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SUPERSTRING PROPAGATION IN CURVED SUPERSPACE  
IN THE PRESENCE OF BACKGROUND  
SUPER YANG-MILLS FIELDS\*

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ABSTRACT

We present a gauge invariant action which describes the propagation of the superstring in curved superspace in the presence of background super Yang-Mills fields. We show that this action possesses the local fermionic world-sheet symmetry needed for a consistent coupling of the string to background fields. Some other aspects of the superspace non-linear  $\sigma$ -model described by this action are also discussed.

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Superstrings seem to offer a consistent approach to the quantum theory of gravity that encompasses the unification of the known forces.<sup>[1]</sup> Mathematical consistency requires that superstring theories be formulated in ten-dimensional spacetime. To make contact with the real world it is, therefore, necessary to investigate compactified solutions of string theory. The proper framework for discussing this issue is the field theory of interacting strings. In the absence of a such a framework\* other approaches have been used to analyse this problem, such as studying the field theory limit of the superstring<sup>[3]</sup> or studying string propagation in background fields corresponding to the massless modes of the string.<sup>[4,5]</sup> In the latter case, quantum conformal invariance requires that the background fields satisfy the classical equations of motion (modified by string induced terms). For the superstring these analyses have been carried out in the Neveu-Schwarz-Ramond (NSR)<sup>[6]</sup> formulation with background space-time fermions set to zero. To study questions of space-time supersymmetry one needs to include the coupling to fermions. This is rather nontrivial in the NSR formulation because of the complicated form of the fermionic vertex.<sup>[7]</sup> In flat spacetime there exists an alternate description of superstring propagation, due to Green and Schwarz (GS),<sup>[8]</sup> that is manifestly space-time supersymmetric. Although in general more complicated than the NSR formulation, it seems better suited to studying questions of space-time supersymmetry.

Crucial to the GS formulation is the existence of a local fermionic world-sheet symmetry, which is needed to gauge away unphysical degrees of freedom. It is necessary to maintain this symmetry while coupling the superstring to background fields. Witten<sup>[9]</sup> has shown how this can be done for the heterotic string when the background consists of N=1, 10-D supergravity.<sup>†</sup> However, phenomenologically interesting vacuum configurations must have nonvanishing Yang-Mills fields; therefore, it is of interest to extend Witten's analysis to include background su-

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\* See, however, E. Witten ref.[2] for a recent proposal. Some references to literature on gauge-invariant free string field theory and light-cone gauge-fixed interacting string field theory are also given in ref.[2].

† See ref. [10] for coupling of the superstring to N=2 supergravity.

per Yang-Mills (SYM) fields. The purpose of this letter is to show how this can be done while preserving the world-sheet symmetries of the superstring.

The basic variables describing the propagation of the heterotic version of the GS superstring are the space-time coordinate  $X^m(\sigma)$  of the string, and its superpartner  $\Theta^\mu(\sigma)$ , a 10-D Majorana-Weyl fermion.<sup>‡</sup> The nature of these variables makes it convenient to formulate this problem in superspace and to use the economical language of superfields. The action for the heterotic string in curved superspace is then given by <sup>[9]</sup>

$$I = \int d^2\sigma \left[ \frac{1}{2} \sqrt{-g} g^{ij} e^{4\phi} v_i^a v_j^b \eta_{ab} + \epsilon^{ij} \partial_i Z^N \partial_j Z^M B_{MN} \right], \quad (1)$$

where

$$v_i^A \equiv \partial_i Z^N e_N^A(Z). \quad (2)$$

$\sigma = (\sigma^0, \sigma^1)$  labels points on the world-sheet;  $g_{ij}$  is the metric on it ( $i, j = 0, 1$ ), with signature  $(+, -)$ , and  $g = \det g_{ij}$ .  $Z^M = (X^m, \Theta^\mu)$  is a coordinate in superspace;<sup>◊</sup>  $M = (m, \mu)$  is the world index ( $m = 0, 1, \dots, 9$  and  $\mu = 1, 2, \dots, 16$ ). Tangent space tensors carry the indices  $A, B, \dots$ ;  $A = (a, \alpha)$  where ‘ $a$ ’ is the bosonic index ( $= 0, 1, \dots, 9$ ) and ‘ $\alpha$ ’ is the fermionic index ( $= 1, 2, \dots, 16$ ).  $e_N^A(Z)$  is the supervielbein,  $\phi(Z)$  is the dilaton superfield and  $B_{MN}(Z)$  is the antisymmetric potential of supergravity.  $\eta_{ab}$  is the 10-D flat metric –  $\text{diag}(+1, -1, \dots, -1)$ .

We shall use two sets of symmetric  $16 \times 16$  10-D  $\gamma$ -matrices  $\Gamma_{\alpha\beta}^a$  and  $\Gamma_a^{\alpha\beta}$ ; the fermionic indices cannot be raised or lowered and a lower fermionic index can only be contracted with an upper index. The Dirac algebra is  $\Gamma_{\alpha\beta}^a \Gamma^b \beta\gamma + \Gamma_{\alpha\beta}^b \Gamma^a \beta\gamma = 2\eta^{ab} \delta_\alpha^\gamma$ . We shall also use  $\Gamma_{abc\dots}$  to represent a totally antisymmetric product of  $\gamma$ -matrices, normalized to unit weight.

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‡ For the moment we shall suppress the gauge degrees of freedom.

◊ We follow the superspace conventions of ref. [12].

In addition to general coordinate invariance in 2-D, the action (1) possesses the following local fermionic world-sheet symmetry <sup>[9]</sup> (henceforth referred to as the  $\kappa$ -symmetry):

$$\delta Z^N e_N^a = 0 \quad (3)$$

$$\delta Z^N e_N^\alpha = 2\sqrt{-g}(\not{\lambda}_i)^{\alpha\beta} \kappa^i_\beta \quad (4)$$

$$\delta(\sqrt{-g}g^{ij}) = 16g P_+^{ik} \{ v_k^\alpha - (\not{\lambda}_k)^{\alpha\beta} \lambda_\beta \} \kappa^j_\alpha, \quad (5)$$

where

$$P_\pm^{ij} = \frac{1}{2} \left( g^{ij} \pm \frac{\epsilon^{ij}}{\sqrt{-g}} \right) \quad \epsilon^{01} = +1 \quad (6)$$

$$(\not{\lambda}_i)^{\alpha\beta} = v_i^a \Gamma_a^{\alpha\beta} \quad (7)$$

$$\lambda_\beta = D_\beta \phi, \quad (8)$$

and  $D_\beta$  is the fermionic covariant derivative in superspace.<sup>[12]</sup>  $\kappa^i_\alpha$  is a 2-D vector and 10-D Majorana-Weyl spinor, satisfying the self-duality condition

$$P_+^{ij} \kappa_{j\alpha} = \kappa^i_\alpha. \quad (9)$$

The proof of the invariance of (1) under (3) – (5) uses the torsion constraints of N=1, 10-D supergravity. With the definition<sup>\*</sup>

$$T^A = D e^A = \frac{1}{2!} e^C e^B T_{BC}^A, \quad (10)$$

the torsion constraints can be written as :

$$\begin{aligned} T_{\alpha\beta}^a &= 2\Gamma_{\alpha\beta}^a & T_{\alpha a}^b &= -T_{a\alpha}^b = 0 \\ T_{a\alpha}^\beta &= -T_{\alpha a}^\beta = (\Gamma_a \psi)_\alpha^\beta & T_{\alpha\beta}^\gamma &= 0, \end{aligned} \quad (11)$$

where  $\psi^{\beta\gamma}$  and the remaining components  $T_{ab}^c$  and  $T_{ab}^\gamma$  are unconstrained. This set is due to Witten;<sup>[9]</sup> although not identical to the torsion constraints of Nilsson,<sup>[13]</sup> the two sets can be shown to be equivalent.

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\* We follow the formulation of superspace differential geometry given in ref.[12].

One also needs an explicit expression for the field strength constructed from the potential  $B_{MN}$ . We define the 2-form potential  $B$  as follows:

$$B \equiv \frac{1}{2!} dZ^N dZ^M B_{MN}. \quad (12)$$

The corresponding 3-form field strength is

$$H \equiv dB = \frac{1}{3!} e^C e^B e^A H_{ABC}. \quad (13)$$

By definition,  $H$  satisfies the Bianchi identity

$$dH = 0. \quad (14)$$

Assuming  $H_{\alpha\beta\gamma} = 0$  and using the torsion constraints (11) one can solve (14) for the components of  $H$ :<sup>[9,15]</sup>

$$\begin{aligned} H_{a\alpha\beta} &= e^{4\phi} \Gamma_{a\alpha\beta} \\ H_{ab\alpha} &= -2e^{4\phi} (\Gamma_{ab})_{\alpha}{}^{\beta} \lambda_{\beta} \\ H_{abc} &= -\frac{3}{2} e^{4\phi} T_{abc}. \end{aligned} \quad (15)$$

Equations (11) and (15), which imply the supergravity equations of motion for the background fields (through the Bianchis), are sufficient to ensure that the action in (1) has the  $\kappa$ -symmetry given in (3) – (5).

To switch on background SYM fields, we first need to restore the gauge degrees of freedom of the string. We use the fermionic representation<sup>[11]</sup> in which the gauge quantum numbers are carried by a 2-D Majorana-Weyl spinor,  $\psi^s$ , where  $s$  is the gauge index. Denoting the 2-D  $\gamma$ -matrices by  $\rho^i$ , the Weyl condition on

$\psi^s$  can be written as

$$P_-^{ij} \rho_j \psi^s = 0. \quad (16)$$

The gauge part of the free string action is:

$$\int d^2\sigma \sqrt{-g} g^{ij} \bar{\psi}^s \rho_i \partial_j \psi^s. \quad (17)$$

To couple background SYM fields to the string one obvious change that one must make is to replace the ordinary derivative in (17) by the gauge covariant derivative. This gives

$$I_{YM} = \int d^2\sigma \sqrt{-g} g^{ij} [\bar{\psi}^s \rho_i (\partial_j \delta_{st} - (A_j)_{st}) \psi^t], \quad (18)$$

where  $(A_j)_{st} \equiv \partial_j Z^N (A_N)_{st}$  is the projection on the world-sheet of the 10-D SYM potential  $A_N(Z)$ .

The action  $I + I_{YM}$  is classically gauge invariant. It also possesses a classical  $\kappa$ -symmetry with a modified transformation law for the world sheet metric given by

$$\delta(\sqrt{-g} g^{ij}) = 16g P_+^{ik} \{ v_k^\alpha - (\rho_k)^{\alpha\beta} \lambda_\beta \} \kappa_\alpha^j + e^{-4\phi} \{ 4g \bar{\psi}^s \rho^j \kappa_\delta^i \chi_{st}^\delta \psi^t \} \quad (19)$$

( $\chi^\delta$  is the Yang-Mills fermion), provided  $\psi^s$  transforms as:

$$\delta\psi^s = (\delta Z^N A_N)_{st} \psi^t. \quad (20)$$

In proving the invariance of the action under (3), (4), (19) and (20), one uses the SYM constraint  $F_{\alpha\beta} = 0$  as well as the solution  $F_{a\alpha} = \Gamma_{a\alpha\beta} \chi^\beta$  of the SYM Bianchis, in addition to (11) and (15).

There are two problems with the system described by the action  $I + I_{YM}$ . The Yang-Mills constraint  $F_{\alpha\beta} = 0$  and the supergravity constraints (11) and (15) do not imply the equations of motion of the coupled SYM–supergravity system; for example, the supergravity equations of motion which they imply do not have the Yang-Mills source terms. This is somewhat surprising since in analogy with the pure supergravity case one would have expected to have used the equations of motion of the coupled background system to ensure the  $\kappa$ -invariance of the action. More seriously,  $I_{YM}$  involves chiral gauge couplings so the gauge symmetry is anomalous. As a consequence of this and the fact that  $\kappa$ -transformation of the  $\psi^s$ , (20), is a gauge transformation, the  $\kappa$ -symmetry is also anomalous. In the rest of this paper we will show that modifying the  $H$ -Bianchi (14) in a particular way eliminates these anomalies. It turns out that this modification also gives the coupled equations of motion of the supergravity – SYM system,\* as has been shown in ref.[15].

We have mentioned that in the case of pure supergravity the torsion constraints imply the existence of a closed 3-form  $H$  in superspace. This means that  $H$  can be written as  $dB$ , at least locally. It is this 2-form that couples to the string. In general, however,  $dH$  need not vanish; it is a 4-form and it could be equal to some linear combination of the two natural 4-forms that exist in curved superspace in the presence of SYM fields, namely,  $\text{tr } F^2$  and  $\text{tr } R^2$ . Here  $F = dA + A^2$  is the SYM field strength 2-form and  $R_A{}^B = d\omega_A{}^B + \omega_A{}^C \omega_C{}^B$  is the super curvature 2-form. Since  $\text{tr } F^2$  and  $\text{tr } R^2$  can locally be written respectively as  $d\omega_{3YM}$  and  $d\omega_{3L}$ , where  $\omega_{3YM}$  is the SYM Chern-Simons 3-form and  $\omega_{3L}$  is the super Lorentz Chern-Simons 3-form, we would in this more general case still be able to define a 3-form  $\tilde{H}$  satisfying  $d\tilde{H} = 0$ † and hence construct a 2-form potential  $\tilde{B}$  (which would now couple to the string). A detailed investigation of

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\* The component field formulation of this theory was studied in [14].

† This does not mean that the theory is pure supergravity. This is because the Bianchi (21) is solved using  $H_{\alpha\beta\gamma} = 0$  and not  $\tilde{H}_{\alpha\beta\gamma} = 0$ .

the Bianchi\*

$$dH = c_1 \operatorname{tr} F^2, \quad (21)$$

along with the Bianchis of supergravity and SYM, shows that it neatly summarizes the coupling of SYM to supergravity in superspace.<sup>[15]</sup> We shall eventually comment on the inclusion of a piece proportional to  $\operatorname{tr} R^2$  in (21). Here we note that using the torsion constraints of supergravity (11), the constraint  $F_{\alpha\beta} = 0$  of SYM, and  $H_{\alpha\beta\gamma} = 0$ , we can solve (21) for the various components of  $H$ . The result is <sup>[15]</sup>

$$\begin{aligned} H_{a\alpha\beta} &= e^{4\phi} \Gamma_{a\alpha\beta} \\ H_{ab\alpha} &= -2e^{4\phi} (\Gamma_{ab})_{\alpha}{}^{\beta} \lambda_{\beta} \\ H_{abc} &= -\frac{3}{2} e^{4\phi} T_{abc} + \frac{c_1}{4} \operatorname{tr}(\chi \Gamma_{abc} \chi). \end{aligned} \quad (22)$$

We see that only the purely bosonic component of  $H$  has changed from the pure supergravity case. Although this change has no effect on the present problem, it has interesting consequences for the compactified solutions of superstring theory. <sup>[16]</sup>

In light of the above modification, we shall now reconsider the coupling of the string to SYM in curved superspace. The action is still given by  $I + I_{YM}$ , with  $B_{MN}$  now satisfying

$$dB = H - c_1 \omega_{3YM}, \quad (23)$$

where

$$\omega_{3YM} = \operatorname{tr}(AF - \frac{1}{3}A^3). \quad (24)$$

Since  $H$  is by definition gauge-invariant, (23) implies that  $B$  is no longer so. In fact, under a gauge transformation,  $\delta_{\Lambda} A = \mathcal{D}\Lambda = d\Lambda - [A, \Lambda]$ ,  $B$  transforms as

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\* In the conventions used here  $c_1$  is a numerical constant which will be fixed later.



$\delta_\Lambda B = -c_1 \text{tr}(\Lambda dA)$ . This change in the transformation property of  $B$  has the welcome feature of removing the anomaly from (18).<sup>‡</sup> <sup>‡</sup> In addition to this (23) also removes the anomaly from the  $\kappa$ -symmetry, as we shall now show.

We start by fixing a gauge for the 2-D metric  $g_{ij}$ . We work in conformal gauge,  $\sqrt{-g}g^{ij} = \eta^{ij}$ , and use the equations of motion for  $g^{ij}$  which can be written as

$$v_\pm^a v_\pm^b \eta_{ab} \equiv v_\pm^2 = 0 \quad (25)$$

where  $v_\pm^a \equiv (v_0^a \pm v_1^a)$ . Let us now integrate out the gauge fermions from (18). In perturbation theory the resulting effective action,  $I_{YM}^{\text{eff}}$ , can be written as a sum of  $n$ -point functions all of which, except for  $n = 2$ , are finite by power counting. Thus, although the gauge field couples to the fermions  $\psi^s$  only through the combination  $A_- \equiv (A_0 - A_1)$  because of (16), the effective action acquires a dependence on  $A_+ \equiv (A_0 + A_1)$  through the regularization needed for the two-point function. Now the gauge variation of  $I_{YM}^{\text{eff}}$  can be shown to be equal to the anomaly

$$\delta_\Lambda I_{YM}^{\text{eff}} = \frac{1}{8\pi} \int d^2\sigma \epsilon^{ij} \text{tr}(\Lambda \partial_i A_j). \quad (26)$$

Since the nonlocal part of the effective action is a functional of  $A_-$  only, the effective action must contain a local piece equal to  $-\frac{1}{16\pi} \int d^2\sigma \text{tr}(A_+ A_-)$  in order to reproduce (26). We may, therefore, write the result of integrating out the fermions from (18) as<sup>◊</sup> :

$$I_{YM}^{\text{eff}} = \frac{-1}{16\pi} \int d^2\sigma \text{tr}(A_+ A_-) + G[A_-]. \quad (27)$$

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‡ For  $\sigma$ -models defined on purely bosonic spaces (as opposed to superspace) an identical situation occurs, which was first discussed in [17]. See also refs. [18] and [19].

‡ This cancellation of gauge anomaly also fixes  $c_1$  to be  $-1/16\pi$ . More precisely, if we restore the gauge coupling constant  $g_{10}$ , Newton's constant  $\kappa_{10}$  and the slope parameter  $\alpha'$  in the action then the anomaly cancellation gives the heterotic string relation  $\frac{g_{10}}{\kappa_{10}} \approx \frac{1}{\sqrt{\alpha'}}$  [11].

◊ We thank A. Sen for discussions on this point.

Now, under a  $\kappa$ -transformation the gauge field  $A_i$  transforms as:

$$\begin{aligned}\delta_\kappa A_i &= \delta_\kappa(\partial_i Z^N) A_N + \partial_i Z^N \delta_\kappa A_N \\ &= \mathcal{D}_i \Lambda_\kappa + \partial_i Z^N \delta_\kappa Z^M F_{MN},\end{aligned}\quad (28)$$

where  $\mathcal{D}_i = \partial_i Z^N \mathcal{D}_N$  and we have used the definition

$$\Lambda_\kappa = \delta_\kappa Z^N A_N. \quad (29)$$

Using (3) and (4), the Yang-Mills constraint  $F_{\alpha\beta} = 0$  and the solution for  $F_{\alpha\alpha}$ , we can simplify (28) to get:

$$\delta_\kappa A_i = \mathcal{D}_i \Lambda_\kappa + \chi^\alpha (\not{\nu}_i \not{\nu}_-)_{\alpha}{}^{\beta} \kappa_{+\beta}. \quad (30)$$

It follows from (30) and (25) that the  $\kappa$ -variation of  $A_-$  is exactly a gauge transformation,  $\delta_\kappa A_- = \mathcal{D}_- \Lambda_\kappa$ , with the field-dependent parameter  $\Lambda_\kappa$ . This observation is crucial for the present analysis since it allows us to compute the  $\kappa$ -variation of (26) without knowing the exact form of  $G$ . In fact, using (26), (27) and (30) we find that

$$\delta_\kappa I_{YM}^{\text{eff}} = \frac{1}{8\pi} \int d^2\sigma \epsilon^{ij} \text{tr}(\Lambda_\kappa \partial_i A_j) - \frac{1}{16\pi} \int d^2\sigma \text{tr}(\chi^\alpha (\not{\nu}_+ \not{\nu}_-)_{\alpha}{}^{\beta} \kappa_{+\beta} A_-). \quad (31)$$

To find the variation of the full action under a  $\kappa$ -transformation we must add the  $\kappa$ -variation of (1) to (31). The variation of the  $B$ -term is:

$$\begin{aligned}\delta_\kappa \int d^2\sigma \epsilon^{ij} \partial_i Z^N \partial_j Z^M B_{MN} &= 3 \int d^2\sigma \epsilon^{ij} \partial_i Z^N \partial_j Z^M \delta_\kappa Z^L \partial_{[L} B_{MN)} \\ &= \int d^2\sigma \epsilon^{ij} \partial_i Z^N \partial_j Z^M \delta_\kappa Z^L [H_{LMN} - 6c_1 (\omega_{3YM})_{[LMN)}]\end{aligned}$$

where in the last equality we have used (23), and  $[ ]$  represents graded antisymmetrisation with unit weight. The  $\kappa$ -variation of the first term in (1) combines

with the  $H$ -term above and vanishes on using (11), (22) and (25), as in the case of pure supergravity. The  $\omega_{3YM}$  piece contributes:

$$2c_1 \int d^2\sigma \epsilon^{ij} \text{tr}[\Lambda_\kappa \partial_i A_j + \chi^\alpha (\not{\nu}_i \not{\nu}_-)_\alpha^\beta \kappa_{+\beta} A_j]. \quad (32)$$

Using  $c_1 = -\frac{1}{16\pi}$  and (25) one can verify that the sum of (31) and (32) vanishes.

In summary, we have constructed a gauge invariant action which describes the propagation of the heterotic string in curved superspace in the presence of background super Yang-Mills fields. We have also demonstrated that the coupled equations of motion of the supergravity-super Yang-Mills system ensure that the string coupling to the background is consistent, i.e. gauge and  $\kappa$ -invariant. We, however, do not expect this to be the end of the story since the action in (1) must have a Lorentz anomaly.\* We strongly suspect that the cancellation of this anomaly will require modification of (21) by a term proportional to  $\text{tr}R^2$ :†

$$dH = c_1 \text{tr}F^2 + c_2 \text{tr}R^2. \quad (33)$$

Preliminary investigation of the superspace Bianchi (33) indicates that the equations of motion for the resulting system are infinite expansions in the parameter  $c_2$ . A detailed investigation of the superspace system described by (33) as well as an investigation of the quantum properties of the action in (1) is in progress.

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\* One can see this by making a  $\theta$ -expansion of the action.

† Similar observations for the  $\sigma$ -model in the purely bosonic case have been made in [17]. See also [18].

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