SLAC – PUB – 3802
October 1985
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GAUGE INVARIANCE OF STRING FIELDS*

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1. STRING SYMMETRIES

This is a report on some work we have done[1] to understand the appearance of gauge bosons and gravitons in string theories. We have constructed an action for free (bosonic) string field theory which is invariant under an infinite set of gauge transformations which include Yang-Mills transformations and general coordinate transformations as special cases. Our work was motivated by some beautiful papers of Siegel[2] in which a covariant, gauge fixed free string action was constructed. At the end of this lecture I will show how to obtain Siegel's action from ours by a straightforward application of the Faddeev-Popov procedure. Two other groups[3] have independently arrived at the form for the gauge invariant string action that we will present here.

The Nambu action:

$$S = \int d^2\xi \sqrt{\epsilon_{ij}\epsilon_{kl} \frac{\partial x^\mu}{\partial \xi_i} \frac{\partial x_\mu}{\partial \xi_k} \frac{\partial x^\nu}{\partial \xi_j} \frac{\partial x_\nu}{\partial \xi_l}} \tag{1}$$

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* Work supported by the Department of Energy, contract DE–AC03–76SF00515.

Invited talk presented at the Unified String Theories, Santa Barbara, California, July 29–Aug 16, 1985; and the 1985 INS International Symposium on Composite Models of Quarks and Leptons, Tokyo, Japan, Aug 13–15, 1985

is not the action for string theory. Its function in string field theory is analogous to that of the relativistic particle action

$$\int \sqrt{\dot{x}^2} \tag{2}$$

in ordinary field theory. That is, the Feynman propagators and vertices in scalar field theory are operators in a Hilbert space of functions $\phi(x)$,which is determined by the quantum dynamics of the Lagrangian of equation (2). Since the Lagrangian is invariant under proper time reparametrizations its dynamics is determined by a Schrodinger-Wheeler-DeWitt (SWD)[4] equation:

$$(p^2 + m^2)\phi(x) = 0 \ . \tag{3}$$

A classical field theory is defined by treating the wave function $\phi(x)$ as a classical field and finding a Lagrangian whose Euler Lagrange equation is the SWD equation.

In order to mimic this formalism in string field theory we introduce the canonical conjugate to $x^\mu(\sigma,\tau)$ in the action of Eq. (1). $(z_0 = \tau, z_1 = \sigma)$

$$P_\mu(\sigma,\tau) = \frac{\partial}{\partial \dot{x}^\mu}\sqrt{\det \frac{\partial x^\mu}{\partial z_i}\frac{\partial x^\mu}{\partial z_j}} \ . \tag{4}$$

The gauge invariant quantum dynamics of the string is constructed by making x^μ and P_μ into operators with canonical commutators:

$$[x^\mu(\sigma), P_\nu(\sigma')] = i\delta^\mu_\nu \delta(\sigma - \sigma') \ . \tag{5}$$

World sheet reparametrization invariance is imposed by constraining the wave functionals $\Phi[x(\sigma)]$ to satisfy:

$$(\pi^2 P^2 + x'^2)\Phi = 0 \tag{6}$$

$$x^{\mu\prime} P_\mu \phi = 0 \ . \tag{7}$$

Since the system is invariant under proper time reparametrizations, these constraints include the SWD equation. Let us introduce some notation to rewrite these constraints in a more convenient form (we also specialize to open strings):

$$\begin{aligned} x^\mu(\sigma) &= x^\mu + \sum_{n>0} \frac{2}{\sqrt{n}} X_n^\mu \cos n\sigma \\ p^\mu(\sigma) &= \frac{1}{\pi}\Big\{ p^\mu + \sum_{n>0} \sqrt{n} P_n^\mu \cos n\sigma \Big\}; \end{aligned} \tag{8}$$

$0 \le \sigma \le \pi$, and $[X_n, P_m] = i\delta_{nm}$. It is convenient to replace

$$X_n = \frac{i}{2\sqrt{n}}(\alpha_n - \alpha_{-n}) \ , \quad P_n = \frac{1}{\sqrt{n}}(\alpha_n + \alpha_{-n}) \ , \tag{9}$$

and to set $\alpha_0^\mu = p^\mu$; then the α_n have the commutation relations:

$$[\alpha_n^\mu, \alpha_m^\nu] = n\,\delta(n+m)\,\eta^{\mu\nu}. \tag{10}$$

$p(\sigma)$ and $x'(\sigma)$ are especially simple functions of the α_n:

$$\Big(\pi p \pm x'\Big) = \sum_{n=-\infty}^{\infty} \alpha_n e^{\mp in\sigma}. \tag{11}$$

The generators of reparametrizations of the string are the local Hamiltonian and momentum densities:

$$\mathcal{H}(\sigma) = \frac{1}{2\pi}\Big(\pi^2 p^2 + (x')^2\Big), \quad \mathcal{P}(\sigma) = p \cdot x'. \tag{12}$$

These quantities are summarized as:

$$\tfrac{1}{2}\Big(\pi p \pm x'\Big)^2 = \sum_{-\infty}^{\infty} L_n e^{\mp in\sigma}, \tag{13}$$

where the L_n are the Virasoro operators[5]

$$L_n = \tfrac{1}{2} \sum_{-\infty}^{\infty} : \alpha_{n+k}^\mu \alpha_{-k}^\mu : . \tag{14}$$

These operators satisfy the algebra:

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{d}{12} n(n^2-1)\delta(n+m), \tag{15}$$

in which the central charge depends on d, the dimensionality of space.

The constraints are then equivalent (after a judicious choice of normal ordering constant in L_0) to:

$$(L_0 - 1)|\Psi >= 0 \tag{16}$$

$$L_n|\Psi >= 0\ . \tag{17}$$

These equations are inconsistent because of the Schwinger term in the Virasoro algebra (15). We replace (17) by:

$$L_n|\Psi >= 0 \text{ for } n > 0 \tag{18}$$

which is sufficient to guarantee the constraint equations as equations for matrix elements between states satisfying (18).

If we were to follow the example of scalar field theory slavishly, we would now seek an action from which all of these constraint equations followed as equations of motion. However, we are searching for a gauge invariant theory, and, guided by hints from the old string literature[6] we will instead interpret only (16) as an equation of motion. Equations (18) will be gauge fixing conditions. Note that in making this artificial separation we are violating the manifest reparametrization invariance of the formalism. In the interacting theory, this will have the consequence that duality is manifest only after all diagrams of a given order are summed.

2. STRING FIELDS

It is now a simple matter to find a gauge invariant action from which (16) and (18) follow. Equation (18) is analogous to the Lorentz gauge condition in electrodynamics. The Maxwell Lagrangian may be written:

$$A_\mu \partial^2 P^{\mu\nu} A_\nu \tag{19}$$

where $P^{\mu\nu} = \eta^{\mu\nu} - \partial^\mu \partial^\nu / \partial^2$ is the projector on states satisfying the Lorentz condition. By analogy we write:

$$S \ = \ -\tfrac{1}{2}\left(\Phi \mid \mathcal{K}\Phi\right). \tag{20}$$

where:

$$\mathcal{K} \ = \ 2(L_0 - 1)P\ , \tag{21}$$

and P is the projector on states satisfying (18). This action has also been proposed by Kaku and Lykken.[7] It is invariant under the gauge transformations

$$\Delta\Phi = \sum L_{-n} C_n\ . \tag{22}$$

The projector P has been studied by Brower and Thorn and Feigin and Fuks[8] We found an infinite product representation of it which is useful for calculational purposes. We also showed that, when restricted to the massless levels of open and closed, bosonic and fermionic strings, our action reproduces the linearized actions and gauge transformations of Yang-Mills theory, general relativity, and supergravity. The spacetime fields arise as coefficients in the expansion of Φ in eigenmodes of L_0, as exemplified by the formula

$$\Phi[x(\sigma)] \;=\; \Big\{\phi(x) - iA^{\mu}(x)\alpha^{\mu}_{-1} - \tfrac{1}{2}\,h^{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1} - iv^{\mu}(x)\alpha^{\mu}_{-2} + \ldots\Big\}\Phi^{(0)} \quad (23)$$

x^{μ} is the center of mass coordinate of the string. As we reported at the Argonne conference[9] the action (20) becomes non-local at the first positive mass squared level. In 26 dimensions a formula for $2(L_0-1)P$ which is valid through level $L_0 - p^2/2 = 3$ (in units where the Regge slope is one half) is

$$\begin{aligned}
\mathcal{K} \;=\; & 2(L_0-1) - L_{-1}L_1 - \tfrac{1}{2}\,L_{-2}L_2 \\
& +\tfrac{1}{2}\,(3L_{-2}+2L^2_{-1})\frac{1}{8L_0+13}(3L_2+2L_1^2) \\
& -\frac{1}{3}L_{-3}L_3 \\
& -(3L_{-2}+2L^2_{-1})L_{-1}\frac{12L_0+37}{48(3L_0+7)(L_0+4)(8L_0+21)}L_1(3L_2+2L_1^2) \\
& -(3L_{-2}+2L^2_{-1})L_{-1}\frac{3}{48(3L_0+7)(L_0+4)}(8L_3+3L_1L_2) \;\; +h.c. \\
& +(8L_{-3}+3L_{-2}L_{-1})\frac{4L_0+7}{48(3L_0+7)(L_0+4)}(8L_3+3L_1L_2) \;\; +\ldots\,.
\end{aligned} \quad (24)$$

Although we have every reason to believe that string theories are indeed nonlocal when written as infinite component field theories in center of mass coordinates, the nonlocality of the *kinetic* term leads to incorrect counting of degrees of freedom at the quantum level. The obvious resolution of this problem is to introduce Stueckelberg[10] fields which make the action local. For example, nonlocality can be removed through the first positive mass squared level by introducing a single string field S with only a scalar component ($S = \int d^dx s(x)|0, x>$) and the action

$$\begin{aligned}
\mathcal{S} \;=\; -\tfrac{1}{2}\,\Big\{ & (\Phi \mid 2(L_0-1)\,\Phi) \;-\; (\mathcal{S} \mid 2(L_0+1)\,\mathcal{S}) \\
& - \big(L_1\Phi + L_{-1}\mathcal{S} \mid L_1\Phi + L_{-1}\mathcal{S}\big) - \tfrac{1}{2}\,\big(L_2\Phi + 3\mathcal{S} \mid L_2\Phi + 3\mathcal{S}\big)\Big\},
\end{aligned} \quad (25)$$

The problem now is to generalize (25) to all mass levels. Such a generalized Stueckelberg action should reduce to (20) when the Stueckelberg fields are integrated out. This will be true if it is gauge invariant and if the Stueckelberg fields decouple from Φ when $L_n\Phi = 0$. This is not sufficient however. The action should also have the same number of degrees of freedom as the light cone gauge action of Kaku and Kikkawa.[11] A direct proof of this latter property for the formalism we will present below will be given in Ref. 12. In our original paper we showed instead that our action is equivalent to the gauge fixed covariant action of Siegel. In Ref. 2, Siegel argued that his formalism was equivalent to the light cone gauge.

Siegel's dynamical variable is a functional field $\Phi[x(\sigma), \theta(\sigma), \widehat{\theta}(\sigma)]$ which depends on two Grassmann variables in addition to $x(\sigma)$. Equivalently, it is an infinite collection of functional fields:

$$\Phi[x, \theta, \widehat{\theta}] = \sum \Phi^{m_1 \ldots m_a}{}_{n_1 \ldots n_b} \theta_{m_1} \ldots \theta_{m_a} \widehat{\theta}_{n_1} \ldots \widehat{\theta}_{n_b} \tag{26}$$

antisymmetric separately in upper and lower indices.

The first idea that we had for reproducing this zoo of fields was based on the concept of ghosts for ghosts.[13] The gauge transformations

$$\Delta\Phi = L_{-n} C^n$$

are redundant in the sense that C^n and

$$C^n + (L_{-m} G^{mn} + \frac{1}{2} V^n_{mk} G^{mk}) \tag{27}$$

are the same transformation of Φ, if G^{mn} satisfies

$$G^{mn} = -G^{nm} \tag{28}$$

and

$$[L_m, L_n] = V^k_{mn} L_k \qquad m, n, k > 0 \ . \tag{29}$$

Consequently, the Faddeev-Popov ghost action will be invariant under the transformations (29) (where C is now interpreted as the Faddeev-Popov ghost field), and we will need ghosts for ghosts. The transformations parametrized by G are themselves redundant and the process continues indefinitely.

The resemblance of all of this to the theory of a p form gauge field motivated us to develop a theory of differential forms in the space of strings. Remarkably, that formalism also leads to a natural solution of the Stueckelberg field problem. We will refer the reader to our Nuclear Physics paper[1] for the details of all of this and content ourselves with recording a few of the relevant equations.

3. DIFFERENTIAL FORMS IN THE SPACE OF STRINGS

It is useful to decompose the commutation relations of the Virasoro algebra as :

$$\begin{aligned} [L_m, L_n] &= V_{mn}{}^p \, L_p \\ [L_{-m}, L_{-n}] &= -\, V_{mn}{}^p \, L_{-p} \\ [L_m, L_{-n}] &= W_{mn}{}^p \, L_p \; + \; W_{nm}{}^p \, L_{-p} \; + \; \eta_{mn}\mathbf{L}(m) \; , \end{aligned} \tag{30}$$

where m, n, p are positive integers. More explicitly,

$$\begin{aligned} V_{mn}{}^p &= \delta\big(p - (m+n)\big)(m-n) \\ W_{mn}{}^p &= \delta\big(m - (n+p)\big)(m+n) \\ \mathbf{L}(m) &= 2L_0 + \frac{13}{6}(m^2 - 1) \; . \end{aligned} \tag{31}$$

The last relations evaluated in $d = 26$. We now think of L_{-n} as covariant derivatives $\bigtriangledown_{z_n}$ on a complex manifold with coordinates $(z_n, \bar{z}_n)$ and L_n as the complex conjugate derivative. Formulas (30) are then statements about the torsion and curvature of the manifold.

An $\binom{a}{b}$-form on a complex manifold is a tensor field

$$\Phi^{m_1 \ldots m_a}{}_{n_1 \ldots n_b} \tag{32}$$

antisymmetric in upper (holomorphic) and lower (antiholomorphic) indices. By analogy with manifolds that have torsion, we can construct exterior derivatives d and $\delta^\star$ satisfying $d^2 = \delta^2 = 0$ from $\bigtriangledown_z$ and$\bigtriangledown_{\bar{z}}$. The formulas are:

$$\begin{aligned} (dC)^{[m_1 \ldots m_a]}{}_{[n_1 \ldots n_{b+1}]} &= L_{[n_1} \, C^{[m_1 \ldots m_a]}{}_{n_2 \ldots n_{b+1}]} + a \, W_{p[n_1}{}^{[m_1} \, C^{p m_2 \ldots m_a]}{}_{n_2 \ldots n_{b+1}]} \\ &\quad - \tfrac{1}{2} \, b \, V_{[n_1 n_2}{}^p \, C^{[m_1 \ldots m_a]}{}_{p n_3 \ldots n_{b+1}]} , \\ (\delta C)^{[m_1 \ldots m_{a-1}]}{}_{[n_1 \ldots n_b]} &= L_{-p} \, C^{[p m_1 \ldots m_{a-1}]}{}_{[n_1 \ldots n_{b+1}]} + b \, W_{[n_1 p}{}^q \, C^{[p m_1 \ldots m_{a-1}]}{}_{q n_2 \ldots n_b]} \\ &\quad - \tfrac{1}{2} \, (a-1) \, V_{kl}{}^{[m_1} \, C^{k l m_2 \ldots m_{a-1}]}{}_{[n_1 \ldots n_b]} . \end{aligned} \tag{33}$$

Here and henceforth, we make the convention that raised indices labeled as (m_i) are antisymmetrized together in the indicated order and lowered indices labeled as (n_i) are antisymmetrized together similarly.

⋆ Our definition of δ differs from the conventional one by a factor $(-1)^b$. Consequently a commutator appears in Eq. (36) instead of an anticommutator

d and δ are adjoints with respect to the scalar product between $\binom{a}{b}$-forms and $\binom{b}{a}$-forms, defined by combining the Hilbert space scalar product with contraction of holomorphic with antiholomorphic indices.

There are also two operations which change a holomorphic into an antiholomorphic index and vice versa:

$$
\begin{aligned}
\left(\Uparrow C\right)^{m_1\ldots m_{a+1}}{}_{n_1\ldots n_{b-1}} &= \eta^{m_1 q}\, C^{m_2\ldots m_{a+1}}{}_{q n_1\ldots n_{b-1}} \\
\left(\Downarrow C\right)^{m_1\ldots m_{a-1}}{}_{n_1\ldots n_{b+1}} &= \eta_{n_1 q}\, C^{q m_1\ldots m_{a-1}}{}_{n_2\ldots n_{b+1}}
\end{aligned}
\tag{34}
$$

$$
\eta_{mp} = p\delta_{mp} \qquad \eta^{mp}\eta_{pq} = \delta^m{}_q \ . \tag{35}
$$

By our convention, the (m_i) are antisymmetrized together, and the (n_i) are antisymmetrized together. $\Uparrow$ and $\Downarrow$ are self adjoint with respect to the scalar product that we have defined.

All of this formalism works for any spacetime dimension (any coefficient of the Schwinger term in the Virasoro algebra). The special role of $d = 26$ becomes apparent when we compute the Laplacian

$$
(d\delta - \delta d)C^{m_1\ldots m_{a-1}}{}_{n_1\ldots n_{b+1}} = \mathbf{K}\, \eta_{[n_1 p}\, C^{p m_1\ldots m_{a-1}}{}_{n_2\ldots n_{b+1}]}, \tag{36}
$$

where

$$
\mathbf{K} = 2(L_0 - 1 + (\text{sum of indices})) \tag{37}
$$

This formula for K is valid only in 26 dimensions. It has the consequence that K commutes with all the other operators in the theory and may be treated as a c-number.

Finally we note that $\Uparrow$ and $\Downarrow$ allow us to symmetrize and antisymmetrize upper with lower indices. Under the symmetric group S_{a+b} of all the indices, an $\binom{a}{b}$-form can be in any of the representations in $a\left\{\begin{smallmatrix}\square\\ \square\\ \square\end{smallmatrix}\times\begin{smallmatrix}\square\\ \square\\ \square\end{smallmatrix}\right\}b$. An $\binom{a}{b}$-form in the representation $k\left\{\begin{smallmatrix}\square\square\\ \square\square\\ \square\square\\ \square\\ \square\end{smallmatrix}\right\}\ell$ of S_{a+b} is said to be (k, ℓ) symmetrized.

The (ghosts)^k of Φ can now be seen to be $\binom{k}{0}$-forms (the (antighosts)^k are $\binom{0}{k}$-forms) while Siegel's action contains $\binom{a}{b}$-forms for all a and b. It is natural to suppose that some of these are the Stueckelberg fields for which we have been searching. Indeed, if we further guess that the kinetic term will be given by K, then a $\binom{1}{1}$-form Φ^m_n will first appear at the first positive mass squared level, just where we first needed a Stueckelberg field.

It is possible to write down an action which contains only this single new string field, and which reproduces Eq. (20) when Φ_n^m is integrated out. This action was described in our paper and has recently been derived in work by Neveu, Schwarz and West.[14] Unfortunately, it does not contain the right number of quantum degrees of freedom, and does not reproduce Siegel's action upon Faddeev-Popov quantization. The problem is that it no longer has redundant gauge symmetries. We found that in order to preserve the redundant gauge structure, we had to introduce $\binom{k}{k}$-forms with (k,k) symmetry for arbitrary integer k. In addition we had to introduce an infinite set of new gauge transformations C_{2k+1} which are $\binom{k+1}{k}$-forms with $(k+1,k)$ symmetry.

The action for this infinite system of fields is $(\Phi = \Phi_0)$

$$
\begin{aligned}
S \;=\; -\tfrac{1}{2} \Big\{ &(-1)^k \left(\Phi_{2k} \,\middle|\, \mathbf{K}\Phi_{2k}\right) \\
&- (-1)^k (k+1)^2 \left(d\Phi_{2k} + \delta\Phi_{2k+2} \,\middle|\Uparrow\, (d\Phi_{2k} + \delta\Phi_{2k+2})\right) \Big\}.
\end{aligned} \tag{38}
$$

The gauge transformations are:

$$
\delta_C\, \Phi_{2k} \;=\; -d\, C_{2k-1} \;+\; \delta\, C_{2k+1}, \tag{39}
$$

where C_{2k+1} is a $\binom{k+1}{k}$-form symmetrized according to $(k+1,k)$. Note that the right hand side must be explicitly (k,k) symmetrized. These gauge transformations are themselves invariant under the redundant transformations

$$
\delta_{\mathcal{G}}\, C_{2k+1} \;=\; d\, \mathcal{G}_{2k} \;+\; \delta\, \mathcal{G}_{2k+2}, \tag{40}
$$

where $\mathcal{G}_{2k+2}$ is a $\binom{k+2}{k}$-form symmetrized according to $(k+2,k)$. The $\mathcal{G}$ transformation law is in turn left invariant by transformations parametrized by $\binom{k+3}{k}$-forms C', symmetrized according to $(k+3,k)$, and so on. Each form that we have introduced has the total number of indices denoted by its subscript and cannot appear at a mass level lower than this number. Thus, at any given mass level, the proliferation of Stueckelberg fields and their successive gauge transformations eventually terminates. Indeed at any finite mass level, the infinite set of symmetrized Stueckelberg string fields, actually generates fewer spacetime fields than the single field Φ_2 without symmetrization. We believe that, in this sense, the Φ_{2k} are the minimal set of Stueckelberg fields for the string theory.

The action we have written has the form of a Feynman gauge kinetic term (each spacetime field has an action given by the appropriate Klein-Gordon operator) plus the square of a gauge fixing term:

$$
\mathcal{F}_{2k-1} \;=\; \left(d\Phi_{2k-2} + \delta\Phi_{2k}\right) . \tag{41}
$$

Thus we can define a gauge (which we call the Feynman-Siegel gauge) in which only the Klein-Gordon terms appear in the action. The Faddeev- Popov action in this gauge is:

$$
\begin{aligned}
S_C \;=(-1)^k \Bigg\{ & \left(\bar{C}_{2k-1} \,\middle|\, \mathbf{K} C_{2k-1}\right) \\
& - \frac{k(k+1)}{2} \left(\delta \bar{C}_{2k+1} - d\bar{C}_{2k-1} \,\middle|\Uparrow\, \left(\delta C_{2k+1} - dC_{2k-1}\right)\right) \Bigg\}.
\end{aligned}
\tag{42}
$$

where C and $\bar{C}$ are the ghost and antighost fields. This action is invariant to the redundant gauge symmetries:

$$
\delta_{\mathcal{G}}\, C_{2k+1} \;=\; d\,\mathcal{G}_{2k} \;+\; \delta\,\mathcal{G}_{2k+2}, \tag{43}
$$

where $\mathcal{G}_{2k+2}$ is a $\binom{k+2}{k}$-form symmetrized according to $(k+2, k)$.

The ghost action has the Feynman-Siegel gauge form plus the square of a gauge fixing term for these redundant transformations. Thus we may fix the gauge and obtain ghosts of ghosts which are $\binom{k}{k+2}$-forms and $\binom{k+2}{k}$-forms with symmetry $(k+2, k)$. We must however be careful to remember the phenomenon of "hidden ghosts"[13], which occurs for any system with redundant gauge transformations. There are only three ghosts of ghosts, rather than the four one might have naively expected. Similarly, when we find that the ghost of ghost action has a gauge invariance, we only need four $(\text{ghosts})^3$ rather than eight. In general we need $p+1$ $(\text{ghosts})^p$.

The systematics of ghost counting is as follows. To each Stueckelberg field Φ_{2k} we associate the gauge transformation C_{2k+1} under which the field transforms as a divergence δ. The ghosts associated with this transformation and its redundancies are called $(\text{ghosts})^p$ of Φ_{2k}. There are $p+1$ $(\text{ghosts})^p$ and they are forms with the symmetry $(k+p, k)$. Table 1 shows how the ghost counting works up to forms with $a+b=4$. The pattern is clear: Φ_0, the Stueckelberg fields, and their descendant ghosts fill out all possible $\binom{a}{b}$-forms with general symmetry. This is precisely the field content of Siegel's action. The gauge fixed action is precisely of the Feynman-Siegel gauge form: each space time field will have a simple Klein-Gordon action.

The formalism we have presented can be simply generalized to closed strings and to open and closed superstrings. The correct treatment of spacetime supersymmetry is not at all straightforward, and is presently under study.[15]

TABLE 1

Number of Indices k	$\Phi_{(2k)}$ fields and their ghosts	Forms with k indices
0	Φ_0	$\binom{0}{0}$
1	$2\ Ghosts\ of\ \Phi_0 = 2 \times \ydiagram{1}$	$\binom{0}{1}\ (= \ydiagram{1}) + \binom{1}{0}\ (= \ydiagram{1})$
2	$3(Ghosts)^2\ of\ \Phi_0 = 3 \times \ydiagram{1,1}$ $\Phi_2 = \ydiagram{2}$	$\binom{0}{2}\ (= \ydiagram{1,1}) + \binom{2}{0}\ (= \ydiagram{1,1}) + \binom{1}{1}\ (= \ydiagram{1,1} + \ydiagram{2})$
3	$4(Ghost)^3\ of\ \Phi_0 = 4 \times \ydiagram{1,1,1}$ $2\ Ghosts\ of\ \Phi_2 = 2 \times \ydiagram{2,1}$	$\binom{0}{3}\ (= \ydiagram{1,1,1}) + \binom{3}{0}\ (= \ydiagram{1,1,1}) + \binom{2}{1}\ (= \ydiagram{1,1,1} + \ydiagram{2,1})$ $+ \binom{1}{2}\ (= \ydiagram{1,1,1} + \ydiagram{2,1})$
4	$5(Ghosts)^4\ of\ \Phi_0 = 5 \times \ydiagram{1,1,1,1}$ $3(Ghost)^2\ of\ \Phi_2 = 3 \times \ydiagram{2,1,1}$ $\Phi_4 = \ydiagram{2,2}$	$\binom{0}{4}\ (= \ydiagram{1,1,1,1}) + \binom{4}{0}\ (= \ydiagram{1,1,1,1}) + \binom{3}{1}\ (= \ydiagram{1,1,1,1} + \ydiagram{2,1,1})$ $+ \binom{1}{3}\ (= \ydiagram{1,1,1,1} + \ydiagram{2,1,1}) + \binom{2}{2}\ (= \ydiagram{1,1,1,1} + \ydiagram{2,1,1} + \ydiagram{2,2})$

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