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A POSSIBLE FINAL FOCUSING MECHANISM FOR LINEAR COLLIDERS*

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ABSTRACT

Large transverse wake fields can be generated via the interaction between a relativistic electron or positron bunch and a plasma, and the bunch will therefore be self-pinched. In this paper we derive a new and general formulation for the plasma wake field in a compact expression. We then suggest that this self-focusing effect can be used as a mechanism to enhance the luminosity for high energy experiments. A plasma lens based on this effect is suggested with a conceptual design and a numerical example. The problem of background noise is discussed at the end.

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1. INTRODUCTION

For future e^+e^- linear colliders, one of the challenges is to increase the luminosity according to the square of the e^+e^- center of mass energy.[1] The hope lies in the increase of repetition rate and the reduction of spot size at the interaction point. Recently, Palmer studied the idea of "super disruption" for this purpose.[2] The scheme employes two pairs of colliding e^+e^- bunches, where the first pair serves as the focusing lenses for the second pair.

It turns out that another type of disruption occurs when a relativistic beam traverses through a plasma. In the study of the plasma wake field accelerator,[3,4] it is shown that accompanying the large acceleration gradient (from the longitudinal wake field excited by a leading charge) there is a transverse wake field with comparable strength. This transverse wake field either focuses or defocuses the trailing particles. In this paper we will derive a general formula for the wake fields generated by an electron or a positron beam with finite longitudinal and transverse extents. We show that for parabolic density profiles in both directions, which is a reasonable approximation to an actual beam profile, the transverse force is always focusing within the bunch. As a result the bunch pinches itself to a smaller effective cross-section.

We suggest that this self-pinching effect can be used as a mechanism for e^+e^- final focusing. Two criteria are imposed and an inequality is derived. We show that in order to satisfy this inequality one should modify the present SLAC beam parameters. A conceptual design of a plasma lens based on the self-pinching effect is then introduced and a numerical example is then given. The problem of background noise due to the lens is discussed at the end.

2. THE PLASMA WAKE FIELDS

The longitudinal plasma wake field at point (r,ς), where $\varsigma \equiv z - ct$, is defined to be the longitudinal electric field of the beam-plasma system, i.e.

$$\vec{W}_{\parallel}(r,\varsigma) \equiv \vec{E}_{1z}(r,\varsigma) \ . \tag{1}$$

The transverse plasma wake field, on the other hand, is the Lorentz force exerting on the unit charge at (r,ς) that experiences the plasma wake and moves with velocity $\vec{\beta}' \equiv \vec{v}\,'/c \simeq 1$, i.e.

$$\vec{W}_{\perp}(r,\varsigma) \equiv \vec{E}_{1r}(r,\varsigma) + \vec{\beta}\,' \times \vec{B}_{1\phi}(r,\varsigma) \quad . \tag{2}$$

Since the particles that generates the plasma wake are also assumed to be relativistic (i.e. $\beta \simeq 1$), we can write

$$\vec{E}_{1z} = (\beta\partial_\varsigma A_{1z} - \partial_\varsigma\phi_1)\hat{z} \simeq \partial_\varsigma(A_{1z} - \phi_1)\hat{z} \ , \tag{3}$$

and

$$\left\{ \begin{aligned} \vec{E}_{1r} &= (\beta\partial_\varsigma A_{1r} - \partial_r\phi_1)\hat{r} \simeq (\partial_\varsigma A_{1r} - \partial_r\phi_1)\hat{r} \\ \vec{\beta}\,' \times \vec{B}_{1\phi} &= -\beta'(\partial_\varsigma A_{1r} - \partial_r A_{1z})\hat{r} \simeq -(\partial_\varsigma A_{1r} - \partial_r A_{1z})\hat{r} \ , \end{aligned} \right. \tag{4}$$

thus the wake fields can be rewritten (with the approximation $\beta \simeq \beta' \simeq 1$) in terms of a common functional, $A_{1z} - \phi_1$:

$$\left\{ \begin{aligned} W_{\parallel}(r,\varsigma) &= \partial_\varsigma(A_{1z} - \phi_1) \\ W_{\perp}(r,\varsigma) &= \partial_r(A_{1z} - \phi_1) \ . \end{aligned} \right. \tag{5}$$

Notice that the Panofsky-Wenzel theorem,[5] $\partial_r W_{\parallel} = \partial_\varsigma W_{\perp}$, is straightforwardly satisfied.

To solve for $W_{\|}$ and $W_{\perp}$ we employ the nonrelativistic fluid theory. Assuming that the unperturbed plasma velocity v_0 be zero and the perturbed plasma density n_1 be much smaller than its unperturbed density n_0, the equation of motion and the equation of continuity can be linearized as:

$$\begin{cases} -\partial_\varsigma \vec{v}_1 = -\dfrac{e}{mc}\vec{E}_1 \\ -c\partial_\varsigma n_1 + n_0 \nabla \cdot \vec{v}_1 = 0 \ , \end{cases} \tag{6}$$

where $\partial_z = \partial_\varsigma$ and $\partial_t = -c\partial_\varsigma$ have been used. The Maxwell's equations in the Coulomb gauge are

$$\nabla^2 \phi_1 = -4\pi\rho_1 \ , \tag{7}$$

and

$$\nabla_\perp^2 \vec{A}_1 = \frac{4\pi}{c}\,\vec{J}_1 - \vec{\nabla}\partial_\varsigma \phi_1 \ , \tag{8}$$

where the charge and current densities are contributed from both the plasma perturbation and the source, $\mp e\sigma(\vec{x})$, for electron $(-)$ and positron $(+)$ bunches, respectively,

$$\begin{cases} \rho_1(\vec{x}) = -en_1(\vec{x}) \mp e\sigma(\vec{x}) \\ \vec{J}_1(\vec{x}) = -en_0\vec{v}_1(\vec{x}) \mp e\vec{c}\sigma(\vec{x}) \ . \end{cases} \tag{9}$$

Notice that in defining the current this way, we have neglected the transverse current in the bunch.[6] This approximation is valid only if the transverse motions of the bunch particles are negligibly small during the beam-plasma interaction, which is the case for a thin plasma lens as we shall discuss in Section 4. Combining the fluid equation and the Poisson equation (Eq. (7)), with the help of $\vec{E}_1 = -\nabla\phi_1 + \partial_\varsigma \vec{A}_1$, we obtain the equation for the plasma density perturbation due to the beam:

$$\partial_\varsigma^2 n_1 + k_p^2 n_1 = \mp k_p^2 \sigma(\vec{x}) \ , \tag{10}$$

where the plasma wave number $k_p \equiv \omega_p/c = (4\pi e^2 n_0/mc^2)^{1/2}$, and the signs in the source term are associated with electron beam $(-)$ and positron beam $(+)$, respectively.

Now we assume the separation of variables in $\sigma(\vec{x})$ and that the bunch is confined to the region $\varsigma \leq 0$ and $r \leq a$:

$$\sigma(\vec{x}) = \rho_b f(r) g(\varsigma) \ , \quad a \leq r, \quad \varsigma \leq 0 \ . \tag{11}$$

Then

$$n_1 = \begin{cases} \pm k_p f(r)\rho_b \int_\varsigma^\infty d\varsigma' g(\varsigma') \sin k_p(\varsigma' - \varsigma) \equiv \pm \rho_b f(r) G(\varsigma), & \varsigma \leq 0 \\ 0, & \varsigma > 0 \end{cases} \tag{12}$$

for electron and positron beams, respectively.

Next we apply ∂_ς to Eq. (8),

$$\nabla_\perp^2 \partial_\varsigma \vec{A}_1 = -\frac{4\pi}{c} \ \partial_\varsigma \vec{J}_1 - \vec{\nabla} \partial_\varsigma^2 \phi_1 \ \ . \tag{13}$$

Evoking Eqs. (6) and (9) we have

$$\nabla_\perp^2 \partial_\varsigma \vec{A}_1 = k_p^2 (\partial_\varsigma \vec{A}_1 - \nabla \phi_1) \pm \frac{4\pi e \rho_b}{c} \ \vec{c} \ f(r) \ \partial_\varsigma \ g(\varsigma) - \nabla \partial_\varsigma^2 \phi_1 \ \ . \tag{14}$$

Concentrating on the z-component and removing the common ∂_ς we obtain

$$(\nabla_\perp^2 - k_p^2) A_{1z} = -(\partial_\varsigma^2 + k_p^2)\phi_1 \pm 4\pi e \rho_b \ f(r) \ g(\varsigma) \ \ , \tag{15}$$

which is equivalent to what we obtained before.[7]

Since A_{1z} and ϕ_1 always appear together in the expressions for the wake fields, we do not need to know them separately. Notice that $\partial_\varsigma^2 = \nabla^2 - \nabla_\perp^2$, we can rewrite the above expression as

$$(\nabla_\perp^2 - k_p^2)(A_{1z} - \phi_1) = -4\pi e n_1 \quad . \tag{16}$$

This is the Master Equation for the plasma wake fields excited by either an electron beam or a positron beam. With n_1 given in Eq. (12), we get

$$\begin{aligned} A_{1z} - \phi_1 = \mp \frac{4\pi e \rho_b}{k_p^2}\, G(\varsigma) \Bigg\{ & k_p^2 \int_0^r r' dr' f(r') I_0(k_p r') K_0(k_p r) \\ & + k_p^2 \int_r^\infty r' dr' f(r') I_0(k_p r) K_0(k_p r') \Bigg\} \\ \equiv \mp \frac{4\pi e \rho_b}{k_p^2}\, & G(\varsigma) F(r) \quad . \end{aligned} \tag{17}$$

Therefore the wake fields can be simply expressed as

$$\begin{cases} W_\parallel = \mp \dfrac{4\pi e \rho_b}{k_p^2}\, \partial_\varsigma G(\varsigma) F(r), \\[2ex] W_\perp = \mp \dfrac{4\pi e \rho_b}{k_p^2}\, G(\varsigma) \partial_r F(r), \end{cases} \tag{18}$$

for electron $(-)$ and positron $(+)$ beams, respectively.

3. THE SELF–FOCUSING EFFECT

Consider the following density distribution for a “standard” electron or positron bunch (see Fig. 1) in most of the presently existing accelerators:

$$\sigma(\vec{x}) = \rho_b f(r) g(\varsigma) = \rho_b \left(1 - \frac{r^2}{a^2}\right)\left(1 - \frac{(\varsigma + b)^2}{b^2}\right) \quad , \tag{19}$$

where $0 \le r \le a$ and $-2b \le \varsigma \le 0$. The parabolic profiles in both r and ς directions are introduced to approximate the Gaussian profiles. The constant, ρ, can be related to the total number of particles N in the bunch:

$$\rho_b = \frac{3N}{2\pi a^2 b} \; . \tag{20}$$

With this density distribution, it is straightforward to find that within the bunch,

$$\begin{aligned} A_{1z} - \phi_1 = \mp \frac{8\pi e \rho_b}{k_p^2} & \left[I_0(k_p r) K_2(k_p a) + \frac{1}{2}\left(1 - \frac{r^2}{a^2}\right) - \frac{2}{k_p^2 a^2} \right] \\ & \times \left[\left(1 - \frac{(\varsigma + b)^2}{b^2}\right) + \frac{2}{k_p b} \sin k_p \varsigma + \frac{2}{k_p^2 b^2} \left(1 - \cos k_p \varsigma\right) \right] \; . \end{aligned} \tag{21}$$

The corresponding transverse wake field within the bunch is thus

$$\begin{aligned} W_\perp = \mp \frac{8\pi e \rho_b}{k_p} & \left[I_1(k_p r) K_2(k_p a) - \frac{r}{k_p a^2} \right] \\ & \times \left[\left(1 - \frac{(\varsigma + b)^2}{b^2}\right) + \frac{2}{k_p b} \sin k_p \varsigma + \frac{2}{k_p^2 b^2} \left(1 - \cos k_p \varsigma\right) \right] \; . \end{aligned} \tag{22}$$

Notice that the transverse force is exerting on the like particles in the same bunch, i.e. $\mathcal{F}_\perp = \mp e W_\perp$, thus $\mathcal{F}_\perp$ has the same sign for both electron and positron bunches. Furthermore, it can be verified that $G(\varsigma)$ in this case is always positive definite, so the force is always focusing.

An interesting case corresponds to the situation where $k_p r \le k_p a \ll 1$. In this limit

$$I_1(k_p r) K_2(k_p a) - \frac{r}{k_p a^2} \simeq -\frac{1}{4} k_p r \quad , \tag{23}$$

and we have a focusing force which is linear in r:

$$\mathcal{F}_\perp \simeq - \left[\frac{3e^2 N}{a^2 b} \, G(\varsigma) \right] r \quad . \tag{24}$$

The requirement that the focusing force be linear in r, i.e. that $k_p a \ll 1$, can be rewritten as

$$n_0 \ll \frac{1}{4\pi r_e a^2} \quad , \tag{25}$$

where r_e is the classical electron radius. On the other hand, self-consistency in the linearized fluid theory that we employed requires that $n_0 \gg n_1$. Combining these two conditions we arrive at a chain inequality which the system must satisfy:

$$\frac{1}{4\pi r_e a^2} \gg n_0 \gg \rho_b \, f(r) G(\varsigma) \ge \rho_b \, G(\varsigma) \quad . \tag{26}$$

This inequality puts a constraint on the bunch length $2b$. Physically, this is a condition imposing on the relative densities between the plasma and the bunch.

In the specific case of a standard bunch, the inequality reads

$$\frac{1}{4\pi r_e a^2} \gg n_0 \gg \frac{3N}{2\pi a^2 b} \left[\left(1 - \frac{(\varsigma + b)^2}{b^2} \right) + \frac{2}{k_p b} \, \sin k_p b + \frac{2}{k_p^2 b^2} (1 - \cos k_p b) \right] \quad . \tag{27}$$

For the present SLAC parameters where $N = 5 \times 10^{10}$ and $b = 1$ mm, this condition is hard to satisfy. However, with slight modifications of the SLAC parameters, the inequality can be easily satisfied in the following two cases.

Case A, Round Beam Limit:

Assuming $k_p b \ll 1$, then the inequality becomes

$$\frac{1}{4\pi r_e a^2} \gg n_0 \gg \frac{3N}{2\pi a^2 b}\left(-\frac{1}{3}\,\frac{k_p^3 \varsigma^3}{k_p b}\right) \;, \quad -2b \le \varsigma \le 0 \;. \tag{28}$$

Notice that the maximum on the right hand side occurs at $\varsigma = -2b$, where the bunch ends. Thus the inequality is further specified to be

$$\frac{1}{4\pi r_e a^2} \gg n_0 \gg \frac{4Nk_p^2 b}{\pi a^2} \;. \tag{29}$$

Together with the previous assumption that $k_p a \ll 1$, this is thus a situation where $a \approx b$, and the beam has roughly the same size in both directions.

When the condition Eq. (29) is satisfied, the transverse force on a particle at (r, ς) within either an electron bunch or a positron bunch is

$$\mathcal{F}_\perp(r, \varsigma) = \mp eW_\perp \simeq \frac{e^2 k_p^2 N}{a^2 b^2}\,\varsigma^3 r \;, \qquad k_p b \ll 1 \;, \tag{30}$$

which is always focusing ($\because\ \varsigma \le 0$) towards the axis of symmetry of the beam. In this case the focusing force is maximum at the tail of the bunch.

Case B, Long Beam Limit:

If N is, for instance, one order of magnitude less than the present SLAC parameter while b remains the same, then the inequality can be straightforwardly satisfied for all $k_p b$. An interesting situation in this case is when $k_p b \gg 1$. Therefore $b \gg \lambda_p/2\pi \gg a$, and we have a long bunch where the longitudinal extent is much larger than the transverse extent. When this is satisfied, the focusing force is

$$\mathcal{F}_\perp(r,\varsigma) \simeq \frac{3e^2N}{a^2b}\left(1-\frac{(\varsigma+b)^2}{b^2}\right)r \quad , \qquad k_pb \gg 1 \ . \tag{31}$$

We see that the maximum of the force is at the mid-point along the bunch.

In either case the focusing force is very strong. For comparison, consider minimal departures from the SLAC parameters: In case A if $N = 1 \times 10^9$, $a = b = 100\ \mu$m, and $k_p \simeq 6 \times 10^{-3}\mu\text{m}^{-1}$, the field gradient $G \sim 173$ KG/cm at the mid-point along the bunch. In case B if $N = 5 \times 10^9$, and $a = 100\ \mu$m, $b = 1$ mm, we find the corresponding $G \sim 720$ KG/cm. In contrast, typical iron magnets ($G \sim 5$ KG/cm) and superconducting magnets ($G \sim 10$ KG/cm) are about $1 \sim 2$ orders of magnitude weaker. Notice that in the case of plasmas, the focusing force is governed by the densities of the beam and the plasma. By properly arranging the densities, the field gradient can be still larger.

Physically, this self-focusing effect arises because the electrons in the plasma are either expelled (for the case of interacting with an electron bunch) or pulled (for the case of interacting with a positron bunch) by the leading particles in the bunch, while on this time scale the ions in the plasma are essentially stationary. As a result the trailing particles in the same bunch experience an attractive force due to the access charges in plasma within the volume of the bunch. Large self-pinching of the beam is thus induced. This effect has been observed in computer simulations.[8]

4. A PLASMA LENS

Although the self-focusing field gradients that we showed in the previous section are high, they unfortunately suffer strong ς dependence in both the round beam limit and the long beam limit for standard bunches. For the purpose of a plasma lens, it is desirable to have a self-focusing force which is independent of particle's longitudinal position in a bunch. To achieve this it is necessary to tailor the charge distribution of the bunch.

Employing a technique developed earlier[7,9] based on the convolution theorem in Laplace transforms, one is able to find the desirable charge distribution which generates a constant transverse wake field. To be explicit, for a given charge distribution in the bunch, $\rho_b g(\varsigma) f(r)$, we have (c.f. Eq. (18))

$$\begin{aligned} W_\perp &= \mp \frac{4\pi e \rho_b}{k_p^2} G(\varsigma) \partial_r F(r) \\ &\equiv \mp \alpha \int_\varsigma^\infty d\varsigma' g(\varsigma') \sin k_p(\varsigma' - \varsigma) \quad . \end{aligned} \tag{32}$$

From the convolution theorem $g(\varsigma)$ can be obtained for the wanted $W_\perp$ by an inverse Laplace transform, i.e.

$$g(\varsigma) = \frac{1}{2\pi\alpha i} \int_{\epsilon - i\infty}^{\epsilon + i\infty} \frac{\mathcal{L}\{W_\perp\}}{\mathcal{L}\{\sin\ k_p \varsigma\}} e^{s\varsigma} ds \ , \tag{33}$$

where $\mathcal{L}\{\ \}$ indicates a Laplace transform. One of the possible arrangements for a constant $W_\perp$ is the following (see Fig. 2):

$$g(\varsigma) = k_p^{-1} \delta(\varsigma) + \theta\left(\varsigma + b + \frac{\pi}{2k_p}\right) - \theta\left(\varsigma + \frac{\pi}{2k_p}\right) , \tag{34}$$

where θ's are the step functions. We see that there are two components in the tailored bunch: an infinitely thin disk, and a "cylinder" with length b which follows behind the disk by one quarter of a plasma wavelength. The transverse density distributions are, however, the same for both components. Under this arrangement, the thin disk serves as a precursor which generates a transverse wake field that grows as a sine function. The transverse wake field reaches its maximum at $\lambda_p/4$ behind the precursor where the main bunch starts. The wake field generated by the main bunch partially balances the sinusoidal wake field excited by the precursor and gives rise to a net constant transverse wake along the bunch (see Fig. 2).

The total charge distribution in this arrangement is therefore

$$\sigma(\vec{x}) = \rho_b \left(1 - \frac{r^2}{a^2}\right) \left[k_p^{-1}\delta(\varsigma) + \theta\left(\varsigma + b + \frac{\pi}{2k_p}\right) - \theta\left(\varsigma + \frac{\pi}{2k_p}\right)\right] , \qquad (35)$$

where

$$\rho_b = \frac{2Nk_p}{\pi a^2(1 + k_p b)} .$$

The self-focusing force along the main bunch is now independent of ς:

$$\mathcal{F}_\perp = -\frac{4Ne^2 k_p r}{a^2(1 + k_p b)} . \qquad (36)$$

Note, however, that the precursor experiences no focusing force. The inequality in this case is

$$\frac{1}{4\pi r_e a^2} \gg n_0 \gg \frac{2Nk_p}{\pi a^2(1 + k_p b)} , \qquad (37)$$

where the denominator on the right hand side is associated with the ratio of charges between the precursor and the main bunch, i.e.

$$\frac{Q_{\text{precursor}}}{Q_{\text{bunch}}} = \frac{1}{k_p b} \ . \tag{38}$$

Assuming that such tailored two-component bunches can be prepared in future e^+e^- linear colliders, consider now the following construction of a plasma lens for final focus (see Fig. 3): At distance s down stream from the e^+e^- interaction point, a non-relativistic, neutral plasma jet (pulsed or continuous) streams in the direction transverse to the beam pipe. The jet speed is chosen such that the plasma which has been perturbed by an incoming tailored bunch can move out of the region before the next bunch enters. Assuming a repetition rate of the e^+e^- bunches in a future linear collider to be $10^3 \sim 10^4$ Hz,[1] and the range of the beam-plasma interaction to be $10 \sim 100$ μm transversely, then the jet speed is supposed to be $1 \sim 100$ cm/sec. Thus the plasma is practically stationary during the transient time of an ultra-relativistic bunch, and all previous formulas are applicable to this situation. The plasma density is chosen such that the tailored e^+e^- bunches would focus to their minimum sizes in distance s after traversing through the plasmas.

In such a plasma lens, the "focusing strength" is

$$K = \frac{\mathcal{F}_\perp / r}{\gamma m c^2} = \frac{4 N k_p r_e}{\gamma a^2(s)(1 + k_p b)} \ , \tag{39}$$

where $a(s)$ is the radius of the bunch at the lens, and γmc^2 is the energy of the particles in the bunch. The field gradient of the plasma lens is

$$G = \frac{1}{r} \, W_\perp = \frac{4 N e k_p}{a^2(s)(1 + k_p b)} \ . \tag{40}$$

The focal length s can be determined if the emittance ϵ_n and the β-function at the interaction point, β^*, are given:

$$a^2(s) = \frac{\epsilon_n}{\gamma}\beta^* \left(1 + \frac{s^2}{\beta^{*2}}\right) \quad . \tag{41}$$

Once s is known, one can evaluate the thickness of the plasma lens (in the direction of the beam pipe) via the thin lens formula,

$$\ell = \frac{1}{Ks} \ . \tag{42}$$

This set of parameters, together with the plasma wave number k_p (or plasma density n_0) completes the conceptual design of the plasma lens.

As a numerical example, we take a non-optimized set of parameters of a 5 TeV + 5 TeV e^+e^- linear collider discussed by Richter[1]:

Each of the colliding e^+ and e^- bunches has 4.1×10^8 particles. At the interaction point the longitudinal and transverse sizes of the bunch are $\sigma_z = 3.4 \times 10^{-3}$ mm and $\sigma_{r_0} = 2.0 \times 10^{-3}$ μm, respectively. The normalized emittance is assumed to be 4×10^{-8} m-rad, and the β-function at the interaction point is $\beta^* = 1$ mm.

For our purpose, we should tailor the bunches, and match their parameters to a reasonable set of plasma parameters such that the inequality in Eq. (37) can be satisfied. We certainly do not need the bunch to have the same transverse dimension σ_{r_0} at the plasma lens, this is actually the reason for the focus. In order not to make the focal length too long we choose $a(s)$ to be about four orders of magnitude larger than σ_{r_0}. However in order that the beam density b satisfies Eq. (37) with a given plasma density n_0, one cannot choose b to be as small as σ_z given above. Let us therefore tailor the bunch such that $b = 100$ μm and $a(s) = 3\ \mu$m and choose the plasma density to be $n_0 = 10^{18}$ cm^{-3}, such that $k_p \simeq (1/5)\ \mu$m^{-1}.

The focusing strength in the case is (c.f. Eq. (39))

$$K \simeq 4.8 \times 10^{-2}\ \mathrm{cm}^{-2} \quad , \tag{43}$$

and the focusing field gradient is

$$G \simeq 830 \quad \mathrm{MG/cm} \quad . \tag{44}$$

To evaluate the focal length, notice that the transverse size is reduced from $a(s)$ to σ_{r_0}, so from Eq. (41) we have

$$s \simeq 4.8\ \mathrm{m} \quad , \tag{45}$$

and the plasma lens thickness is

$$\ell \simeq 23\ \mathrm{cm} \quad , \tag{46}$$

which is consistent with the thin lens assumption (i.e. $s \gg \ell$) and justifies our approximation on neglecting the transverse current within the beam (c.f. Eq. (9)) during the beam-plasma interaction time.

5. DISCUSSION

The phenomena of self-focusing during beam-plasma interaction was described and the effect was shown to be very strong. The idea of a plasma lens employing this effect was introduced with bunch density properly tailored. An interesting point is that in the two-component tailored bunch discussed above, the precursor not only serves to provide ς-independent focusing force for the main bunch but also serves to further focus the main bunch of the opposite beam at the interaction point after the two precursors from e^+ and e^- beams passing each other. This effect is a non-optimized 'super disruption' discussed by Palmer,[2] which will further enhance the luminosity.

From the numerical example given it seems, however, that the plasma lens may be located within the detector at the interaction point; and since e^+e^- will interact with the plasma, a background noise seems unavoidable. To estimate the noise let us notice that the high energy physics event rate at the plasma lens is

$$R(s) = \rho_b f b \pi a^2(s) n_0 \ell \; \sigma(s) \quad , \tag{47}$$

where ρ_b is the beam density, f the incoming beam repetition rate, $\pi a^2(s) n_0 \ell$ the number of plasma electrons within the interaction volume, and $\sigma(s)$ the interaction cross section. On the other hand, neglecting the non-optimized super disruption effect, the e^+e^- event rate at the interaction point is

$$R(0) = \mathcal{L}(0) \; \sigma(0) = \left(\frac{N^2 f H_D}{4\pi \sigma_{r_0}^2} \right) \; \sigma(0) \ , \tag{48}$$

where $\mathcal{L}(0)$ is the luminosity, H_D the enhancement factor,[10] and $\sigma(0)$ the cross section at the interaction point.

In terms of known parameters, Eq. (47) can be rewritten as

$$R(s) = \frac{\gamma f b \; n_0 \; a^2(s)\sigma(s)}{2 r_e s} \quad . \tag{49}$$

Notice that the scattering cross section generally varies as the inverse of the center of mass energy squared. Thus $\sigma(s)/\sigma(0) = 2\gamma$, and the background noise is

$$\eta = \frac{R(s)}{R(0)} = \frac{4\pi\gamma^2 b n_0 \sigma_{r_0}^2 a^2(s)}{N^2 H_D r_e s} \quad . \tag{50}$$

In our particular numerical example, H_D is of order unity. Thus

$$\eta \simeq 2.0 \times 10^3 \quad . \tag{51}$$

This is certainly a non-negligible noise; however, the effect is really not so harmful. Firstly, contrasting to the beam-beam events, the beam-plasma interaction is a stationary target interaction, thus the event products travel mostly along the beam pipe and escape from detection. Secondly, since the center of mass energy of the beam-plasma events is a factor $\sqrt{2\gamma}$ lower than those of the beam-beam events, it should be straightforward to distinguish the two types of events. Therefore the important issue should really be the absolute event rate, rather than the relative event rate of the background, which would potentially damage the detector. The fact that the high-energy event rates decrease as the square of the center of mass energy ensures very low absolute event rates in future colliders, thus the noise from the plasma lens should not be a problem.

There are however several important issues yet to be addressed. In the conceptual design of the lens we assumed a perfect uniform plasma slab. In practice there may be local density fluctuations inside plasma and inhomogeneity across the face of the plasma. Both effects would contribute to the aberration and need to be further studied.

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REFERENCES

1. B. Richter, in *Laser Acceleration of Particles*, ed. C. Joshi and T. Katsouleas, AIP Conf. Proc. No. 130, pp. 8, 1985 and B. Richter, IEEE Trans. Nucl. Sci. No. 130, pp. 8, 1985.
2. R. B. Palmer, SLAC-PUB-3688, May 1985.
3. P. Chen, R. W. Huff and J. M. Dawson, UCLA Preprint PPG-802, July 1984 and Bull. Am. Phys. 29, 1355 (1984); P. Chen, J. M. Dawson, R. W. Huff and T. Katsouleas, Phys. Rev. Lett. 54, 693 (1985); Erratum: Phys. Rev. Lett. 55, 1537 (1985).
4. R. D. Ruth, A. W. Chao, P. L. Morton and P. B. Wilson, Particle Accel. 17, 171 (1985).
5. W. K. H. Panofsky and W. A. Wenzel, Rev. Sci. Instrum. 27, 967 (1956).
6. The author appreciates helpful comments from B. Lippmann in this regard.
7. P. Chen and J. M. Dawson, in *Laser Acceleration of Particles*, ed. C. Joshi and T. Katsouleas, AIP Conf. Proc. No. 130, pp. 201, 1985.
8. T. Katsouleas, UCLA Preprint PPG-881, August 1985.
9. K. L. F. Bane, P. Chen, P. B. Wilson, IEEE Trans. Nucl. Sci. NS-32, 3524 (1985).
10. P. B. Wilson, SLAC-PUB-3674, November 1985.

FIGURE CAPTIONS

Figure 1: A 'standard' electron bunch traversing a plasma. The radius and half-length of the bunch are a and b, respectively. The parabolic dashed curves within the bunch indicate its longitudinal density distribution. For the plasma, the squares represent ions and circles represent electrons.

Figure 2: The density $g(\varsigma)$ and the wake function $G(\varsigma)$ as functions of distance for a 'tailored' bunch. A thin-disk precursor is followed by a constant (longitudinal) density main bunch by one quarter of a plasma wavelength.

Figure 3: A conceptual design of a plasma lens. The plasma (in squares and circles) is ejected from a pipe perpendicular to the beam pipe, and absorbed by a low pressure pipe across the beam's trajectory. The speed of the plasma, $v_\perp$, only needs to be large enough such that a chain of incoming bunches can experience fresh plasmas.

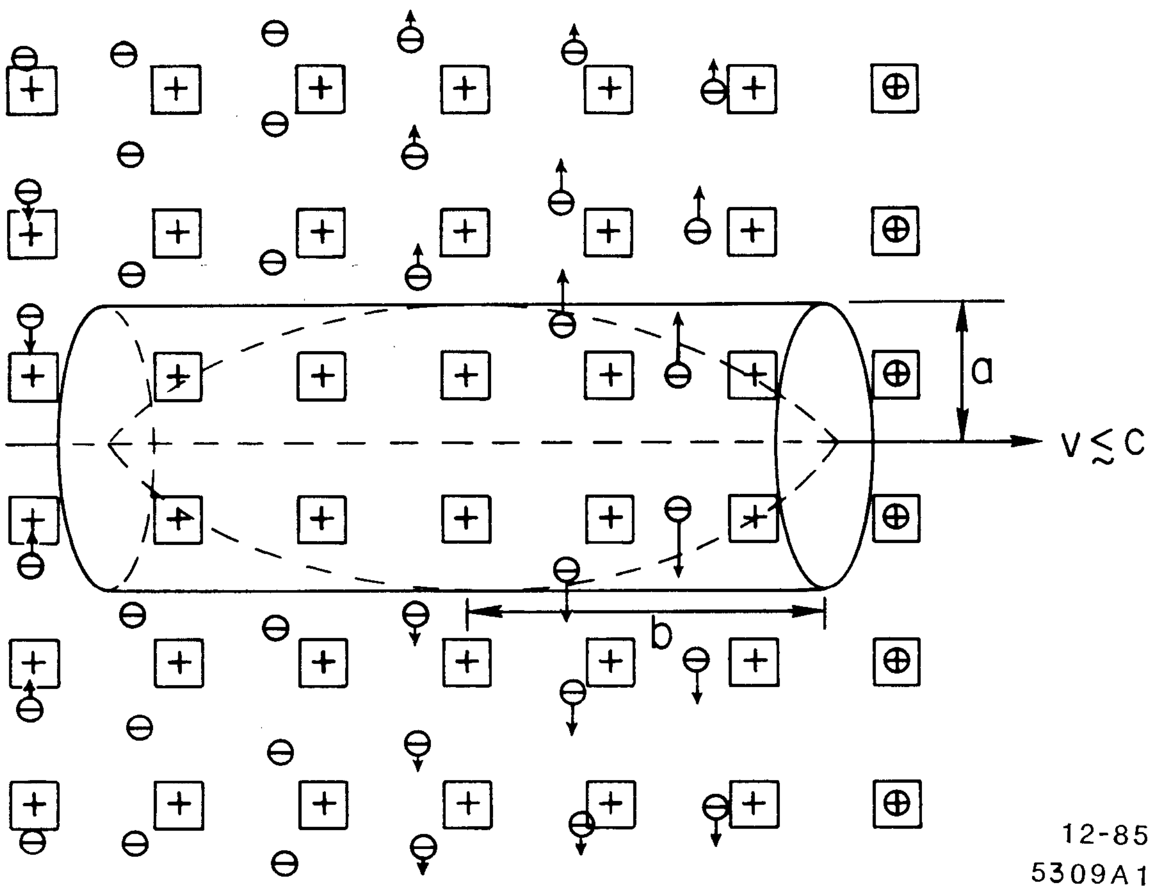
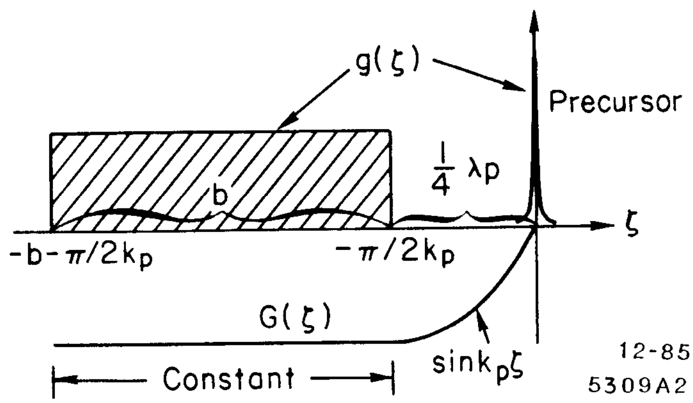


Fig. 1



g(ζ)
Precursor
$\frac{1}{4}$ λp
b
ζ
-b-π/2kp
-π/2kp
G(ζ)
sinkpζ
Constant
12-85
5309A2

Fig. 2

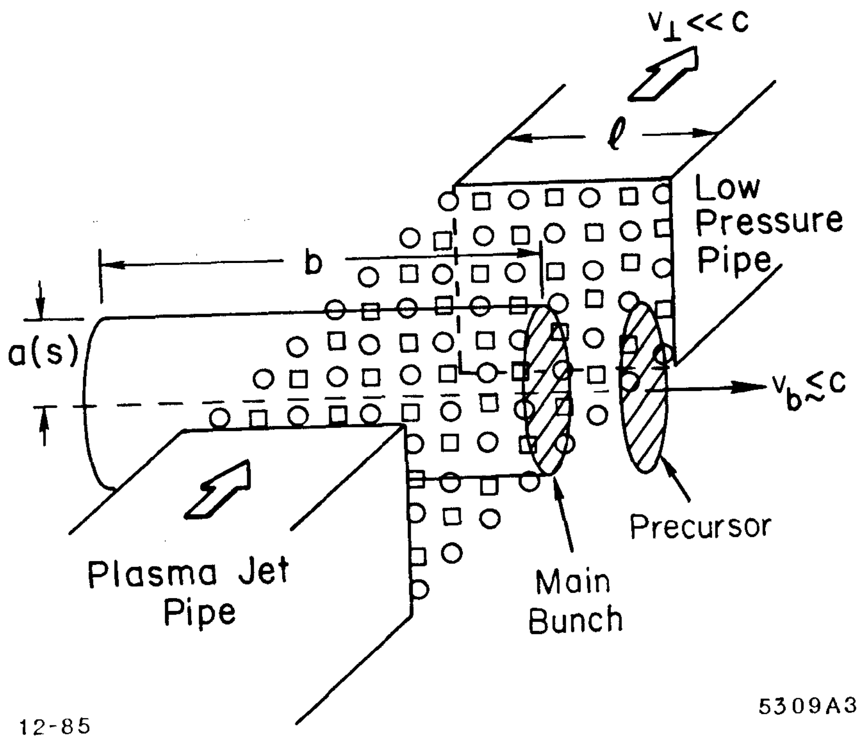

v⊥ << c
ℓ
Low
Pressure
Pipe
b
a(s)
$v_b \lesssim c$
Precursor
Plasma Jet
Pipe
Main
Bunch
12-85
5309A3

Fig. 3