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MULTIQUARK EVOLUTION IN QCD*

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ABSTRACT

We present a formalism for the evolution in Q^2 of multiquark systems as an application of perturbative quantum chromodynamics (QCD) to asymptotic, exclusive nuclear amplitudes. To leading terms in log Q^2 our formalism is equivalent to solving the renormalization group equations for these amplitudes. Completely antisymmetric multiquark color-singlet representations are constructed and their evolution is investigated from the one-gluon exchange kernel. We argue that the evolution equation, together with a cluster decomposition, demonstrates a transition from the traditional meson and nucleon degrees of freedom of nuclear physics to quark and gluon degrees of freedom with increasing Q^2, or at small internucleon separations. As an example, we derive an evolution equation for a completely antisymmetric six-quark distribution amplitude and solve the evolution equation for a deuteron S-wave amplitude. The leading anomalous dimension and the corresponding eigensolution are found for the deuteron in order to predict the asymptotic form of the deuteron distribution amplitude (*i.e.*, light-cone wave function at short distances). The fact that the six-quark state is 80 percent hidden color at small transverse separation implies that the deuteron form factor cannot be described at large Q^2 by meson-nucleon degrees of freedom alone. Furthermore, since the N–N channel is very suppressed under these conditions, the effective nucleon-nucleon potential is naturally repulsive at short distances.

1. INTRODUCTION

Nuclear chromodynamics is concerned with the application of quantum chromodynamics (QCD) to nuclear physics.[1] Its goal is to give a fundamental description of nuclear dynamics and nuclear properties in terms of quark and gluon fields at short distance, and to obtain a synthesis at long distances with the normal

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nucleon, isobar, and meson degrees of freedom. If we define Q^2 as a scale of momentum square involved in a certain reaction, then there are two complementary approaches to nuclear chromodynamics: one for high Q^2 region and the other for low Q^2 region. The essential criteria for the high and the low Q^2 regions are based on two principles in QCD; asymptotic freedom and confinement respectively. Whereas many models such as Skyrmions and soliton bags, *etc.*, are available as viable approaches in the low Q^2 region,[2] perturbative QCD is the only consistent approach in high–Q^2 region. Therefore, it is worthwhile looking at the implications of exact perturbative QCD predictions on various effective nuclear phenomena.

One of the main ingredients in the perturbative QCD approach is the factorization theorem for both inclusive and exclusive processes which separates the hadronic bound state physics from perturbative dynamics. The processes which are easily analyzed are those in which all final particles are measured at large invariant masses compared to each other, i.e.: large momentum transfer exclusive reactions. This includes form factors of hadrons and nuclei at large momentum transfer Q and large angle scattering reactions such as photoproduction $\gamma p \to \pi^+ n$, nucleon-nucleon scattering, photodisintegration $\gamma d \to np$ at large c.m. angles and energies, *etc.*, which can be analyzed in terms of a simple picture for exclusive processes based on light-cone perturbation theory. A key result is that such amplitudes factorize at large momentum transfer in the form of a convolution of a hard scattering amplitude T_H which can be computed perturbatively from quark-gluon subprocesses multiplied by process-independent "distribution amplitudes" $\phi(x, Q)$ which contain all of the bound-state non-perturbative dynamics of each of the interacting hadrons.[3] For example, the baryon form factor at large Q^2 is represented by the factorized form [see fig. 1(a)][4]

$$\begin{aligned} F_B(Q^2) &= \int_0^1 [dx] \int_0^1 [dy]\ \phi^*(y_i, \check{Q}_y)\, T_H(x_i, y_i, Q)\, \phi(x_i, \check{Q}_x) \left[1 + \mathcal{O}\left(\frac{m_i^2}{Q^2}\right)\right] \\ &= \frac{32\pi^2}{9}\, \frac{\alpha_s^2(Q^2)}{Q^4} \sum_{n,m} b_{nm} \left(\ell n\, \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n-\gamma_m} \left[1 + \mathcal{O}\left(\alpha_s(Q^2), \frac{m_i^2}{Q^2}\right)\right] \\ &\to C \left(\frac{\alpha_s(Q^2)}{Q^2}\right)^2 \left(\ell n\, \frac{Q^2}{\Lambda^2}\right)^{-2\gamma_0} \quad (\text{as } Q^2 \to \text{large}) \quad , \end{aligned} \tag{1.1}$$

where x_i is the light-cone longitudinal momentum fraction of i^{th} quark $x_i = (k_i^0 + k_i^3)/(p^0 + p^3)$, $[dx] \equiv dx_1\, dx_2\, dx_3\, \delta\left(1 - \sum_i x_i\right)$ and $\check{Q}_x \equiv \min_i(x_i Q)$. The dominant Q^2 dependence $(\alpha_s(Q^2)/Q^2)^2$ is derived from the hard scattering amplitude $T_H(x_i, y_i, Q)$ of the subprocess $\gamma^* + 3q \to 3q$ [see fig. 1(b)] with the only weak (logarithmic) Q^2 dependence coming from quark distribution amplitude $\phi(x_i, Q)$ (γ_0 is the leading anomalous dimension) [see fig. 1(c)]. The essential feature of eq. (1.1) is that a very complicated process can be simply represented by the factorization into product of three amplitudes.

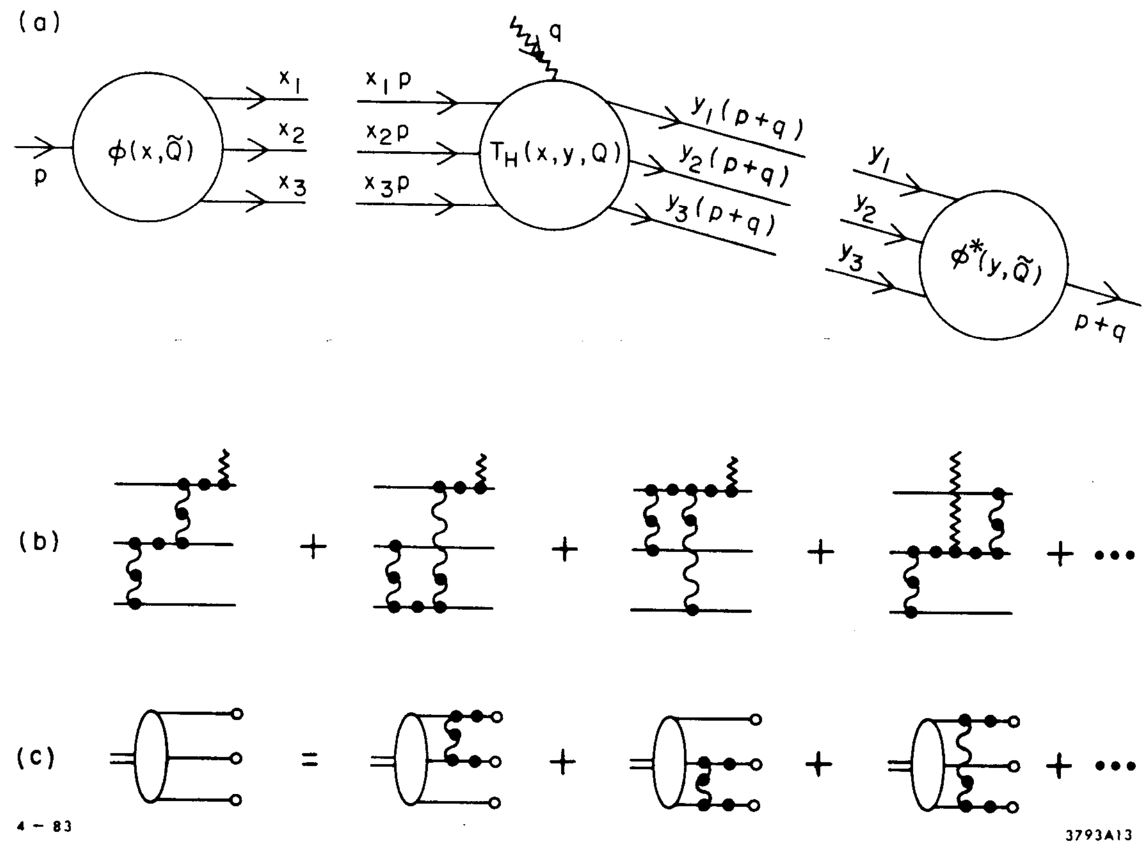

Fig. 1. (a) Factorization of the nucleon form factor at large Q^2 in QCD. (b) The leading order diagrams for the hard scattering amplitude T_H. The dots indicate insertions which enter the renormalization of the coupling constant. (c) The leading order diagrams which determine the Q^2 dependence of $\phi_B(x, Q)$.

The quark distribution amplitude $\phi(x_i, Q)$ is the amplitude for converting the baryon into three valence quarks at impact separation $b_\perp \sim O(1/Q)$. It is related to the equal $\tau = t + z$ hadronic wave function $\psi(x_i, \vec{k}_{\perp i})$[5]:

$$\phi(x_i, Q) \propto \int^{Q} \prod_{i=1}^{3} d^2\vec{k}_{\perp i}\, \delta^2\left(\sum_i \vec{k}_{\perp i}\right) \psi(x_i, \vec{k}_{\perp i}) \quad , \tag{1.2}$$

and contains the physics of that part of the hadronic wave function which affects exclusive processes at large momentum transfer. Therefore, constructing $\phi(x_i, Q)$ is an essential part of developing the perturbative QCD approach to the nuclear chromodynamics.

In this talk we will present a generalized method of constructing $\phi(x_i, Q)$ for multibaryon systems of $3n$ quarks[6] which satisfies the evolution equation derived basically from the Bethe–Salpeter equation [see, *e.g.*, fig. 1(c)]. For the baryon (three-quark) system the evolution equation has been derived and solved by Brodsky and Lepage.[3] However, their method of solving the evolution equation cannot be simply extended to multibaryon systems. Recently, Brodsky and I[7] developed a new method in order to extend the simple baryon analysis to the case of multibaryon systems.[8,9] As a starting point, in the next section we use a simple scalar field model and derive an evolution equation for a two-body bound state. Then we extend it to a realistic three-quark system and describe our new method of solving the evolution equation in section 3. In section 4 we present an analysis of a six-quark system as an example of the extension to multibaryon systems. We focus on calculating a leading anomalous dimension for a deuteron S-wave amplitude. In section 5 we present an application of our formalism. We derive rigorous constraints on the short distance effective force between two baryons, using an evolution in Q^2 of a toy dibaryon system. Discussions and conclusions follow in section 6.

2. SOLUTIONS OF THE BOUND STATE EQUATION AND THE EVOLUTION EQUATION

In this section and as an introduction, we will use $(\phi^3)_6$-type theory, which shares the asymptotic freedom property of QCD. The model Lagrangian density which we consider in this section is

$$\mathcal{L} = \frac{1}{2} \left(\partial_\mu\phi\,\partial^\mu\phi + \partial_\mu\chi\,\partial^\mu\chi\right) - \frac{1}{2}\, m^2\phi^2 - g\phi^2\chi \; , \tag{2.1}$$

where μ runs from 1 to 6, and ϕ and χ are "quark" and "gluon" fields respectively.

A conventional tool for dealing with the relativistic two-body problem in quantum field theory is the Bethe–Salpeter formalism[10] utilizing the Green functions of covariant perturbation theory. However this formalism has difficulties with the relative time dependence especially for multiparticle states and in systematically including higher order irreducible kernels such as cross diagrams and vacuum polarizations.[11]

An alternative approach which can remove these difficulties and restore a systematic perturbative calculation for obtaining higher accuracy is the reformulation of the covariant Bethe–Salpeter equation using the light-cone coordinate.[12] This is equivalent to expressing the Bethe–Salpeter equation in the infinite momentum frame.[13] The light-cone quantization method[14] provides a Fock–state representation at equal light-cone time $\tau = t + z/c$ for a bound state $|B\rangle$

$$|B\rangle = \langle \phi\overline{\phi} | B\rangle \; |\phi\overline{\phi}\rangle + \langle \phi\overline{\phi}\chi | B\rangle \; |\phi\overline{\phi}\chi\rangle + \ldots \; , \tag{2.2}$$

and removes the difficulty of the relative time dependence of the covariant formalism. The light-cone wavefunctions $\langle \phi\overline{\phi} | B\rangle$, $\langle \phi\overline{\phi}\chi | B\rangle$, ... provide a physically transparent description of a bound state, since the vacuum fluctuations are suppressed in the light-cone frame and all constituents are on the mass shell where

$$k_i^- = \frac{k_{\perp i}^2 + m_i^2}{x_i}\,, \tag{2.3}$$

with $x_i = k_i^+/P^+$. Furthermore, cross diagrams can be included systematically when higher Fock-state contributions such as $|\phi\bar{\phi}\chi\chi\rangle$ are taken into account.

2.1 Bound State Equation

By taking into account only two– and three–body sectors, we arrive at the effective equation for the two-body wavefunction ($x_1 = x$, $x_2 = 1 - x$, $\vec{k}_{\perp 1} = -\vec{k}_{\perp 2} = \vec{k}_\perp$):

$$\begin{aligned}\left[M^2 - \frac{k_\perp^2 + m^2}{x(1-x)}\right] \psi(x, \vec{k}_\perp) &= g^2 \int_0^1 \frac{dy}{y(1-y)} \int \frac{d^4\vec{l}_\perp}{16\pi^3} \\ &\times \left\{ \frac{\theta(y-x)}{y-x} \cdot \frac{1}{M^2 - \frac{k_\perp^2+m^2}{x} - \frac{l_\perp^2+m^2}{1-y} - \frac{(\vec{k}_\perp - \vec{l}_\perp)^2}{y-x}} + (x \leftrightarrow y) \right\} \\ &\times \psi\left(y, \vec{l}_\perp\right)\,,\end{aligned} \tag{2.4}$$

which we call the light-cone ladder approximation (LCLA).[14,15] This equation provides an eigenvalue problem and the eigensolution is a nonperturbative solution in the sense that it includes the effect by summing all orders of ladder diagrams. The eigenvalue of the bound state equation is the binding energy in terms of the coupling constant. An approximate solution of eq. (2.4) to the ground state has been suggested by Karmanov:[16]

$$\psi(x, \vec{k}_\perp) = \frac{N}{\left(M^2 - \frac{\vec{k}_\perp^2 + m^2}{x(1-x)}\right)^2 (1 + |2x - 1|)}\,, \tag{2.5}$$

where N is a normalization constant. The corresponding eigenvalue (binding energy)[17] is given by the following relation between the coupling constant $\alpha = g^2/(16\pi m^2)$ and the binding energy $\beta^2 = m^2 - (M^2/4)$:

$$\frac{\pi}{\alpha} = \frac{z-2}{\sqrt{z-1}} \left(\frac{\pi}{2} - \tan^{-1} \frac{1}{\sqrt{z-1}}\right) + \ell n\, \frac{4}{z}\,, \tag{2.6}$$

where $z = m^2/\beta^2$.

2.2 Evolution Equation

The distribution amplitude in the $(\phi^3)_6$ model is given by

$$\phi(x,Q) = \int^{|\vec{k}_\perp|<Q} \left[d^4\vec{k}_\perp\right] \psi^{(Q)}(x,\vec{k}_\perp) , \tag{2.7}$$

where $[d^4\vec{k}_\perp] = 1/[2(2\pi)^5]\ d^4\vec{k}_\perp = \vec{k}_\perp^2/(64\pi^3)\ d\vec{k}_\perp^2$. The variation of ϕ with Q comes from the upper limit of the integration as well as from renormalization scale dependence of the wave function

$$\psi^{(Q)}(x,\vec{k}_\perp) = \frac{Z_2(Q)}{Z_2(Q_0)} \psi^{(Q_0)}(x,\vec{k}_\perp) , \tag{2.8}$$

where $Z_2(Q) \simeq e^{1/6\ \xi(Q)}$ with $\xi(Q) = \int_Q^\infty [d^4\vec{k}_\perp]\ [g^2(\vec{k}_\perp^2)]/\vec{k}_\perp^4$ because of vertex and self-energy insertions.[14] Therefore, the differentiation of eq. (2.7) yields

$$Q^2 \frac{\partial}{\partial Q^2} \phi(x,Q) = Q^2 \frac{\partial \ell n\, Z_2(Q)}{\partial Q^2} \phi(x,Q) + \left(Q^2\pi\right)^2 \frac{Z_2(Q)}{2(2\pi)^5} \psi^{(Q)}(x,Q) . \tag{2.9}$$

By taking asymptotic limit ($|\vec{k}_\perp| = Q \to \infty$) of eq. (2.4), one can compute $\psi^{(Q)}(x,Q)$ in terms of the distribution amplitude ϕ which combines with eq. (2.9) and ends up with the following evolution equation:

$$Q^2 \frac{\partial}{\partial Q^2} \phi(x,Q) = -\frac{1}{64\pi^3} g^2(Q) \left\{\frac{1}{6}\phi(x,Q) - \int_0^1 dy\, V(x,y)\, \phi(y,Q)\right\} , \tag{2.10}$$

where

$$V(x,y) = \Theta(y-x)\, \frac{x}{y} + \Theta(x-y) \frac{1-x}{1-y} . \tag{2.11}$$

This is an integro-differential equation and again provides an eigenvalue problem. The general solution of eq. (2.10) is given by a linear combination of the Gegenbauer polynomials:

$$\phi(x,Q) = x(1-x) \sum_{n=0}^{\infty} A_n C_n^{3/2}(2x-1)\, e^{-\gamma_n \xi(Q^2)} ,$$

where each eigensolution $C_n^{3/2}(2x-1)$ is directly related to a term in the operator-product expansion of the wave function evaluated near the light cone and the eigenvalues are the corresponding anomalous dimensions, γ_n given by

$$\gamma_n = \frac{1}{6} - \frac{1}{(n+1)(n+2)} .$$

To leading terms in $\ell n\, Q^2$, the evolution formalism is equivalent to solving the renormalization group equation for the distribution amplitude.

3. EVOLUTION OF THE COLOR–SINGLET THREE–QUARK SYSTEM

We now extend the evolution formalism to the realistic three-quark system using QCD. The evolution equation for the color-singlet three-quark system has been derived by Brodsky and Lepage[3]

$$x_1 x_2 x_3 \left(\frac{\partial}{\partial \xi} + \frac{3C_F}{2\beta}\right) \widetilde{\phi}(x_i, Q) = \frac{C_B}{\beta} \int_0^1 [dy]\, V(x_i, y_i)\, \widetilde{\phi}(y_i, Q) , \tag{3.1}$$

where $C_B = 2/3$, $C_F = 4/3$ and $\beta = 11 - 2n_f/3$. The leading order kernel is computed from the single-gluon exchange diagram [see fig. 1(c)] and $V(x_i, y_i)$ in this case is given by

$$\begin{aligned} V(x_i, y_i) &= 2x_1 x_2 x_3 \sum_{i \neq j} \Theta(y_i - x_i)\, \delta(x_k - y_k) \frac{y_j}{x_j} \left(\frac{\delta_{h_i \bar{h}_j}}{x_i + x_j} + \frac{\Delta}{y_i - x_i}\right) \\ &= V(y_i, x_i) , \qquad (k \neq i, j) , \end{aligned} \tag{3.2}$$

where $\Delta \phi(y_i) = \phi(y_i) - \phi(x_i)$, and $\delta_{h_i \bar{h}_j} = 0(1)$ when the helicities of constituents are antiparallel (parallel). The general solution of eq. (3.1) is

$$\phi(x_i, Q) = x_1 x_2 x_3 \sum_{n=0}^{\infty} A_n\, \widetilde{\phi}_n(x_i)\, e^{-\gamma_n \xi(Q)} , \tag{3.3}$$

where the anomalous dimensions γ_n and the eigenfunctions $\widetilde{\phi}_n(x_i)$ satisfy the characteristic equation

$$x_1 x_2 x_3 \left(-\gamma_n + \frac{3C_F}{2\beta}\right) \widetilde{\phi}_n(x_i) = \frac{C_B}{\beta} \int_0^1 [dy]\, V(x_i, y_i)\, \widetilde{\phi}_n(y_i) . \tag{3.4}$$

Therefore, the rest of this section is just describing the method to solve the eigenvalue problem of eq. (3.4).

Let's introduce first the Brodsky-Lepage's method:[3]

(1) Take the basis for $\widetilde{\phi}_n(x_i)$ as $\{x_1^k\, x_3^\ell\}_{k,\ell=0}^{\infty}$.

(2) Construct the kernel matrix under the above basis by integrating over y_i.

(3) Diagonalize the kernel matrix and find the eigenvalues γ_n and the eigenfunctions $\widetilde{\phi}_n(x_i)$.

In the simple three-quark case, the color singlet property guarantees all three quarks have different quantum numbers. However, if we consider multibaryon systems of $3n$ quarks,[6] then the color singlet requirement does not guarantee that all the quarks of the system have different quantum numbers. Thus, we have to antisymmetrize the system according to Pauli's principle and $\widetilde{\phi}_n(x_i)$ cannot be derived by expanding $V(x_i, y_i)$ on a simple polynomial basis $\{x_1^k\, x_3^\ell\}_{k,\ell=0}^{\infty}$.

The new method which we developed[7] is basically the same as the Brodsky–Lepage method except for replacing the basis $\{x_1^k\, x_3^\ell\}_{k,\ell=0}^{\infty}$ by {completely antisymmetric color-isospin–spin-index power $\left(x_1^k\, x_2^\ell\, x_3^m\right)$ representations$\}_{k,\ell,m=0}^{\infty}$. The index power $x_1^k\, x_2^\ell\, x_3^m$ is analogous to the orbital dependence of nonrelativistic wave functions. The new method can be extended to multibaryon systems of $3n$ quarks[8,9] and predicts the correct distribution amplitudes of multiquark systems since the basis of the new method is a set of completely antisymmetric representations. Furthermore, it has several additional advantages. Among them,

(1) Even in the three quark system, one can classify the baryon system by observing the isospin multiplet (*e.g.*, nucleon and isobar belong to $T = 1/2$ and $3/2$ respectively). Therefore, we predict the difference of $\ell n\, Q^2$ behavior between the form factors of the nucleon (N) and the isobar (Δ).[7]

(2) Combining the results obtained by the new method for multiquark systems with the fractional parantage technique[18] which can decompose the systems into clusters, one can derive constraints on the effective force among baryons at short distances. The analysis for toy dibaryon systems of four quarks[8] and the rigorous constraints on the effective force between baryons derived from first principle QCD will be presented in section 5.

4. GENERALIZATION TO MULTIQUARK SYSTEMS

As we have discussed, the eigenvalue problem for the evolution formalism is generically given by

$$K\,|\phi_A\rangle \;=\; \gamma\,|\phi_A\rangle \;, \tag{4.1}$$

where K, γ and $|\phi_A\rangle$ represent the kernel, the eigenvalue (*e.g.*, anomalous dimensions) and the eigenfunction which is given by a linear combination of the antisymmetric representations respectively, and the integration over y_i [see *e.g.*, eq. (3.1)] is understood. If we consider only the single-gluon exchange and factorize the color matrices, then the kernel K can be written as

$$K = \sum_{i \neq j} \left(\frac{\vec{\lambda}_i}{2} \cdot \frac{\vec{\lambda}_j}{2} \right) V_{ij} \ , \tag{4.2}$$

where V_{ij} is the dynamical part obtained by calculating the one-gluon exchange diagrams. For simplicity (but without loss of generality), let us consider a six-quark case as an example of multiquark systems.

The six-quark system has five orthogonal color singlet states $|(222)\alpha\rangle$ with $\alpha = 1, 2, \ldots, 5$ (see below) and the evolution kernel becomes 5×5 matrix:

$$K_{\alpha\beta} = \sum_{i \neq j} C_{\alpha\beta}(i,j) \ V_{ij} \ , \tag{4.3}$$

where

$$C_{\alpha\beta}(i,j) = \left\langle (222)\alpha \middle| \frac{\vec{\lambda}_i}{2} \cdot \frac{\vec{\lambda}_j}{2} \middle| (222)\beta \right\rangle . \tag{4.4}$$

Therefore, the general formula of the evolution equation in terms of a color-singlet basis becomes

$$K_{\alpha\beta} \left| \phi^{\beta}_{A} \right\rangle = \gamma \left| \phi^{\alpha}_{A} \right\rangle . \tag{4.5}$$

The key observation to simplify the above matrix equation is that $K_{\alpha\beta}$ can be written in terms of K_{fY} which has a well-defined permutation symmetry[19] represented by a Young–tableau f and a certain Yamanouchi–label Y;

$$K_{\alpha\beta} = \sum_{f} \sum_{Y} \langle (222)\alpha, fY | (222)\beta \rangle \ K_{fY} \ . \tag{4.6}$$

From the Clebsch–Gordan coefficients of the S_6 group, we know that only two Young–tableaus are possible for the six-quark one-gluon exchange kernel and they are given by (see table I)

$$K_{(6)} = -\frac{C_F}{5} \sum_{i \neq j} V_{ij} \ , \tag{4.7a}$$

$$K_{(42)Y} = \frac{9}{5} \sum_{\alpha} \sum_{\beta} \sum_{i \neq j} \langle (222)\alpha, (42)Y | (222)\beta \rangle \ C_{\alpha\beta}(i,j) \ V_{ij} \ , \tag{4.7b}$$

where only one Yamanouchi–label is allowed in $f = (6)$, and $f = (42)$ has nine different Yamanouchi–labels.

TABLE I

K_{fY}	N_{fY}	(56)	(46)	(45)	(36)	(35)	(26)	(25)	(16)	(15)	(23)	(24)	(34)	(14)	(13)	(12)
$K_{(6)[111111]}$	$-\frac{4}{15}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$K_{(42)[221111]}$	$\frac{\sqrt{6}}{20}$	12	-3	-3	-3	-3	-3	-3	-3	-3	2	2	2	2	2	2
$K_{(43)[212111]}$	$\frac{\sqrt{10}}{20}$		9	-3	-3	1	-3	1	-3	1	1	-2	-2	-2	2	2
$K_{(42)[211211]}$	$\frac{\sqrt{5}}{10}$				6	-2	-3	1	-3	1	-1	1	-2	1	-1	-2
$K_{(43)[211121]}$	$\frac{\sqrt{15}}{10}$						3	-1	-3	1	-1	-1	0	1	1	
$K_{(42)[122111]}$	$\frac{1}{\sqrt{5}}$			3		-1		-1		-1	1	-1	-1	-1	1	1
$K_{(42)[121211]}$	$\frac{1}{\sqrt{10}}$					4		-2		-2	-1	1	-2	1	-1	2
$K_{(42)[121121]}$	$\sqrt{\frac{3}{10}}$							2		-2	-1	-1		1	1	
$K_{(42)[112211]}$	$\sqrt{\frac{2}{10}}$										-1	-1	2	-1	-1	2
$K_{(42)[112121]}$	$\sqrt{\frac{3}{10}}$										-1	1		-1	1	

As an example, let us calculate the leading anomalous dimension of the deuteron state. Since the total index-power is $n = 0$ for the leading anomalous dimension and the isospin is fixed as a singlet for the deuteron state, two antisymmetric representations are possible and defined by the inner-product of Young–tableaus[19]:

$$|A_1\rangle = |(222)\rangle_C \times |(33)\rangle_T \times |(6)\rangle_S \times |(6)\rangle_O \ , \tag{4.8a}$$

$$|A_2\rangle = |(222)\rangle_C \times |(33)\rangle_T \times |(42)\rangle_S \times |(6)\rangle_O \ , \tag{4.8b}$$

The eigensolutions will be linear combinations of $|A_1\rangle$ and $|A_2\rangle$,

$$|E_1\rangle = \cos\theta\,|A_1\rangle + \sin\theta\,|A_2\rangle \ , \tag{4.9a}$$

$$|E_2\rangle = -\sin\theta\,|A_1\rangle + \cos\theta\,|A_2\rangle \ , \tag{4.9b}$$

where $|E_1\rangle$ and $|E_2\rangle$ have eigenvalues e_1 and e_2 respectively,

$$K_{\alpha\beta}\left|E_1^{\beta}\right\rangle = e_1\,|E_1^{\alpha}\rangle \ , \tag{4.10a}$$

$$K_{\alpha\beta}\left|E_2^{\beta}\right\rangle = e_2\,|E_2^{\alpha}\rangle \ . \tag{4.10b}$$

As we have pointed out, an essential simplification can be obtained by replacing $K_{\alpha\beta}$ with K_{fY}. Projecting out a certain state which has common color (C), isospin (T) and index-power (O) representations, we get a set of equations for spin states:

$$K_{(6)}\,|(6)\rangle_S = \left(e_1 cos^2\theta + e_2 sin^2\theta\right)|(6)\rangle_S \ , \tag{4.11a}$$

$$K_{(42)Y}\,|(6)\rangle_S = (e_1 - e_2)\cos\theta\sin\theta\,|(42)Y\rangle_S \ , \tag{4.11b}$$

$$\frac{\sqrt{53}}{12}\sum_{Y_S}\sum_{Y_K}\langle(42)Y_S,(42)Y_K \mid (43)Y\rangle\, K_{(42)Y_K}\,|(42)Y_S\rangle_S = (e_1 - e_2)\left(sin^2\theta - cos^2\theta\right)|(42)Y\rangle_S \ . \tag{4.11c}$$

Since the kernel of each equation has a definite symmetry and its explicit representation is known (see table I), we can determine relative weighting factors among the independent equations (4.11a),(4.11b) and (4.11c) by counting the number of spin annihilation terms [see *e.g.*, $\delta_{h_i \bar{h}_j}$ term in eq. (3.2)] in the kernel:

$$K_{(6)}\,|(6)\rangle_S \;=\; \gamma_0\,|(6)\rangle_S \ , \tag{4.12a}$$

$$K_{(42)Y}\,|(6)\rangle_S \;=\; \frac{3\sqrt{6}}{16}\gamma_0\,|(42)Y\rangle_S \ , \tag{4.12b}$$

$$\frac{\sqrt{53}}{12}\sum_{Y_S}\sum_{Y_K}\langle(42)Y_S,(42)Y_K \ \mid \ (42)Y\ \rangle\,K_{(42)Y_K}\,|(42)Y_S\rangle_S \;=\; \frac{5}{48}\,\gamma_0\,|(42)Y\rangle_S \ , \tag{4.12c}$$

where γ_0 is the eigenvalue of eq. (4.12a). Comparing eqs. (4.11) and (4.12), we find

$$\tan\theta \;=\; \frac{\sqrt{6}}{2}\ , \quad e_1 \;=\; \frac{25}{16}\,\gamma_0\ , \quad e_2 \;=\; \frac{8}{5}\,\gamma_0\ , \tag{4.13}$$

and the only equation which we have to solve explicitly is eq. (4.12a), which has the symmetric kernel $K_{(6)}$. Solving eq. (4.12a),[9] we find

$$\gamma_0 \;=\; \frac{6}{5}\,\frac{C_F}{\beta} \qquad S_Z \;=\; 0\ , \tag{4.14a}$$

$$\;=\; \frac{7}{5}\,\frac{C_F}{\beta} \qquad\quad =\pm 1\ . \tag{4.14b}$$

Therefore, the leading anomalous dimension for a deuteron state is given by

$$\min(e_1,e_2) \;=\; \frac{3}{4}\,\frac{C_F}{\beta} \qquad \text{for} \quad S_Z \;=\; 0 \ , \tag{4.15a}$$

$$\;=\; \frac{7}{8}\,\frac{C_F}{\beta} \qquad\qquad\quad =\ \pm 1 \ . \tag{4.15b}$$

Using the result of Eq. (4.15), one can calculate the asymptotic deuteron form factor $F_d(Q^2)$. The QCD prediction for the asymptotic Q^2–behavior of the deuteron reduced form factor[20] $f_d(Q^2)$ defined by

$$f_d(Q^2) \;=\; \frac{F_d(Q^2)}{F_N^2\left(\frac{Q^2}{4}\right)}$$

is given by

$$f_d(Q^2) \;\sim\; \frac{\alpha_s(Q^2)}{Q^2}\left(\ell n\,\frac{Q^2}{\Lambda^2}\right)^{C_F/2\beta} \quad . \tag{4.16}$$

5. APPLICATION: THE EFFECTIVE FORCE BETWEEN BARYONS

In Section 4, we have shown how we can solve QCD evolution equations in order to predict the short distance behavior of multiquark systems using Young diagrammatic methods. Since the eigensolutions obtained in this way have definite permutation symmetry, we can apply the fractional parentage technique[18] for the multibaryon system in order to relate the eigensolutions to cluster representations which have physical baryon, or alternatively, "hidden-color" degrees of freedom.

For example, if we apply this technique to the simple case of the four-quark system under $\mathrm{SU}(2)_\mathrm{C}$,[8] then we find the transition matrix given by Table II ($\mathrm{T}=\mathrm{S}=0$ case) which relates the symmetry basis represented by four-quark eigensolutions and the physical basis represented by "toy"-dibaryon and hidden-color degrees of freedom. From this table we can expand the distribution amplitudes of the physical basis in terms of eigensolutions:

$$\begin{aligned}
\phi_{NN}(x_i,Q) &= 0.07\phi_1(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{0.13\,C_F/\beta} - 0.64\phi_2(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{-0.06\,C_F/\beta} + \ldots \\
\phi_{\Delta\Delta}(x_i,Q) &= -0.07\phi_1(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{0.13\,C_F/\beta} - 0.59\phi_2(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{-0.06\,C_F/\beta} + \ldots \\
\phi_{CC}(x_i,Q) &= -0.70\phi_1(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{0.13\,C_F/\beta} - 0.35\phi_2(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{-0.06\,C_F/\beta} + \ldots
\end{aligned} \tag{5.1}$$

where $C_F = 3/4$ in this case.

Table II. The relationship between four-quark antisymmetric SU(2) color representations and effective two-cluster representations (T = S = 0 case). Isospin singlet and triplet states both with color singlet are denoted N and Δ, while color triplet state is represented by C. The square and curly brackets represent orbital (O) and spin-isospin (TS) symmetries separately.

	[4] {22}	[22] {22}	[22] {4}
NN	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
$\Delta\Delta$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
CC	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0

Thus, we find that the NN, $\Delta\Delta$ and CC states have completely different Q^2 evolution. As Q^2 goes to infinity, the NN and $\Delta\Delta$ components are negligible but the CC components are large. In other word, the dominant degrees of freedom at the origin of the dibaryon system at zero impact separation are hidden-color states rather than physical baryon states. This indicates that the physical dibaryons have a repulsive core at the origin[21] while the colorful hidden-color clusters behave as in an attractive well. In this way, we derive constraints on the effective force between two baryons.[22] We discuss the results for the six-quark states using the realistic $SU(3)_C$ in the next section.

6. DISCUSSIONS AND CONCLUSIONS

By using a new method based on completely antisymmetric representations, we have analyzed the quark distribution amplitudes $\phi(x_i, Q)$ in QCD in order to predict the short distance behavior of multiquark systems. Since the new method is based on permutation symmetry, we can readily classify the multiquark systems. In the 3-quark case, we can resolve the N and Δ form factors. In the multibaryon system, this method is essential since it cannot be guaranteed that all quarks have different quantum numbers.

We have also decomposed the multiquark systems into multibaryon physical components and hidden color components, and expanded each component in terms of the QCD eigensolutions. Through the evolution of each components we can derive constraints on the effective force between the clusters. Using the toy–$SU(2)_C$–dibaryon analysis, we find that colorless clusters tend to be repulsive but colorful clusters are attractive at short distances.

The deuteron state which has the leading anomalous dimension is related to the NN, $\Delta\Delta$, and hidden color (CC) physical bases, for both the $(TS) = (01)$ and (10) cases with Young symmetry of {33}, by the formula[17]

$$\psi_{[6]\{33\}} = \sqrt{\frac{1}{9}}\,\psi_{NN} + \sqrt{\frac{4}{45}}\,\psi_{\Delta\Delta} + \sqrt{\frac{4}{5}}\,\psi_{CC} \quad .$$

Thus the physical deuteron state, which is mostly ψ_{NN} at large distance, must evolve to the $\psi_{[6]\{33\}}$ state when the six-quark transverse separations $b_\perp^i \leq O(1/Q) \to 0$. Since this state is 80% hidden color, the deuteron wave function cannot be described solely by the meson-nucleon isobar degrees of freedom in this domain. The fact that the six-quark color singlet state inevitably evolves in QCD to a dominantly hidden color configuration at small transverse separation also has implications for the form of the nucleon-nucleon ($S_z = 0$) potential, which can be considered as one interaction component in a coupled scattering channel system. As the two nucleons approach each other, the system must do work in order to change the six-quark state to a dominantly hidden color configuration; *i.e.*, QCD requires that the nucleon-nucleon potential must be repulsive at short distances [see Fig. 2].[20] Finally, we note that the evolution equation for the six-quark system suggests that the distance where this change occurs is in the domain where $\alpha_s(Q^2)$ most strongly varies.

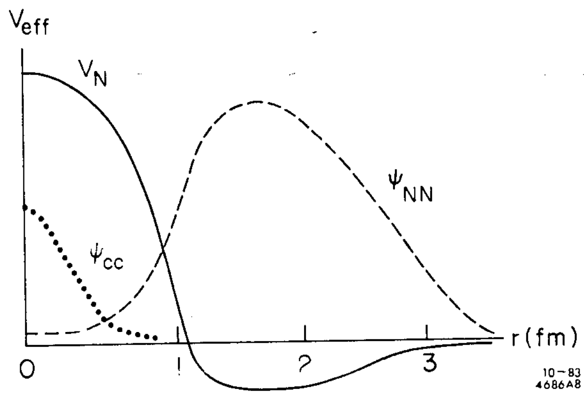

Fig. 2. Schematic representation of the deuteron wave function in QCD indicating the presence of hidden color six-quark components at short distances.

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