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STUDY OF A HIGH GRADIENT PULSED LINAC STRUCTURE*

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ABSTRACT

Analytical calculation of the accelerating gradient in a radial line transformer linac will be presented.

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Introduction

We will describe a high gradient structure capable of accelerating extremely short electron bunches.

The structure proposed is a very simple induction linac; this induction linac is similar to Radlac[1] in one way, and to the wakefield accelerator in another. The closest kin is the switched power linac proposed by Willis.[2]

The Structure

A set of parallel metallic disks with a small hole in the center is alternatively charged at a potential $+V_0$ with respect to the adjacent disks, which are at ground (Fig. 1). An ideal switch distributed around the periphery of the disks is closed at $t = 0$, generating a wave that moves towards the center of the disks. The wave grows in amplitude because of the increasing impedance, just like in the pillbox wakefield transformer. The timing is adjusted so that the high field appears at the central hole when a short electron beam bunch to be accelerated enters the gap between the two disks. The beam gains energy traversing this gap; the voltage applied is the wave voltage, V, minus the DC voltage, V_{DC} and $V \gg V_{DC}$. In the following gap the electric field is in the "right" direction, so that the total energy gain is V, neglecting beam loading.

The following is an attempt to calculate the accelerating gradient.

Calculation of the Accelerating Gradient

Two parallel disks of radius b form a capacitor initially charged at V_0 (see Fig. 2). At $t = 0$ the outer edge of the disks is shorted, *i.e.* a wave $-V_0$ is injected from the periphery towards the center.

The electric and magnetic field at a radius r and time t are given by the following expressions:[3]

$$E_z\left(\frac{r}{b}\,,t\right) = 2\,E_0 \sum_{e=1}^{\infty} \frac{J_0\left(x_e \frac{r}{b}\right)\,\cos\,x_e\,\frac{ct}{b}}{x_e\,J_1\left(x_e\right)} \tag{1}$$

$$B_\varphi\left(\frac{r}{b}\,,t\right) = -\frac{2\,E_0}{c} \sum_{e=1}^{\infty} \frac{J_1\left(x_e \frac{r}{b}\right)\,\sin\,x_e\,\frac{ct}{b}}{x_e\,J_1\left(x_e\right)} \tag{2}$$

$J_0(x_e\frac{r}{b})$ and $J_1(x_e\frac{r}{b})$ are the Bessel functions calculated at $x_e\frac{r}{b}$, x_e being the solutions of $J_0(x_e) = 0$.

A plot of the electric field as a function of time for different positions along the radius is in Fig. 3. The E field grows as $\sqrt{r/b}$; a simple energy conservation argument can be made for this dependence: since the pulse length does not change while the pulse travels towards the center of the disks, then

$$\frac{V_1^2}{Z_1} = \frac{V_2^2}{Z_2}$$

where (1) and (2) refer to two different voltages at different values of radii; therefore

$$\frac{V_1}{V_2} = \sqrt{\frac{Z_1}{Z_2}} = \sqrt{\frac{R_2}{R_1}}$$

since $Z(R)$ is proportional to $1/R$.

The solution contains an impedance pole at $r = 0$, so that one sees a reflected wave following the initial step. This distance in time between the reflection and the front step decreases as one approaches the center, *i.e.* the two waves merge,

Fig. 3 through 6. The expression given in Eq. (1) allows us to calculate more precisely the energy gain of an electron crossing the gap, under the following assumptions:

(1) The wakefield induced by the accelerated charged is neglected.

(2) No beam loading.

(3) The E_z field is not modified by the presence of the center holes, *i.e.* we use the analytical solution even though the boundary conditions are slightly different in reality.

(4) The E_z field is ideally fast, *i.e.* no dispersion is applied to $E_z(t)$.

At a certain position r one electron enters the gap, the time t_0. The electron will exit the gap after a time g/c. The increase in momentum Δ will be:

$$\Delta = q \int_{t_0}^{t_0+\frac{g}{c}} E_z\left(\frac{r}{b}\,,t\right)\, dt \tag{3}$$

Integrating Eq. (13) term by term:

$$\Delta\left(\frac{r}{b}\,,t_0\right) = 2q\,\frac{b}{c}\,E_0 \sum_{e=1}^{\infty} \frac{J_0\left(x_e\frac{r}{b}\right)}{x_e^2 J_1(x_e)}\left[\sin\,\frac{x_e c t_0}{b} - \sin\frac{x_e c\left(t_0+\frac{g}{c}\right)}{b}\right] \tag{4}$$

The momentum gain calculated by Eq. (4) can be thought of as a time average, $\bar{E}$, of the electric field acting upon the charge q

$$\bar{E} = \frac{\Delta c}{qg}$$

The value of $\bar{E}$ has been calculated for two fixed radii of 1 meter and 4 meters, varying the gap from 2 to 14 mm in 1mm increments. The results are plotted in Fig. 7 and summarized in Table I. Each curve represents the value of $\bar{E}$ for a fixed gap length, as seen by an electron appearing in the gap at different phases from the wave. The electron is always injected in the disk's geometrical center. These calculations show that the peak value of $\bar{E}$ is related to the ratio R/g ($R \equiv b$)

$$\bar{E} \cong E \cdot 2\sqrt{\frac{R}{.55\,g}} \tag{5}$$

Table I summarizes the values of peak field *vs* gap, and the ratio $\bar{E}/E \cdot \sqrt{R/g}$ for the 1 m radius and 4 m radius disks: G is the gain, *i.e.* the ratio between the pulsed and the DC field. For instance, a radius of 1 m with a 2 mm gap will provide a time averaged field $\bar{E}$ about 62 times larger than the DC field. The peak field however will be much higher for a very short time.

Table I

g, mm	$\sqrt{R/g}$ 1m	G 1m	$\bar{E}/\sqrt{R/g}$ 1m	$\sqrt{R/g}$ 4m	G 4m	$\bar{E}/\sqrt{R/g}$ 4m
2	22.36	61.69	2.758	44.72	117.5	2.63
3	18.26	49.73	2.723	36.51	98.7	2.70
4	15.81	42.62	2.696	31.62	86.4	2.73
5	14.14	38.06	2.691	28.28	77.7	2.74
6	12.91	34.79	2.695	25.82	71.2	2.76
7	11.95	32.18	2.692	23.90	66.0	2.76
8	11.18	29.98	2.681	22.36	61.7	2.75
9	10.54	28.15	2.670	21.08	58.0	2.75
10	10.54	28.15	2.670	21.08	58.0	2.75
11	9.535	25.41	2.665	19.07	52.1	2.73
12	9.13	24.30	2.661	18.25	49.7	2.72
13	8.77	23.28	2.654	17.54	47.6	2.71
14	8.451	22.38	2.648	16.90	45.8	2.71

Zero Risetime

Equation (4) gives the accelerating field for a zero risetime pulse. In practice, the switch across the outer edge of the radial line will have a finite risetime. Using Eq. (1) for the electric field, (and Eq. (2) for B_φ) we have introduced the effect of finite risetime τ by calculating the value of the $\bar{E}$ as the super position of infinitesimal successive steps varying in time as

$$E(t) = E_0 e^{-t/\tau} \tag{6}$$

so that Eq. (1) becomes

$$E_z\left(\frac{r}{b}\,, t, \tau\right) = 2 \sum_{e=1}^{\infty} A_e \int_0^{\infty} \left(-\frac{dE}{dt'}\right) \cos \frac{x_e c}{b}\,(t - t')\, dt' \tag{7}$$

where A_e is

$$A_e = \frac{J_0\left(x_e \frac{r}{b}\right)}{x_e\, J_1\left(x_e\right)}$$

Substituting (6) into (7) we obtain

$$E_z\left(\frac{r}{b}\,, t, \tau\right) = \frac{2\,E_0}{\tau} \sum_{e=1}^{\infty} A_e \int_0^{\infty} e^{-t'/\tau} \cos \frac{x_e c}{b}\,(t - \tau')\; dt' \tag{8}$$

Setting $d = c\tau/b$

$$E_z\left(\frac{r}{b}\,, t, d\right) = 2\,E_0 \sum_{e=1}^{\infty} A_e \left[\frac{1}{1 + x_e^2 d^2} \cos \frac{x_e c t}{b} + \frac{x_e d}{1 + x_e^2 d^2} \sin \frac{x_e c t}{b}\right] \tag{9}$$

Note that Eq. (9) is identical to Eq. (1) for $\tau = 0$.

Using Eq. (9) we calculate $\bar{E}$ as it was done in Eq. (4). Intuitively the finite risetime effects should be similar to extending the gap g to larger values.

Fitting the calculated values of $\bar{E}$ we obtain a relation similar to (5):

$$\bar{E} \cong 2\,E_0\,\sqrt{\frac{R}{.48g + .577\tau_R c}} \tag{10}$$

where τ_R is the 10% to 90% risetime. A convenient approximation of (10) is:

$$\bar{E} \cong 2\,E_0\,\sqrt{\frac{2R}{g + \tau_R c}} \tag{11}$$

Conclusions

The radial line transformer technique is capable of providing a high gradient for very short electron beam bunches. Such linac requires a very fast (few picoseconds risetime) and very efficient switch to become a reality.

References

1. K. R. Prestwich *et al.*, *IEEE* Trans. on Nucl. Sci., *NS-30, 4*, Nov. 1983, (3155).
2. W. Willis, "Switched Power Linac," Proceedings of the ECFA/INFN Workshop, CERN 85/07, 10 June 1985.
3. R. Cooper, LANL, private communication.

Figure Captions

1. Schematic arrangement of the linac structure.
2. One set of electrodes. The variables of Eq. (1) and (2) are indicated.
3. Electric field at different radii as a function of time, as calculated by Eq. (1).
4. Same as in Fig. 3, moving closer to the center ($r/b = .01$), *i.e.* this is the electric field seen by an observer 1 cm away from the disk's center, when the disk's radius is 1 meter. Remember that this is a step response, and yet the pulse is already extremely short.
5. Same as in Fig. 4, 2.5 mm from the center.
6. Closer yet, 1 mm from the center, the peak field is larger but the "width" has decreased.
7. Value of $\bar{E}$, as defined by Eq. (15), for a 1 m radius disk, with gap varying from 2 mm to 14 mm. The value is calculated in the geometrical center. The horizontal axis is the position of the wave front at the time at which the test electron enters the gap.

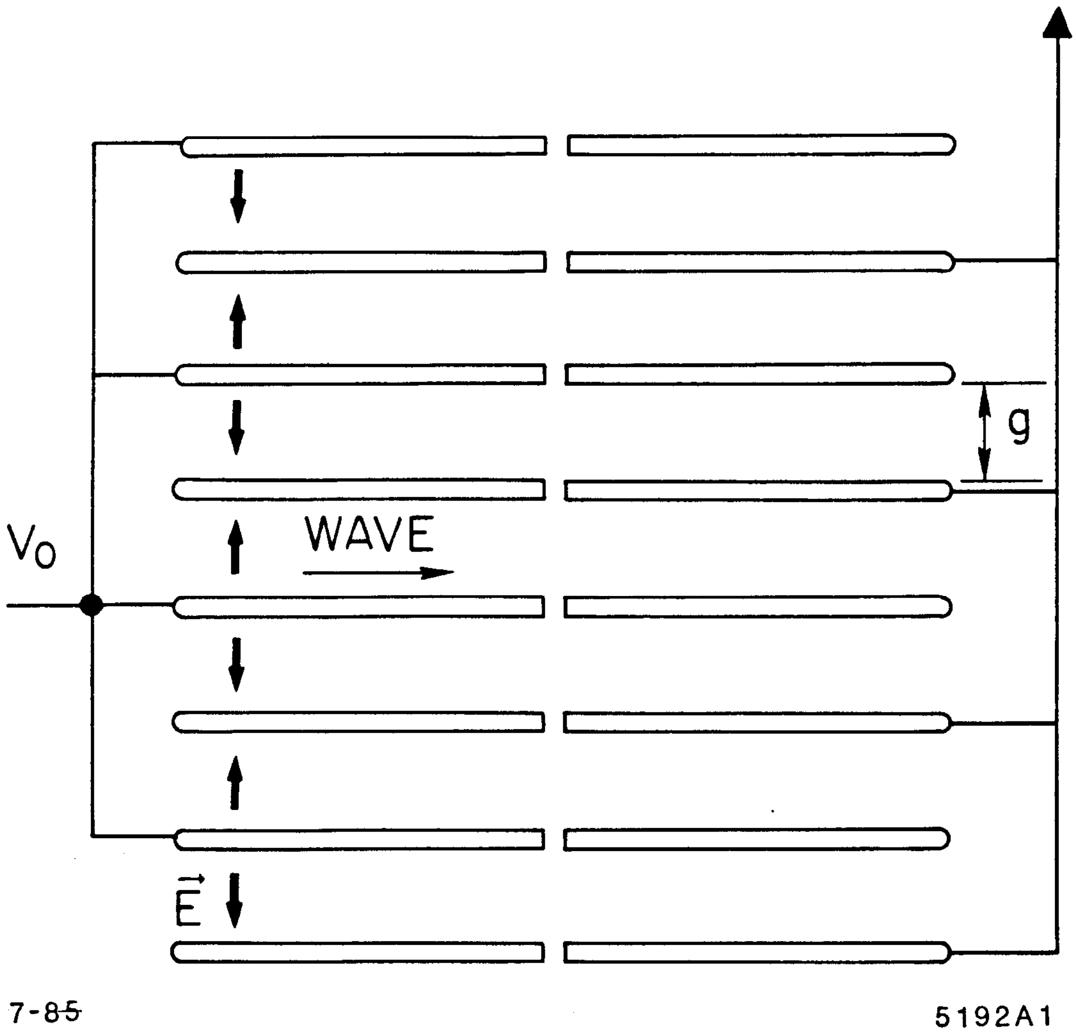


Fig. 1

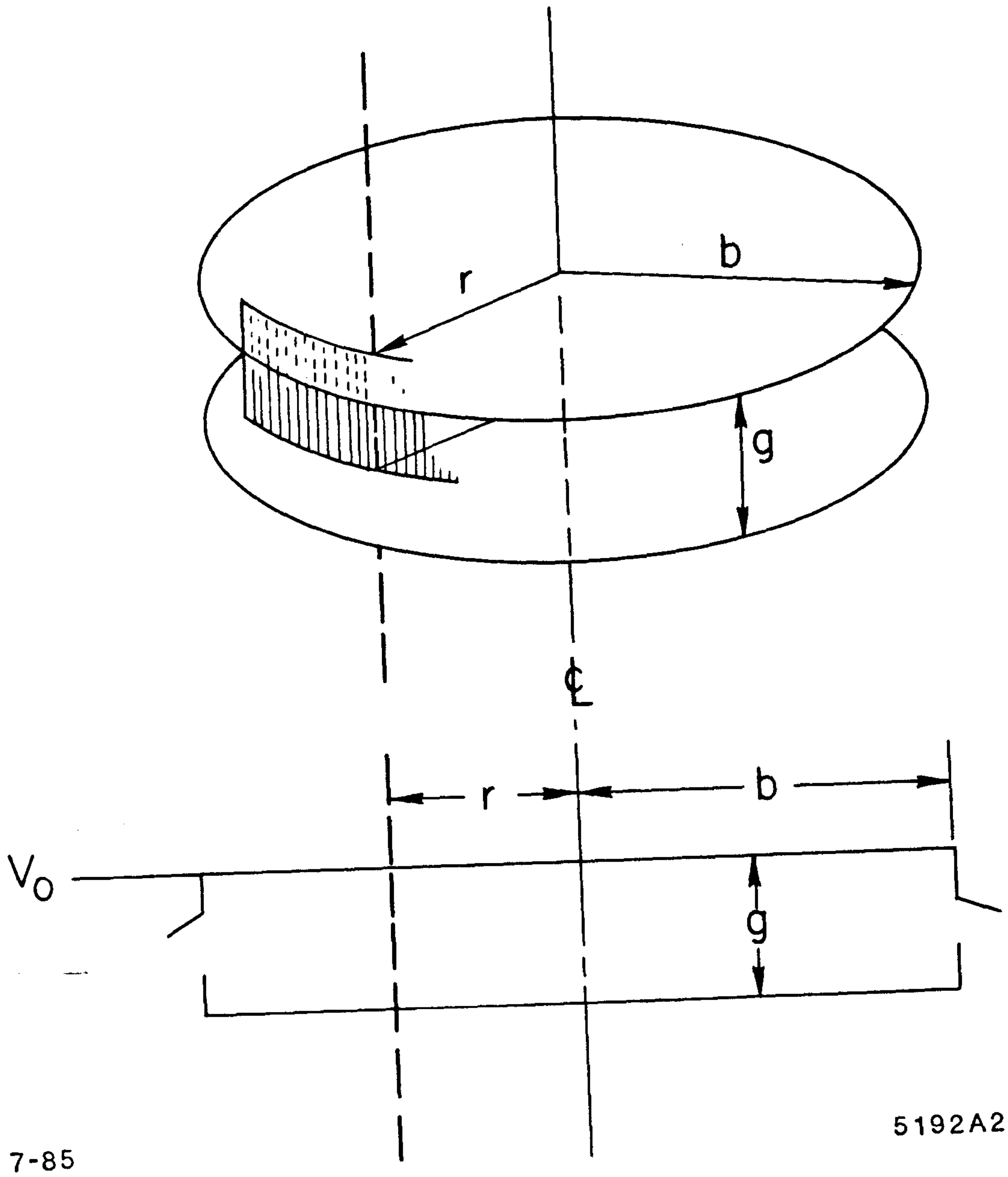


Fig. 2

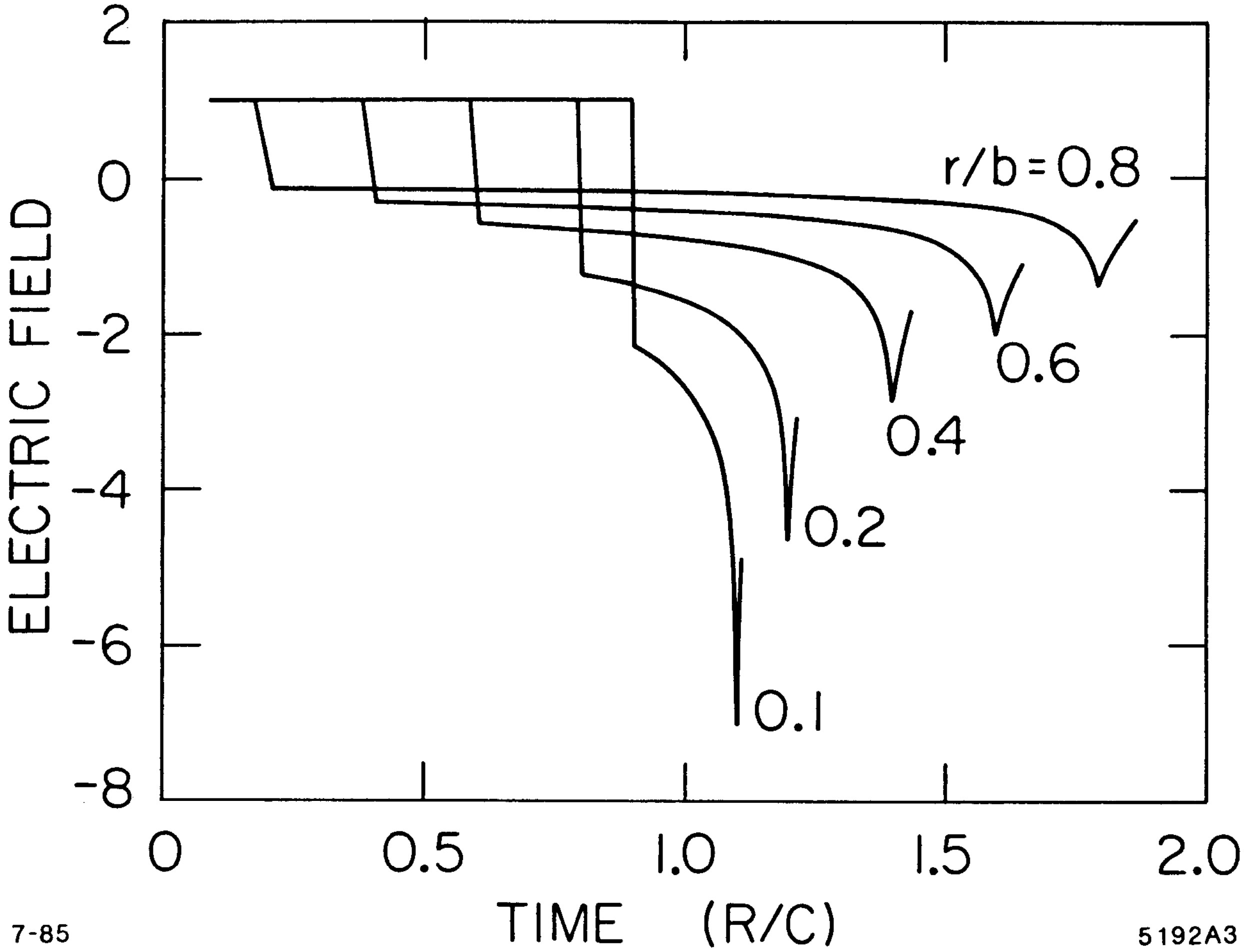

2
0
-2
-4
-6
-8
ELECTRIC FIELD
r/b=0.8
0.6
0.4
0.2
0.1
0
0.5
1.0
1.5
2.0
TIME (R/C)
7-85
5192A3

Fig. 3

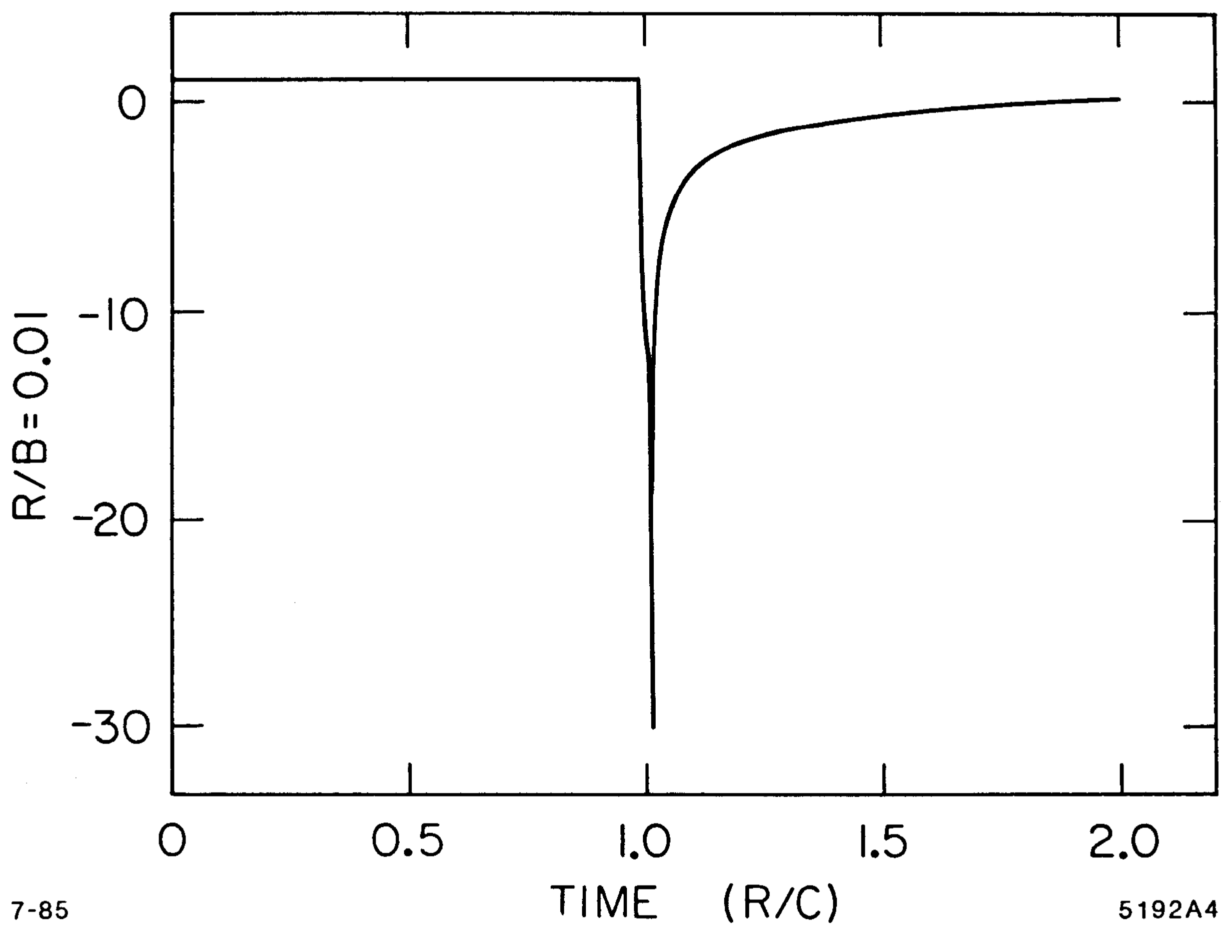


Fig. 4

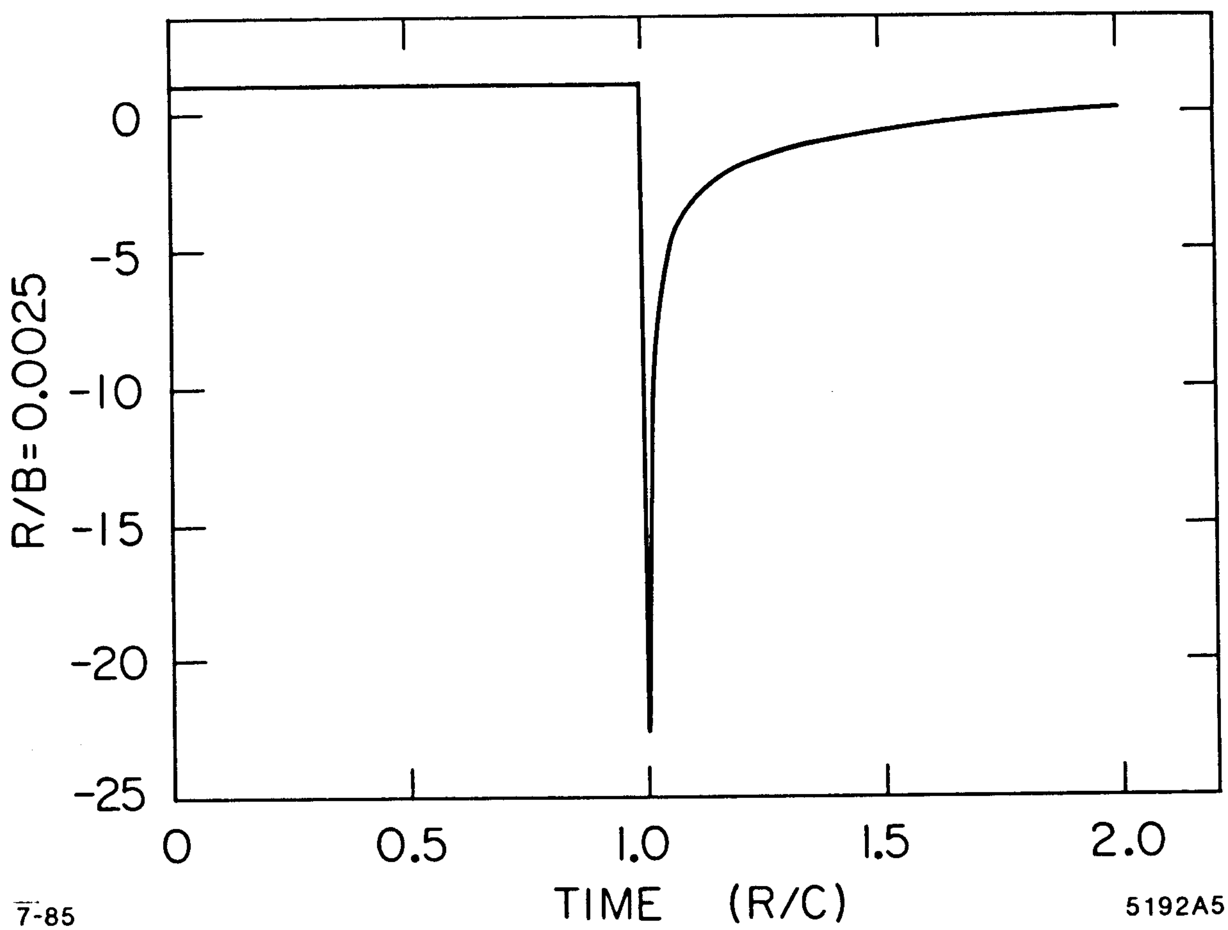


Fig. 5

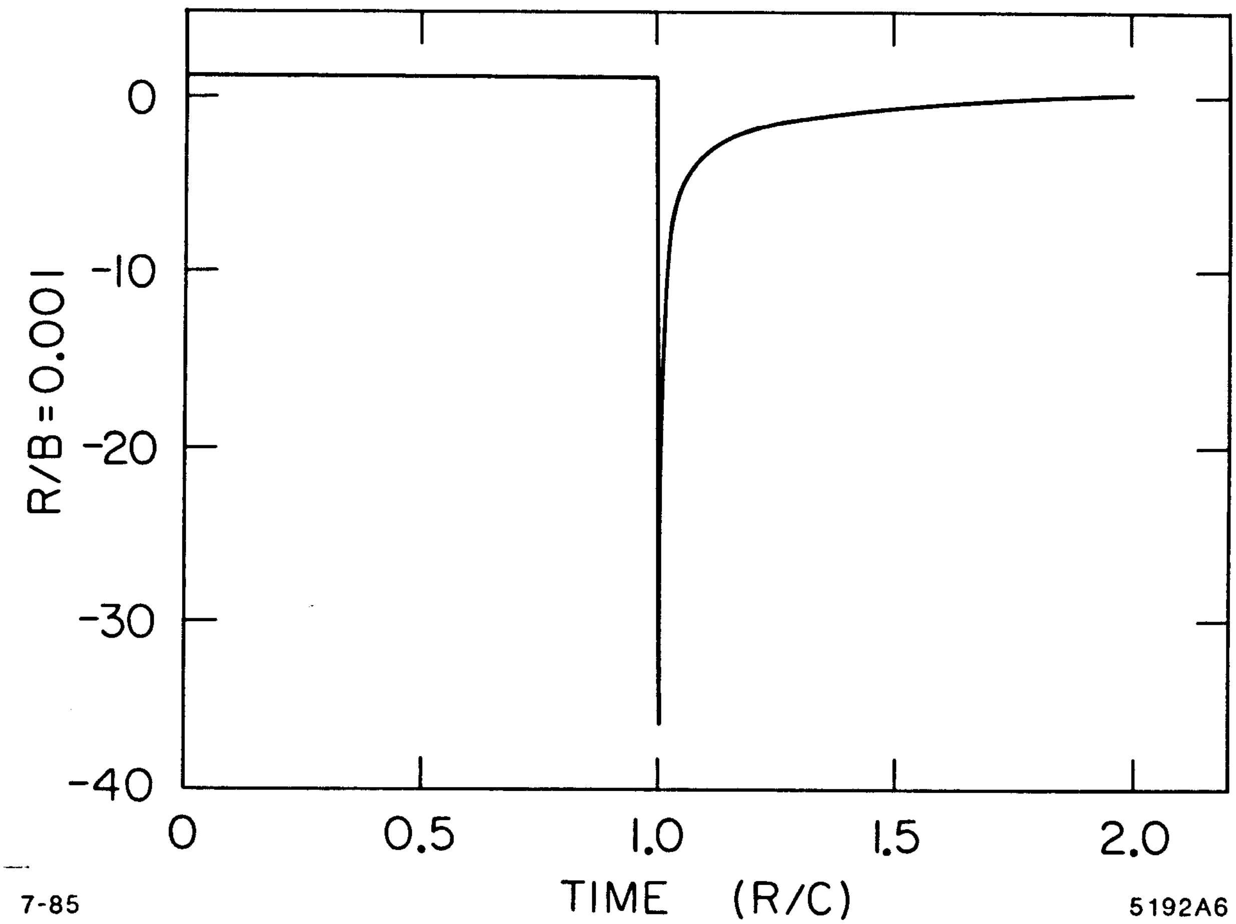
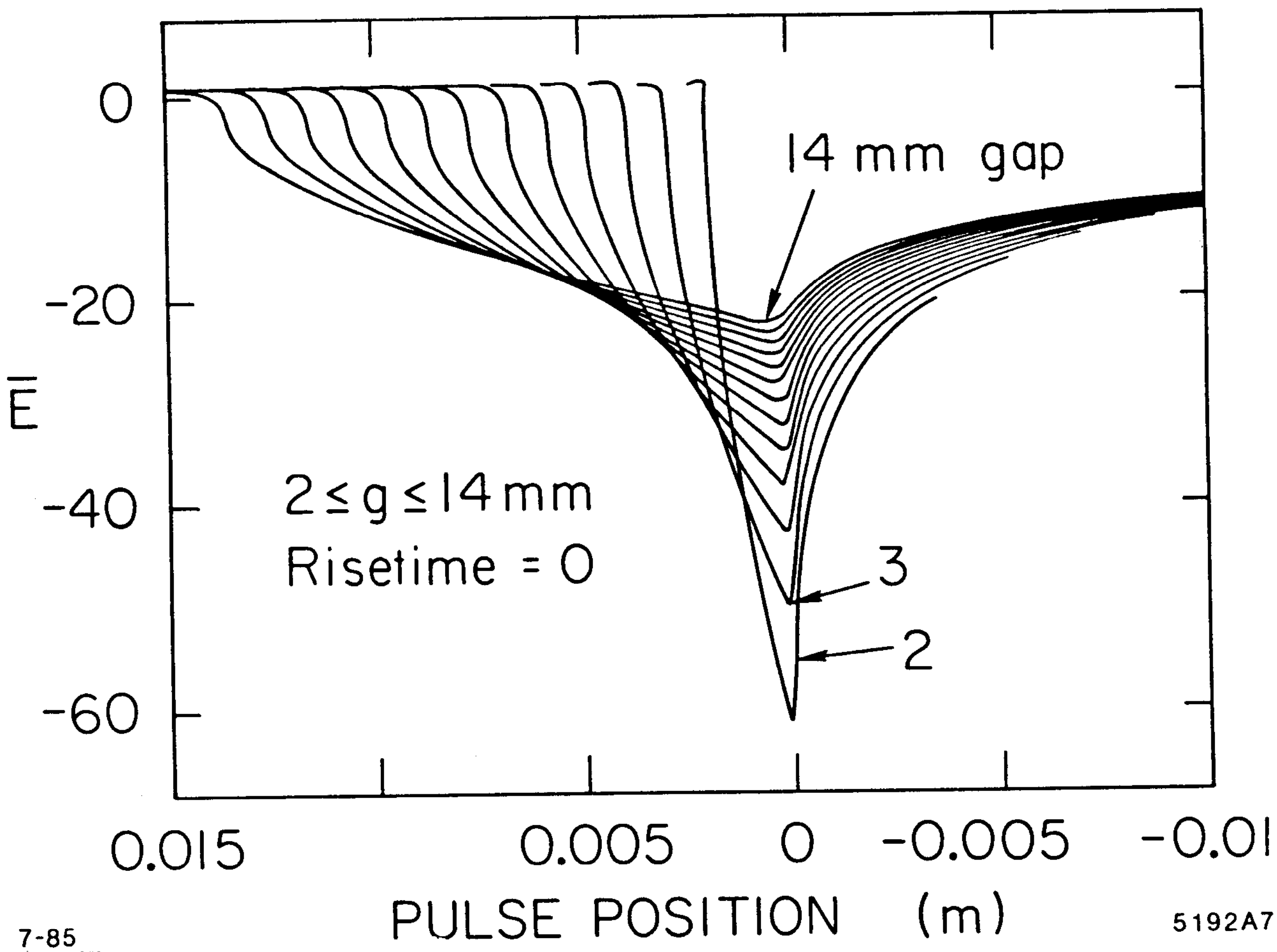


Fig. 6



0
-20
-40
-60
Ē
14 mm gap
2 ≤ g ≤ 14 mm
Risetime = 0
3
2
0.015
0.005
0
-0.005
-0.01
PULSE POSITION (m)
7-85
5192A7

Fig. 7