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THE $SO(10)_V \times SU(3)_H$ MODEL*

PEI–YOU XUE†

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

ABSTRACT

The $SO(10)_V \times SU(3)_H$ model, in which the quark mass hierarchy is induced through their mixing with horizontal superheavy fermions, is discussed. The success of the Fritzsch mass-matrices is explained using the mass matrices of horizontal superheavy up-quarks and down-quarks closely proportional to each other, which is natural in the model.

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† On leave from the Institute of High Energy Physics, Academi Sinica, P. O. Box 918(4), Peking, The People's Republic of China.

Recently, a new mechanism of quark and lepton mass generation via their mixing with hypothetical superheavy fermions is proposed for the model with the $SU(5)_V \times SU(3)_H$ symmetry.[1] Although the minimal standard SU(5) grand unified model by Georgi and Glashow[2] is simple and beautiful, but it is in disagreement with the proton decay experiments.[3] Using standard analysis of the renormalization group equations, it is obvious that this serious problem still exists in Ref. 1. On the other hand, SO(10) grand unified model is in agreement with proton decay experiments.[4] In the standard SO(10) model, each fermion generation put in an irreducible 16 spinorial representation which is automatically anomaly free. It is known that the following three generations exist, which repeatedly put in only 16 spinorial representations in the ordinary SO(10) model,

$$(u_i,\, d_i,\, \nu_e,\, e), \qquad (c_i,\, s_i,\, \nu_\mu,\, \mu), \qquad (t_i(?),\, b_i,\, \nu_\tau,\, \tau) \qquad , \tag{1}$$

where $i = 1, 2, 3$ is color index. So it is hard to understand about the replication of generations, fermion mass hierarchy and the structure of weak mixing.

In this paper, we propose a model with the $SO(10)_V \times SU(3)_H$ symmetry. Here SO(10) is a vertical grand unified group, $SU(3)_H$ is a local generation symmetry which unified the generations [Eq. (1)] in a horizontal triplet. At the same mass scale M_H, the $SU(3)_H$ is breaking. To suppress the flavor-changing neutral currents,[5] generated by the horizontal gauge bosons, $M_H > 10^6$ GeV. In our model, the hierarchy between the masses of three generations [Eq. (1)] and the weak mixing angles are due to the spontaneous breaking of the $SU(3)_H$ symmetry. The quark mass matrices are obtained, which have the form suggested by Fritzsch.[6] The relations between the quark masses and mixing angles, which are derived from our model, are in good agreement with recent experiments. The value of CP–violation parameter $\epsilon \sim s_2\, s_3 \sin\delta$ is $O(10^{-3})$.

Now let the quarks and leptons of Eq. (1) put in the representations $(16_V, \bar{3}_H)$ of $SO(10)_V \times SU(3)_H$ symmetry. After the reduction of 16_V^α under the $[SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_x]_V$, we have

$$
\begin{aligned}
16_V^\alpha = & \left(3,\, 2,\, 1,\, \sqrt{\frac{1}{24}}\right)^\alpha + \left(\bar{3},\, 1,\, 2,\, \sqrt{-\frac{1}{24}}\right)^\alpha \\
& + \left(1,\, 2,\, 1,\, \sqrt{-\frac{3}{8}}\right)^\alpha + \left(1,\, 1,\, 2,\, \sqrt{\frac{3}{8}}\right)^\alpha \quad ,
\end{aligned}
\tag{2}
$$

where $\alpha = 1, 2, 3$ are $SU(3)_H$ indices. At the grand unified mass scale M_x of $SO(10)_V$, we introduce the $(45;1)$ Higgs representation, which decomposes under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_x \times SU(3)_H$,

$$
\begin{aligned}
(45;\,1) \; = & \; (8,\, 1,\, 0;\, 1) + \left(3,\, 2,\, 2,\, -\sqrt{\frac{1}{6}};\, 1\right) + \left(\bar{3},\, 2,\, 2,\, \sqrt{\frac{1}{6}};\, 1\right) \\
& + \left(3,\, 1,\, 1,\, \sqrt{\frac{2}{3}};\, 1\right) + \left(\bar{3},\, 1,\, 1,\, -\sqrt{\frac{2}{3}};\, 1\right) + (1,\, 3,\, 1,\, 0;\, 1) \\
& + (1,\, 1,\, 3,\, 0;\, 1) + (1,\, 1,\, 1,\, 0;\, 1) \quad .
\end{aligned}
\tag{3}
$$

So the scheme for symmetry breaking is the following

$$
\begin{aligned}
& SO(10)_V \times SU(3)_H \; \xrightarrow[M_x]{(45,\,1)} \\
& \qquad G = \; [SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_x]_V \; \times SU(3)_H \; .
\end{aligned}
\tag{4}
$$

and the quarks, leptons cannot obtain masses without the breaking of $SU(3)_H$ symmetry.

Using the color index i, generation index α, the isospin I_L, I_R and hypercharge Y of G, we can explicitly indicate the particles as the following,

$$\begin{aligned} \begin{pmatrix} u_{i\alpha} \\ d_{i\alpha} \end{pmatrix}_L &\equiv q_{Li\alpha}\left(\frac{1}{2}, 0\,\frac{1}{3}\right) \qquad & \begin{pmatrix} u_i^{\alpha} \\ d_i^{\alpha} \end{pmatrix}_R &\equiv q_{Ri}^{\alpha}\left(0\,\frac{1}{2}, \frac{1}{3}\right) \\ \begin{pmatrix} \nu\alpha \\ e\alpha \end{pmatrix}_L &\equiv \ell_{L\alpha}\left(\frac{1}{2}, 0\,-1\right) \qquad & \begin{pmatrix} \nu^{\alpha} \\ e^{\alpha} \end{pmatrix}_R &\equiv \ell_R^{\alpha}\left(0\,\frac{1}{2}, -1\right) \quad . \end{aligned} \tag{5}$$

The number in the bracket of Eq. (5) is I_L, I_R and hypercharges Y. The electric charge of fermions is $Q = I_L^3 + I_R^3 + Y/2$.

Notice that the assignment of quarks and leptons [see Eqs. (2) or (5)] have $SU(3)_H$ triangle anomalies. To cancel these anomalies, we need to introduce also the hypothetical superheavy horizontal quarks and leptons which put in the $(1, \bar{3})$ representations of $SO(10)_V \times SU(3)_H$ symmetry and the assignment of their hypercharge as the following,

$$\begin{aligned} &U_{Ri\alpha}\left(0, 0, \frac{4}{3}\right) \qquad && D_{Ri\alpha}\left(0, 0, -\frac{2}{3}\right) \\ &E_{R\alpha}\ (0, 0, -2) \qquad && N_{R\alpha}\ (0, 0, 0) \quad . \end{aligned} \tag{6}$$

$U_{Ri\alpha}$ and $D_{Ri\alpha}$ are the superheavy *up-quarks* and *down-quarks*, respectively, and $E_{R\alpha}$ and $N_{R\alpha}$ are the superheavy *charged leptons* and *neutrinos*, respectively. For the superheavy quarks and leptons, the weak isospin $I_L^3 = I_R^3 = 0$.

To break the horizontal symmetry $SU(3)_H$, we introduce also the following Higgs scalars,[1] which put in the representations $2(1, \bar{3})$ and $(1, 6)$ of $SO(10)_V \times SU(3)_H$ symmetry, respectively, namely,

$$\xi^{\alpha}(1, \bar{3}), \qquad \eta^{\alpha}(1, \bar{3}), \qquad \chi_{\{\alpha\beta\}}(1, 6) \quad .$$

Their vacuum expectation values are

$$\langle \xi^\alpha \rangle = p\,\delta_3^\alpha\,, \qquad \langle \eta^\alpha \rangle = q\,\delta_1^\alpha\,, \qquad \langle \chi_{\{\alpha\beta\}} \rangle = r\,\delta_\alpha^3\,\delta_\beta^3 \quad . \tag{7}$$

We also need to break $[\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_x]_V$ down to low energy standard model $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, which we can obtain by using $(126,1)$ Higgs representation of $\mathrm{SO}(10)_V \times \mathrm{SU}(3)_H$at mass scale M'. Because after the reduction of $(126,1)$ under $[\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_x]_V \times \mathrm{SU}(3)_H$, we have

$$\begin{aligned}(126,1) = {} & (8,\,2,\,2,\,0;\,1) + \left(6,\,3,\,1,\,\sqrt{\tfrac{1}{6}};\,1\right) + \left(\bar{6},\,1,\,3,\,-\sqrt{\tfrac{1}{6}};\,1\right) \\ & + \left(3,\,3,\,1,\,-\sqrt{\tfrac{1}{6}};\,1\right) + \left(3,\,2,\,2,\,\sqrt{\tfrac{2}{3}};\,1\right) + \left(\bar{3},\,2,\,2,\,-\sqrt{\tfrac{2}{3}};\,1\right) \\ & + \left(\bar{3},\,1,\,3,\,\sqrt{\tfrac{1}{6}};\,1\right) + \left(3,\,1,\,1,\,-\sqrt{\tfrac{1}{6}};\,1\right) + \left(\bar{3},\,1,\,1,\,\sqrt{\tfrac{1}{6}};\,1\right) \\ & + \left(1,\,3,\,1,\,-\sqrt{\tfrac{3}{2}};\,1\right) + (1,\,2,\,2,\,0;\,1) + \left(1,\,1,\,3,\,\sqrt{\tfrac{3}{2}};\,1\right) \quad . \end{aligned} \tag{8}$$

Of course, we further need to break $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ down to $\mathrm{SU}(3)_C \times \mathrm{U}(1)_{em}$. In fact, from the view of the subgroup $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_x \times \mathrm{SU}(3)_H$, we can use the standard Higgs doublets [see Ref. 7] $\phi_L(1;\,0,\,1/2,\,0;\,1)$ and $\phi_R(1;\,0,\,0,\,1/2,;\,1)$ which are embedded in the representations of $\mathrm{SO}(10)_V \times \mathrm{SU}(3)_H$. The vacuum expectation values are

$$\langle \phi_L \rangle = \begin{pmatrix} 0 \\ v_L \end{pmatrix}, \qquad \langle \phi_R \rangle = \begin{pmatrix} 0 \\ v_R \end{pmatrix} \quad . \tag{9}$$

To suppress the right charged current, we require $v_R \gg v_L$. But our model is a grand unified model with horizontal, which is different from that in Ref. 7. Here, v_R can be obtained by using renormalization group equations of the evolution coupling constants in the $\mathrm{SO}(10)_V \times \mathrm{SU}(3)_H$model,[8]

$$\ell n\left(\frac{M_x}{v_R}\right) = \frac{\frac{1}{e^2(\mu)} - \frac{8}{3g_3^2(\mu)} - 2\left(\frac{5}{3}\,b_1 + b_2^L - \frac{8}{3}\,b_3\right)\,\ell n\left(\frac{v_R}{\mu}\right)}{2\left(b_2^L + b_2^R - \frac{8}{3} + \frac{2}{3}\,b_1^x\right)}$$

$$\sin^2\theta_W(\mu) = \frac{3}{8} - \frac{5}{4}\,e^2(\mu)$$

$$\times\left[\left(\frac{3}{5}\,b_2^R + \frac{2}{5}\,b_1^x - b_2^L\right)\,\ell n\left(\frac{M_x}{v_R}\right) + \left(b_1 - b_1^L\right)\,\ell n\left(\frac{v_R}{\mu}\right)\right] . \qquad (10)$$

where

$$b_1 = \frac{1}{16\pi^2}\,\frac{2}{3}\,f\,,$$

$$b_2^L = b_2^R = \frac{1}{16\pi^2}\left(-\frac{22}{3} + \frac{2}{3}\,f\right)\,,$$

$$b_3 = \frac{1}{16\pi^2}\left(-11 + \frac{2}{3}\,f\right)\,,$$

b_1^x is the coefficient of the $\mathrm{U}(1)_x$ β–function, and f is the number of quark flavors. Neglecting Higgs contributions, from Eqs. (1), (4) and (10), for $\sin^2\theta_W = 0.21$, $v_r \simeq 10^{10}$ GeV, $M_x \simeq 10^{18}$ GeV. It is enough to suppress the right charged current and the life of proton decay is very long.

Now, from eqs. (5), (6), (7) and (9), the Yukawa coupling allowed by the symmetry of the model is [Notice the point is there are no bare mass terms for ordinary quarks and leptons],

$$
\begin{aligned}
&\Big[\ \big(C_0\chi_{\{\alpha\beta\}} + C_1\xi_{[\alpha\beta]} + C_2\eta_{[\alpha\beta]}\big)\,\overline{D}_{R\alpha}\,D_L^{\beta} + +\text{h.c.} \\
&+ \big(C_0'\chi_{\{\alpha\beta\}} + C_1'\xi_{[\alpha\beta]} + C_2'\eta_{[\alpha\beta]}\big)\,\overline{U}_{Ra}\,U_L^{\beta} + +\text{h.c.}\ \Big] \\
&+ \Big[\, g\bar{g}_{L\alpha}\,D_{R\alpha}\,\phi_L + g_R\bar{q}_R^{\alpha}\,D_L^{\alpha}\,\phi_R + +\text{h.c.}\ \Big] \\
&+ \Big[\, g_L'\,\bar{g}_{L\alpha}\,U_{R\alpha}\,(i\tau_2\,\phi_L^*) + g_R'\,\bar{q}_R^{\alpha}\,U_L^{\alpha}\,(i\tau_2\,\phi_R^* + +\text{h.c.}\ \Big] \quad .
\end{aligned}
\tag{11}
$$

Inserting Eqs. (7) and (9) into Eq. (11), 6×6 quark mass matrices can be obtained, which has been discussed in Ref. 9, for the down quarks,

$$
\begin{pmatrix}
0 & 0 & 0 & g_L v_L & 0 & 0 \\
0 & 0 & 0 & 0 & g_L v_L & 0 \\
0 & 0 & 0 & 0 & 0 & g_L v_L \\
g v_R & 0 & 0 & M_D & 0 & 0 \\
0 & g v_R & 0 & 0 & -M_S & 0 \\
0 & 0 & g v_R & 0 & 0 & M_B
\end{pmatrix} \quad , \tag{12}
$$

where

$$
\begin{pmatrix}
M_D & 0 & 0 \\
0 & M_S & 0 \\
0 & 0 & M_B
\end{pmatrix}
= U_D
\begin{pmatrix}
C_0 r & C_1 p & 0 \\
C_1 p & 0 & C_2 q \\
0 & C_2 q & 0
\end{pmatrix}
U_D^+ \quad . \tag{13}
$$

From Eqs. (12) and (13), we see that the ordinary down quark (d, s, b) mass arises by mixing with superheavy down-quark (D, S, B) mass. If $M_B \gg |g v_R|$,[9] then

$$
\begin{aligned}
m_d &= \frac{g_L\ v_L\ g_R\ v_R}{M_D} \\
m_s &= \frac{g_L\ v_L g_R\ v_R}{M_S} \quad . \\
m_b &= \frac{g_L\ v_L g_R\ v_R}{M_B}
\end{aligned}
\tag{14}
$$

Similarly, for the up-quark

$$
\begin{aligned}
m_u &= \frac{g_L' \, v_L \, g_R' \, v_R}{M_U} \\
m_c &= \frac{g_L' \, v_L g_R' \, v_R}{M_C} \qquad , \\
m_t &= \frac{g_L' \, v_L g_R' \, v_R}{M_T}
\end{aligned}
\tag{15}
$$

where

$$
\begin{pmatrix} M_U & & \\ & M_C & \\ & & M_R \end{pmatrix} = U_u \begin{pmatrix} C_0'r & C_1'p & 0 \\ C_1' & 0 & C_2'q \\ 0 & C_2 q & 0 \end{pmatrix} U_u^+ \quad . \tag{16}
$$

It is interesting to note that the Fritzsch mass-matrix ansatz can be explained by using the mass matrices of superheavy horizontal up-quark (U, C, T) and down-quark (D, S, B) closely proportional to each other. The symmetric Fritzsch form for the quark mass matrices is given by

$$
\begin{aligned}
m_u &= \begin{pmatrix} 0 & |f|e^{i\phi_1} & 0 \\ |f|e^{i\phi_1} & 0 & |k|e^{i\phi_2} \\ 0 & |k|e^{i\phi_1} & |\ell|e^{i\phi_3} \end{pmatrix} \quad , \\
m_d &= \begin{pmatrix} 0 & |f'|e^{i\phi_1'} & 0 \\ |f'|e^{i\phi_1'} & 0 & |k'|e^{i\phi_2'} \\ 0 & |k'|e^{i\phi_1'} & |\ell'|e^{i\phi_3'} \end{pmatrix} \quad ,
\end{aligned}
\tag{17}
$$

i.e., only the third generation gets a diagonal mass, but the first and second generation masses arise through mixings between neighboring generations. Fritzsch mass matrices are interesting because they explain the smallness of KM mixing angles in terms of the flavor mass hierarchy, and three weak mixing angles and one CP–violating phase can be predicted from the quark mass eigenvalues and two linear combination of phase ϕ_1, ϕ_2, ϕ_3 and ϕ_1', ϕ_2', ϕ_3' in Eq. (17). How can we obtain the Fritzsch mass matrices in our model? From the structure of mass matrices in Eqs. (13) and (16), we see that up-superquark matrices have the same form as down-superquark matrices. So it is natural to assume that up-superquark mass and down-superquark mass ratio in each generation should be the same:

$$\frac{M_D}{M_U} = \frac{M_S}{M_C} , \quad \frac{M_S}{M_C} = \frac{M_B}{M_T} \quad , \tag{19}$$

From Eqs. (14), (15) and (18), we get, for the ordinary quarks,

$$\frac{m_u}{m_d} = \frac{m_c}{m_s} , \tag{19}$$

$$\frac{m_t}{m_b} = \frac{m_c}{m_s} \quad , \tag{20}$$

i.e., the ordinary charge 2/3 and −1/3 quark mass ratio in each generation is generation independent. The known ordinary quark values[10] are

$$\begin{aligned} m_u &\simeq 5.1 \text{ MeV} , & m_c &\simeq 1.35 \text{ GeV} , & m_t &= 30-50 \text{ GeV (?)} , \\ m_d &\simeq 8.9 \text{ MeV} , & m_s &\simeq 175 \text{ MeV} , & m_b &\simeq 5.3 \text{ GeV} . \end{aligned} \tag{21}$$

So Eq. (20) is a better approximation than Eq. (19). We need to correct the relations of Eq. (18). We expect that the correction of $M_S/M_C = M_B/M_T$ is smaller than that of $M_D/M_U = M_S/M_C$, for example,

$$\left(\frac{M_D}{M_S}\right)^{1/2} - \left(\frac{M_U}{M_C}\right)^{1/2} = \mathcal{O}(\Delta) \ ,$$
$$\left(\frac{M_S}{M_B}\right)^{1/2} - \left(\frac{M_C}{M_T}\right)^{1/2} = \mathcal{O}(\Delta^2) \ , \tag{22}$$

where Δ is a small correction, $\Delta = \mathcal{O}(0.1)$, then from Eqs. (14) and (15) we have

$$\left(\frac{m_d}{m_s}\right)^{1/2} - \left(\frac{m_u}{m_c}\right)^{1/2} = \mathcal{O}(\lambda\Delta) \ ,$$
$$\left(\frac{m_s}{m_b}\right)^{1/2} - \left(\frac{m_c}{m_t}\right)^{1/2} = \mathcal{O}[\lambda(\Delta^2)] \ ; \tag{23}$$

$\lambda(\Delta)$ is also a small correction. Now, we think of λ as the Wolfenstein parameter,[11] $\lambda = U_{us} \doteq 0.225$. Cheng and Li[12] have shown that Eqs. (23) are the key ingredient needed for Fritzsch ansatz and from Eqs. (11)–(13), $s_1 \simeq (m_d/m_s)^{1/2}$, $s_2 \doteq (m_c/m_t)^{1/2}$, $s_3 \doteq (m_s/m_b)^{1/2}$, and the value of CP–violation parameter $\epsilon \sim s_2 s_3 \sin\delta$ is $O(10^{-3})$, which is in good agreement with the recent experiments.

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References

1. Z. G. Berezhiani, *Phys. Lett.* **150B**, 177 (1985).
2. H. Georgi and S. L. Glashow, *Phys. Rev. Lett.* **32**, 438 (1974).
3. See *Proceedings of the International Colloquium on Baryon Nonconservation, Salt Lake City, January 1984.*
4. H. Fritzsch and P. Minkovski, *Ann. of Phys.* **93**, 193 (1975); M. S. Chanowitz, J. Ellis and M. K. Gaillard, *Nucl. Phys.* B **124**, 506 (1977); H. Georgi and D. V. Nanopoulos, *Nucl. Phys.* B **155**, 52 (1979);
5. S. L. Glashow and S. Weinberg, *Phys. Rev.* *D* **15**, 1958 (1977);
6. H. Fritzsch, *Nucl. Phys.* B **155**, 189 (1979); *Phys. Lett.* **73B**, 317 (1978). L. F. Li, *Phys. Lett.* **84B**, 461 (1979).
7. R. Mohapatra and J. C. Pati, *Phys. Rev.* *D* **11**, 566 (1975); 2558 (1975).
8. H. Georgi, H. R. Quinn and S. Weinberg, *Phys. Rev.* *D* **22**, 2157 (1980).
9. Z. G. Berezhiani and J. L.Chkareuli, *Pis'ma Zh. Eksp. Theor. Fiz.* **35**, 494 (1982); *Yad. Fiz.* **37**, 1043 (1983); Z. G. Berezhiana, *Phys. Lett.* **129** 99 (1983).
10. J. Gasser and H. Leutwyler, *Phys. Rep.* **87** 77 (1982).
11. L. Wolfenstein, *Phys. Rev. Lett.* **51**, 1945 (1983). C. Jarlskog, *Phys. Rev. Lett.* **55**, 1039 (1985).
12. T. P. Cheng and Ling-Fong Li, *Phys. Rev. Lett.* **55**, 2249 (1985).