SLAC – PUB – 3801
October 1985
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Equivalence of the Light-Cone Formulation and the Gauge-Invariant Formulation of String Dynamics

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ABSTRACT

We show how to fix the newly discovered gauge-invariant string field theory to the light-cone gauge. We prove that this procedure leads to the well-known light-cone formulation of string field theory, with no additional propagating degrees of freedom.

Submitted to *Nuclear Physics B*

* Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.
† Work supported in part by the Department of Energy, contract DE–AS05–81ER40008.

1. Introduction

The internal consistency of dual string models depends on the scattering amplitudes satisfying a huge class of Ward-like identities[1] which can be expressed as the decoupling from any physical scattering process of on-mass-shell single string states of the form $L_{-n}|\Psi\rangle$, $n > 0$, where the L_n are the generators of the Virasoro algebra. For example, the no-ghost theorem shows that this decoupling mechanism removes all negative-norm single-string states from physical processes if space-time has dimension $d \leq 26$[2,3] . For the massless vector and tensor states, these Ward-like identities (which involve only L_{-1}) give precisely the restrictions on S-matrix elements which guarantee the proper Lorentz transformation properties for massless spin-1 and spin-2 particles. The above decoupling mechanism generalizes to the fermionic string[4–8] , with the L_{-n} replaced by their appropriate graded extension.

At the tree level, decoupling of the states created by the L_{-n} is a consequence of the (super)conformal invariance of world-sheet dynamics. At the one loop level, the Ward-like identities may be anomalous, as is true in the chiral superstring[9,10] unless the gauge group is $SO(32)$, Spin$(32)/Z_2$, or $E_8 \times E_8$[11,12] . It is interesting to note that these anomalies break the $SO(2,1)$ subalgebra which is associated with one of the most fundamental properties of string models—duality[13] .

As long as one limits consideration to S-matrix elements, the on-mass-shell decoupling of states is an adequate statement of the underlying invariances necessary for the consistency of the theory. However, this is not sufficient for formulating an action principle for string dynamics. To date, the only action principle which has been formulated for interacting strings is the one based on light-cone dynamics[14,15] and re-expressed in terms of a string field theory through the machinery of second quantization[16] ; this formalism makes use of a preferred frame and a fixed gauge. But for the free string theory, the problem of formulating a Lorentz-covariant, gauge-invariant action principle has recently been solved. The zero-slope analysis of the open and closed string theories[17,18] suggested that one should search for exact off-shell gauge invariances. An important step toward the realizaton of these invariances was the development of a manifestly Lorentz-covariant, though still gauge-fixed, action principle[19] based on the BRST (first) quantization of the string[20,21] . Later, off-mass-shell gauge transformation laws were postulated[22,23] and several groups constructed free-string Lagragians with these symmetries as invariances[23,24] . In particular, three groups[22,25,26] succeeded in constructing a linearized local string field theory from which the formulation of ref. 19 could be recovered by gauge-fixing.

In this article, we will show how the gauge-invariant formulation of refs. 22, 25, 26 reduces upon suitable gauge-fixing to the light-cone formulation of refs. 14, 15, 16. We believe it is important to understand this question, because the most pressing problem for the gauge-invariant formulation—the construction of the interaction terms—has already been solved in the light-cone formulation[15] . It is likely that an understanding of the precise relation of these two formulations will offer insight toward the construction of a gauge-invariant interaction.

The reduction of the gauge-invariant formulation to the light-cone gauge is not entirely straightforward, since the proper local gauge-invariant string action contains new degrees of freedom, beyond those supplied by the string modes of oscillation, and additional gauge transformations which act only on these new states. This means that the argument for the sufficiency of the transverse light-cone states[3] , even as recently recast[27] as a gauge-fixing prescription for an earlier, nonlocal form of the action[22,23] , needs to be re-examined and perhaps modified in this new context. From the point of view of counting propagating degrees of freedom, the covariant formulation has already been shown to agree with the light-cone formulation[28] . What we will do here is explain the explicit manner in which the reduction to the light-cone gauge is achieved.

We will, in fact, present two different arguments which connect the two formulations. The first, presented in Section 3, generalizes the argument of ref. 27 in proving that all solutions to the equations of motion of the gauge-invariant free string theory are gauge-equivalent to transverse oscillations of strings. This establishes the connection even at the quantum level, though by a somwhat indirect argument. The second, presented in Section 4, proceeds by a direct and brutal gauge reduction of the action.

We will present our arguments in a unified notation which makes clear that they apply to open and closed bosonic strings, and to open and closed fermionic strings in the Neveu-Schwarz-Ramond formulation. This notation, which was introduced in ref. 22, is reviewed in Section 2. Whenever a formula must be written explicitly, we will use, for simplicity, the example of the open bosonic string.

2. Formal Preliminaries

In this section, we will review the structure of the gauge-invariant free string theory to the extent necessary for our analysis. We will use the notation of ref. 22; the reader should note that the free-string actions of refs. 22, 25, and 26, while somewhat different in appearance, are completely equivalent. We will also set up our conventions for the light-cone formalism.

Let us first review the gauge-invariant action for the open bosonic string, in the formulation of ref. 22. The basic objects used in this formulation are string differential forms. The simplest string form is a string field, a general functional of a string position $x(\sigma)$. Such a functional may be expanded in eigenstates of the single-string Hilbert space. Let α_n^μ denote creation and annihilation operators for string modes:

$$[\alpha_n^\mu, \alpha_m^\nu] \;=\; n\,\eta^{\mu\nu}\,\delta(n+m) \tag{2.1}$$

(In general, positive (negative) indices will denote annihilation (creation) operators.) Let $\Phi^{(0)}$ denote the single-string vacuum, the state annihilated by α_n for

$n > 0$. Then we may expand:

$$\Phi[x(\sigma)] \;=\; \Big\{\phi(x) - iA_\mu(x)\alpha^\mu_{-1} - \frac{1}{2}h_{\mu\nu}(x)\alpha^\mu_{-1}\alpha^\nu_{-1} - iv_\mu(x)\alpha^\mu_{-2} + \dots\Big\}\Phi^{(0)} \quad (2.2)$$

Notice that the center-of-mass coordinate x^μ is retained in the coefficient functions; thus, these coefficient functions are local fields of increasing spin.

String differential forms are string fields which, additionally, carry two sets of indices (upper and lower). These indices take values which run over positive integers; these are in 1-to-1 correspondence with the annihilation operator generators of the Virasoro algebra: $\{L_n\colon\ n > 0\}$. Each set of indices is completely antisymmetrized. Indices are raised and lowered with the metric

$$\eta_{mn} \;=\; m\delta_{mn}\,. \quad (2.3)$$

We denote a form with a upper and b lower indices as an $\binom{a}{b}$-form. Exterior differentiation is defined on these forms as follows: View the Virasoro operators as the basic differential operators on the space of strings. Write the commutation relations of the Virasoro algebra symbolically in the form

$$\begin{aligned}
[L_m, L_n] \;&= V_{mn}{}^p\, L_p \\
[L_{-m}, L_{-n}] \;&= \;-\,V_{mn}{}^p\, L_{-p} \\
[L_m, L_{-n}] \;&= W_{mn}{}^p\, L_p \;+\; W_{nm}{}^p\, L_{-p} \;+\; \eta_{mn}\mathbf{L}(m)\ ,
\end{aligned} \quad (2.4)$$

where m, n, p are positive integers. Then for C an $\binom{a}{b}$-form, define dC as the operation of differentiating by L_n, appropriately covariantized, and δC as the operation of differentiating with respect to L_{-n}:

$$(dC)^{[m_1\dots m_a]}{}_{[n_1\dots n_{b+1}]} = L_{[n_1}\, C^{[m_1\dots m_a]}{}_{n_2\dots n_{b+1}]} + a\, W_{p[n_1}{}^{[m_1}\, C^{pm_2\dots m_a]}{}_{n_2\dots n_{b+1}]}$$
$$- \frac{1}{2} b\, V_{[n_1 n_2}{}^{p}\, C^{[m_1\dots m_a]}{}_{pn_3\dots n_{b+1}]},$$
$$(\delta C)^{[m_1\dots m_{a-1}]}{}_{[n_1\dots n_b]} = L_{-p}\, C^{[pm_1\dots m_{a-1}]}{}_{[n_1\dots n_{b+1}]} + b\, W_{[n_1 p}{}^{q}\, C^{[pm_1\dots m_{a-1}]}{}_{qn_2\dots n_b]}$$
$$- \frac{1}{2}(a-1)\, V_{kl}{}^{[m_1}\, C^{klm_2\dots m_{a-1}]}{}_{[n_1\dots n_b]}. \tag{2.5}$$

The inner product of two forms may be defined to include the contraction of the upper indices of the first with the lower indices of the second. Then, an $\binom{a}{b}$-form C_A would have a nonzero inner product with a $\binom{b}{a}$-form C_B. With this definition, d is the adjoint of δ:

$$\big(dC_A \mid C_B\big) \;=\; \big(C_A \mid \delta C_B\big). \tag{2.6}$$

Let us define three auxiliary operations, raising and lowering operators $\Uparrow$ and $\Downarrow$ which preserve the separate antisymmetry of raised and lowered indices

$$\Big(\Uparrow C\Big)^{m_1\dots m_{a+1}}{}_{n_1\dots n_{b-1}} \;=\; \eta^{[m_1 q}\, C^{m_2\dots m_{a+1}]}{}_{qn_1\dots n_{b-1}}$$
$$\Big(\Downarrow C\Big)^{m_1\dots m_{a-1}}{}_{n_1\dots n_{b+1}} \;=\; \eta_{[n_1 q}\, C^{qm_1\dots m_{a-1}}{}_{n_2\dots n_{b+1}]}, \tag{2.7}$$

and a generalized kinetic energy operator

$$\mathbf{K} = 2(L_0 - 1 + (\text{sum of indices})), \tag{2.8}$$

which, in the free string theory of ref. 22, equals the free-field action $p^2 + \mathcal{M}^2$ on each string mode. Using these operations, we can write the algebra of d and δ:

$$d^2 C \;=\; 0\,, \quad \delta^2 C \;=\; 0, \tag{2.9}$$

$$d\,\delta \;-\; \delta\, d \;=\; \mathbf{K} \Downarrow\,. \tag{2.10}$$

For the bosonic string, eq. (2.10) is true only in 26 dimensions. The exterior derivatives and auxiliary operators satisfy a number of other useful identities. We will require, in particular, the relations

$$\{\Downarrow, d\} \;=\; \{\Downarrow, \delta\} \;=\; 0, \tag{2.11}$$

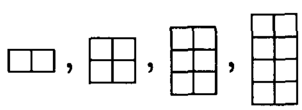
and, if C is an $\binom{a}{b}$-form,

$$a(b+1) \Uparrow\Downarrow C \;-\; b(a+1) \Downarrow\Uparrow C \;=\; (a-b)\, C\ . \tag{2.12}$$

Some further identities are given in Section 5 of ref. 22.

Our ability to raise and lower indices with the metric (2.3) allows us to bring indices to the same level and then to symmetrize or antisymmetrize them. It is, in fact, useful to consider $\binom{a}{b}$-forms of definite symmetry under general permutations of their $a+b$ indices. The symmetry property may be represented by a Young tableau with $a+b$ boxes. Since the upper and lower indices are each completely antisymmetrized among themselves, the full permutation symmetry must be given by a Young tableau with no more than two columns. We will refer to such a tableau, with columns of length k, ℓ, by writing (k, ℓ); the notation is illustrated in Fig. 1. (Also shown there is a convenient notation for some other tableaux which will arise in the course of our discussion.) An $\binom{a}{b}$-form, with $a > b$, may then be symmetrized according to (a, b), $(a+1, b-1)$, ..., $(a+b, 0)$. Of these, the structure (a, b) with the longest possible second column will play a special role. Let us refer to an $\binom{a}{b}$-form with such a symmetry as *maximally symmetrized.*

The free-string action of ref. 22 is constructed from a string field Φ_0 (a $\binom{0}{0}$-form) and a sequence $\{\Phi_{2k}\}$ of maximally symmetrized $\binom{k}{k}$-forms. Thus, the basic fields are string forms symmetrized according to

1 ,  , ... (2.13)

In terms of these fields, the action takes the form

$$S = -\frac{1}{2}\Big\{(-1)^k \left(\Phi_{2k} \mid \mathbf{K}\Phi_{2k}\right) - (-1)^k (k+1)^2 \left(d\Phi_{2k} + \delta\Phi_{2k+2} \,|\!\Uparrow\, (d\Phi_{2k} + \delta\Phi_{2k+2})\right)\Big\}. \tag{2.14}$$

The action (2.14) is constructed to be gauge-invariant under the transformations

$$\delta_C\,\Phi_{2k} = -d\,C_{2k-1} + \delta\,C_{2k+1}, \tag{2.15}$$

where C_{2k+1} is a $\binom{k+1}{k}$-form symmetrized according to $(k+1,k)$, and the right-hand side should be understood to be symmetrized according to (k,k). It is important to note that some of the transformations (2.15) are redundant, since the right-hand side of this equation is invariant to the second-level transformation

$$\delta_{\mathcal{G}}\,C_{2k+1} = d\,\mathcal{G}_{2k} + \delta\,\mathcal{G}_{2k+2}, \tag{2.16}$$

where $\mathcal{G}_{2k+2}$ is a $\binom{k+2}{k}$-form symmetrized according to $(k+2,k)$. The $\mathcal{G}$ transformation law is in turn left invariant by transformations parametrized by $\binom{k+3}{k}$-forms C', symmetrized according to $(k+3,k)$, and so on.

This formalism generalizes straightforwardly to other string theories[29]. For closed strings, one must extend the range of the indices of string forms to run over the positive integers for both the left- and right-moving Virasoro operators, and then project onto states such that $\mathbf{K}_L = \mathbf{K}_R$⋆. For fermionic strings, one must extend the range of the indices to run over the positive-index generators of the Neveu-Schwarz and Ramond algebras. It is also necessary to convert Young symmetrization to graded Young symmetrization, in which one supplies an extra (-1) for each interchange of indices corresponding to anticommuting generators,

⋆ The free closed-string field theory was derived earlier, in a slightly different form, in ref. 25.

and to change the dimensionality of space-time to 10. Both of these modifications produce definitions of d and δ which satisfy the identities (2.9) and (2.10). Thus, the action (2.14), with the new definitions of forms and of d and δ, is again a gauge-invariant free string action. The arguments we will give to fix the light-cone gauge in the case of the bosonic string go through without change for these other string theories.

The main objective of this paper will be to show how to fix the gauge freedom just described to cast the action (2.14) into the light-cone gauge. In preparation for that study, we should set out explicitly our notation for light cone gauge dynamics. First of all, we will use the metric

$$x \cdot y \;=\; \vec{x} \cdot \vec{y} \;-\; x^0 y^0 \tag{2.17}$$

In the light-cone quantization, we regard

$$x^+ \;=\; \frac{1}{\sqrt{2}}\,(x^0 + x^1) \tag{2.18}$$

as the evolution parameter. The variable conjugate to x^+ is

$$p^- \;=\; \frac{1}{\sqrt{2}}\,(p^0 - p^1) \;\Leftrightarrow\; i\,\partial_+ \;=\; i\frac{\partial}{\partial x^+}. \tag{2.19}$$

x^-, p^+, and ∂_- are purely kinematical quantities. Transverse directions are indexed by $i = 2, \ldots, d-1$.

In a gauge theory of ordinary local fields, one fixes the light-cone gauge by setting to zero the lowered $(-)$ components of tensors. In QED, for example, where the basic field is A_μ, one places the gauge condition $A_- = 0$. Then A_+ may be eliminated by kinematical equations, leaving the transverse components of A as the dynamical fields. In string field theory, local tensor fields appear as component fields in the expansion (2.2). Since a typical term in this expansion has the structure

$$\left\{ A_{\mu_1 \mu_2 \ldots \mu_b} \, \alpha^{\mu_1}_{-n_1} \cdots \alpha^{\mu_b}_{-n_b} \right\} \Phi^{(0)} \, , \tag{2.20}$$

fields with $-$ $(+)$ indices multiply operators α^{-}_{-n} (α^{+}_{-n}). Let us, then define

$$K_n \;=\; \alpha^{+}_{n} \, , \qquad M_n \;=\; \alpha^{-}_{n} \, . \tag{2.21}$$

These operators obey the algebra

$$[K_n, M_m] \;=\; -n \, \delta(n+m). \tag{2.22}$$

The direct analogue of the light-cone gauge condition for string fields would be that no terms appear in $\Phi[x(\sigma)]$ in which an M_n acts on $\Phi^{(0)}$. The condition is equivalent to

$$K_n \, \Phi \;=\; 0. \tag{2.23}$$

The analogue of the statement that only the transversely polarized states propagate is the statement that the propagating fields in the string action belong to the subspace $\bar{\mathcal{T}}$ defined by

$$\bar{\mathcal{T}} \;=\; \left\{ \Phi \, : \, K_n \Phi \;=\; M_n \Phi \;=\; 0 \, , \; n > 0 \right\}. \tag{2.24}$$

We will show in Section 4 that, by imposing a gauge condition on the full set of fields $\{\Phi_{2k}\}$ which is slightly weaker than (2.23), we can show that only components of Φ_0 belonging to $\bar{\mathcal{T}}$ are propagating states. This accords with the formalism of refs. 14 and 16.

In Section 3, we will make use of a slightly different characterization of the transversely-polarized states which is more naturally related to the d and δ operations. Following ref. 3, let us define the subspace $\mathcal{T}$ of the single-string Hilbert space:

$$\mathcal{T} = \left\{ \Phi : K_n \Phi = L_n \Phi = 0\,,\; n > 0 \right\}, \tag{2.25}$$

where the L_n are the Virasoro operators. The subspace $\mathcal{T}$ is isomorphic to the subspace $\bar{\mathcal{T}}$, as can be easily recognized by observing that $\mathcal{T}$ is generated from the Fock vacuum by the DDF operators[30] A_n^i. These operators obey an algebra isomorphic to that of the α_n^i (eq. (2.1)) corresponding to transverse directions in space-time. We will make repeated use of the basis for the single-string Hilbert space given by the vectors

$$L_{-1}^{\lambda_1} L_{-2}^{\lambda_2} \dots L_{-n}^{\lambda_n} K_{-1}^{\mu_1} K_{-2}^{\mu_2} \dots K_{-n}^{\mu_n} \big|\tau\big) \;, \tag{2.26}$$

where the vectors $\big|\tau\big)$ form an orthonormal basis for $\mathcal{T}$. For any $p^+ \neq 0$, the states (2.26) are linearly independent for arbitrary complex values of p^- [3,31]. It is of course well known that light-cone dynamics is singular on states for which $p^+ = 0$; we will, then, restrict our attention to states with $p^+ \neq 0$. With this restriction, the vectors (2.26) span the single-string Hilbert space. Because L_n and K_n $(n > 0)$ annihilate $\big|\tau\big)$, the action of the L's and K's on the states (2.26) is completely determined by their algebra

$$[L_n, K_m] = -mK_{n+m}\,, \quad [K_n, K_m] = 0 \;, \tag{2.27}$$

and eq. (2.4).

3. Equivalence via the Equations of Motion

In this section, we will establish the equivalence of the gauge-invariant and light-cone formulations of the free string by making use of the equations of motion of the gauge-invariant theory. Our method will be to extend the arguments of ref. 27 to prove that *all* solutions to the equations of motion of the gauge-invariant free string theory are gauge-equivalent to the solutions for which Φ_0 is in $\mathcal{T}$ and the higher Φ_{2k} vanish. First, though, we will explain why this statement suffices to prove the equivalence for the full quantum theory.

We begin by reviewing the conclusion of ref. 27 that the gauge-invariant Lagrangian reduces to the correct form when restricted to states in $\mathcal{T}$. Since the components T_0 of Φ_0 lying in $\mathcal{T}$ are annihilated by L_n for $n > 0$, they decouple from the higher Φ_{2k}'s; their contribution to the action is then just

$$\begin{aligned} S_{LC} &= -\frac{1}{2}\left(T_0 \mid L_0 - 1 \mid T_0\right) \\ &= -\frac{1}{2}\left(T_0 \mid (p^i)^2 - 2p^+p^- + \sum_{n=1}^{\infty} \alpha^i_{-n}\alpha^i_n \mid T_0\right). \end{aligned} \tag{3.1}$$

Since $|T_0)$ is also annihilated by K_n for $n > 0$, the α^i_n in (3.1) may be replaced by the DDF operators A^i_n. Since the algebra of the A^i_n is isomorphic to that of the α^i_n, eq. (3.1) takes exactly the form of the light-cone gauge action of ref. 16.

Now let us discuss the utility of identifying the solutions to the equations of motion. Write

$$S = S_{LC} + \hat{S}\,, \tag{3.2}$$

where $\hat{S}$ involves only the fields which must eventually be eliminated. Call these $\hat{\Phi}_{2k}$; these include all components of Φ_{2k} for $k > 0$ and all components of Φ_0 which are elements of the subspace spanned by the vectors (2.26) with $\sum_k k\lambda_k +$

$\sum_k k\mu_k > 0$. From the gauge transformation law for Φ_0

$$\delta_C \Phi_0 \;=\; L_{-p}\, C^p\,, \tag{3.3}$$

we see that all components of Φ_0 with $\sum_k k\lambda_k > 0$ may be set to zero by a gauge transformation, and that this completely fixes the gauge for Φ_0. (Note that the T_0 component of Φ_0 in the decomposition with respect to (2.26) is gauge invariant.)

$\hat{S}$ is a bilinear form in the fields $\hat{\Phi}_{2k}$. We will show in a moment that, for any fixed complex value of p^-, the only zero eigenvectors of this bilinear form are pure gauge degrees of freedom. This means that after we fix the remaining gauge invariance, there are no zero eigenvectors of the bilinear form for any *complex* value of p^-. The bilinear form $\hat{S}$ has no coupling between components with different eigenvalues of $\mathbf{K}$ and hence can be broken into a sum of finite bilinear forms, each at most quadratic in p^-:

$$\hat{S} \;=\; \sum_{p^-} \sum_{\ell=0}^{\infty} \big(\hat{\Phi}^{(\ell)} \mid M^{(\ell)}(p^-) \mid \hat{\Phi}^{(\ell)}\big). \tag{3.4}$$

Each $M^{(\ell)}$ is a quadratic polynomial in p^- with no zero eigenvectors for any complex p^-. Integrating out the $\hat{\Phi}^{(\ell)}$ fields yields a factor

$$\prod_{p^-} \prod_{\ell=0}^{\infty} \det{}^{-\frac{1}{2}} M^{(\ell)}(p^-)\ . \tag{3.5}$$

Since $\det M^{(\ell)}(p^-)$ is a polynomial in p^- with no zeroes, it must be the trivial polynomial, *i.e.*

$$\prod_{\ell=0}^{\infty} \det{}^{-\frac{1}{2}} M^{(\ell)}(p^-) \;=\; \prod_{\ell=0}^{\infty} \det{}^{-\frac{1}{2}} M^{(\ell)}(0)\ . \tag{3.6}$$

This establishes the nondynamical nature of integrating out the redundant fields.

Then

$$\int D\Phi_{2k}\,\delta_{G.F.}e^{-S} \;=\; \mathbf{N}\cdot\int DT_0\,e^{-S_{LC}}\,, \tag{3.7}$$

where $\mathbf{N}$ is a normalization factor, the product of the factors (3.6). This is a statement of the equivalence we sought.

Now, we return to the proof of our assertion that the bilinear form $\hat{S}$ has no zero eigenvectors after gauge fixing. The problem of finding zero eigenvectors of $\hat{S}$ is just the problem of finding the solutions of the classical equations of motion for the fields $\hat{\Phi}$. The classical equations of motion are just

$$\left[\mathbf{K}-(k+1)^2\delta\Uparrow d+k^2 d\Uparrow\delta\right]\big|\Phi_{2k}\big) \;=\; (k+1)^2\delta\Uparrow\delta\,\big|\Phi_{2k+2}\big)\;-\;k^2 d\Uparrow d\,\big|\Phi_{2k-2}\big)\,. \tag{3.8}$$

Our analysis of these equations will make essential use of the property that the space of free string fields possesses trivial cohomology, in particular, that the statement $\delta\mathcal{C}=0$ implies that there exists a $\mathcal{D}$ such that $\delta\mathcal{D}=\mathcal{C}$. Were it not for the structure constant terms in the definition of δ, this latter statement would be obvious. We have proved it for the simplest case that $\mathcal{C}$ is a $\binom{1}{0}$-form; this will suffice to prove that Φ_0 is restricted to $\mathcal{T}$. To show that the higher Φ_{2k}'s are pure gauges, one needs this property also for $\binom{k}{k-1}$-forms and for $\binom{k}{k}$-forms, for arbitrary k. We believe that all free string forms have trivial cohomology, but we do not yet have a proof.

Begin with the equation for Φ_0,

$$\mathbf{K}\,\Phi_0 \;=\; \delta\Uparrow d\,\Phi_0 \;+\; \delta\Uparrow\delta\,\Phi_2\,. \tag{3.9}$$

After using the gauge freedom to set all components of Φ_0 with $\sum_k k\lambda_k > 0$ to zero, (3.9) becomes a set of two equations, since then the left-hand side of (3.9) has $\sum_k k\lambda_k = 0$ while the right-hand side has $\sum_k k\lambda_k > 0$. Thus,

$$\mathbf{K}\Phi_0 = 0 , \tag{3.10}$$

$$\delta \Uparrow d\,\Phi_0 + \delta \Uparrow \delta\,\Phi_2 = 0 . \tag{3.11}$$

The components T_0 appear only in (3.10) (since $dT_0 = 0$); they do not appear either in (3.11) or in the equations (3.9) for $k > 0$.

A zero eigenvector of $\hat{S}$ must therefore be a non-trivial solution of (3.11) and the $k > 0$ components of (3.8). Let us first explore the constraint of (3.11). This equation implies

$$\Uparrow d\Phi_0 + \Uparrow \delta\Phi_2 = \delta E_2 , \tag{3.12}$$

where E_2 is a 2-form with 2 upper indices. Applying $\Downarrow$ to this equation and using the identities (2.11), (2.12) yields

$$d\,\Phi_0 + \delta\left(\Phi_2 + \Downarrow E_2\right) = 0 . \tag{3.13}$$

Call $(\Phi_2 + \Downarrow E_2) = \Lambda_2$ (a $\binom{1}{1}$-form), and consider (3.13) in component form:

$$L_n\Phi_0 + \sum_{p=1}^{\infty} L_{-p}\Lambda^p{}_n + \sum_{p=1}^{n-1} (n+p)\Lambda^p{}_{n-p} = 0 , \tag{3.14}$$

for $n > 1$, and

$$L_1\Phi_0 + \sum_{p=1}^{\infty} L_{-p}\Lambda^p{}_1 = 0 . \tag{3.15}$$

Since $L_1\Phi_0$ has $\sum_k k\lambda_k = 0$, (3.15) implies

$$L_1\Phi_0 = 0 \tag{3.16}$$

and

$$\sum_{p=1}^{\infty} L_{-p}\Lambda^{p}{}_{1} \;=\; 0\,. \tag{3.17}$$

Regarding $\Lambda^{p}{}_{1}$ as the p component of a 1 form, $\mathcal{C}^p$, eq. (3.17) is just

$$\delta\,\mathcal{C} \;=\; 0, \tag{3.18}$$

which implies that there is a $\mathcal{G}_2$ such that

$$\Lambda^{p}{}_{1} \;=\; \mathcal{C}^p \;=\; (\delta\,\mathcal{G}_2)^p \;=\; \sum_q L_{-q}\mathcal{G}^{qp} \;-\; \frac{1}{2}\sum_{k<p}(2k-p)\,\mathcal{G}^{k,p-k}\,. \tag{3.19}$$

The $p = 1$ component of (3.19) is just

$$\Lambda^{1}{}_{1} \;=\; \sum_q L_{-q}\mathcal{G}^{q1}\;, \tag{3.20}$$

so the $n = 2$ component of (3.14) is just

$$\begin{aligned} 0 \;&=\; L_2\Phi_0 \;+\; \sum_{p=1}^{\infty} L_{-p}\Lambda^{p}{}_{2} \;+\; 3\Lambda^{1}{}_{1} \\ &=\; L_2\Phi_0 \;+\; \sum_{p=1}^{\infty} L_{-p}\big(\Lambda^{p}{}_{2} \;+\; 3\mathcal{G}^{p1}\big)\,. \end{aligned} \tag{3.21}$$

Since $L_2\Phi_0$ has $\sum_k k\lambda_k = 0$, (3.21) implies

$$L_2\Phi_0 \;=\; 0\;, \tag{3.22}$$

$$\sum_{p=1}^{\infty} L_{-p}\big(\Lambda^{p}{}_{2} \;+\; 3\mathcal{G}^{p1}\big) \;=\; 0\,. \tag{3.23}$$

The pair (3.16), (3.22) imply $L_n\Phi_0 = 0$. In other words, $\Phi_0 = T_0$, or $\hat{\Phi}_0 = 0$.

Plugging this conclusion into (3.11) shows that

$$\delta \Uparrow \delta \, \Phi_2 \; = \; 0 \; . \tag{3.24}$$

To proceed, we need the following relation:

$$\delta \Uparrow \delta \, \Phi_{2k} \; = \; 0 \quad \text{implies} \quad \Phi_{2k} \; = \; \left[\delta C \right]_{(k,k)} \; , \tag{3.25}$$

for some $\binom{k+1}{k}$-form C, where the bracket denote symmetrization corresponding to the Young tableau (k,k) (in the notation of Fig. 1). We shall prove this relation in a moment. First, though, note that (3.25), together with (3.24), implies that

$$\Phi_2 \; = \; \left[\delta C_3 \right]_{(1,1)} \; , \tag{3.26}$$

This means that Φ_2 is a pure gauge; it can be set to zero by a gauge transformation (2.15). Once this choice is made, the $k = 1$ component of (3.8) reads

$$\delta \Uparrow \delta \, \Phi_4 \; = \; 0 \; . \tag{3.27}$$

(3.25) then implies that Φ_4 also may be gauged to zero. Continuing in this way we learn that all of the Φ_{2k} with $k > 0$ may be gauged away . Thus after gauge fixing there are no zero eigenvectors of the bilinear form $\hat{S}$ for any complex p^-.

Finally, we return to the proof of the relation (3.25). $\delta \Uparrow \delta\Phi_{2k} = 0$ implies that there exists a $\binom{k+1}{k-1}$-form E such that

$$\Uparrow \delta \, \Phi_{2k} \; = \; \delta \, E \; . \tag{3.28}$$

Applying $\Downarrow$ to (3.28) and using (2.11), (2.12), and that fact that $\Downarrow$ annihilates

Φ_{2k}, we find

$$k^2 \delta \Phi + \delta \Downarrow E = 0 . \tag{3.29}$$

This relation implies that there exists a $\binom{k+1}{k}$-form F such that

$$k^2 \Phi + \Downarrow E = \delta F . \tag{3.30}$$

But consider the Young symmetrization of the terms in (3.30). Φ_{2k} has the symmetry (k,k). However, $\Downarrow E$ cannot have this maximal symmetry. If we symmetrize (3.30) according to (k,k), the $\Downarrow E$ term drops out and we obtain the desired conclusion

$$\Phi_{2k} = \frac{1}{k^2} \left[\delta F\right]_{(k,k)} . \tag{3.31}$$

This completes our analysis of the equations of motion of the free string theory. Two points are worth reiterating. First, the analysis does depend on our unproved assertion that the space of free strings has trivial cohomology. Second, modulo this first objection, our analysis and conclusions apply for all complex p^-. This allows us to extend a classical analysis of the equations of motion to the fully quantum statement (3.7): integrating out the redundant fields yields the light-cone formulation of the theory, corrected only by *non-dynamical* determinants.

4. Direct Descent to the Light-Cone Gauge

Now let us turn to a second line of argumentation, one somewhat closer in spirit to the conventional light-cone quantization of a gauge theory. In this section, we will explain how to apply gauge-fixing directly to the free-string action, eq. (2.14). In this section, we will work in the basis of states formed by applying K_{-n}'s, M_{-n}'s, and transverse α_{-n}'s to the Fock vacuum. The argument given at the beginning of the previous section can be repeated in this basis to prove that the correct light-cone action arises if all states are removed except those in Φ_0

which belong to $\bar{\mathcal{T}}$. We will call this space of states $\bar{\mathcal{T}}_0$. We will show that the nonredundant gauge transformations in (2.15) give precisely the freedom required to reduce the full Fock space to $\bar{\mathcal{T}}_0$

Our analysis will concentrate on removing the (lowered) – components of the tensor fields of the free string theory; this is equivalent to removing Fock space states in which some M_{-n} is acting on the Fock vacuum. However, one must exercise some care in this reduction process. Because of the commutation relation (2.22), the M annihilation operators M_n destroy excitations created by the K creation operators K_{-n}, and vice versa. Thus, a state of the form $K^a M^b |0\rangle$ has a nonzero inner product only with a state of the form $K^b M^a |0\rangle$. If we gauge away or otherwise remove states with more M's than K's, states with more K's than M's can appear in the action only when their surfeit of K's is explicitly compensated. Often, these states can appear only in terms with no ∂_+'s; they are then nondynamical and may, in fact, serve as Lagrange multipliers to eliminate states which cannot be removed by gauge transformations.

To understand how this works in more concrete terms, we will first perform the gauge-fixing of the first two mass levels very explicitly, counting carefully the K and M creation operators which appear. We should note that the operators d and δ which appear throughout (2.14) can contribute extra factors of M and K, since, from the definition of L_n,

$$L_n \;=\; p^+ M_n \;+\; p^- K_n \;+\; \ldots \tag{4.1}$$

At the first mass level, the free-string action is simply the action of an Abelian gauge boson. We can remove the component A_- by using the gauge motion

$$\delta_C \Phi_0 \;=\; L_{-1} C^1 \;=\; \left(p^+ M_{-1} + \ldots\right) C^1 \tag{4.2}$$

to remove the component of Φ_0 of the form $M_{-1}|0\rangle$. (The corresponding Faddeev-Popov determinant contains p^+ but not p^- and so is nondynamical.) After this gauge-fixing, the component of Φ_0 proportional to $K_{-1}\big|0\big)$ can survive in the action only if it appears in a term with two explicit M's. The only such term comes from the second piece of (2.14) (after integrating by parts):

$$\big(\Phi_0 \,\big|\, \delta \Uparrow d \,\big|\, \Phi_0\big) \;=\; \big(\Phi_0 \,\big|\, p^+M_1 \cdot p^+M_{-1} + \ldots \,\big|\, \Phi_0\big) \; . \tag{4.3}$$

The term with two M's contains no ∂_-, so A_+, the coefficient field of $K_{-1}\big|0\big)$, is nondynamical. It is, of course, just the Coulomb potential.

At the second mass level, we can divide the states which appear into five classes as subsets of the following sets of states:

(1). $\bar{\mathcal{T}}_0$

(2). $K_{-1}\bar{\mathcal{T}}_0$, $M_{-1}\bar{\mathcal{T}}_0$

(3). $K_{-2}\bar{\mathcal{T}}_0$, $M_{-2}\bar{\mathcal{T}}_0$

(4). $K^2_{-1}\bar{\mathcal{T}}_0$, $M_{-1}K_{-1}\bar{\mathcal{T}}_0$, $M^2_{-1}\bar{\mathcal{T}}_0$

(5). $\bar{\mathcal{T}}_2$

Set (5) contains the first Stueckelberg fields. Sets (2) and (3) can be eliminated just as explained in the previous paragraph. Concentrate, then, on the classes (4) and (5). States of the form $M_{-1}K_{-1}\bar{\mathcal{T}}_0$ and $M^2_{-1}\bar{\mathcal{T}}_0$ can be eliminated by (4.2), using the components $K_{-1}\bar{\mathcal{T}}_0$ and $M_{-1}\bar{\mathcal{T}}_0$ of the string field $\mathcal{C}^1$; again, the gauge-fixing determinant is nondynamical. After gauge-fixing, states in $K^2_{-1}\bar{\mathcal{T}}_0$ can only appear in terms of the action with two explicit M annihilation operators. The only such term is

$$\big(\Phi_2 \,\big|\, d \Uparrow d \,\big|\, \Phi_0\big) \;=\; \big(\Phi_2 \,\big|\, p^+M_1 \cdot p^+M_1 + \ldots \,\big|\, \Phi_0\big) \; . \tag{4.4}$$

From the structure of this term, we can see, first, that the states $K^2_{-1}\bar{\mathcal{T}}_0$ are nondynamical and, second, that they are Lagrange multipliers which eliminate the states $\bar{\mathcal{T}}_2$, leaving behind only a nondynamical determinant. We have now eliminated all states except those of class (1); this is the desired result.

Let us now introduce some notation that will allow us to generalize this argument to all mass levels. Consider, first of all, the labelling of states. It is clear that we will need to keep track of the K and M creation operators used to form each state; the Lagrange multiplier mechanism tells us that we must also keep track of the indices on each Stueckelberg form. More generally, consider an arbitrary string form, symmetrized according to the Young tableau (k,ℓ), in the notation of Fig. 1. Let us denote the class of Fock states within this form with a K's and b M's acting on the Fock vacuum by writing

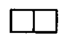
$$K^a M^b \left|k,\ell\right) . \tag{4.5}$$

We can count the number of such states at any given level by considering the indices on the K and M operators, as well as the explicit indices of the form, to be indices of the state. All of these indices run over the same set of values (the positive integers, for the case of the open bosonic string). Since the K and M creation operators commute with one another, the states (4.5) belong to the following representation of the group of permutations of all of these indices:

$$\{a\} \times \{b\} \times (k,\ell) . \tag{4.6}$$

This representation is displayed diagrammatically in Fig. 2.

Now let us turn to the operators d and δ. In the explicit manipulations just given, the only pieces of these operators which are relevant are the terms involving M creation and annihilation operators. These terms appear when we wish to determine the piece of the gauge variation of a Φ_{2k} which contains an additional M_{-n}, and when we wish to evaluate the term in the action linking the Lagrange multiplier field, with two extra K_{-n}'s, to a Stueckelberg field. The action of d and δ which produces these terms can be described very simply: δ, which removes one upper index from a string form, replaces this index with an index on an M_{-n}. d, which adds a lower index to the string form, acts an M_n on the state and so removes one index carried by a K_{-n}. In each case, the total number of K, M, and string form indices is conserved. The permutation symmetry of these indices is also preserved. Our discussion of the gauge-fixing at the second mass level gives the simplest illustration of this rule: Of the classes of states defined there, classes (2) and (3) correspond to states with one index; these are gauge-fixed separately from (4) and (5), which are states with two indices. Note that the Lagrange multipliers $K^2_{-1}\bar{\mathcal{T}}_0 = K^2\big|0,0\big)$ and the Stueckelberg states $\bar{\mathcal{T}}_2 = \big|1,1\big)$ both have indices symmetrized according to

 ; (4.7)

otherwise, they could not form a nonzero matrix element of $\delta \upuparrows \delta$.

Now we are ready to present the general counting argument for fixing the light-cone gauge. Let us define a tower of states as the sequence

$$K^a M^b\big|0,0\big)\,,\ K^{a-1}M^{b-1}\big|1,1\big)\,,\ K^{a-2}M^{b-2}\big|2,2\big)\,,\ \ldots\ . \tag{4.8}$$

The sequence terminates when either the K's or the M's are exhausted. Associated with this sequences is a tower of gauge parameters $\mathcal{C}_{2k+1}$:

$$K^a M^{b-1}\big|1,0\big)\,,\ K^{a-1}M^{b-2}\big|2,1\big)\,,\ K^{a-2}M^{b-3}\big|3,2\big)\,,\ \ldots\ . \tag{4.9}$$

Imagine both the two towers to be presented vertically, and interleaved, so that $K^aM^b\big|0,0\big)$ stands at the top, with $K^aM^{b-1}\big|1,0\big)$ just below, $K^{a-1}M^{b-1}\big|1,1\big)$ below that, and so forth. Then, acting δ on any state in the $\mathcal{C}$ tower adds an M and so gives a gauge transformation of the Φ state just above; and acting d on any state in the $\mathcal{C}$ tower removes a K and so gives a gauge transformation of the Φ state just below.

Our strategy will be to gauge away as many states as possible in the towers with $b \geq a$. The remaining states in these towers must then be removed by the Lagrange multiplier mechanism. The Lagrange multiplier for a state $K^cM^d\big|k,k\big)$ is the state which completes a nonzero matrix element of $\delta \upuparrows \delta$; this is a state which has a nonzero inner product with that state after two K's have been removed. Thus, the Lagrange multiplier for a state $K^c M^d |k,k)$ has the form

$$K^{d+2} M^c |k-1, k-1) \ . \tag{4.10}$$

The Lagrange multipliers for the states in the tower (4.8) ($b \geq a$) live in another tower whose top member is

$$K^{b+1} M^{a-1} |0,0) \ . \tag{4.11}$$

We must then show that the second tower contains, after an appropriate gauge-fixing, exactly the states needed to be Lagrange multipliers for the first.

Let us begin by working out what states in a tower with $b \geq a$ remain after gauge-fixing. To do this, we must first work out the number of *nonredundant* gauge parameters available. This means that we must subtract from the states in $\mathcal{C}_{2k+1}$ the number of redundant parameters in $\mathcal{G}_{2k+2}$, then add back the parameters in $\mathcal{C}'_{2k+3}$, and so forth. The simplest example is given by the state $M^{b-1}|1,0)$ in $\mathcal{C}_1$. The redundant components are those in $\delta\mathcal{G}_2$, where the state in $\mathcal{G}_2$ has the form $M^{b-2}|2,0)$; these have redundancies given by $\delta\mathcal{C}'_3$, using states of the form $M^{b-3}|3,0)$, etc. We can count the number of nonredundant gauge transformations by writing each of theses states as the corresponding representation of the permutation group and performing the sum:

$$\{b-1\} \times (1,0) \ - \ \{b-2\} \times (2,0) \ + \ \{b-3\} \times (3,0) \ - \ \dots \ . \tag{4.12}$$

The value of the sum is given by the Young tableau identity shown in Fig. 3. (The identity is readily proved by systematically decomposing the products of representations.) The result is:

$$\text{nonredundant part of } M^{b-1}|1,0) \ = \ \{b\} \ . \tag{4.13}$$

This is precisely the gauge freedom necessary to gauge away $M^b|0,0)$. Since all of our manipulations have involved the use of δ, which adds an M_{-n}, they are unaffected by adding K's to the state. Thus, we can immediately conclude:

$$\text{nonredundant part of } K^a M^{b-1}\big|1,0\big) \;=\; \{a\} \times \{b\} \;, \tag{4.14}$$

which is precisely what is required to gauge away the component $K^a M^b\big|0,0\big)$ of Φ_0. Our counting, combined with the correspondence between M_n and L_n, has then recovered the result, familiar from ref. 3, that the gauge freedom in $\mathcal{C}_1$ is precisely what is required to remove every state in the basis (2.26) which contains an L_{-n} acting on $\big|\tau\big)$.

This analysis generalizes straightforwardly to the higher $\mathcal{C}_{2k+1}$'s. The redundancies, redundancies of redundancies, etc. of the gauge parameters in $M^{b-1}\big|k+1,k\big)$ are given by:

$$M^{b-1}\big|k+1,k\big) \;-\; M^{b-2}\big|k+2,k\big) \;-\; M^{b-3}\big|k+3,k\big) \;+\; \ldots \tag{4.15}$$

The series can be summed by the identity shown in Fig. 4. The result is:

$$\text{nonredundant part of } M^{b-1}\big|k+1,k\big) \;=\; \{b\} \times (k,k) \;-\; (k,k;b) \;. \tag{4.16}$$

Both sides of this relation can be multiplied by K^a, as before. Note that in this case, not all of the components of the Φ state $K^a M^b\big|k,k\big)$ can be gauged away by the nonredundant components of $\mathcal{C}$. A residue, with the symmetry $\{a\} \times (k,k;b)$, is left behind.

If we exhaust the gauge freedom in this way, we can reduce the tower (4.8) $(b \geq a)$ to its components:

$$0 + \{a-1\} \times (1,1;b-1) + \{a-2\} \times (2,2;b-2) + \{a-3\} \times (3,3;b-3) + \ldots \tag{4.17}$$

These last components must be removed by Lagrange multipliers belonging to the tower starting with (4.11). The first nonvanishing term in (4.17) is precisely

$\{a-1\} \times \{b+1\}$, and so the (4.11) is exactly its Lagrange multiplier. At lower levels of the towers, only a subset of the states on the Lagrange multiplier side is needed to remove the corresponding components of (4.17). We proceed, then, as follows: Use the C_1 state in the Lagrange multiplier tower to gauge away as many states as possible in the Φ_2 component of this tower (using the transformation law $\delta_C \Phi_2 = dC_1$). Then extract from the states remaining in Φ_2 a multiplet of the symmetry $\{a-2\} \times (2,2;b-2)$, which will be the Lagrange multiplier for the third term in (4.17). Gauge away the rest of Φ_2 using C_3. Now use the remaining states in C_3 to gauge away part of Φ_4, extract the required Lagrange multipler, and gauge away the rest of Φ_4 using C_5. Proceeding in this way, one can exhaust the gauge freedom in the Lagrange multiplier tower, hopefully leaving over the states which will be the Lagrange multipliers for (4.17). Let us check that the correct states are indeed left over. The counting of states minus (nonredundant) gauge parameters in the Lagrange multiplier tower goes as follows

$$\begin{aligned}
&\{b+1\} \times \{a-1\} \ + \ \{b\} \times \{a-2\} \times (1,1) \ + \ \{b-1\} \times \{a-3\} \times (2,2) \ + \ \dots \\
&\quad - \ \{b+1\} \times (\{a-1\}) - \ \{b\} \times (\{a-2\} \times (1,1) \ - \ (1,1;a-2)) \\
&\qquad\qquad - \ \{b-1\} \times (\{a-3\} \times (2,2) \ - \ (2,2;a-3)) \ - \ \dots \\
&= \ \{b\} \times (1,1;a-2) \ + \ \{b-1\} \times (2,2;a-3) \ + \ \{b-2\} \times (3,3;a-3) \ + \ \dots
\end{aligned} \tag{4.18}$$

One can now check that the last line of (4.18) is exactly equal to (4.17), by virtue of one further Young tableau identity, shown in Fig. 5.

Thus, for any tower of states (4.8) with $b \geq a$, all states that cannot be gauged away can be removed by a set of Lagrange multipliers given precisely by the states in the tower (4.11) which cannot be gauged away. Every tower with $b < a$ can be considered such a tower of Lagrange mulitipliers, except for the tower with $a - b = 1$. Once all the other states are eliminated, however, these states may appear only in terms in the action with two explicit M's. Such terms, as we have already discussed, contain p^+ but not p^-; thus, these fields are nondynamical. They are, in fact, the generalized Coulomb potentials.

We have now proven, in two different ways, that the light-cone formulation of string theories can be derived from the new gauge-invariant formulation by fixing the light-cone gauge. No additional propagating states appear. We hope that these direct demonstrations of the equivalence of these two formalisms will be useful in the construction of gauge-invariant interactions.

ACKNOWLEDGEMENTS

We are grateful to Tom Banks, Thom Curtright, and Emil Martinec for many illuminating conversations. One of us (MEP) also thanks Peter Arnold, Mirjam Cvetic, and Christian Preitschopf for help in understanding the Young tableau identities.

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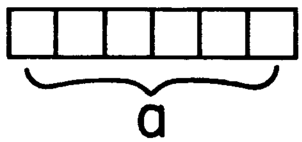
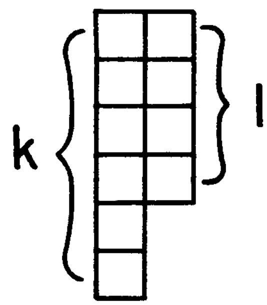
FIGURE CAPTIONS

1. Conventions for naming Young tableaux which will arise in our analysis.
2. The permutation symmetry of the full set of indices of the state (4.5).
3. The Young tableau identity needed to identify the nonredundant gauge parameters in C_1.
4. The Young tableau identity needed to identify the nonredundant gauge parameters in C_{2k+1}.
5. Another Young tableau identity, needed to establish the equivalence of (4.17) and (4.18).

$\{a\} =$



$(k,l) =$



$(k,k;a) =$

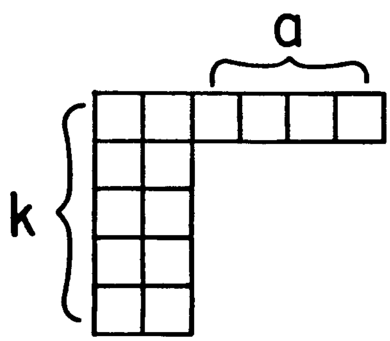


Fig. 1

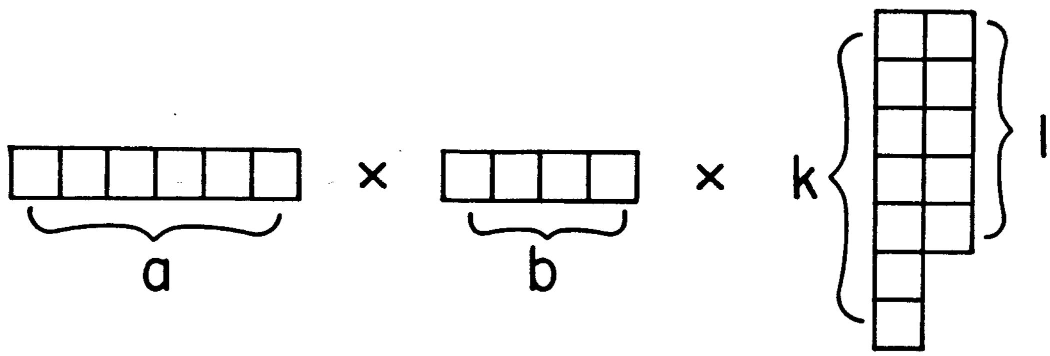


Fig. 2

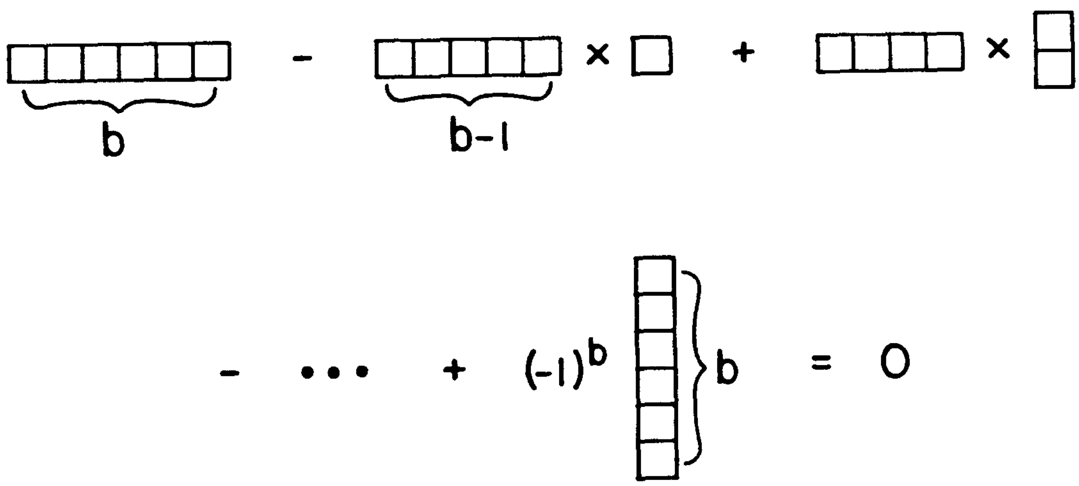


Fig. 3

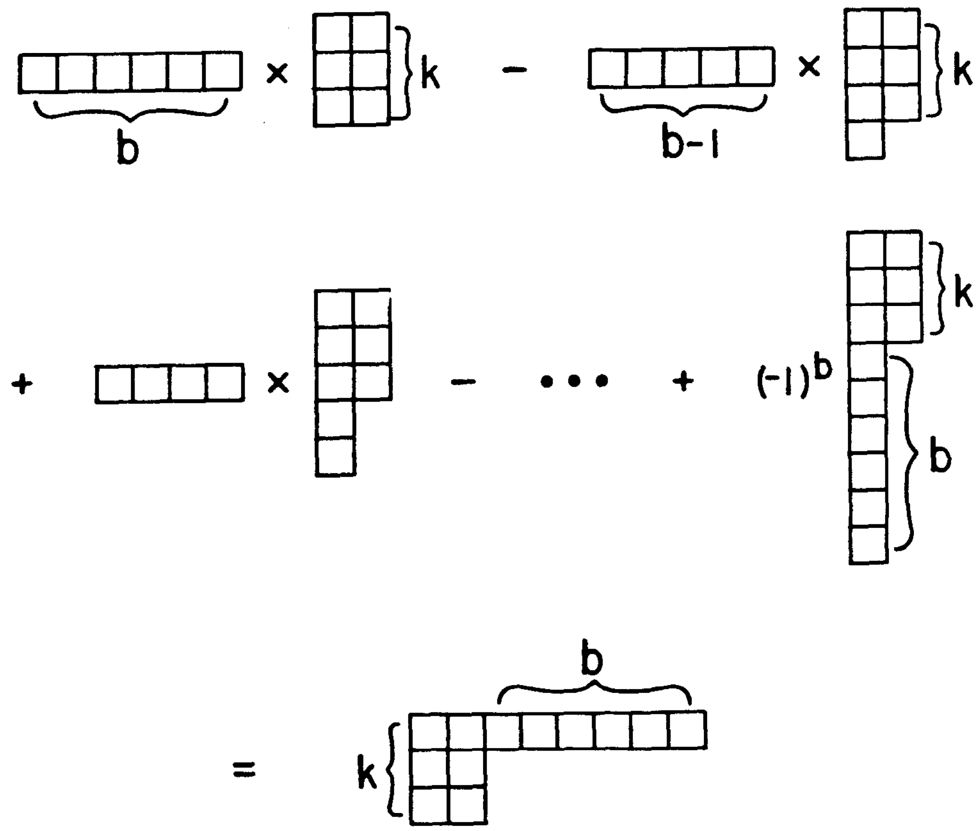


Fig. 4

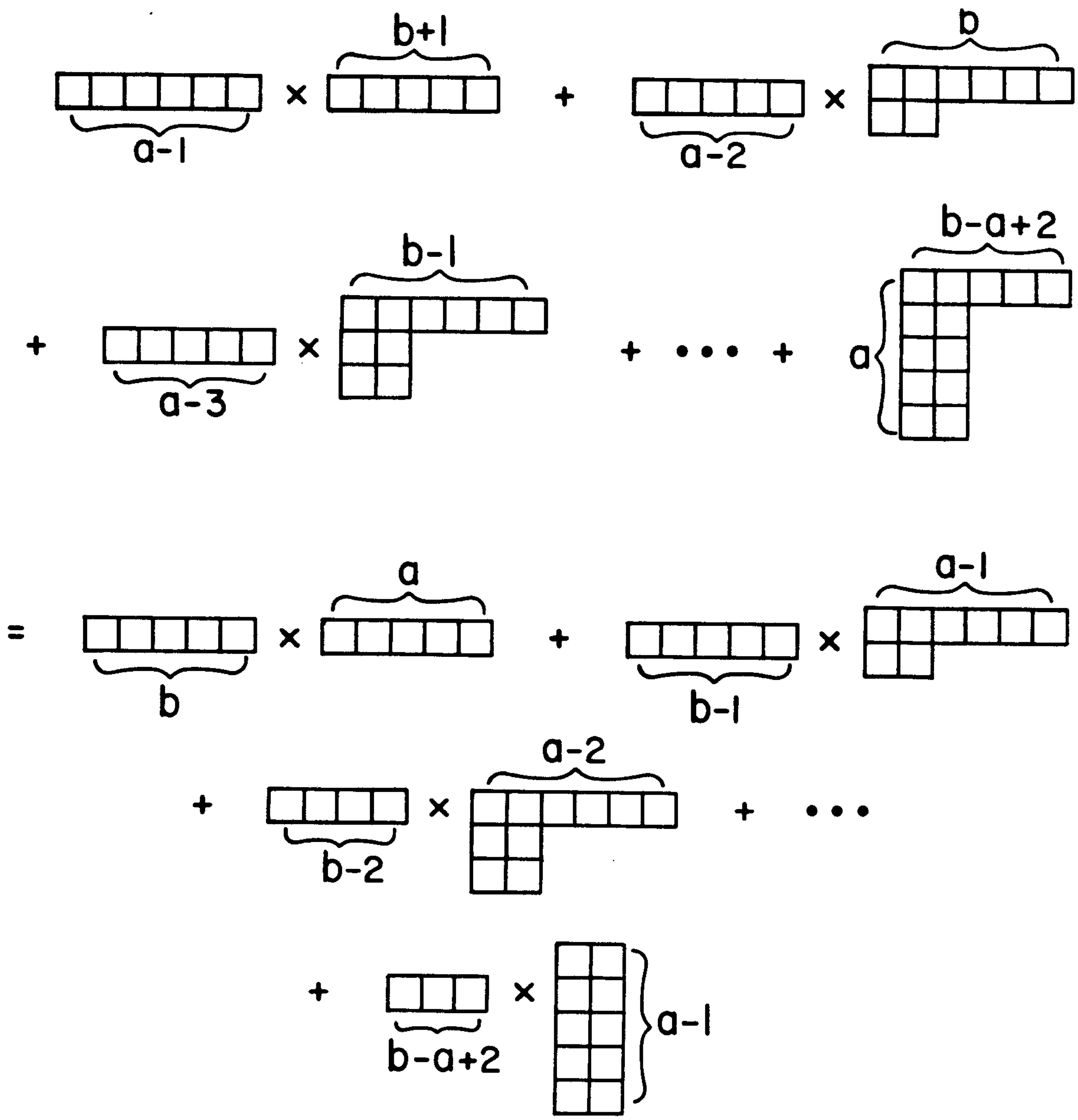


Fig. 5