substitute for N, an expression using the fraction η of energy that is extracted from the cavity by the bunch, divided by the total stored energy in the cavity: ($\eta = 4eNC_0/\lambda^2 E_a$):

$$N = \frac{\eta\lambda^2 E_a}{e4C_0} \tag{2.7}$$

then

$$\frac{\Delta E}{E} = \frac{B(\sigma_z/\lambda)}{4}\,\eta \tag{2.8}$$

which for $\sigma_z/\lambda \to 0$

$$\frac{\Delta E}{E} \approx \eta\ . \tag{2.9}$$

Thus we see that in order to control the longitudinal wakefields we have to limit the fraction η of energy extracted from the cavity.

Transverse Wake

The transverse wake scaling seems at first to be more complicated. If the bunch is perfectly on the structure axis, no transverse wake exists, but if the bunch entering a structure is displaced by ϵ then transverse fields are excited that can cause a sideways displacement of the tail of the bunch by a distance δ given by[9]:

$$A = \frac{\delta}{\epsilon} \propto \frac{N\beta z\, u_0(\sigma_z/\lambda)}{4E\lambda^3} \tag{2.10}$$

where β is the average focusing strength in the structure, z the distance along the structure and u_0 a variable dependent on the structure and the bunch length. For small values of the variable σ_z/λ, u_0 will be proportional to σ_z/λ.

The above equation is true providing the β is the same for the head and tail of the bunch. Luckily this is unlikely to be the case. The longitudinal wake effect will cause the tail to have a lower energy than the head and this will result in the β's for head and tail being different. In this case the amplitude A does not rise linearly with z but oscillates with a maximum amplitude given by:

$$A = e\, \frac{N\beta^2\, u_0(\sigma_z/\lambda)}{4E\lambda^3\, \Delta p/p} \,. \tag{2.11}$$

It is now instructive to substitute for N as above and also substitute for β assuming that the focusing is provided by RFQ fields and that the magnitude of these fields is some fixed fraction of the accelerating fields E_a. In this case

$$\beta = B_0(\lambda E/E_a)^{1/2} \tag{2.12}$$

where B_0 is a constant for the cavity geometry. Substituting into the equation for the transverse wake gives

$$A = \left[\frac{1}{8}\, \frac{u_0(\sigma_z/\lambda)\cdot B_0}{c_0}\right] \frac{\eta}{dp/p} \tag{2.13}$$

and for small σ_z/λ

$$A \approx \text{ const } \cdot \frac{\sigma_z}{\lambda} \cdot \frac{\eta}{dp/p} \,. \tag{2.14}$$

As in the case of the longitudinal wake we find that the transverse wake, expressed in this way, is independent of λ (except for the fraction σ_z/λ) and scales only with the fraction of energy taken from the cavity: η.

Consequences for Average Power

We have seen that wakefield considerations will set a limit on the number of particles per bunch which is related to the structure, the fraction η of energy extracted, the wavelength and the accelerating gradient by Eq. (2.7). Combining this with Eq. (2.1) we obtain

$$P_B \propto \mathcal{L} \, \frac{\epsilon_n \beta C_0}{E_a \lambda^2 \eta} \,. \tag{2.15}$$

If we insert a value for C_0 equal to that for a SLAC structure:

$$\begin{aligned} C_0 = k_0\lambda^2 &= 20 \times 10^{12} \cdot (.1)^2 \\ &= 2 \times 10^{11} \text{ Volt meters/coulomb} \end{aligned}$$

then

$$P_B = 1 \times 10^{-19} \mathcal{L} \, \frac{\epsilon_n \beta}{E_a \lambda^2 \eta} \qquad (\text{mks}) \,. \tag{2.16}$$

If as an example I choose $\eta = 5\%, \lambda = 1.4$ mm and $E_a = .5$ GeV/m (we will see why I choose such a small λ later) then for $\epsilon_n = 1.2 \times 10^{-8}$ meters, $\beta = 1$ mm and $\mathcal{L} = 10^{33}$ cm^{-2} sec^{-1}(10^{37} m^{-2} sec^{-1}), I obtain

$$\begin{aligned} P_B &= .3 \text{ megawatt} \qquad && (\text{per beam}) \\ \text{and} \quad N &= 4 \times 10^8 && (\text{per bunch}) \,. \end{aligned}$$

If we take 10% efficiency RF-to-beam, then, for this example, we need 6 megawatts average RF power. The capital cost associated with such power will depend on the technology. For SLAC klystrons it can be as little as \$2 per watt, but for a new short wavelength source such as an FEL, it is probably a factor of ten higher: say \$20 per watt. Thus for 6 MW the capital costs might be in the range of 120 million dollars, quite reasonable.

For a laser power source, the cost is higher ($\sim$ \$100 per average watt) yielding an estimated capital cost of 600 million dollars. Whether such a high cost for average power can be offset by the laser's peak power advantage remains to be seen.

3. Cost Optimization

In the above section we have discussed the possible costs for a collider of $\sqrt{S} = 3$ TeV center of mass energy, luminosity of 10^{33} cm^{-2} sec^{-1}, accelerating gradient of 500 MeV/m and wavelength of 1.4 mm. The parameters and costs of the example are summarized in Table I. The choice of gradient and wavelength was justified only on qualitative grounds. In this section I wish to examine the dependency of overall cost on their choice in a more quantitative manner.

Making all the same assumptions as above but allowing the energy $\sqrt{S}$, the accelerating gradient E_a and the wavelength λ to be variables then one obtains the following contributions to the total machine cost. I have assumed that the luminosity must rise linearly with S.

$$\$(\text{length}) \approx 30\text{M\$} \cdot \frac{(S(\text{TeV}^2))^{1/2}}{E_a(\text{GeV/m})} \propto \frac{1}{E_a} \tag{3.1}$$

$$\$(\text{Ave. Power}) \approx 13\text{M\$} \cdot \frac{S(\text{TeV}^2)}{E_a(\text{GeV/m}) \cdot (\lambda(\text{mm}))^2} \propto \frac{1}{E_a \lambda^2} \tag{3.2}$$

Table I

Example parameters of a linear collider with energy 1.5 plus 1.5 TeV and Luminosity 10^{33} cm^{-2} sec^{-1}.

	Conventional (SLC)	Example
Gradient, E_a	20 MeV/m	500 MeV/m
Length, l		6 km
Cost per Meter	$\sim$ 30 K$	$\sim$ 30 K$
Total Length Cost		$\sim$ 180 M$
Invariant Emittance, ϵ_n	3×10^{-5}	1.2×10^{-8} m
Focus Parameter, β	5 mm	1 mm
Spot radius, $\sigma_x = \sigma_y$		2 nm
Bunch Length, σ_z	1 mm	10 μm
Beamstrahlung, $\Delta E/E = \delta$		.3
Ave. Beam Power (both), P_b		.6 MW
Wall Plug to Beam Eff., η_w	$\sim$ .1%	1%
Wall Power		60 MW
RF Source to Beam Eff., η_{RF}	2%	10%
RF Ave. Power		6 MW
Capital Cost per Ave. Watt	~2$	$\sim$ 20$
Capital Cost per Ave. Power		$\sim$ 120$
Particle per Bunch, N	5×10^{10}	4×10^{8}
Bunch Energy (Both), E_b		200 Joules
Stored RF to Bunch Eff., η	$\sim$ 5%	5%
RF Stored Energy, E_{RF}		4000 Joules
RF Source - RF Stored Eff., η_t	$\sim$ 50%	50%
RF Energy/Fill, E_f		8000 Joules
Source Frequency, f_s	120 Hz	750 Hz
RF Source Cost/Joule	$\sim$ 1 K$/J	$\sim$ 10 K$/J
RF Source Stored Energy Cost		$\sim$ 80 M$
Wavelength	10 cm	1.4 mm
Fill time	.8 μsec	1.3 nsec
RF Peak Power		6 TW
Cost per Peak Watt	$\sim 700 \times 10^{-6}$	$\sim 35 \times 10^{-6}$ $/watt
Peak Power Cost		$\sim$ 200 M$

$$\$(\text{RF energy}) \approx 30\text{M}\$(S(\text{TeV}^2))^{1/2} E_a(\text{GeV/m})(\lambda(\text{mm}))^2 \propto E_a\lambda^2 \tag{3.3}$$

$$\$(\text{Peak Power}) \approx 100\text{M}\$(S(\text{TeV}^2))^{1/2} E_a(\text{GeV/m})(\lambda(\text{mm}))^{1/2} \propto E_a\lambda^{1/2} \tag{3.4}$$

Note always that the constants were obtained on assumptions that may turn out to be unreliable and that they should be considered to have errors of about a factor of three both up and down. Note also that the above does not include costs for the cooling and injector systems (that may be very large), nor for experimental areas, labs, contingency etc.

The dependence of cost on E_a and λ can be illustrated graphically as shown in Fig. 2(a). Lines of constant component cost are shown as a function of the variables E_a and λ. The minimum cost for the sum of the four components will be at the geometrical center of the four-sided inner figure. We see that this cost minimum (λ = 1.6 mm and E_a = .4 GeV/m) is very close to the example of Table I (not, of course, a coincidence).

We can also examine the sensitivity of the minimum to our cost assumptions. If the cost of average power were ten times higher, or that of RF energy ten times lower, then the wavelength for minimum cost only rises by a factor of 1.8 to 2.9 mm. If the linear cost rises by a factor of three, or the cost of peak power falls by the same amount, then the accelerating gradient rises by 1.7 to .7 GeV/m. Clearly it will require quite radical changes in our assumptions to change the general conclusion that the accelerating for minimum cost should be of order .5 GeV/m, and that the wavelength for minimum cost is of the order of a few millimeter.

As the energy increases all components rise linearly except that for average power, which rises as the energy squared. As a result the cost minimum moves to

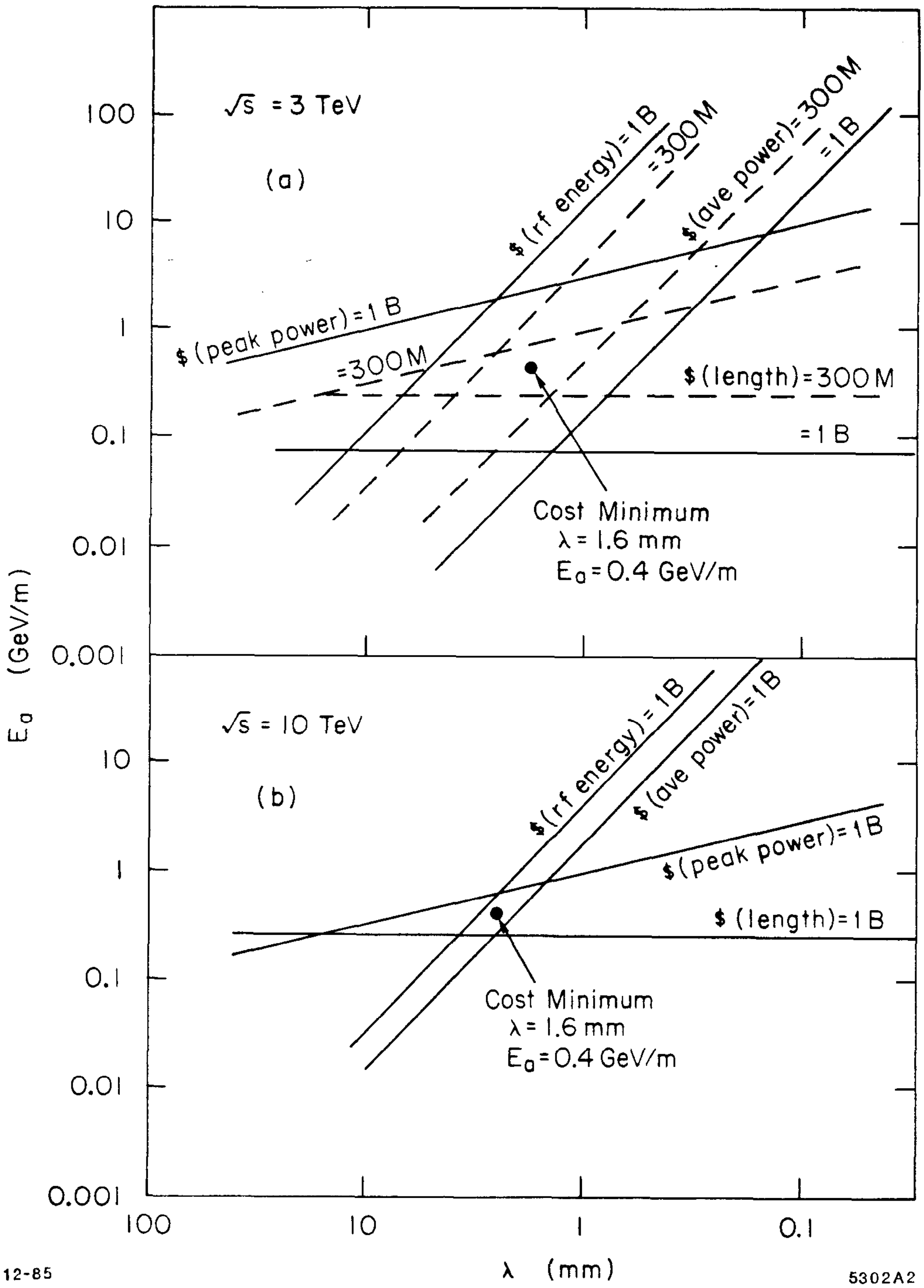

Fig. 2. Plot of lines of constant component costs as a function of accelerating gradient (E_a) and the microwave wavelength (λ). (a) Is for a 1.5 on 1.5 TeV collider and (b) is for a 5 on 5 TeV collider.

somewhat higher wavelengths. Figure 2(b) shows the situation for $\sqrt{S} = 10$ TeV where λ for cost minimum is 2.5 mm.

4. Beamstrahlung Considerations

So far we have ignored consideration of beamstrahlung at the final collision. Beamstrahlung is the name given to synchrotron radiation emitted by the particles of one bunch as they are deflected by the fields (both electric and magnetic) within the opposite bunch.

If the bunches are sufficiently long and the energy sufficiently low then the normal classical synchrotron radiation formulae apply and the average fractional energy loss δ_c of one bunch passing through the other is given[10] by:

$$\delta_c \approx 5 \times 10^{-45} \, \frac{N^2 \gamma^2}{\epsilon_n \beta \sigma_z} \qquad \text{(mks)} \tag{4.1}$$

where σ_z is the rms bunch length.

The spectrum of radiation emitted has the familiar synchrotron form with a critical photon energy of

$$E_{\text{crit}} \approx (\sqrt{3} \hbar c \; r_e) \cdot \frac{\gamma^2 N(r)}{r \sigma_z} \tag{4.2}$$

where $N(r)$ is the number of particles inside the radius r.

As σ_z gets less or γ increases eventually E_{crit} becomes larger than the initial electron energy. At this point the classical formulation breaks down and one enters a quantum beamstrahlung region where the fractional energy loss is given[11] by:

$$\delta_{qm} \approx 7 \times 10^{-9} \left(\frac{N^2 \sigma_z}{\epsilon_n \beta} \right)^{1/3} \quad . \tag{4.3}$$

In the example listed in Table I we had $N = 4 \times 10^8$, $\epsilon_n = 1.2 \times 10^{-8}$, $\beta = 1$ mm and $\gamma = 3 \times 10^6$ and for these parameters we get δ as a function of σ_z as shown in Fig. 3. At very large values of σ_z the classical formula holds and δ decreases with increasing σ_z; at small σ_z the quantum formula is applicable and the reverse is true. The dotted transition between the regions was added by eye.

We see that the fractional energy loss δ is reasonably, i.e. less than 30%, for bench lengths less than 10 μm, or longer than about 1 mm. Unfortunately the long bunch solution has too great a disruption parameter and is not practical. In addition, of course, it is not possible to accelerate a 1 mm long bunch in a structure employing a wavelength of the same order. Thus we find ourselves compelled to use a short bunch and operate in the quantum beamstrahlung region. We note also that at energies above the SSC equivalent, the clasical beamstrahlung even at $\sigma_z = 1$ mm is excessive and the use of short bunches and the quantum regime is imperative.

We have in this analysis chosen the wavelength to minimize cost and obtained from those considerations a number of particles per bunch. With this we have calculated beamstrahlung and chosen a bunch length to give a reasonable energy spread. We could have gone the other way.

If we had fixed the acceptable energy spread δ then we could have expressed the number of particles per bunch:

$$N \approx 1.7 \times 10^{12} \left(\frac{\epsilon_n \beta \delta^3}{\sigma_z} \right)^{1/2} \quad (\text{mks}) \; , \tag{4.4}$$

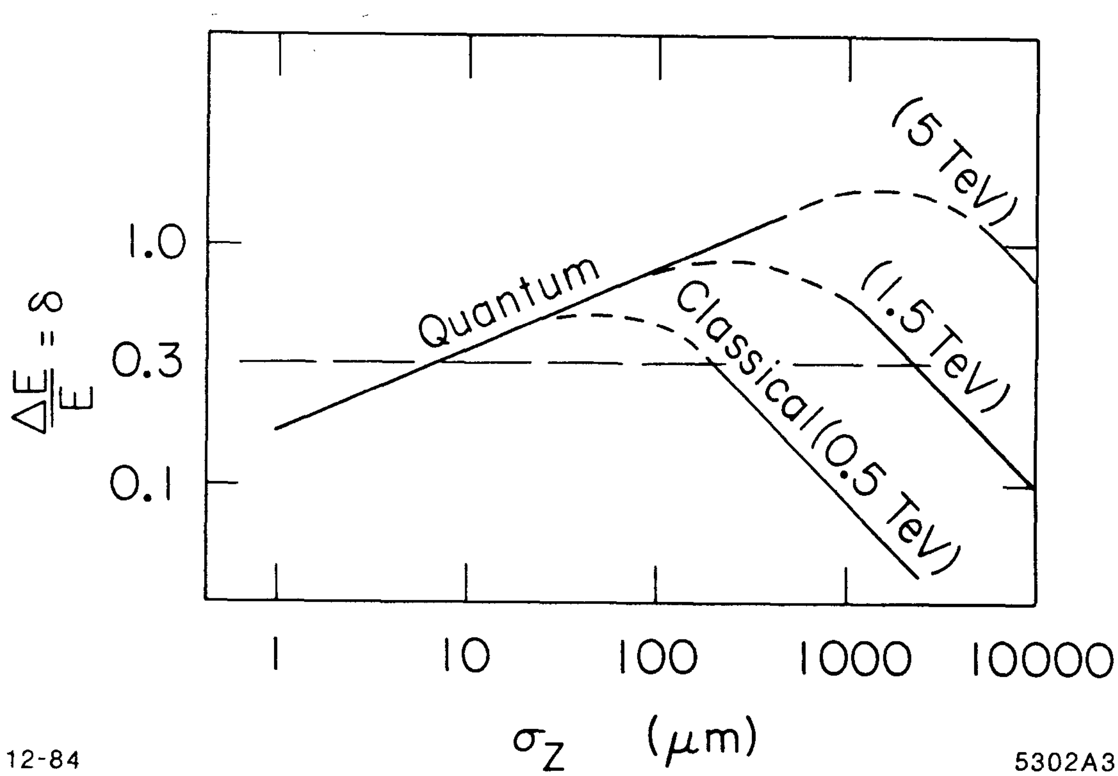

Fig. 3. Fractional energy loss due to beamstrahlung ($\Delta E/E = \delta$) for the example given in Table I, plotted versus the rms bunch length σ_z.

And then combining this with Eq. (2.5) obtain:

$$P_{\text{beam}} \approx .5 \times 10^{-24} \, \frac{\mathcal{L}(\epsilon_n \beta \sigma_z)^{1/2}}{\delta^{3/2}} \quad (\text{mks}) \ . \tag{4.5}$$

This is a particularly interesting relationship. As the machine energy rises, then the costs related to the average power rise faster than all other costs (assuming the need for luminosity to rise as the square of energy). Eventually these average power costs will dominate and Eq. (4.5) indicates the importance for the three-dimensional emittance $\epsilon_n \sigma_z$ and strong focusing, i.e. a low β.

5. Focusing Optics

5.1 INTRODUCTION

As has been discussed above there is a relation (Eq. (4.5)) between the average beam power and, among other things the beam emittance (ϵ_n) and final focusing strength (β). The parameter β defines the "depth of focus" or distance along the axis from the focal center to the point when the spot has increased in radius by a factor of two. It is a convenient way of expressing the focusing strength and gives the spot radius $\sigma_{x,y}$ for a given emittance:

$$\sigma_{x,y} = \left(\frac{\epsilon_n \beta}{\gamma} \right)^{1/2} \ . \tag{5.1}$$

We see that reduction of either ϵ_n or β will reduce the σ and thus the cross-sectional area of the spot. This in turn increases, for a given beam current, the rate of interactions when two beams collide; i.e. increases the "luminosity".

Focusing of particle beams is done by magnetic fields which bend the particles towards the axis. If the fields rise linearly with their distance from the axis, i.e. have a gradient, then the resulting bends will result in focusing the beam to a spot, just as a conventional lens focuses an optical beam. Unfortunately it is not possible in a vacuum to magnetically focus simultaneously in both horizontal and vertical directions. "Quadrupole" magnets are used whose fields focus in one direction but defocus by an equal amount in the other. By the "strong focusing principle" when two or more of such magnets are employed with finite spacing focusing in both planes can be achieved.

5.2 CONVENTIONAL

The current SLC focusing system employs quadrupoles with gradients of the order of 10 kG per cm and achieves a final beta of about 5 mm. A fairly sophisticated chromatic correction system cancels first and second order effects and achieves, with .5% momentum spread, a final spot little larger than that given by geometric considerations. If such a design is scaled in length by the root of gamma/gradient then the beta scales by the same factor:

$$\beta \propto \left(\frac{\gamma}{\mathrm{gradient}}\right)^{1/2} \tag{5.2}$$

and the relative correction of chromatic effects remains unchanged. Unfortunately this implies that with the same gradients, the final beta for a 1.5 TeV collider will have risen to 3 cm. With exotic superconductors a gradient of the order of 40 kG/cm might be possible. This would reduce the beta to about 1 cm. More efficient packing and correction of higher order chromatic effects might be able to further lower this to 1 mm[12] (Erickson), but the required higher order correction remains to be demonstrated. Alternatively, as we will see below, one could use very small "conventional" quads and obtain about the same value.

Discussions in earlier sections have indicated that if the average beam power is to be limited in a high luminosity collider then the emittance, as well as beta will have to be very small. If we assume such a very small emittance (10^{-8} m) then the beam size, even at its largest point in the focus system, is less than a tenth of a millimeter. It is tempting then to fix the pole tip fields (B) and scale the quadrupole apertures to a fixed factor over this beam size. Doing this, we obtain for the same chromatic correction and the same quadrupole packing:

$$\beta \propto \left(\frac{\gamma \epsilon_n}{B^2}\right)^{1/3} . \tag{5.3}$$

For an emittance (epsilon) of 10^{-8}, and 10 kG pole tip field, then at 1.5 TeV one would again obtain $\beta = 1$ mm, but this is now obtained without third order chromatic correction. The required quads are indeed very small, but no smaller than the gaps in recording heads, and there do seem to be possible ways to build them to the required optical precision. Once again however it has yet to be demonstrated.

5.3 LASER FOCUSING

With the short wavelengths and high powers of lasers one hopes to be able to obtain accelerating gradients of perhaps as high as 5 GeV per meter. Whatever the economics of such fields for acceleration they might be useful for focusing. In a radio frequency quadrupole (RFQ) such fields would correspond to pole tip bending fields of 150 kG and, by the above scaling Eq. (5.3), reduce the final beta by another factor of six. This option has not however been fully studied.

5.4 SUPER DISRUPTION

It has been known for some time that when two bunches of finite length collide the fields induced by one will focus or pinch the other. The result is an effective reduction of beta and a consequent enhancement of the luminosity by a factor of the order of four. Unfortunately the use of such long bunches is inconsistent with the need to limit the energy loss from beamstrahlung.

It has been suggested, however, that the enhancement could be recovered by the use of two or more short bunches[13] (Leith/Palmer). In this case the earlier bunches can be used to focus the oncoming final bunch and greatly enhance the luminosity achieved when these two final bunches collide. The luminosity enhancement (h) for bunches with uniform current distribution is given approximately by:

$$h \approx .5 \times 10^{30} \left(\frac{N}{\epsilon_n}\right)^2 \quad \text{mks} . \tag{5.4}$$

Note: new beta = old beta/($4h$).

In this equation N is the total number of particles, divided half and half in the two bunches and epsilon is the emittance in meters. For an N chosen to give a beamstahlung energy loss of .3, and an emittance of 10^{-8}, h would be approximately 25, and the beta reduced by 100. Even higher gains are calculated when more than two bunches are used. The enhancements are more limited when a less ideal current distribution is assumed, but a significant factor is still obtained even with Gaussian distributions.

A second advantage that comes with this "self focusing" is that the two beams do not need to be aligned to the same accuracy as would be required if the same final spot were obtained by external focusing. This relaxation in tolerance is by a factor of four for two bunches and nearly seven with three. There is of course no relaxation on the tolerance on the alignment between the first and second bunches.

One must note from Eq. (5.4) that neither the enhancement nor the reduction in tolerance can be obtained without first obtaining the very small emittance. It cannot be used as a substitute.

5.5 CONCLUSION

There seems at first sight to be several ways of obtaining a beta of 1 mm, or even less, provided a very small emittance is available. There is however at least one possible problem that has not been looked at. After the collision the bunches will be greatly refocused by their passage through one another and these "debris" may well not pass through the apertures of the opposite focusing quads. Will this produce unacceptable background? Or even damage the magnets? Clearly much more study is required.

6. Conclusion

The main conclusion of this discussion is that the best wavelength for an SSC equivalent electron positron collider is in the few millimeter region. If conventional wavelengths of a few centimeter are used then the likely cost of providing the microwave energy is excessive. If a laser is employed the likely cost of providing the average beam power is excessive. Efforts should be made to develop efficient sources in the mm range.

A second conclusion might be, providing the right choice of wavelength is made and providing an FEL and bunch compression scheme is practical, that the likely cost of an SSC equivalent electron positron collider would be in the region of 500 million dollars (see Table I). We might conclude therefore that the overall cost of an SSC-equivalent linear collider could, in principle, be lower than that of the SSC. The problem is much more complex than this indicates. The considerations of luminosity and beam power have led to a requirement for extremely high particle density at interactions (of the order of 10^{27} electrons per cc). This, in turn, requires more difficult mechanical tolerances in the accelerating structure beyond that attained in the past and establishes a need for novel methods of achieving final focusing. The formal optimization based on unit costs indicates the necessity to use wavelengths shorter than those at SLAC. This, in combination with the increased mechanical tolerances, brings us into a new region of technology whose cost implications have not been studied. Thus, although cost scaling laws applied to the different parameters give a useful guide as to how an overall cost minimum for a large linear collider might be attained, one should be loath to simply add the costs associated with those parameters. There are still so many cost elements associated with things we do not as yet know how to do at all, such that overall cost estimates would not be meaningful.

I am nevertheless encouraged by this study and, despite all the reservations, believe that we will, in time, learn how to make linear colliders at costs that will enable us to go well beyond the SSC. The emphasis should however be on the "in time". There is much work to be done.

I have freely used ideas from many sources. In particular I wish to acknowledge the contribution of W. Panofsky (some of whose prose has been used), and of P. Wilson.

References and Notes

1. B. Richter, "Requirements for Very High Energy Accelerators," SLAC-PUB-3630 (1985).

2. N. Kroll, "Surface Heating by Short Bunches of Radiation," 'Laser Acceleration of Particles,' *AIP Conference Proceedings #130*, p. 296 (1985).

3. P. B. Wilson, 'Lineaaccelerators for TeV Colliders'; IBID, p. 560, and also SLAC AAS-Note 2.

4. For the purposes of this analysis I have taken the cost of a SLAC klystron to be \$100,000 and its modulator and feeders to cost \$200,000. I have further taken the klystron cost to be associated with peak power need, and divided the modulator cost into \$100,000 for average power and \$100,000 for stored energy. I was guided in this by conversations with Greg Loew of SLAC. The performance assumed, including SLED, was: stored energy 120 Joules, pulse length .8 μsec, and peak power 150 M watts. This yields \$800/Joule (rounded to 1000), 7×10^{-4} dollars per peak watt, and \$4 /average watt. Without SLED the cost per average watts is \$2.

5. For induction linac driven FEL costs I have had to rely on conversations with members of the Two Beam Accelerator group who have demonstrated such a source. I have assumed that a peak power of 1 GW can be obtained (as reported in the Wall Street Journal). I have assumed a pulse length of 20 nsec and thus a stored energy of 20 Joules. I have assumed that such a system could be made to cycle at 1 kHz and that it would cost 1.4 million dollars. If I divide this cost into .2 million dollars for stored energy, .4 millon dollars for average power and .8 million dollars for peak power; then

I obtain costs of \$10/joule, \$20/ave. watt, and $.8 \times 10^{-3}$ dollar/peak watt. Despite the Wall Street Journal publication the performance of the system is classified and thus no reference is available for the 1 GW operation. A reference to a lower performance is: T. J. Orzechowski et al., Phys. Rev. Lett. **54** p. 889 (1985). See also J. S. Wurtele, p. 305, (1985).

6. For average power costs I used some numbers quoted by D. D. Lowenthal of Spectra Technology Inc. from a study by that firm for another application. Values as low as \$50 per watt and as high as \$1,000 per watt were given depending on optimism and time scale. I have taken \$100/watt. For peak power I note that Corkum achieved, for 2 psec pulses, power densities of the order of 10^{12} watts/cm^2. Assuming one tenth of this density, and taking \$100,000 as the cost of a 1 cm^2 final amplifier, I obtain 10^{-6} dollar/watt for peak power. The stored energy cost of \$10,000 per Joule is obtained by considering the cost of lasers designed for high storage capacity. It must again be emphasized that these estimates have errors of a factor three or so either up or down. The Corkum reference is: P. B. Corkum "High Power, sub-psec ten μm Pulse Generation", Optics Letters **8**, 514 (1983). See also D. Lowenthal and J. Slater, 'Laser Acceleration of Particles,' *AIP Conference Proceedings #130*, p. 818 (1985).

7. Z. D. Farkas, "Binary Power Multiplier" SLAC-PUB-3694.

8. The 'switched power linac' was proposed first by W. Willis, 'Laser Acceleration of Particles' *AIP Conference Proceedings #130*, p. 421 (1985); see also *Proceedings of the ECFA/INFN Workshop*, CERN 85/07 (1985). F. Villar has a similar idea: SLAC-PUB-3804. In general, these papers concern a single pulse of radiation generated by a pulse of current from a wire

photocathode to a high voltage anode. The idea can be extended to the excitation of a resonant standing wave in a small cavity by fast pulsing a photocathode wire within such a cavity. This idea, which I refer to as a "microlasertron" may be compared with a conventional lasertron in which a photocathode is again pulsed, but in which the electrons produced are focused into a bunched beam and energy extracted as in a klystron by ring cavities about the beam. See E. L. Garwin et al., "An Experimental Program to Build a Multimegawatt Lasertron for Super Lineacolliders", 1985 Particle Accelerator Conference (to be published), IEEE Trans. Nucl. Sci. NS-32; also SLAC-PUB-3650.

9. P. B. Wilson, "High Energy Electron Linacs" SLAC-PUB-2884 (1982); also Ref. 3.

10. M. Bassetti and M. Gygi-Hanney, LEP-Note-221, CERN, Geneva (1980).

11. T. Erber and G. B. Baumgartner Jr., *Proc. 12th International Conference on High Energy Accelerators* (Fermilab, August 1983), p. 372; T. Himel and J. Siegrist, "Quantum Effects in Linear Collider Scaling Laws," 'Laser Acceleration of Particles,' *AIP Conference Proceedings #130*, p. 602 (1985). See also Ref. 3.

12. Roger Erickson, "Final Focus", SLAC AAS Note #6 (1985).

13. R. Palmer "Super Disruption", SLAC-PUB-3688.