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SUPERSPACE FORMULATION OF TEN-DIMENSIONAL $N = 1$ SUPERGRAVITY COUPLED TO $N = 1$ SUPER YANG-MILLS THEORY*

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ABSTRACT

We present an on-shell superspace formulation of ten-dimensional $N = 1$ supergravity coupled to $N = 1$ super Yang-Mills theory. The coupling is completely specified in superspace by the Bianchi $dH = c_1 \mathrm{tr} F^2$, where H is the gauge-invariant 3-form field strength of supergravity and F is the 2-form super Yang-Mills field strength. We also briefly discuss the theory that results from modifying this Bianchi by the addition of a piece proportional to the square of the super curvature 2-form.

1. INTRODUCTION

A striking feature of ten-dimensional $N = 1$ supergravity coupled to $N = 1$ super Yang-Mills (SYM) theory[1] is that the gauge invariance of the lagrangian requires that the antisymmetric potential $B_{mn}(x)$ transform anomalously under gauge transformations. The demonstration, by Green and Schwarz [2], of anomaly cancellation in superstring theory has shed further light on this curious feature of the SYM-supergravity theory. They showed that the anomalous transformation of B_{mn} under gauge transformations (and similarly under local Lorentz transformations) is required for anomaly cancellation in the field theory limit of superstrings. More recently, studies of two-dimensional non-linear σ-models have revealed an unexpected connection between these anomalous transformation laws of B_{mn} and world-sheet properties of the string. Hull and Witten[3] have shown that the nonlinear σ-model describing string propagation in background fields (belonging to the massless sector of the string spectrum) has gauge and local Lorentz anomalies which can, however, be cancelled by postulating that B_{mn} transform anomalously.

The present work reveals another interesting aspect of this feature of the SYM-supergravity theory. We present a superspace formulation of on-shell ten-dimensional N=1 supergravity coupled to SYM; the coupling is succinctly summarized by the superspace Bianchi

$$dH = c_1 \mathrm{tr} F^2, \tag{1.1}$$

where H is the gauge invariant 3-form field strength of the 2-form potential B of supergravity and F is the 2-form SYM field strength. Eqn. (1.1), of course, implies that B transforms anomalously under gauge transformations.

This work grew out of an attempt to include background SYM fields in Witten's analysis[4] of the propagation of the heterotic version[5] of the Green-Schwarz superstring[6] in curved superspace. Crucial to the Green-Schwarz formulation is the existence of a local fermionic world-sheet symmetry, the κ-symmetry, which is needed to gauge away unphysical degrees of freedom. It is necessary to maintain this symmetry while coupling the superstring to background fields. For the heterotic string propagating in curved superspace[7] Witten has shown that the κ-symmetry is ensured if the background fields satisfy the supergravity torsion constraints[8] (which imply the supergravity equations of motion). We have shown[9] that a naive coupling of background SYM fields to this system possesses a classical κ-symmetry, but that both it and the gauge symmetry of the resulting superspace σ-model[10] are anomalous. These anomalies have, however, been shown to be absent if the modified superspace Bianchi for H, (1.1), is used instead of the pure supergravity Bianchi $dH = 0$. Since the torsion constraints of supergravity together with the Bianchi (1.1) ensure the existence of the κ-symmetry, one might, in analogy with the pure supergravity case, then suspect that the resulting background system describes the fully coupled SYM-supergravity theory in superspace. The purpose of this paper is to show that this is indeed the case.

The organization of this paper is as follows. Sec. 2 is devoted to establishing our notation and discussing some technical preliminaries. The latter are essentially a paraphrashing of ref. [11] and are included here only for completeness. In Sec. 3 we motivate and discuss the coupling of SYM to supergravity from another point of view. In Sec. 4 solutions of the Bianchis of the coupled system are exhibited and some of their more interesting features are discussed. The detailed derivation of these solutions is relegated to the appendices. We conclude in Sec.

5 with a brief discussion of the theory resulting from modifying (1.1) by adding a piece proportional to the square of the supercurvature 2-form.

2. TECHNICAL PRELIMINARIES

We consider a curved superspace with points parametrized, in local coordinates, by $Z^M = (X^m, \Theta^\mu)$ where X^m $(m = 0, 1, 2, ..., 9)$ are ten ordinary bosonic world coordinates and Θ^μ $(\mu = 1, 2, ..., 16)$ are 16 anticommuting fermionic world coordinates. At each point in superspace we introduce a set of basis 1-forms $\{e^A\}$:

$$e^A = dZ^M e_M{}^A, \tag{2.1}$$

where $e_M{}^A$ is the superveilbein. We shall denote its inverse by $E_A{}^M$, so

$$e_M{}^A E_A{}^N = \delta_M^N, \qquad E_A{}^M e_M{}^B = \delta_A^B. \tag{2.2}$$

The tangent space indices $A, B...$, can either be bosonic $a, b, ..(= 0, 1, ..., 9)$ or fermionic $\alpha, \beta..(= 1, 2, ..., 16)$. A basis for p-forms is constructed from the set $\{e^A\}$, in the usual way, by forming wedge products, except that the wedge product is now graded, *i.e.*,

$$e^A e^B = -(-)^{[A][B]} e^B e^A, \tag{2.3}$$

where $[a] = 0$ and $[\alpha] = 1$. We have omitted an explicit wedge symbol.

Vectors transform under the tangent space group as follows :

$$\delta V^A = V^B L_B{}^A, \qquad \delta V_A = -L_A{}^B V_B. \tag{2.4}$$

We choose the tangent space group to be SO(1,9) with ordinary (bosonic) vectors transforming as the 10 and spinors as the 16 or $\bar{16}$ depending on their chirality .

This implies that the Lie algebra-valued matrices $L_A{}^B$ must satisfy

$$L_\alpha{}^a = 0 = L_a{}^\alpha, \qquad L_\alpha{}^\beta = \frac{1}{4} L_{ab} (\Gamma^{ab})_\alpha{}^\beta. \tag{2.5}$$

We work with a bimodular representation of the ten-dimensional γ- matrices. Thus there are two sets of (symmetric) 16×16 γ- matrices, $\Gamma^a_{\alpha\beta}$ and $\Gamma^{a\alpha\beta}$. The fermionic indices cannot be raised or lowered and an upper fermionic index can only be contracted with a lower one. The Dirac algebra is $\Gamma^a_{\alpha\beta}\Gamma^{b\beta\gamma} + \Gamma^b_{\alpha\beta}\Gamma^{a\beta\gamma} = 2\eta^{ab}\delta_\alpha{}^\gamma$. $\Gamma^{abc..}$ is used to denote a totally antisymmetric product of γ- matrices, normalized to unit weight.

The covariant exterior derivative, $D = dZ^M D_M = e^A D_A$ may be defined by its action on vector-valued p-forms :

$$DV^A = dV^A + V^B \omega_B{}^A, \tag{2.6}$$

$$DV_A = dV_A - (-)^p \omega_A{}^B V_B, \tag{2.7}$$

where $\omega_A{}^B = dZ^M \omega_{MA}{}^B = e^C \omega_{CA}{}^B$ is the superconnection 1-form. The operator d is the exterior derivative defined by $d = dZ^M \partial_M$. It satisfies $d^2 = 0$ and obeys the Leibnitz rule with the sign convention of ref. [11]. From the connection and veilbein one can construct the torsion 2-form T^A and the curvature 2-form $R_A{}^B$ defined as :

$$T^A = De^A, \tag{2.8}$$

$$R_A{}^B = d\omega_A{}^B + \omega_A{}^C \omega_C{}^B. \tag{2.9}$$

In terms of components

$$T^A = \frac{1}{2} dZ^N dZ^M T_{MN}{}^A = \frac{1}{2} e^C e^B T_{BC}{}^A, \tag{2.10}$$

$$R_A{}^B = \frac{1}{2} dZ^N dZ^M R_{MNA}{}^B = \frac{1}{2} e^D e^C R_{CDA}{}^B. \tag{2.11}$$

As a result of our choice for the tangent space group $R_A{}^B$ satisfies

$$R_\alpha{}^a = 0 = R_a{}^\alpha, \qquad R_\alpha{}^\beta = \frac{1}{4} R_{ab} (\Gamma^{ab})_\alpha{}^\beta. \tag{2.12}$$

Similar conditions are also satisfied by $\omega_A{}^B$.

In the presence of SYM fields one also needs to consider the field stength F which can be written as the Lie algebra-valued (in the gauge group) 2-form :

$$F = \frac{1}{2} e^B e^A F_{AB}, \tag{2.13}$$

where we have suppressed the gauge indices. It is defined in terms of the 1-form potential $A = dZ^M A_M = e^B A_B$ as

$$F = dA + A^2 \tag{2.14}$$

All the 'field strengths' introduced above satisfy Bianchis by virtue of their definition in terms of 'potentials'. These can be obtained from (2.8), (2.9) and (2.14) by using $d^2 = 0$ and are

$$DT^A - e^B R_B{}^A = 0, \tag{2.15}$$

$$DR_A{}^B = 0, \tag{2.16}$$

$$\mathcal{D} F = 0, \tag{2.17}$$

where $\mathcal{D}$ is the gauge and superspace covariant derivative. Its action on a Lie algebra-valued scalar superfield Λ is $\mathcal{D}\Lambda = d\Lambda - [A, \Lambda]$.

At this stage it is appropriate to remark that the Bianchis (2.15) and (2.16) are not independent. Dragon [12] has shown that, for the choice of the tangent space group made here, (2.16) is in fact identically satisfied by virtue of (2.15). Thus the only independent Bianchis are (2.15) and (2.17). In component form these are:

$$D_{[A}T_{BC)}{}^{D} + T_{[AB}{}^{E}T_{\hat{E}C)}{}^{D} - R_{[ABC)}{}^{D} = 0 \tag{2.18}$$

$$\mathcal{D}_{[A}F_{BC)} + T_{[AB}{}^{D}F_{\hat{D}C)} = 0, \tag{2.19}$$

where [ ) represents graded antisymmetrization normalized to unit weight. Also, [ ] and ( ) will be used to represent ordinary antisymmetrization and symmetrization with the same normalization. Indices with a caret are excluded from these operations. Henceforth (2.18) and (2.19) will be called the T and F-Bianchis respectively.

3. COUPLING OF SYM TO SUPERGRAVITY

A basic feature of the superspace formulation of a supersymmetric theory is that the number of ordinary (*i.e.*, X-space) fields is usually far greater than the number of dynamical fields required to describe the theory. This makes it necessary to impose constraints on some of the superfields to eliminate redundant X-space fields. For pure supergravity these constraints are usually imposed on components of the torsion tensor $T_{AB}{}^{C}$. In view of Dragon's result[12], this is a natural thing to do, since the supercurvature can be related to the supertorsion and its covaiant derivatives through the T-Bianchis (2.18). In the presence of

SYM fields one has, in addition, to constrain the field strength F_{AB}. Once constraints are imposed the Bianchis are no longer identically satisfied. In fact, for an appropriate set of constraints they determine all the unconstrained superfields in terms of the dynamical fields. They also provide equations of motion for these fields.

The superspace formulation of ten-dimensional N=1 supergravity along these lines was first presented by Nilsson[13]. In his formulation the $\Theta = 0$ components of the torsion and curvature tensors contain all but the antisymmetric tensor degree of freedom. In order to accomodate this degree of freedom at the $\Theta = 0$ level Nilsson introduced a super 2-form B by constructing a closed 3-form H using a suitable set of constraints. Since H is closed it can be written as the exterior derivative of a 2-form, at least locally; it is this 2-form that Nilsson identified with B.

In formulating supergravity coupled to (SYM) we introduce the 2-form B using a natural generalization of Nilsson's procedure. We require that the 3-form H satisfy a Bianchi but will not insist on it being closed (it is not necessary for H to be closed to interpret it as the field strength of B). The Bianchi that H now obeys must relate dH to other closed 4-forms in the system. Even though this Bianchi can be reexpressed as $d\tilde{H} = 0$ in terms of a new 3-form, $\tilde{H}$, related to H, this is more general than requiring that H be closed. This is because this Bianchi is solved using suitable constraints on H not $\tilde{H}$. Now, there are only two 4-forms in this system that are naturally closed, namely, $\mathrm{tr}R^2$ and $\mathrm{tr}F^2$ (the trace in the first term is over tangent space indices and in the second term over the group indices). So, in general, the Bianchi for H takes the form

$$dH = c_1 \mathrm{tr}F^2 + c_2 \mathrm{tr}R^2, \tag{3.1}$$

where c_1 and c_2 are apriori arbitrary. Since $d\mathrm{tr}F^2 = d\mathrm{tr}R^2 = 0$ we may write them as

$$\mathrm{tr}F^2 = d\omega_{3YM}, \tag{3.2}$$

$$\mathrm{tr}R^2 = d\omega_{3L}, \tag{3.3}$$

where ω_{3YM} is the SYM Chern-Simons 3-form

$$\omega_{3YM} = \mathrm{tr}(A\ F - \frac{1}{3}A^3), \tag{3.4}$$

and ω_{3L} is the super Lorentz Chern-Simons 3-form

$$\omega_{3L} = \mathrm{tr}(\omega R - \frac{1}{3}\omega^3). \tag{3.5}$$

This implies the following relation between H and B:

$$dB = H - c_1\ \omega_{3YM} - c_2\ \omega_{3L}. \tag{3.6}$$

Since H is, by definition, gauge and local Lorentz invariant, (3.6) implies that B is no longer so. In fact it transforms as:

$$\delta B = -c_1\mathrm{tr}(d\Lambda\ A) - c_2\mathrm{tr}(d\Omega\ \omega) \tag{3.7}$$

where Λ and Ω are the superfield parameters of gauge and local Lorentz transformations. As mentioned in the introduction, it is precisely this anomalous gauge transformation of B that is required for a consistent coupling of the string to background SYM fields in curved superspace[9]. The superspace σ-model discussed in ref. [9] must also have a Lorentz anomaly, as can be seen by expanding the action in powers of Θ. We expect that the above anomalous Lorentz transformation property of B will be required to cancel this anomaly. In most of what follows we shall, however, restrict ourselves to the case with $c_2 = 0$, *i.e.*, we will assume that H satisfies the Bianchi (1.1). In the next section we will solve the T, F and H-Bianchis using an appropriate set of torsion constraints. The resulting equations of motion and supersymmetry transformation laws describe coupled SYM-supergravity theory.

It is important to realize that (1.1) introduces an arbitrary parameter in the coupled theory. If we restore the gauge coupling constant in the definition of F and work with fields of canonical dimensions in (1.1), then c_1 can be seen to be of length dimension four. This is precisely the dimension of the ten-dimensional gravitational coupling constant. It is, perhaps, appropriate that the gravitational constant first appears explicitly in (1.1), since it is this equation that is responsible for coupling matter to gravity. (This should be contrasted with the pure supergravity Bianchis where no arbitrary parameter appears explicitly.) Interestingly, as we shall see in the next section, c_1 can actually be removed from all equations by appropriate rescalings of the various fields, reflecting the fact that in the Chapline-Manton theory the coupling constants can be scaled away from the lagrangian.

We end this section by giving the H-Bianchi, (1.1), in component form:

$$D_{[A}H_{BCD)} + \frac{3}{2}T_{[AB}{}^{F}H_{\hat{F}CD)} - \frac{3}{2}c_1 \mathrm{tr}(F_{[AB}F_{CD)}) = 0 \tag{3.8}$$

4. SOLUTIONS OF THE BIANCHIS

In this section we discuss the solutions of the T, F and H-Bianchis. These Bianchis are solved using the following set of constraints on the torsion tensor:

$$\begin{aligned} &T_{\alpha\beta}{}^{a} = 2\Gamma^{a}_{\alpha\beta} \qquad T_{\alpha a}{}^{b} = -T_{a\alpha}{}^{b} = 0 \\ &T_{a\alpha}{}^{\beta} = -T_{\alpha a}{}^{\beta} = (\Gamma_a\psi)_\alpha{}^{\beta} \qquad T_{\alpha\beta}{}^{\gamma} = 0, \end{aligned} \tag{4.1}$$

where the superfield $\psi^{\alpha\beta}$ and the components $T_{ab}{}^{c}$ and $T_{ab}{}^{\alpha}$ are unconstrained. This set is due to Witten[4]; although not identical to that used by Nilsson[13], the two sets can be shown to be equivalent. We use this set since we find it simpler to work with. We also use the following constraints on the superfields H_{ABC} and F_{AB}:

$$H_{\alpha\beta\gamma} = 0, \qquad F_{\alpha\beta} = 0. \tag{4.2}$$

Since the algebra is rather involved, a detailed derivation of the solutions is relegated to the appendices. Here we present the solutions and discuss some of their more interesting features.

<u>T-Bianchis</u>

Using (4.1) and (2.18) one obtains a number of equations for the unconstrained components of torsion and curvature . These have been listed in Appendix A, eqns. $(A1) - (A7)$. An immediate consequence of these equations is that the superfield $T_{abc} \equiv T_{ab}{}^{d}\eta_{dc}$, which is apriori antisymmetric in its first two indices only, is actually totally antisymmetric. This result is interesting because it makes T_{abc} have symmetry properties identical to those of H_{abc}. In fact, as we will see when we discuss the solutions of the H-Bianchis, these two superfields are simply related.

There are a number of additional results that can be obtained from (A1)–(A7). One can determine the superfields $\psi^{\alpha\beta}$ and $R_{\alpha\beta ab} \equiv R_{\alpha\beta a}{}^{d}\eta_{db}$ completely in terms of T_{abc}:

$$\psi^{\alpha\beta} = -\frac{1}{24}T_{abc}(\Gamma^{abc})^{\alpha\beta} \tag{4.3}$$

$$R_{\alpha\beta ab} = \frac{1}{6}T_{cde}(\Gamma_{ab}{}^{cde})_{\alpha\beta} + 3T_{abc}\Gamma^{c}_{\alpha\beta} \tag{4.4}$$

Also, using the following decomposition of $T_{ab}{}^{\alpha}$ in terms of $SO(1,9)$ irreducibles

$$T_{ab}{}^{\alpha} = J_{ab}{}^{\alpha} + 2J_{\beta[a}\Gamma_{b]}{}^{\beta\alpha} + J^{\beta}(\Gamma_{ab})_{\beta}{}^{\alpha} \tag{4.5}$$

where

$$J_{ab}{}^{\alpha}\Gamma^{b}_{\alpha\beta} = 0 \tag{4.6}$$

$$J_{\beta a}\Gamma^{a\beta\alpha} = 0 \tag{4.7}$$

one can show that

$$J^{\beta} = -\frac{13}{90}D_{\alpha}\psi^{\alpha\beta} \tag{4.8}$$

$$J_{\beta a} = -\frac{1}{56}[D_{\alpha}(\Gamma_{a}\psi)^{\alpha}{}_{\beta} + 288J^{\alpha}\Gamma_{a\alpha\beta}]. \tag{4.9}$$

From (4.7) and (4.9) one finds that ψ must satisfy the equation $D_{\alpha}\psi^{\alpha\beta} = 0$, which implies

$$J^{\beta} = 0. \tag{4.10}$$

This equation will eventually turn out to be the equation of motion for the supergravity 'spin-$\frac{1}{2}$' field. The remaining irreducible component $J_{ab}{}^{\alpha}$ can be related to a fermionic derivative on T_{abc}, as in (A25). This expression satisfies (4.6) identically and so does not lead to any further constraints. The Rarita-Schwinger equation is obtained from

$$T_{ab}{}^{\alpha}(\Gamma^{abc})_{\alpha\beta} = 16 J_{\beta d}\eta^{dc}, \tag{4.11}$$

once we have solved for $J_{\beta a}$ using the H-Bianchis.

Finally, one can also relate $R_{\alpha abc}$ and R_{abcd} to T_{abc} and its fermionic derivatives. The latter relation leads to the following two results; the equation

$$D_a T_{bc}{}^{a} = 0 \tag{4.12}$$

which will turn out to be the equation of motion for the field strength H_{abc}, and an expression for the Ricci tensor

$$R_{acbd}\eta^{cd} \equiv R_{ab} = D_\alpha J_{\beta a}\Gamma_b{}^{\alpha\beta} + \frac{1}{4}\eta_{ab}T^2 - \frac{3}{2}T^2_{ab}, \tag{4.13}$$

where we have used the notation $T^2 \equiv T_{abc}T^{abc}, T^2_{ab} \equiv T_{acd}T_b{}^{cd}$. Although not evident in (4.13), R_{ab} is actually symmetric, as shown in Appendix C.

In summary, the T-Bianchis enable us to relate all the unconstrained components of the torsion tensor and all the components of the curvature tensor to the single superfield T_{abc}. They also give us a number of equations which will eventually turn out to be the equations of motion for some of the dynamical fields of the theory.

$\underline{F\text{-Bianchis}}$

Using (4.1) and (4.2) in (2.19) these Bianchis can be written out in components as in Appendix B, eqns. $(B1) - (B4)$. It follows from $(B2)$ that $F_{a\alpha}$ is of the form

$$F_{a\alpha} = \Gamma_{a\alpha\beta}\chi^{\beta}, \tag{4.14}$$

where the 'spin-$\frac{1}{2}$' superfield χ is a 16 of $SO(1,9)$ and transforms in the adjoint representation of the gauge group. We identify it as the gluino, the superpartner of the gauge field. It is now relatively straightforward to obtain the following equations:

$$\mathcal{D}_{\alpha}\chi^{\beta} = \frac{1}{2}F_{ab}(\Gamma^{ab})_{\alpha}{}^{\beta} \tag{4.15}$$

$$\mathcal{D}_{\alpha}F_{ab} = 2\Gamma_{[a\hat{\alpha}\hat{\beta}}D_{b]}\chi^{\beta} - T_{abc}\Gamma^{c}_{\alpha\beta}\chi^{\beta} - 2(\Gamma_{[a}\not{\psi}\Gamma_{b]})_{\alpha\beta}\chi^{\beta} \tag{4.16}$$

$$\Gamma^{a}_{\alpha\beta}\mathcal{D}_{a}\chi^{\beta} = 0. \tag{4.17}$$

The first two of these are essentially the variations of the gluino and the gauge field strength under a supersymmetry transformation and the last is the equation of motion for the gluino. The simplicity of this equation is deceptive—we remind the reader that all our covariant derivatives are torsionful. There is one more result that can be derived from $(B1) - (B4)$. It is the Yang-Mills equation,

$$\mathcal{D}^{b}F_{bc} = \frac{1}{2}\Gamma_{c\alpha\beta}\chi^{\alpha}\chi^{\beta} - T_{abc}F^{ab} - 8J_{\alpha c}\chi^{\alpha}. \tag{4.18}$$

In summary, the F-Bianchis can be completely solved using the results of the T-Bianchis. One obtains in this way the supersymmetry variations of the SYM fields and the equations of motion for them.

H-Bianchis

So far the scalar and 'spin-$\frac{1}{2}$' degrees of freedom of supergravity have not appeared in our discussion. As we shall see below, the solutions of the H-Bianchis contain these missing degrees of freedom. In addition, we will be able to relate H_{abc} and T_{abc}, solve for a fermionic derivative on T_{abc} and obtain an expression for $J_{\alpha a}$ in terms of the other superfields whose $\Theta = 0$ components are directly related to the dynamical fields of the theory. This will enable us to obtain all the equations of motion and also show that, on-shell, all the superfields can be expressed in terms of the dynamical fields of the SYM-supergravity system.

Using (4.1) and (4.2) in (3.8) one can write the H-Bianchis in components as in Appendix C, eqns. $(C1) - (C5)$. Eqn. $(C2)$ is solved by[15]

$$H_{a\alpha\beta} = \phi \Gamma_{a\alpha\beta} \tag{4.19}$$

where ϕ is a scalar superfield, whose $\Theta = 0$ component is just the dilaton. Its superpartner is the $\Theta = 0$ component of the 'spin- $\frac{1}{2}$' superfield λ, which is defined by

$$\lambda_\alpha \equiv D_\alpha \phi \tag{4.20}$$

Using (4.19) eqns. $(C3)$ and $(C5)$ can be solved for the other components of H_{ABC}. We obtain

$$H_{ab\alpha} = -\frac{1}{2}(\Gamma_{ab})_\alpha{}^\beta \lambda_\beta \tag{4.21}$$

$$H_{abc} = -\frac{3}{2}\phi T_{abc} + \frac{c_1}{4}(\Gamma_{abc})_{\alpha\beta} tr(\chi^\alpha \chi^\beta). \tag{4.22}$$

One other important equation that can be obtained from $(C5)$ is

$$D_\alpha \lambda_\beta = -\Gamma^a_{\alpha\beta} D_a \phi + \frac{1}{9}(\Gamma^{abc})_{\alpha\beta}[H_{abc} + \frac{c_1}{8}(\Gamma_{abc})_{\gamma\delta} tr(\chi^\gamma \chi^\delta)]. \tag{4.23}$$

This equation is essentially the supersymmetry variation of λ.

Eqns. (4.22) and (4.23) are extremely interesting. The first tells us that the field strength H_{abc} is proportional to the spacetime torsion in the absence of coupling to SYM[16]. When SYM fields are present this relation is modified by the appearance of the gluino bilinear. Since all our equations, with the exception of (4.23), are written in terms of T_{abc}, they will involve H_{abc} only in this specific combination with the gluino bilinear. The fact that H_{abc} always appears in a specific combination with the gluino bilinear, except in the supersymmetry variation of λ, has interesting consequences for the compactified solutions of superstring theory. As argued in ref. [17] this means that it might be possible to have vacuum solutions with a vanishing cosmological constant even when supersymmetry is broken. Equation (4.22) also explains the 'perfect square' of ref. [17]; this appears through the T^2 terms in (4.13).

Returning to eqns. $(C1)-(C5)$, there is another important result that we can obtain from them. This result, given in $(C13)$, expresses a fermionic derivative of T_{abc} in terms of the other superfields. A number of relations follow from this equation. First of all, imposing the restriction (4.10) gives the λ equation of motion :

$$\Gamma^{a\alpha\beta} D_a \lambda_\beta = 2\psi^{\alpha\beta}\lambda_\beta + \frac{c_1}{3}(\Gamma^{ab})^\alpha{}_\beta tr(F_{ab}\chi^\beta). \tag{4.24}$$

From this one can obtain the equation of motion for ϕ:

$$D^a D_a \phi = -\frac{1}{2}\phi T^2 - \frac{c_1}{6} T_{abc}(\Gamma^{abc})_{\alpha\beta} tr(\chi^\alpha \chi^\beta) + \frac{c_1}{3} tr(F_{ab}F^{ab}) \tag{4.25}$$

Finally, one can obtain the following expression for $J_{\alpha a}$

$$J_{\alpha a} = \frac{\phi^{-1}}{16}[D_a\lambda_\alpha - (\Gamma_a\psi + 2\psi\Gamma_a)_\alpha{}^\beta\lambda_\beta + \frac{c_1}{6}(3\Gamma^{bc}\Gamma_a - 2\Gamma_a\Gamma^{bc})_{\alpha\beta}tr(F_{bc}\chi^\beta)]. \tag{4.26}$$

Eqn. (4.11) then gives us the Rarita-Schwinger equation while the Ricci tensor can be obtained from (4.13). The latter is

$$\begin{aligned} R_{ab} = &-\frac{1}{2}\phi^{-2}(\lambda\Gamma_{(a}D_{b)}\lambda) + c_1\phi^{-1}\ \mathrm{tr}(\chi\Gamma_{(a}\mathcal{D}_{b)}\chi) \\ &-\phi^{-1}D_{(a}D_{b)}\phi + \frac{1}{4}\ \eta_{ab}T^2 - 2\ T^2_{ab} \\ &+\frac{c_1}{2}\phi^{-1}\ \mathrm{tr}(\chi\Gamma_{jk(a}\chi)T_{b)}{}^{jk} \\ &-\frac{c_1}{36}\phi^{-1}\ \eta_{ab}\mathrm{tr}(\chi\Gamma_{cde}\chi)T^{cde} \\ &+\frac{c_1}{6}\ \phi^{-1}\mathrm{tr}(4F_{ac}F^c{}_b + 9\eta_{ab}F_{cd}F^{cd}) \\ &-\frac{1}{4}\phi^{-2}\ (\lambda\Gamma_{jk(a}\lambda)T_{b)}{}^{jk} \\ &+\frac{1}{48}\phi^{-2}\ \eta_{ab}(\lambda\Gamma_{cde}\lambda)T^{cde} \\ &-\frac{c_1}{12}\phi^{-2}\ \mathrm{tr}[F_{hj}\chi(\Gamma^{hj}\eta_{ab} + 12\delta^h_{(a}\Gamma^j\Gamma_{b)})\lambda] \end{aligned} \tag{4.27}$$

We have used an obvious compact notation in this equation. All expected source terms appear in it, though in a noncanonical form. The last equation of motion, that for H_{abc}, is obtained from (4.12) and (4.22). Having obtained all the equations of motion we can now see that the parameter c_1 can be removed from them by the field rescalings $\phi \to c_1\phi$ (which also implies $\lambda \to c_1\lambda$ through (4.21)) and $H_{abc} \to c_1H_{abc}$. (In the $\Theta \to 0$ limit, this corresponds to rescaling the antisymmetric potential B_{mn}.)

We should mention here that in solving the three sets of Bianchis one comes across a number of consistency conditions. We have checked that they are all satisfied. For example, one might have thought that one could solve for $T_{ab}{}^{\alpha}$ in terms of the other superfields since it is related to a fermionic derivative of T_{abc} for which an expression has been obtained in ($C13$). It turns out that this is not the case, as explained in Appendix C. This is as it should be since the $\Theta = 0$ component of $T_{ab}{}^{\alpha}$ involves a dynamical field, the Rarita-Schwinger field. However, fermionic derivatives of $T_{ab}{}^{\alpha}$ can be expressed in terms of the other fields. In fact, from the solutions we have obtained it is not difficult to see that this is true of all the superfields. Hence the constraints (4.1) and (4.2) are sufficient to determine the on-shell system completely.

A detailed comparison of this theory with the Chapline-Manton theory[1] entails working out the $\Theta \to 0$ limit of the equations of motion and supersymmetry transformations for the various fields and then finding the appropriate field redefinitions. We shall not attempt to do this here but only remark that qualitatively all our equations of motion and transformation laws agree with those of the Chapline-Manton theory, except for the presence of extra terms quartic in χ. These terms are necessary for the theory to be supersymmeteric, as was first noted in ref. [17].

5. CONCLUDING REMARKS

In the preeceding sections we have discussed the coupling of SYM to supergravity, which was achieved by considering (3.1) with only c_1 nonzero. However, for reasons mentioned earlier, we expect that a consistent treatment of superstring propagation in curved superspace in the presence of background SYM fields would require H to satisfy the full Bianchi (3.1). It is therefore of interest to extend the previous analysis to this case. Another reason for doing so is that SYM-supergravity theory is known to be anomalous, and, as demonstrated by Green and Schwarz[2], a modification in the definition of the field strength H similar to (3.6), in X-space is required for anomaly cancellation. In this concluding section we will briefly investigate the effect of this modification on our previous results.

To see what this modification entails it is necessary to look at the Bianchi (3.1) in components. These are:

$$D_{[e}\ H_{abd]} + \frac{3}{2}\ T_{[ea}{}^{f}\ H_{\hat{f}bd]} - \frac{3c_1}{2}\ \mathrm{tr}(F_{[ea}\ F_{bd]}) - \frac{9c_2}{2}\ R_{[ea\hat{f}}{}^{g}\ R_{bd]g}{}^{f} = 0 \tag{5.1}$$

$$\Gamma^{f}_{(\epsilon\alpha}\ H_{\hat{f}\beta\delta)} - \frac{3c_2}{2}\ R_{(\epsilon\alpha\hat{f}}{}^{g}\ R_{\beta\delta)g}{}^{f} = 0 \tag{5.2}$$

$$D_{(\epsilon}\ H_{\alpha\beta)d} + 2\Gamma^{f}_{(\epsilon\alpha}\ H_{\hat{f}\beta)d} - 6c_2\ R_{(\epsilon\alpha\hat{f}}{}^{g}\ R_{\beta)dg}{}^{f} = 0 \tag{5.3}$$

$$\begin{aligned} &3D_{[e}\ H_{ab]\delta} - D_{\delta}\ H_{eab} + 3T_{[ea}{}^{F}\ H_{\hat{F}b]\delta} \\ &\qquad + 3\psi^{\epsilon\gamma}\ \Gamma_{[e\hat{\epsilon}\hat{\delta}}H_{ab]\gamma} - 6c_1\ \mathrm{tr}(F_{[ea}\ \Gamma_{b]\alpha\delta}\ \chi^{\alpha}) \\ &\qquad - 18c_2\ R_{[ea\hat{f}}{}^{g}\ R_{b]\delta g}{}^{f} = 0 \end{aligned} \tag{5.4}$$

$$
\begin{aligned}
D_{[e}\, H_{a]\beta\delta} + D_{(\beta}\, H_{\delta)ea} + \frac{1}{2}\, T_{ea}{}^{f}\, H_{f\beta\delta} + \Gamma^{f}_{\beta\delta}\, H_{fea} \\
- \psi^{\epsilon\gamma}\, \Gamma_{[e\hat{\epsilon}\hat{\beta}}\, H_{\hat{\gamma}a]\delta} - \psi^{\epsilon\gamma}\, \Gamma_{[e\hat{\epsilon}\hat{\delta}}\, H_{\hat{\gamma}a]\beta} \\
+ 2c_1\, \Gamma_{[e\hat{\beta}\hat{\alpha}}\, \Gamma_{a]\delta\gamma}\, \mathrm{tr}(\chi^{\alpha}\, \chi^{\gamma}) \\
- 3c_2\, R_{eaf}{}^{g}\, R_{bdg}{}^{f} + 3c_2\, R_{[e\hat{\beta}\hat{f}}{}^{g}\, R_{a]\delta g}{}^{f} \\
+ 3c_2\, R_{[e\hat{\delta}\hat{f}}{}^{g}\, R_{a]\beta g}{}^{f} = 0.
\end{aligned}
\tag{5.5}
$$

Since the T and F-Bianchis do not change, their solutions in terms of the superfields T_{abc} , F_{ab} and χ^{α} are unchanged and can still be used in (5.1)–(5.5) to solve for the various components of H_{ABC}. However, the presence of curvature squared terms in these equations now makes them harder to solve. Assuming that a consistent set of solutions exists, it is almost certain that it cannot be obtained in a closed form. However, it seems feasible to obtain the solutions in a power series in the parameter c_2.

To see how this can be done, we first note that since the curvature component $R_{\alpha\beta ab}$ is simply related to T_{abc} through (4.4), eqn. (5.2) can be solved for $H_{a\alpha\beta}$. The solution is modified from (4.19) by terms proportional to the square of T_{abc}. To solve (5.3) and (5.5) for $H_{ab\alpha}$ and H_{abc} to first order in c_2 it suffices to substitute the zeroth order solution for $D_{\alpha}T_{abc}$ in these equations. This is because all terms involving $D_{\alpha}T_{abc}$ appear either through $H_{a\alpha\beta}$ or the curvature squared terms and so are alredy first order in c_2. Substituting these solutions in (5.4), $D_{\alpha}T_{abc}$ can be determined to first order in c_2. This procedure can obviously be iterated to generate series solutions of (5.1)–(5.5).

It is clear that the equations of motion of this theory obtained by the above procedure will be infinite series in the parameter c_2. Since the theory is manifestly supersymmetric and anomaly free (for specific values of c_1 and c_2) it is tempting to conclude that it is some kind of low-energy field theory approximation to superstring theory. Precisely in what sense, if at all, it arises from superstring theory is, however, far from clear. In any case, the iterative procedure outlined above provides a systematic way of obtaining an anomaly free SYM- supergravity field theory. In this connection we mention the recent attempts[18] that have been made using the component field formalism. The superspace approach presented here is technically more efficient, but a detailed analysis is required to establish its consistency. Work in this direction is in progress.

Note added After this work was completed, L. Mezincescu brought to our attention the recent preprint of R. Kallosh and B. Nilsson (CERN-TH-4300/85) which also discusses some of the issues studied here.

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APPENDIX A: THE T-BIANCHIS

In this appendix we discuss the solutions of the T-Bianchis (2.18). In component form these Bianchis are:

$$D_{[a}\, T_{bc]}{}^{d} - T_{[ab}{}^{e}\, T_{c]e}{}^{d} - R_{[abc]}{}^{d} = 0 \tag{A1}$$

$$D_{[a}\, T_{bc]}{}^{\delta} - T_{[ab}{}^{e}\, T_{c]e}{}^{\delta} - T_{[ab}{}^{\gamma}\, \Gamma_{c]\gamma\epsilon}\, \psi^{\epsilon\delta} = 0 \tag{A2}$$

$$R_{(\alpha\beta\gamma)}{}^{\delta} = 0 \tag{A3}$$

$$D_{\beta}\, T_{bc}{}^{d} + 2T_{bc}{}^{\gamma}\, \Gamma^{d}_{\gamma\beta} - 2R_{\beta[bc]}{}^{d} = 0 \tag{A4}$$

$$\begin{aligned} &D_{\beta}\, T_{bc}{}^{\delta} + 2D_{[b}\, \psi^{\epsilon\delta}\, \Gamma_{c]\beta\epsilon} + T_{bc}{}^{e}\, \psi^{\epsilon\delta}\, \Gamma_{e\beta\epsilon} \\ &+ 2\psi^{\epsilon\gamma}\, \psi^{\alpha\delta}\, \Gamma_{[b\hat{\beta}\hat{\epsilon}}\, \Gamma_{c]\alpha\gamma} - R_{bc\beta}{}^{\delta} = 0 \end{aligned} \tag{A5}$$

$$2\psi^{\epsilon\delta}\, \Gamma_{a\epsilon(\beta}\, \Gamma^{d}_{\gamma)\delta} - \frac{1}{2}\, R_{\gamma\beta a}{}^{d} + \Gamma^{b}_{\gamma\beta}\, T_{ba}{}^{d} = 0 \tag{A6}$$

$$R_{a(\beta\gamma)}{}^{\delta} + D_{(\gamma}\, \psi^{\epsilon\delta}\, \Gamma_{a\beta)\epsilon} - \Gamma^{e}_{\gamma\beta}\, T_{ea}{}^{\delta} = 0. \tag{A7}$$

We first study the algebraic equations (A3) and (A6). From (A3) we find:

$$R_{\alpha(\beta\gamma)}{}^{\alpha} = 0. \tag{A8}$$

Using this in (A6) we get:

$$T_{ab}{}^{b} = 0. \tag{A9}$$

Contracting a with d in (A6) and using (A9) we find:

$$\psi^{\epsilon\delta}\, \Gamma_{a\epsilon(\beta}\, \Gamma^{a}_{\gamma)\delta} = 0, \tag{A10}$$

or, equivalently,

$$\psi^{(\epsilon\delta)}\,\Gamma_{a\epsilon\beta}\,\Gamma^{a}_{\gamma\delta}=0. \tag{A11}$$

Also, multiplying (A10) with $\Gamma^{c\beta\gamma}$ we get:

$$\psi^{\epsilon\delta}\,\Gamma^{c}_{\epsilon\delta}=0. \tag{A12}$$

Using this equation and the part of (A6) symmetric in a and d we find

$$T_{a(bc)}=0. \tag{A13}$$

Since T_{abc} in antisymmetric in the first two indices this tells us that T_{abc} is totally antisymmetric.

Now $\psi^{\epsilon\delta}$ may be expanded in $SO(1,9)$ irreducibles as:

$$\psi^{\epsilon\delta}=G_a(\Gamma^a)^{\epsilon\delta}+G_{abc}(\Gamma^{abc})^{\epsilon\delta}+G_{abcde}(\hat{\Gamma}^{abcde})^{\epsilon\delta} \tag{A14}$$

where G_{abc} is antisymmetric and G_{abcde} , as well as $\hat{\Gamma}^{abcde}$, are both antisymmetric and self-dual. Equation (A12) forces G_b to vanish while (A13) and the part of (A6) symmetric in a and d implies that the 126 is absent. So,

$$\psi^{\epsilon\delta}=G_{abc}\,(\Gamma^{abc})^{\epsilon\delta}. \tag{A15}$$

To determine G_{abc} we proceed as follows. Multiplying (A3) with $\Gamma^{a\beta\gamma}$ and contracting δ with α we find:

$$R_{\alpha\beta cd}\,\Gamma^{d\beta\alpha}=0 \tag{A16}$$

while multiplying it with $\Gamma^{a\beta\gamma}(\Gamma^{ef})_\delta{}^\alpha$ gives us:

$$\begin{aligned} &R_{\alpha\beta cd}\ (\Gamma^{acdef})^{\alpha\beta} - 18R_{\alpha\beta}{}^{ef}\ \Gamma^{a\alpha\beta} \\ &\qquad - 2R_{\alpha\beta}{}^{ea}\ \Gamma^{f\alpha\beta} + 2R_{\alpha\beta}{}^{fa}\ \Gamma^{e\alpha\beta} = 0. \end{aligned} \tag{A17}$$

Similarily, multiplying (A6) with $(\Gamma^{acdef})^{\gamma\beta}$ we get:

$$24 \times 16 \times 42\ G^{aef} + (\Gamma^{acdef})^{\alpha\beta}\ R_{\alpha\beta cd} = 0, \tag{A18}$$

while multiplying it with $\Gamma_e^{\beta\gamma}$ we find

$$6 \times 64\ G_{aed} - R_{\gamma\beta ad}\ \Gamma_e^{\beta\gamma} + 32T_{ead} = 0. \tag{A19}$$

Using (A17)–(A19) we can derive (4.3). Substituting this in (A6) we get (4.4).

We now study the remaining equations and show that all the other unknown superfields can be related to T_{abc} and its fermionic derivative. From (A4) we see:

$$R_{\beta cbd} = \frac{1}{2}\ D_\beta\ T_{cbd} + T_{cb}{}^\gamma\ \Gamma_{d\gamma\beta} + T_{db}{}^\gamma\ \Gamma_{c\gamma\beta} + T_{dc}{}^\gamma\ \Gamma_{b\gamma\beta}. \tag{A20}$$

Multiplying (A7) by $\Gamma_e^{\beta\gamma}$ and (A20) by $\frac{1}{4}\ (\Gamma_e\Gamma^{bd})^{\beta\delta}$ and eliminating $R_{\beta cbd}$ we can derive the equation:

$$\begin{aligned} &6T_{ec}{}^\delta - D_\gamma\ \psi^{\epsilon\delta}\ (\Gamma_c\ \Gamma_e)_\epsilon^\gamma \\ &\qquad = \frac{1}{8}\ D_\beta\ T_{bcd}\ (\Gamma_e\ \Gamma^{bd})^{\beta\delta} + \frac{9}{2}\ T_{cb}{}^\gamma\ (\Gamma^b\ \Gamma_e)_\gamma{}^\beta \\ &\qquad + \frac{1}{4}\ T_{bd}{}^\gamma(\Gamma^{bd}\ \Gamma_c\ \Gamma_e)_\gamma{}^\delta - T_{ed}{}^\gamma\ (\Gamma^d\ \Gamma_c)_\gamma{}^\delta \end{aligned} \tag{A21}$$

Using (4.5) this becomes:

$$6T_{ec}{}^{\delta} - D_{\alpha}\ \psi^{\epsilon\delta}\ (\Gamma_c\ \Gamma_e)_{\epsilon}{}^{\alpha} - \frac{1}{8}\ D_{\beta}T_{bcd}\ (\Gamma_e\ \Gamma^{bd})^{\beta\delta}$$

$$= 36J_{\alpha c}\ \Gamma_e^{\alpha\delta} - 8J_{\alpha e}\ \Gamma_c^{\alpha\delta} \tag{A22}$$

$$+ 27J^{\beta}\ (\Gamma_c\ \Gamma_e)_{\beta}{}^{\delta} - 18J^{\delta}\ \eta_{ec}.$$

Contracting c and e in this equation we get (4.8), while multiplying it with $\Gamma^c_{\delta\gamma}$ gives (4.9). Using the constraint (4.7) on $J_{\alpha e}$ and (4.8) we find:

$$D_{\beta}\ \psi^{\beta\delta} = 0. \tag{A23}$$

This implies that the 16 in $T_{bd}{}^{\alpha}$ vanishes while the 144 is given by:

$$J_{\gamma e} = -\frac{1}{56}\ D_{\beta}\ (\Gamma_e\ \psi)^{\beta}{}_{\gamma}. \tag{A24}$$

Using (4.5) and (A22) we can obtain an expression for the remaining irreducible, the 560:

$$J_{ec}{}^{\delta} = \frac{1}{16}\ D_{\beta}\ T_{jk[e}\ (\Gamma^{jk}_{c]})^{\beta\delta} + \frac{1}{8}\ D_{\beta}\ T_{eck}\ \Gamma^{k\beta\delta} \tag{A25}$$

Combining these results we can relate the superfield $T_{ec}{}^{\delta}$ to fermionic derivatives of T_{abc}:

$$T_{ec}{}^{\delta} = \frac{1}{14}\ D_{\beta}\ T_{jk[e}\ (\Gamma^{jk}_{c]})^{\beta\delta} + \frac{3}{28}\ D_{\beta}\ T_{eck}\ \Gamma^{k\beta\delta}. \tag{A26}$$

From (A20), we see that the same is true of $R_{\beta cbd}$. Finally, we obtain an expression for the Ricci tensor in terms of T_{abc} and its fermionic derivatives. Multiplying

(A5) by $(\Gamma_e\,\Gamma_f)_\delta{}^\beta$ and evaluating some traces we find:

$$R_{cbef} = \frac{1}{8}\,D_\beta\,T_{bc}{}^\delta\,(\Gamma_e\,\Gamma_f)_\delta{}^\beta + D_{[b}\,T_{c]ef}$$
$$+\frac{1}{2}\,T_{bc}{}^d\,T_{efd} + \frac{1}{12}\,\eta_{e[c}\,\eta_{b]f}\,T^2 \tag{A27}$$
$$+\frac{1}{4}\,\eta_{e[b}\,T_{c]}{}^{jk}\,T_{fjk} - \frac{1}{4}\,\eta_{f[b}\,T_{c]}{}^{jk}\,T_{ejk}.$$

The Ricci tensor is then:

$$R_{ce} = \frac{1}{8}\,D_\beta\,T_{bc}{}^\delta\,(\Gamma_e\,\Gamma^b)_\delta{}^\beta - \frac{1}{2}\,D_b\,T_{ec}{}^b$$
$$+\frac{1}{4}\,\eta_{ec}\,T^2 - \frac{3}{2}\,T_{ejk}\,T_c{}^{jk}, \tag{A28}$$

where $R_{ce} \equiv \eta^{bf} R_{cbef}$. This expression can be simplified by using

$$D_\beta\,T_{bc}{}^\beta = 0 \tag{A29}$$

which follows from (A5). The first term in (A28) may then be expressed in terms of a fermionic derivative on the 144 of $T_{bc}{}^\delta$. The antisymmetric (in c and e) part of the resulting equation is

$$R_{[ce]} = D_\beta\,J_{\gamma[c}\Gamma_{e]}{}^{\beta\gamma} - \frac{1}{2}D_a\,T_{ec}{}^a. \tag{A30}$$

On the other hand, from (A1) we get:

$$D_a\,T_{bc}{}^a + 2R_{[bc]} = 0. \tag{A31}$$

Comparing (A30) and (A31) we see

$$D_\beta\,J_{\gamma[a}\Gamma_{b]}{}^{\beta\gamma} = -D_e\,T_{ab}{}^e. \tag{A32}$$

Now using (A26) and (A29) and the result

$$D_\alpha\ D_\beta\ T_{ab}{}^c\ \Gamma_c{}^{\beta\alpha} = -16 D_e\ T_{ab}{}^e, \tag{A33}$$

which can be obtained by using the relation for the anticommutator of fermionic derivatives, we get

$$D_\alpha\ D_\beta\ T_{jk[a}\ (\Gamma_{b]}{}^{jk})^{\beta\alpha} = 24 D_e\ T_{ab}{}^e. \tag{A34}$$

This equation along with (A24) and (A33) gives us:

$$D_\beta\ J_{\gamma[a}\Gamma_{b]}{}^{\beta\gamma} = -4 D_e\ T_{ab}{}^e. \tag{A35}$$

Comparing (A32) and (A35) gives us the equation of motion for T_{abc}, (4.12). Also, this result can be used to simplify (A28) to obtain the expression for the Ricci tensor given in (4.13).

APPENDIX B: THE F-BIANCHIS

In this appendix we discuss the solutions of the F- Bianchis (2.19). In component form these Bianchis are:

$$\mathcal{D}_{[c}\ F_{ba]} - T_{[cb}{}^{d}\ F_{a]d} - T_{[cb}{}^{\delta}\ F_{a]\delta} = 0 \qquad (B1)$$

$$\Gamma^{d}_{(\gamma\beta}\ F_{\alpha)d} = 0 \qquad (B2)$$

$$2\mathcal{D}_{[c}\ F_{b]\alpha} + \mathcal{D}_{\alpha}\ F_{cb} + T_{cb}{}^{d}\ F_{d\alpha} - 2T_{\alpha[c}{}^{\delta}\ F_{b]\delta} = 0 \qquad (B3)$$

$$\mathcal{D}_{(\gamma}\ F_{\beta)a} + \Gamma^{d}_{\gamma\beta}\ F_{da} = 0. \qquad (B4)$$

Writing $F_{\alpha d}$ in terms of irreducibles we may use (B2) to show that the 144 is absent and so (4.14) follows. Writing $\mathcal{D}_{\alpha}\ \chi^{\delta}$ in irreducibles we may use (B4) to show that the 1 and 210 are absent and thus derive (4.15). Eqn. (B3) directly gives us (4.16).

To obtain the gluino equation of motion, (4.17), we use the anticommutation relation:

$$\{\mathcal{D}_{\beta},\ \mathcal{D}_{\alpha}\}\ \chi^{\delta} = -2\Gamma^{c}_{\beta\alpha}\ \mathcal{D}_{c}\ \chi^{\delta} - R_{\beta\alpha}{}^{\delta}{}_{\gamma}\ \chi^{\gamma}. \qquad (B5)$$

Contracting α and δ and using (4.16) we get:

$$7\Gamma^{a}{}_{\beta\delta}\mathcal{D}_{a}\ \chi^{\delta} = -\frac{1}{2}\ T_{bcd}\ (\Gamma^{bcd})_{\beta\delta}\ \chi^{\delta} + 9\psi^{\gamma\delta}\ \Gamma^{b}_{\beta\delta}\ \Gamma_{b\delta\epsilon}\ \chi^{\epsilon} + R_{\beta\alpha}{}^{\alpha}{}_{\gamma}\chi^{\gamma} \qquad (B6)$$

Using (4.3) and (4.4) this simplifies to (4.17). Finally, the Yang-Mills equation is obtained by using the commutation relation:

$$[\mathcal{D}_b,\ \mathcal{D}_\alpha]\ \chi^\delta = F_{b\alpha}\ \chi^\delta - R_{b\alpha}{}^\delta{}_\gamma\ \chi^\gamma - T_{b\alpha}{}^\beta\ \mathcal{D}_\beta\ \chi^\delta. \tag{B7}$$

Multiplying this with $(\Gamma^{eb})_\delta{}^\alpha$, using (4.15) in the first term and commuting $\mathcal{D}^e$ through $\mathcal{D}_\alpha$ in the second term on the left hand side we find:

$$\begin{aligned} 16\mathcal{D}_b\ F^{be} &= 8\ \chi^\alpha\ \Gamma^e_{\alpha\beta}\ \chi^\beta \\ &- \frac{1}{8}\ D_\alpha\ T_{bcd}\ (\Gamma^{bcd}\ \Gamma^e)_\delta{}^\alpha\ \chi^\delta \\ &- 16\ T^e{}_{bd}\ F^{bd} \\ &- \frac{9}{2}\ T_{bc}{}^\delta (\Gamma^c\ \Gamma^{eb})_{\gamma\delta}\ \chi^\gamma \\ &+\ T_{bc}{}^\delta (\Gamma^{ce}\ \Gamma^b)_{\gamma\delta}\ \chi^\gamma \\ &+ 3\ T_{cd}{}^\delta (\Gamma^{cd}\ \Gamma^e)_{\gamma\delta}\ \chi^\gamma \\ &+ \frac{9}{2}\ T^e{}_c{}^\delta \Gamma^c{}_{\gamma\delta}\ \chi^\gamma \end{aligned} \tag{B8}$$

Using (A24) we may simplify this to:

$$\mathcal{D}_b\ F^{be} = \frac{1}{2}\Gamma^e{}_{\alpha\beta}\chi^\alpha\chi^\beta -\ T^e{}_{bd}\ F^{bd} - 8\ J_\gamma{}^e\ \chi^\gamma \tag{B9}$$

APPENDIX C: THE H-BIANCHIS

In this appendix we discuss the solutions of the H-Bianchis (3.8). In component form, these Bianchis are:

$$D_{[e}\ H_{abd]} + \frac{3}{2}\ T_{[ea}{}^{f}\ H_{\hat{f}bd]} - \frac{3c_1}{2}\ \mathrm{tr}(F_{[ea}\ F_{bd]}) = 0 \tag{C1}$$

$$\Gamma^{f}_{(\epsilon\alpha}\ H_{\hat{f}\beta\delta)} = 0 \tag{C2}$$

$$D_{(\epsilon}\ H_{\alpha\beta)d} + 2\Gamma^{f}_{(\epsilon\alpha}\ H_{\hat{f}\beta)d} = 0 \tag{C3}$$

$$\begin{aligned} 3D_{[e}\ H_{ab]\delta} - D_{\delta}\ H_{eab} + 3T_{[ea}{}^{F}\ H_{\hat{F}b]\delta} \\ + 3\psi^{\epsilon\gamma}\ \Gamma_{[e\hat{\epsilon}\hat{\delta}}H_{ab]\gamma} - 6c_1\ \mathrm{tr}(F_{[ea}\ \Gamma_{b]\alpha\delta}\ \chi^{\alpha}) = 0 \end{aligned} \tag{C4}$$

$$\begin{aligned} D_{[e}\ H_{a]\beta\delta} + D_{(\beta}\ H_{\delta)ea} + \frac{1}{2}\ T_{ea}{}^{f}\ H_{f\beta\delta} + \Gamma^{f}_{\beta\delta}\ H_{fea} \\ - \psi^{\epsilon\gamma}\ \Gamma_{[e\hat{\epsilon}\hat{\beta}}\ H_{\hat{\gamma}a]\delta} - \psi^{\epsilon\gamma}\ \Gamma_{[e\hat{\epsilon}\hat{\delta}}\ H_{\hat{\gamma}a]\beta} \\ + 2c_1\ \Gamma_{[e\hat{\beta}\hat{\alpha}}\ \Gamma_{a]\delta\gamma}\ \mathrm{tr}(\chi^{\alpha}\ \chi^{\gamma}) = 0 \end{aligned} \tag{C5}$$

Eqn. (C2) is solved by (4.19). Using this (C3) can be solved for $H_{fd\beta}$. To obtain the solution we multiply this equation with $\Gamma^{c\alpha\beta}$ and find:

$$8\lambda_{\epsilon}\ \delta^{c}_{d} + \lambda_{\alpha}(\Gamma_{d}\ \Gamma^{c})_{\epsilon}{}^{\alpha} + 2(\Gamma^{f}\ \Gamma^{c})_{\epsilon}{}^{\beta}\ H_{f\beta d} + 16\ H^{c}{}_{\epsilon d} = 0. \tag{C6}$$

We may write $H_{fd\beta}$ in terms of irreducibles as:

$$H_{fd\beta} = \mathcal{H}_{fd\beta} + 2\mathcal{H}_{[f}{}^{\alpha}\ \Gamma_{d]\alpha\beta} + \mathcal{H}_{\alpha}\ (\Gamma_{fd})_{\beta}{}^{\alpha} \tag{C7}$$

where the superfields $\mathcal{H}_{fd\beta}$ and $\mathcal{H}_{f}{}^{\alpha}$ satisfy the constraints:

$$\mathcal{H}_{fd\beta}\ \Gamma^{d\beta\alpha} = 0 \tag{C8}$$

$$\mathcal{H}_f{}^{\alpha}\ \Gamma^f_{\alpha\beta} = 0 \tag{C9}$$

Equation (C6) then requires that the 560 in $H_{fd\beta}$ be absent. Contracting c and d in this equation we can solve for the 16:

$$\mathcal{H}_{\beta} = -\frac{1}{2}\ \lambda_{\beta}. \tag{C10}$$

Substituting this in (C6) we find that the 144 vanishes, thus resulting in (4.21).

To solve for H_{abc}, we multiply (C5) with $\Gamma^{d\beta\delta}$ to get:

$$\begin{aligned} H_{dea} &= -\phi\ T_{dea} + \frac{1}{32}\ (\Gamma_{dea})^{\beta\epsilon}\ D_{\beta}\ \lambda_{\epsilon} \\ &+ \frac{c_1}{8}\ \ (\Gamma_{dea})_{\alpha\beta}\ \mathrm{tr}\ (\chi^{\alpha}\chi^{\beta}), \end{aligned} \tag{C11}$$

while multiplying (C5) with $(\Gamma^{bcdea})^{\beta\delta}$ and using the expression for the anticommutator of two fermionic derivatives on ϕ, we get, after some algebra:

$$\begin{aligned} D_{\beta}\ \lambda_{\epsilon} &= -\Gamma^b_{\beta\epsilon}\ D_b\ \phi - \frac{\phi}{6}\ T_{abc}\ (\Gamma^{abc})_{\beta\epsilon} \\ &+ \frac{c_1}{24}\ \mathrm{tr}\ (\chi^{\alpha}\chi^{\gamma})\ \ (\Gamma_{abc})_{\alpha\gamma}\ (\Gamma^{abc})_{\beta\epsilon}. \end{aligned} \tag{C12}$$

Equations (C11) and (C12) lead to (4.22) and (4.23). Finally, an expression for a fermionic derivative on T_{abc} can be obtained from (C4) by using (C11):

$$\begin{aligned} D_{\gamma}\ T_{abc} &= 2T_{[ab}{}^{\alpha}\Gamma_{c]\alpha\gamma} + \phi^{-1}D_{[a}\lambda_{\hat{\beta}}(\Gamma_{bc]})_{\gamma}{}^{\beta} \\ &- \phi^{-1}T_{abc}\lambda_{\gamma} - \phi^{-1}T_{[ab}{}^{d}(\Gamma_{c]d})_{\gamma}{}^{\beta}\lambda_{\beta} \\ &- \phi^{-1}(\Gamma_{[a}\psi\Gamma_{bc]})_{\gamma}{}^{\beta}\lambda_{\beta} + \frac{c_1}{6}\phi^{-1}(\Gamma^{ef}\Gamma_{abc})_{\gamma\beta}\mathrm{tr}(F_{ef}\chi^{\beta}) \\ &+ 4c_1\phi^{-1}\Gamma_{[a\hat{\gamma}\hat{\beta}}\mathrm{tr}(F_{bc]}\chi^{\beta}) \end{aligned} \tag{C13}$$

This completes the set of solutions of the H-Bianchis. We shall now derive the equations of motion for λ and ϕ. Multiplying (C13) by $(\Gamma^{abc})^{\delta\gamma}$ and using (4.10) we get (4.24). Taking the fermionic derivative of (4.24) and using the expression for the commutator of a bosonic and a fermionic derivative on λ and several of the previous results we obtain the equation of motion for ϕ, (4.25). The expression, (4.26), for the 144 in $T_{ab}{}^{\alpha}$ may be derived by multiplying (C13) by $(\Gamma^{bc})_{\beta}{}^{\gamma}$.

We may now evaluate the first source term in the Einstein equation; this is a little tedious. Using (4.26), (4.24), the expression for the commutator of a bosonic and a fermionic derivative on λ, the expression for the commutator of two bosonic derivatives on ϕ, (A20), (A26) and (B3), we find:

$$
\begin{aligned}
D_\beta\ J_{\gamma a}\Gamma_b{}^{\beta\gamma} = &-\frac{1}{2}\phi^{-2}(\lambda\Gamma_{(a}D_{b)}\lambda) + c_1\phi^{-1}\ \mathrm{tr}(\chi\Gamma_{(a}\mathcal{D}_{b)}\chi) \\
&-\phi^{-1}D_{(a}D_{b)}\phi - \frac{1}{2}\ T_{ajk}\ T_b{}^{jk} \\
&+\frac{c_1}{2}\phi^{-1}\ \mathrm{tr}(\chi\Gamma_{jk(a}\chi)T_{b)}{}^{jk} \\
&-\frac{c_1}{36}\phi^{-1}\ \eta_{ab}\mathrm{tr}(\chi\Gamma_{cde}\chi)T^{cde} \\
&+\frac{c_1}{6}\ \phi^{-1}\mathrm{tr}(4F_{ac}F^c{}_b + 9\eta_{ab}F_{cd}F^{cd}) \\
&-\frac{1}{4}\phi^{-2}\ (\lambda\Gamma_{jk(a}\lambda)T_{b)}{}^{jk} \\
&+\frac{1}{48}\phi^{-2}\ \eta_{ab}(\lambda\Gamma_{cde}\lambda)T^{cde} \\
&-\frac{c_1}{12}\phi^{-2}\ \mathrm{tr}[F_{hj}\chi(\Gamma^{hj}\eta_{ab} + 12\delta^h_{(a}\Gamma^j\Gamma_{b)})\lambda]
\end{aligned}
\qquad (C14)
$$

We have used an obvious compact notation in this equation. This expression is symmetric in a and b, thus satisfying the constraint on it, from (A32) and (A35), identically. Also, the Ricci tensor is, therefore, symmetric.

Finally, we show that (C13) and (A26) do not determine $T_{ab}{}^{\delta}$ in terms of the other superfields. Substituting (C13) in (A26) we see that all terms involving the 560 of $T_{ab}{}^{\alpha}$ cancel and the resulting equation just determines the 144.

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