SLAC-PUB-3771
RU85/B/115
September 1985
(M)

ON THE STABILITY OF BACKGROUND SOLUTIONS IN CONFORMAL GRAVITY.*

By

I. Antoniadis †
Stanford Linear Accelerator Center
Stanford University, Stanford, CA. 94305

J. Iliopoulos ††
Laboratoire de Physique Theorique, Ecole Normale
Superieure, Paris, France

T. N. Tomaras †††
Physics Department
Rockefeller University

Abstract:
In the context of conformal gravity with spontaneously broken local conformal invariance we prove a non-renormalization theorem for any fixed value of the cosmological constant and we study the classical stability of the background solutions which correspond to maximally symmetric spaces.

(Published in Nuclear Physics B261 (1985) pp. 157-171)

* Work supported in part by the Department of Energy Contract DE-AC028ER40033B.000.

† On leave of absence from Centre de Physique Theoreque, Ecole Polytechnique, Palaiseau, France. Supported in part by the Department of Energy, contract DE-AC03-76SF00515.

†† Laboratoire propre du CNRS associe a l'Ecole Normale Superieure et a l'Universite de Paris Sud.

††† Address after August 1, 1985, Physics Department, University of Crete, Iraklion, Crete, Greece.

I. Introduction

Einstein's theory of general relativity is a highly successful theory of gravity. Its only problem, at the classical level, is that it offers no hint to understand the vanishingly small value of the cosmological constant. Indeed, the study of clusters of galaxies with average mass density $\rho \sim 10^{-29} gr/cm^3$ puts an upper bound on the physical value of $|\Lambda| < 10^{-29}$ gr/cm^3 or 10^{-57} cm^{-2} in units $G_N = c = 1$ [1].

At the classical level, one usually tries to explain the absence of a certain term by finding an exact symmetry which forbids its appearance in the lagrangian. No such symmetry is known to exist for the cosmological constant in general relativity. A second approach is to study whether the theory for some values of Λ, becomes classically unstable leading to tachyonic modes in fluctuations around the corresponding classical solutions. For the case of pure gravity with $\mathcal{L} = -\sqrt{-g}\,(R+\Lambda)$ we are interested in solutions corresponding to maximally symmetric spaces since they seem to be the only sensible ones, having a ten-dimensional isometry group. The result [2] is negative: All such spaces (de Sitter, Minkowski or anti-de Sitter) are stable in the above sense and there is nothing special with the value $\Lambda=0$.

At the quantum level the situation is much more problematic. Einstein's theory, because of its bad ultraviolet behaviour, is non-renormalizable and has resisted all attempts to quantization. The problem of the cosmological constant cannot be meaningfully formulated because the vacuum energy is usually divergent in quantum field theory. Global supersymmetry gives naturally vanishing vacuum energies but it is not clear how to exploit this property in the presence of gravity and after supersymmetry breaking.

Because of all these problems people occasionally study alternative theories of gravity and a prominent one among them is Weyl's conformally invariant theory. In this paper we want to report on some results pertaining to the above questions. We show that in theories in which local conformal invariance is spontaneously broken, a certain region of values of the cosmological constant which includes the value $\Lambda=0$, may be privileged because every value in it does not get renormalized by higher order corrections and all other values give, formally, classically unstable solutions.

The organization of the paper is as follows: In section II we describe the theory, its symmetries as well as its well-known defects. In section III we generalize the discussion of Ref. [3] and derive the Ward identities for arbitrary background space-times. We show that maximally symmetric backgrounds are the only ones which are stable against higher order corrrections and we prove for them a non-renormalization theorem which gives $\langle h_{\mu\nu}\rangle=0$ for the graviton tadpole. In other words the value of the cosmological constant does not get renormalized. Section IV contains the analysis of the classical stability of the solutions. The result is that stability against exponentially fast growing fluctuations implies bounds for the value of the cosmological constant of the form $\Lambda_0 \leq \Lambda \leq 0$ i.e. the stable backgrounds are the Minkowski flat space as well as anti-de Sitter spaces with not too large value of the cosmological constant. Λ_0 is given in terms of the other parameters of the theory.

Section II. The Theory.

The theory we will be discussing in this paper is, essentially, Weyl gravity[4] coupled to matter, with spontaneously broken local conformal invariance. The field content of the gravity sector is the metric $g_{\mu\nu}(x)$ (or the vierbein $e^{\alpha}{}_{\mu}(x)$) and a scalar, $\phi(x)$, called "the dilaton". The introduction of the latter enables us to write a lagrangian with manifest local conformal symmetry. Since any theory of scalar, spinor and vector fields can be coupled to $g_{\mu\nu}$ and ϕ in a way consistent with general coordinate transformation and local conformal invariances we do not have to specify its exact content in matter fields. Thus for our purposes they will be denoted collectively by $\Psi(x)$ and we will occasionally refer to them only for completeness, since they are not going to affect our reasoning and conclusions in any way.

The dynamics is described by the lagrangian:

$$\mathcal{L} = \sqrt{-g}\left\{-\frac{1}{G^2}C_{\mu\nu\lambda\rho}{}^2 + \frac{\gamma}{72}\left(R - 6\frac{\Box\phi}{\phi}\right)^2 - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - \right. \qquad (2.1)$$
$$\left. - \frac{1}{12}R\phi^2 + \lambda\phi^4 \right\} + \mathcal{L}_{\text{Matter}}$$

where the Weyl tensor $C^{\mu}{}_{\alpha\beta\gamma}$ is defined in n-dimensions by

$$C^{\mu}{}_{\alpha\beta\gamma} \equiv R^{\mu}{}_{\alpha\beta\gamma} - \frac{1}{n-2}\left(\delta^{\mu}_{\beta}R_{\alpha\gamma} - \delta^{\mu}_{\gamma}R_{\alpha\beta} - g_{\alpha\beta}R^{\mu}_{\gamma} + g_{\alpha\gamma}R^{\mu}_{\beta}\right) - \qquad (2.2)$$
$$- \frac{1}{(n-1)(n-2)}R\left(\delta^{\mu}_{\gamma}g_{\alpha\beta} - \delta^{\mu}_{\beta}g_{\alpha\gamma}\right).$$

and $\Box\phi$ denotes the covariant box acting on $\phi(x)$.[f1] In order for (2.1) to reproduce Einstein's theory at large distances, we must choose $\gamma>0$.

The two invariants C^2 and R^2 exhaust the list of independent, quadratic in curvature terms. One might imagine that $C^2{}_{\mu\nu\alpha\rho}$, R^2, $R_{\mu\nu}{}^2$ and $R_{\mu\nu\alpha\rho}{}^2$ should all be included in $\mathcal{L}$. But, using the identity $C_{\alpha\beta\gamma\delta}^2 = R_{\mu\nu\alpha\gamma}{}^2 - 2R^2{}_{\mu\nu} + 1/3\ R^2$ (or its generalization in n dimensions) as well as the Gauss-Bonnet theorem to express $\int\sqrt{-g}\ R_{\mu\nu\lambda\rho}{}^2\ d^4x = \int\sqrt{-g}\ (4R_{\mu\nu}{}^2 - R^2)\ d^4x$ [f2] we conclude that only two of them are independent and $\mathcal{L}$ in (2.1) is the most general lagrangian with up to four derivatives and with the two invariances which will mostly concern us here, namely, general coordinate transformation (GCT) and local conformal (LC).

The action of the GCT with parameters $\omega^\mu(x)$ and the LC with parameter $\Omega(x)$ is given by

$$\Delta g_{\mu\nu} = - g_{\mu\rho}\partial_\nu\omega^\rho - g_{\nu\rho}\partial_\mu\omega^\rho - \omega^\rho\partial_\rho g_{\mu\nu}$$

$$\Delta\phi = -\omega^\rho\partial_\rho\phi \tag{2.3}$$

and

$$\delta g_{\mu\nu} = 2\Omega(x)\, g_{\mu\nu}$$

$$\delta\phi = -\Omega(x)\,\phi \tag{2.4}$$

Let us assume, for simplicity, that $\mathcal{L}_M$ does not contain scalar fields or if it does they have zero VEV. (In the opposite case the role of the dilaton would be played by a linear combination of $\phi(x)$ and the scalar field(s) with VEV $\neq 0$ from the matter sector). Setting $\Psi(x) = 0$ we trivially satisfy the $\delta S/\delta\Psi(x) = 0$ equations of motion, while it is straightforward, although a little tedious, to derive the remaining field equations:

$$\frac{\delta S}{\delta g_{\mu\nu}(x)} = \sqrt{-g}\Big\{-\frac{1}{G^2}\Big[2\left(R^{\mu\alpha;\nu}{}_{\alpha} + R^{\nu\alpha;\mu}{}_{\alpha}\right) - 2R^{\mu\nu;\alpha}{}_{\alpha} - \frac{4}{3}R^{;\mu\nu} +$$

$$+ \frac{1}{3}g^{\mu\nu}R^{;\alpha}{}_{\alpha} - 4R^{\mu\alpha}R^{\nu}{}_{\alpha} + \frac{4}{3}RR^{\mu\nu} + g^{\mu\nu}\left(R_{\alpha\beta}{}^2 - \frac{1}{3}R^2\right)\Big] +$$

$$+\gamma\Big[\frac{1}{36}\left(R^{;\mu\nu} - g^{\mu\nu}R^{;\alpha}{}_{\alpha} - RR^{\mu\nu} + \frac{1}{4}g^{\mu\nu}R^2\right) + \frac{1}{6}R^{\mu\nu}\frac{\Box\phi}{\phi} - \frac{1}{6}\left(\frac{\Box\phi}{\phi}\right)^{;\mu\nu} +$$

$$+\frac{1}{6}g^{\mu\nu}\left(\frac{\Box\phi}{\phi}\right)^{;\alpha}{}_{\alpha} - \frac{1}{4}g^{\mu\nu}\left(\frac{\Box\phi}{\phi}\right)^2 + \frac{1}{12}g^{\mu\nu}\partial^{\alpha}\left(\frac{R}{\phi}\right)\partial_{\alpha}\phi -$$

$$-\frac{1}{12}\left(\partial^{\mu}\frac{R}{\phi}\partial^{\nu}\phi + \partial^{\nu}\frac{R}{\phi}\partial^{\mu}\phi\right) - \frac{1}{2}g^{\mu\nu}\partial^{\alpha}\left(\frac{\Box\phi}{\phi^2}\right)\partial_{\alpha}\phi +$$

$$+\frac{1}{2}\left(\partial^{\mu}\left(\frac{\Box\phi}{\phi^2}\right)\partial^{\nu}\phi + \partial^{\nu}\left(\frac{\Box\phi}{\phi^2}\right)\partial^{\mu}\phi\right)\Big] +$$

$$+\frac{1}{2}\partial^{\mu}\phi\,\partial^{\nu}\phi - \frac{1}{4}g^{\mu\nu}(\partial_{\alpha}\phi)^2 + \frac{1}{12}\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right)\phi^2 +$$

$$+\frac{1}{12}g^{\mu\nu}(\Box\phi^2) - \frac{1}{12}(\phi^2)^{;\mu\nu} + \frac{1}{2}\lambda g^{\mu\nu}\phi^4\Big\} = 0 \qquad (2.5)$$

and

$$\frac{\delta S}{\delta\phi(x)} = \sqrt{-g}\Big\{\gamma\Big[\frac{R}{3}\frac{\Box\phi}{\phi^2} - \frac{R}{3}\frac{(\partial_{\mu}\phi)^2}{\phi^3} - \frac{1}{6}\frac{\Box R}{\phi} + \frac{1}{3\phi^2}\partial_{\alpha}R\,\partial^{\alpha}\phi - \qquad (2.6)$$

$$-\frac{3}{\phi^3}(\Box\phi)^2 + \frac{6}{\phi^4}(\partial_{\mu}\phi)^2\Box\phi - \frac{4}{\phi^3}\partial_{\mu}\Box\phi\,\partial^{\mu}\phi + \frac{1}{\phi^2}\Box^2\phi\Big] + \Box\phi - \frac{1}{6}R\phi + 4\lambda\phi^3\Big\} = 0$$

and to check that they satisfy the identity

$$g_{\mu\nu}\frac{\delta S}{\delta g_{\mu\nu}} = \frac{1}{2}\phi\frac{\delta S}{\delta\phi} \qquad (2.7)$$

as they should, due to the local conformal symmetry of $\mathcal{L}$.

Among the solutions of eqs. (2.5) and (2.6) there exists the one-parameter family of maximally symmetric spaces which can be parametrized by the constant v and are given by:

$$\phi(x) = v = \text{constant} \quad , \quad g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) \quad \text{with}$$

$$\bar{R}^{\mu\nu}(\bar{g}) = \frac{1}{4}\bar{R}(\bar{g})\,\bar{g}^{\mu\nu}(x) \quad , \quad \bar{R} = 24\lambda v^2 \tag{2.8}$$

Thus, depending on the value of λ, the "vacuum" state (2.8) is a de Sitter ($\lambda<0$), anti-de Sitter ($\lambda>0$) or Minkowski ($\lambda=0$) space-time and, for every $v \neq 0$, it spontaneously breaks the local conformal symmetry of (2.1). Any other solution, not equivalent by a coordinate reparametrization and/or a local conformal transformation to one in the family (2.8), defines a non-maximally symmetric space with fewer isometries.

We define the fluctuating fields $h_{\mu\nu}$ and $\sigma(x)$ by

$$g_{\mu\nu}(x) \equiv \bar{g}_{\mu\nu}(x) + G\,h_{\mu\nu}(x) \quad \text{and} \quad \phi(x) \equiv v + \sigma(x) \tag{2.9}$$

and expand the action in powers of them. For any $v \neq 0$ we thus avoid the singularity $\phi=0$ and obtain a well-defined theory for the small fluctuations.

Quantization of the present theory requires gauge fixing for the local symmetries (2.3) and (2.4) whose action on $h_{\mu\nu}$ and σ is

$$G\Delta h_{\mu\nu} = -\bar{D}_\mu \omega_\nu - \bar{D}_\nu \omega_\mu - G\left(h_{\mu\rho}\partial_\nu \omega^\rho + h_{\nu\rho}\partial_\mu \omega^\rho + \omega^\rho \partial_\rho h_{\mu\nu}\right) \tag{2.3'}$$

$$\Delta\sigma = -\omega^\rho \partial_\rho \sigma$$

$$G\delta h_{\mu\nu} = 2\Omega(x)\left(\bar{g}_{\mu\nu} + G h_{\mu\nu}\right) \quad ; \quad \delta\sigma = -\Omega(x)(v+\sigma) \tag{2.4'}$$

where we have defined $\omega_\mu \equiv \bar{g}_{\mu\nu}\omega^\nu$ and $\bar{D}_\mu\omega_\nu = \partial_\mu\omega_\nu + \bar{\Gamma}(\bar{g})^\lambda{}_{\mu\nu}\omega_\lambda$ for the covariant derivative on ω_μ corresponding to the background $\bar{g}_{\mu\nu}$, out of which we construct the Christoffel symbols $\bar{\Gamma}(\bar{g})^\lambda{}_{\mu\nu}$. The gauge-fixing conditions, symbolically $\Phi_\mu(h_{\alpha\beta})$ and $\Phi(h_{\alpha\beta})$, can always be chosen in such a way that the effective lagrangian

$$\begin{aligned} \mathcal{L}_{eff.} &= \mathcal{L} + \mathcal{L}_{GF} \\ \mathcal{L}_{GF} &= \frac{1}{2\zeta}\Phi_\mu\Phi_\nu\bar{g}^{\mu\nu} + \frac{1}{2\xi}\Phi^2 - \bar{c}_\mu\, s\Phi_\nu\bar{g}^{\mu\nu} - \bar{c}\, s\Phi \end{aligned} \tag{2.10}$$

leads to an action invariant under the usual BRS transformations [5] for the combined GCTs and LCTs

$$\begin{aligned} G\, s h_{\mu\nu} &= -\bar{D}_\mu c_\nu - \bar{D}_\nu c_\mu - G\left(h_{\mu\rho}\partial_\nu c^\rho + h_{\nu\rho}\partial_\mu c^\rho + c^\rho\partial_\rho h_{\mu\nu}\right) + 2c\left(\bar{g}_{\mu\nu} + G h_{\mu\nu}\right) \\ s\sigma &= -c^\rho\partial_\rho\sigma - c\,(v+\sigma) \\ s\bar{c}_\mu &= \frac{1}{\zeta}\Phi_\mu \quad , \quad s c_\mu = -G\, c^\rho\partial_\rho c_\mu \\ s\bar{c} &= \frac{1}{\xi}\Phi \quad , \quad s c = -G\, c^\rho\partial_\rho c \end{aligned} \tag{2.11}$$

as well as under the so-called "background gauge transformations" with parameters $\omega^\mu(x)$ and $\Omega(x)$

$$\begin{aligned} &\underline{\delta}\,\bar{g}_{\mu\nu} = -\bar{D}_\mu\omega_\nu - \bar{D}_\nu\omega_\mu + 2\Omega\,\bar{g}_{\mu\nu} \, , \quad \underline{\delta}\, h_{\mu\nu} = -h_{\mu\rho}\partial_\nu\omega^\rho - h_{\nu\rho}\partial_\mu\omega^\rho - \omega^\rho\partial_\rho h_{\mu\nu} + 2\Omega\, h_{\mu\nu} \\ &\underline{\delta}\, v = -\Omega\, v \quad , \quad \underline{\delta}\,\sigma = -\omega^\rho\partial_\rho\sigma - \Omega\,\sigma \\ &\underline{\delta}\, c_\mu = -\partial_\mu\omega^\nu c_\nu - \omega^\nu\partial_\nu c_\mu \quad , \quad \underline{\delta}\,\bar{c}_\mu = -\partial_\mu\omega^\nu\,\bar{c}_\nu - \omega^\nu\partial_\nu\,\bar{c}_\mu \\ &\underline{\delta}\, c = -\omega^\rho\partial_\rho c \quad , \quad \underline{\delta}\,\bar{c} = -\omega^\rho\partial_\rho\,\bar{c} \end{aligned} \tag{2.12}$$

In the above formulas $c_\mu(\bar{c}_\mu)$ and $c(\bar{c})$ are the ghosts (anti-ghosts) associated with the general coordinate and conformal gauge transformations. As always, the BRS transformation (2.11) on the original fields $h_{\mu\nu}$ and σ is of the same form as (2.3') and (2.4') with parameters equal to the corresponding ghost fields. It is straightforward to extend (2.10) to (2.12) so as to include matter fields as well as more gauge symmetries. But since, as it will become apparent, these considerations do not affect any of the conclusions of this paper, they will be omitted in order to simplify our pesentation. Notice that the "background gauge transformations" act as if every index $\mu, \nu, \ldots$ carried by any field $\bar{g}_{\mu\nu}$, $h_{\mu\nu}$, $c_\mu, \ldots$ is a vector index, so that quantities like for instance, $\bar{c}_\mu c_\nu \bar{g}^{\mu\nu}$ are invariant under the $\omega\rho$ part of (2.12). Finally, an example of gauge conditions Φ_μ, Φ which achieve what we claimed above is

$$\Phi_\mu = \bar{D}_\nu h^\nu{}_\mu \quad , \quad \Phi = \bar{g}^{\mu\nu} h_{\mu\nu} \tag{2.13}$$

Before we proceed to the investigation of the stability of the ground state (2.8), we would like to make a few comments about the theory. First of all, it is known [6,3] that for $\bar{g}_{\mu\nu}=\eta_{\mu\nu}$ it is power-counting renormalizable. Since renormalizability has to do with the short distance behaviour of the theory, we expect it to persist even in the presence of any smooth background $\bar{g}_{\mu\nu}(x)$, in particular, in the "vacuum"(2.8). The large distance behaviour of (2.1) dominated by the $\sqrt{-g}\, R\langle\phi\rangle^2$ term, leads to agreement with the Einstein theory if and only if we take $\langle\phi\rangle \sim M_{Pl}$. In a realistic theory of nature, more scalar fields with VEVs much smaller than the Planck mass are present and the requirement for this to happen naturally, leads to the celebrated gauge hierarchy or fine tuning problem. Extension of the theory to conformal supergravity might lead to some kind of a solution, but we won't discuss it any further here.

The most important drawback of Weyl's theory, which has prevented it from having any physical application, is the fact that renormalizability has been achieved at the price of introducing unphysical degrees of freedom with negative metric. Both the dilaton field and the graviton propagators contain ghost poles. Perturbation expansion does not lead to a unitary theory. In spite of this, some non-perturbative arguments [7,8] tend to indicate that the spectrum may consist of only positive metric states, implying unitarity for the resulting S-matrix. Finally, if one introduces N matter fields and considers the theory in the 1/N expansion [9,3], one finds that the negative metric massive pole of the graviton propagator is plit into a pair of complex conjugate poles in the first sheet. Application of the Lee-Wick method [10] may again yield a unitary theory. Although we do not consider this procedure as a satisfactory solution to the uitarity problem, we may appeal to it occasionally as an example of a prescription of how to deal with the unphysical poles in the theory. It is obvious that, unless a physically acceptable solution to the unitarity problem is found, all investigations of the properties of conformally invariant gravity theories are little more than academic exercises.

Section III. Ward identities and a non-renormalization theorem

In this section we will prove that, as a consequence of the GCT and the spontaneously broken LC symmetries of the theory, the quantum corrections to the $h_{\mu\nu}$ tadpole vanish automatically if and only if the background space-time $(\bar{g}_{\mu\nu}(x), \bar{\phi}(x))$ we expand around is maximally symmetric. This result has been established in [3] for the Minkowski case ($\lambda=0$) and here we generalize it to arbitrary de Sitter or anti-de Sitter ground states. Our discussion is going to be formal in two ways: (i) It will apply only to background solutions around which a consistent perturbation expansion can be established. (ii) We shall rely upon the validity of the Ward identities. There exists a regularization scheme which preserves both coordinate and local conformal invariance [11], but its consistency has not been proven to all orders of perturbation.

We begin by deriving the Ward identities associated with the transformations (2.11) and (2.12). The regularization scheme of ref [11] amounts to going to n<4 dimensions and, with the help of the dilaton field, writing $\mathcal{L}_{eff}$ in a way invariant under local conformal transformations. In n-dimensions the latter take the form:

$$G\,\delta h_{\mu\nu} = 2\,\Omega(x)\left(\bar{g}_{\mu\nu} + G\,h_{\mu\nu}\right) \tag{3.1}$$
$$\delta\sigma = -\frac{n-2}{2}\,\Omega(x)\,(v+\sigma)$$

The BRS transformations (2.11) change, accordingly, to

$$G\,s h_{\mu\nu} = -\bar{D}_\mu c_\nu - \bar{D}_\nu c_\mu - G\left(h_{\mu\rho}\partial_\nu c^\rho + h_{\nu\rho}\partial_\mu c^\rho + c^\rho\partial_\rho h_{\mu\nu}\right) + 2c\left(\bar{g}_{\mu\nu} + G\,h_{\mu\nu}\right) \tag{3.2}$$
$$s\sigma = -c^\rho\partial_\rho\sigma - \frac{n-2}{2}c\,(v+\sigma)$$
$$s\bar{c}_\mu = \frac{1}{\eta}\Phi_\mu\,,\quad s c_\mu = -G c^\rho\partial_\rho c_\mu\,,\quad s\bar{c} = \frac{1}{\xi}\Phi\,,\quad s c = -G c^\rho\partial_\rho c\,.$$

Furthermore, the use of renormalization conditions which respect the symmetries implies that the Ward identities we shall derive below for the bare Green functions are valid for the renormalized ones as well. For the form of $\mathcal{L}_{eff}$ in n-dimensions ($\mathcal{L}^{(n)}{}_{eff}$) we refer the reader to refs [6 and 3§4].

As usual, to derive Ward identities one starts with the generating functional $W[J_F, J_F^s]$ for the connected Green functions

$$e^{iW[J_F,J_F^s]} \equiv \int\Big(\prod_F dF\Big)\, e^{i\int d^n x\left(\mathcal{L}^{(n)}_{eff} + \sum_F J_F \cdot F + \sum_{F\neq \bar{c}_a} J_F^s \cdot sF\right)} \tag{3.3}$$

where generically, J_F and J_F^s are the sources for the field F and its BRS transform sF respectively. A Legendre transformation leads to the generating functional $\tilde{\Gamma}[\hat{F}, J_F^s]$ of the one-particle irreducible (1-PI) Green functions. Instead of $\tilde{\Gamma}$ and in order to simplify the form of the Ward identities we define the related quantity[f3]

$$\Gamma[\hat{F}, J_F^s] \equiv W[J_F, J_F^s] - \int d^n x\left\{\sum_F J_F \hat{F} + \frac{1}{2J}\hat{\Phi}_\mu \hat{\Phi}_\nu \bar{g}^{\mu\nu} + \frac{1}{2\xi}\hat{\Phi}^2\right\} \tag{3.4}$$

with the classical field $\hat{F}(x)$ defined by

$$\hat{F}(x) \equiv \frac{\delta}{\delta J_F(x)} W[J_F, J_F^s] \tag{3.5}$$

The equations of motion are

$$\frac{\delta W}{\delta J_F^s} = \frac{\delta \Gamma}{\delta J_F^s} \;;\; J_F = -\frac{\delta \Gamma}{\delta \hat{F}} - \frac{1}{J}\frac{\delta \hat{\Phi}_\mu}{\delta \hat{F}}\hat{\Phi}_\nu \bar{g}^{\mu\nu} - \frac{1}{\xi}\frac{\delta \hat{\Phi}}{\delta \hat{F}}\hat{\Phi} \;,\; F \neq \bar{c}_a, c_a \tag{3.6}$$

$$J_{c_a} = \frac{\delta \Gamma}{\delta \hat{c}_a} \;;\; J_{\bar{c}_a} = \frac{\delta \Gamma}{\delta \hat{\bar{c}}_a}$$

($c_a, \bar{c}_a$ stand for all the ghosts and antighosts $c_\mu, \bar{c}_\mu, c, \bar{c}$).

We now change the variables of integration in (3.3) à la BRS from F to $F+sF\eta$(η is the anticommuting parameter of the BRS transformation). The integral (3.3) remains the same. The Jacobian of this particular transformation is one, while using the fact that $s^2F=0$ for any $F\neq\bar{c}_a$, we conclude that only the source terms $\sum J_F F$ in the integrand are affected by the above change of variables. We are thus led to the Ward identities

$$\int d^n x \left\langle \sum_F J_F\, sF \right\rangle \equiv \int \left(\prod_F dF\right) \int d^n x \left(\sum_F J_F\, sF\right) e^{iS_{eff}^{(n)} + i\int d^n x \left(\sum_F J_F F + \sum_{F\neq\bar{c}_a} J_F^s\, sF\right)} = 0 \tag{3.7}$$

The combination of (3.7) with the antighost equations of motion

$$\left\langle \frac{\delta}{\delta\bar{c}_a(x)} \right\rangle = \left\langle \frac{\delta S_{eff}^{(n)}}{\delta\bar{c}_a(x)} + J_{\bar{c}_a}(x) \right\rangle = 0 \tag{3.8}$$

yields the more familiar form:

$$\int d^n x \sum_{F\neq\bar{c}_a} \frac{\delta\Gamma}{\delta\hat{F}(x)} \frac{\delta\Gamma}{\delta J_F^s(x)} = 0 \tag{3.9a}$$

$$\int d^n y \left(\sum_{F\neq c_b, \bar{c}_b} \frac{\delta\hat{\Phi}_a(y)}{\delta\hat{F}(x)} \frac{\delta\Gamma}{\delta J_F^s(y)} \right) + \frac{\delta\Gamma}{\delta\hat{\bar{c}}_a(x)} = 0 \tag{3.9b}$$

Finally, we note that the ghost number N_c with $N_c(\hat{c}_a)=1$, $N_c(\hat{\bar{c}}_a)=-1$, $N_c(J^s_{c_a})=-2$, $N_c(\hat{F})=0$ and $N_c(J^s_F)=-1$ for $F\neq c_a, \bar{c}_a$ is conserved.

We differentiate (3.9a) with respect to $\hat{\bar{c}}(y)$, we set all $\hat{F}$'s and J_F^s's equal to zero and use the ghost number conservation, to obtain

$$\int d^4x \left\{ \frac{\delta\Gamma}{\delta\hat{h}_{\mu\nu}(x)} \frac{\delta^2\Gamma}{\delta\hat{\bar{c}}(y)\,\delta J^{s\mu\nu}(x)} + \frac{\delta\Gamma}{\delta\hat{\sigma}(x)} \frac{\delta^2\Gamma}{\delta\hat{\bar{c}}(y)\,\delta J_\sigma^s(x)} \right\}_{\hat{F}=J_F^s=0} = 0$$

By definition, $\hat{\sigma}=0$ is the value of $\hat{\sigma}$ for which the dilaton tadpole vanishes. Furthermore, the 2-point function $\delta^2\Gamma/\delta\hat{\bar{c}}(y)\,\delta J^{s\mu\nu}(x)\,\big|_{\hat{F}=J_F^s=0}$ due to the specific form of $sh_{\mu\nu}$ is equal to $2\,\bar{g}_{\mu\nu}(x)\,\delta^4(x-y)$ plus another term which we will call $f_{\mu\nu}(x,y)$ and which vanishes at the tree level. We are thus led to

$$\left\{ \frac{\delta\Gamma}{\delta\hat{h}_{\mu\nu}(x)}\, 2\,\bar{g}_{\mu\nu}(x) + \int d^4y\, \frac{\delta\Gamma}{\delta\hat{h}_{\mu\nu}(y)}\, f_{\mu\nu}(x,y) \right\}_{\hat{F}=J_F^s=0} = 0 \qquad (3.10)$$

The last important ingredient that we shall need in our discussion is the intuitively obvious fact that for any maximally symmetric ground state (2.8) and only for them

$$\left.\frac{\delta\Gamma}{\delta\hat{h}_{\mu\nu}(x)}\right|_{\hat{F}=J_F^s=0} = A\sqrt{-\bar{g}(x)}\;\bar{g}^{\mu\nu}(x) \quad , A=\text{constant} \qquad (3.11)$$

which we prove next. (3.11) expresses the fact that our theory has the "background gauge transformation" invariance (2.12).

We start with the generating functional W' of the connected Green's function with only graviton and dilaton external lines.

$$e^{iW'} \equiv \int \Big(\prod_F dF\Big)\, e^{i S_{eff} + i\int d^n x\,(J^{\mu\nu}h_{\mu\nu} + J\sigma)} \qquad (3.12)$$

A change of the variables of integration according to (2.12) leads to the Ward identity for the renormalized Green's functions.

$$\int d^4x \left\langle J^{\mu\nu}(x)\left(\partial_\nu \omega^\rho h_{\mu\rho} + \partial_\mu \omega^\rho h_{\nu\rho} + \omega^\rho \partial_\rho h_{\mu\nu}\right) + J\,\omega^\rho \partial_\rho \sigma \right\rangle = 0$$

for any set of functions $\omega^\mu(x)$.

In terms of the classical fields $\hat{F}(x)$ and the generating functional $\tilde{\Gamma} = W' - \int d^n x\, J_F \hat{F}$ $\left(\hat{F} \equiv \frac{\delta W'}{\delta J_F}\right)$ of the 1-PI vertices of the theory, the above identity becomes

$$\int d^4x \left\{ \frac{\delta\tilde{\Gamma}}{\delta \hat{h}_{\mu\nu}(x)} \left(\partial_\nu \omega^\rho \hat{h}_{\mu\rho} + \partial_\mu \omega^\rho \hat{h}_{\nu\rho} + \omega^\rho \partial_\rho \hat{h}_{\mu\nu}\right) + \frac{\delta\tilde{\Gamma}}{\delta\hat{\sigma}(x)}\,\omega^\rho \partial_\rho \hat{\sigma} \right\} = 0 \qquad (3.13)$$

We functionally differentiate (3.13) with respect to $\hat{h}_{\alpha\beta}$ and evaluate the result for $\hat{h}_{\mu\nu} = \hat{\sigma} = 0$. This leads to the following identity for the tensor field $t^{\alpha\beta}(x)$ defined by $\left.\frac{\delta\tilde{\Gamma}}{\delta\hat{h}_{\alpha\beta}(x)}\right|_{\hat{h}_{\alpha\beta}=0=\hat{\sigma}} \equiv \sqrt{-\bar{g}}\; t^{\alpha\beta}(x)$

$$\partial_\rho \omega^\alpha t^{\beta\rho} + \partial_\rho \omega^\beta t^{\alpha\rho} - \omega^\rho \partial_\rho t^{\alpha\beta} = t^{\alpha\beta} \frac{1}{\sqrt{-\bar{g}}}\, \partial_\rho\left(\sqrt{-\bar{g}}\,\omega^\rho\right), \quad \forall\, \omega^\mu(x) \qquad (3.14)$$

For a maximally symmetric background space-time like (2.8), there exist 10 Killing vector fields $\xi_{(i)}{}^\rho$ $i=1,2,\ldots,10$, which by definition satisfy $\xi^{(i)}{}_{\mu;\nu} + \xi^{(i)}{}_{\nu;\mu} = 0$ $(\xi_\nu \equiv \bar{g}_{\mu\nu}\xi^\mu$ and the semicolon denotes covariant differentiation for the metric $\bar{g}_{\mu\nu})$. Consequently, $\xi^{(i)\mu}{}_{;\mu} = (1/\sqrt{-\bar{g}})\,\partial_\mu(\sqrt{-\bar{g}}\;\xi^{(i)\mu}) = 0$ and for $\omega^\mu = \xi_{(i)}{}^\mu$ (3.14) reduces to

$$\partial_\rho \xi_{(i)}{}^\alpha t^{\beta\rho} + \partial_\rho \xi_{(i)}{}^\beta t^{\alpha\rho} - \xi_{(i)}{}^\rho \partial_\rho t^{\alpha\beta} = 0 \quad \forall i = 1,2,\ldots,10 \qquad (3.15)$$

Eqn.(3.15) is precisely the statement that $t^{\alpha\beta}(x)$ is a maximally form invariant, 2-index symmetric tensor field on the space-time (2.8), which is equivalent to [12]

$t^{\alpha\beta}(x) = A\ \bar{g}^{\alpha\beta}(x)$, A=constant

The meaning of (3.11) is that in a maximally symmetric space the tensors $\bar{R}_{\mu\nu}$, $\bar{R}_{\mu\nu\lambda\rho}\ldots$, are all expressible in terms of the $\bar{g}_{\mu\nu}$ alone [12] and since, furthermore, $\bar{R}$=constant, a symmetric tensor $t_{\mu\nu}$ has to be proportional to $\bar{g}_{\mu\nu}$. This is not the case for a non-maximally symmetric space-time and this concludes the proof of statement (3.11).

For a maximally symmetric background we now use (3.11) into (3.10) to obtain:

$$A\left\{ 8\sqrt{-\bar{g}(x)} + \int d^4y \sqrt{-\bar{g}(y)}\ \bar{g}^{\mu\nu}(y)\ f_{\mu\nu}(x,y) \right\}_{\hat{F}=J^s_F=0} = 0 \qquad (3.16)$$

The quantity in curly brackets is non-zero. The reason is that the second term starts at the 1-loop level and cannot cancel the first. (3.16) then implies that A must be zero, i.e. that

$$\left.\frac{\delta\Gamma}{\delta\hat{h}_{\mu\nu}(x)}\right|_{\hat{F}=J^s_F=0} = 0 \qquad (3.17)$$

to all orders of any consistent perturbation expansion.

At any order of perturbation theory the $h_{\mu\nu}$-tadpole vanishes in the ground state (2.8) of our theory. We do not have to "shift" the background $\bar{g}_{\mu\nu}$ corresponding to some value of the cosmological constant (curvature), order-by-order to achieve $\langle h_{\mu\nu}\rangle=0$.

As the title of this section indicates, this is a non-renormalization theorem. At this stage, every value of the cosmological constant is possible. Perturbation theory does not favor any-one in particular. What we have shown is that there exists a way to organize the expansion, given by the regularization scheme of ref. [11], such that quantum effects do not alter the value we choose at the classical level. This result does not solve the problem of the vanishing cosmological constant we stated in the introduction. However, if this theory is proven to be physically acceptable, it may offer the framework to stabilize such a solution if one is found. We shall address this question again in the next section.

Notice finally that the above conclusion is not true in a non-maximally-symmetric background. Now $\langle h_{\mu\nu}\rangle \neq 0$ and any such ground state cannot be technically natural . But as we explained in the introduction, non-maximally symmetric solutions are not physically interesting. The only sensible ground states of our world must be Einstein spaces with at least seven isometries corresponding to the locally observed energy, momentum and angular momentum conservation laws. All such spaces are necessarily maximally symmetric [12].

IV. Classical stability

The non-renormalization theorem of the previous section states that any particular choice of the curvature $\bar{R}=24\lambda v^2$ in (2.8) is a technically natural one. In this section we shall study the classical stability of any maximally symmetric solution against small fluctuations of the dilaton and graviton fields. The hope is that by doing so we shall be able to discriminate among the possible values of $\bar{R}$.

Our starting point is the classical lagrangian (2.1). It depends on the fields $g_{\mu\nu}(x)$ and $\phi(x)$ as well as their first and second derivatives. We prefer to work with lagrangians containing only first derivatives and this can be easily achieved with the introduction of two auxiliary fields. We start with the dilaton field $\phi(x)$. Using one auxiliary field $F(x)$ the part of (2.1) which contains $\phi(x)$ can be written as:

$$\mathcal{L}_\phi = \sqrt{-g}\left\{-\frac{1}{2}(\partial_\mu\varphi)^2 + \frac{1}{2}(\partial_\mu F)^2 - \frac{R}{12}(\varphi^2 - F^2) - \frac{1}{2\gamma}F^2(\varphi+F)^2 + \right. \\ \left. + \lambda(\varphi+F)^4 \right\} \tag{4.1}$$

where we have put $\varphi = \phi - F$ and the F equation of motion yields:

$$F = -\frac{\gamma}{\phi^2}\left(\Box\phi - \frac{R}{6}\phi\right) \tag{4.2}$$

The two theories (2.1) and (4.1) are classically equivalent. At the quantum level the equivalence is not maintained because the Jacobian of the transformation in the functional integral is not equal to one. We can restore the equivalence by adding a term equal to $\bar{z}(\varphi+F)z$ with z and $\bar{z}$ being ghost fields. However, for the discussion of the classical stability, this term is irrelevant and it will be dropped.

In terms of the new fields a maximally symmetric solution is given by:

$$g_{\mu\nu}(x)=\bar{g}_{\mu\nu}(x)\ ;\quad \varphi=\bar{\varphi}=(1-4\lambda\gamma)v\ ;\quad F=\bar{F}=4\lambda\gamma v \tag{4.3}$$

with, as before, $\bar{R}=24\lambda v^2$. The fluctuating fields $h_{\mu\nu}$, χ and f are defined by:

$$g_{\mu\nu}=\bar{g}_{\mu\nu}+G\,h_{\mu\nu}\quad ;\quad \varphi=\bar{\varphi}+\chi\quad ;\quad F=\bar{F}+f \tag{4.4}$$

We now want to expand the complete lagrangian around the classical solution and keep terms up to second order in the fluctuating fields. A simplifying observation is that in the gauge:

$$\bar{D}^{\mu}h_{\mu\nu}\equiv h_{\mu\nu}{}^{;\mu}=0\quad ;\quad \bar{g}^{\mu\nu}h_{\mu\nu}=0 \tag{4.5}$$

both $\sqrt{-g}$ and $\sqrt{-g}\,R$ have no linear terms in $h_{\mu\nu}$. Therefore, $\mathcal{L}_{\phi}$ as well as $\mathcal{L}_W=-(1/G^2)\sqrt{-g}\,C_{\mu\nu\alpha\rho}{}^2$ contain no non-diagonal terms of the form h-χ or h-f at the quadratic level. The small χ and f fluctuations are, thus, decoupled from the $h_{\mu\nu}$ ones and their behaviour will be studied separately. The quadratic part of the χ-f lagrangian is:

$$\mathcal{L}_{\chi,f}=\sqrt{-\bar{g}}\left[-\frac{1}{2}(\partial_\mu\chi)^2+\frac{1}{2}(\partial_\mu f)^2+\frac{v^2}{2}(\chi\ f)\,M\begin{pmatrix}\chi\\ f\end{pmatrix}\right] \tag{4.6}$$

where the matrix elements of M are:

$$M_{11}=8\lambda-16\lambda^2\gamma\ ,\quad M_{12}=M_{21}=4\lambda-16\lambda^2\gamma\ ,\quad M_{22}=-\frac{1}{\gamma}-16\lambda^2\gamma \tag{4.7}$$

All differentiations are covariant with respect to the background metric $\bar{g}_{\mu\nu}$ which is also used for raising (or lowering) space-time indices.

Before going on with the analysis we wish to outline the method and discuss some of the problems we encounter. The stability against small fluctuations of the $\varphi=0$ solution of a scalar field theory given by $\mathcal{L}_{\varphi}=1/2(\partial_\mu\varphi)^2 - 1/2\ m^2\varphi^2$ in flat space-time is based on the well-known energy considerations. The result is that for $m^2>0$ the solution is stable while for $m^2<0$ we obtain the Goldstone or Higgs model instability. This analysis can be extended to an anti-de Sitter background ($\lambda>0$) [13]. $m^2>0$ is again stable but $m^2<0$ is unstable only if a certain condition is satisfied [13] which reads $12m^2/\bar{R} \geq 9/4$. For de Sitter backgrounds ($\lambda<0$), one cannot rely on energy cvonsiderations but one can study the nature of the extrema of the corresponding Eucledian action. This is the method used in the stability analysis of inflation models. The result again is that $m^2>0$ corresponds to a stable solution while $m^2<0$ to an unstable one. Our problem is that the field χ has a negative sign in front of its kinetic energy. Although one might argue that this may not be a problem since the dilaton is not a physical field, the same situation appears with the massive pole of the graviton, so sooner or later the problem should be faced. Already in flat space-time, if we quantize the theory with positive metric we shall have negative energy states and hence instability. Alternatively we can quantize with negative metric in which case the same analysis shows [14] that stability requires the Hamiltonian to be bounded from above. Needless to say that this does not solve the unitarity problem and we can only repeat here the remarks we made in the introduction . We hope that a physically sensible theory along those lines can be constructed and the negative metric fields will turn out to be harmless. The Lee-Wick prescription could be used as an example. With this in mind we shall try to make our analysis in such a way that it is independent of the ultimate fate of the negative metric states. For this we shall first diagonalize the quadratic forms, thus identifying positive and negative metric fields, and then apply the criteria of small oscillations.

After this clarification, we proceed with the analysis. In order to diagonalize the mass-matrix in (4.6) we distinguish two cases:

(i) $8\lambda > \gamma^{-1} \geq 0$

$$\mathcal{L}_{\chi' f'} = \sqrt{-g}\left\{-\frac{1}{2}(\partial_\mu \chi')^2 + \frac{v^2}{2\gamma}\chi'^2 + \frac{1}{2}(\partial_\mu f')^2 - \frac{8\lambda v^2}{2} f'^2\right\} \tag{4.8}$$

According to our discussion above, the configuration $\chi'=f'=0$, which implies $\chi=f=0$, is a stable one

(ii) $8\lambda < \gamma^{-1}$

The diagonalization now yields:

$$\mathcal{L}_{\chi'' f''} = \sqrt{-g}\left\{-\frac{1}{2}(\partial_\mu \chi'')^2 + \frac{1}{2}8\lambda v^2 \chi''^2 + \frac{1}{2}(\partial_\mu f'')^2 - \frac{v^2}{2\gamma} f''^2\right\} \tag{4.9}$$

Anti-de Sitter space ($\lambda>0$) is again stable and the de Sitter one ($\lambda<0$) is unstable. Finally, flat space ($\gamma=0$) has a neutral stability.

We now turn to the graviton part and study small oscillations of the metric around the background solution $\bar{g}_{\mu\nu}$. We are only interested in $\lambda \geq 0$ since the de Sitter space has been shown to be unstable by the oscillations of the dilaton field. The method is the same. We introduce an auxiliary field $H_{\mu\nu}$ and diagonalize the resulting mass matrix. We get:

$$\mathcal{L}_{h'H} = \sqrt{-\bar{g}}\left\{-\frac{1}{2}\left(\bar{D}_\lambda h'_{\mu\nu}\right)^2 + \frac{1}{2}\left(\bar{D}_\lambda H_{\mu\nu}\right)^2 + \frac{v^2}{2}\left(h'_{\mu\nu}\ H_{\mu\nu}\right)\tilde{M}\begin{pmatrix} h'_{\mu\nu} \\ H_{\mu\nu}\end{pmatrix}\right\} \qquad (4.10)$$

where $H_{\mu\nu}$ is the auxiliary field, $h'_{\mu\nu}=h_{\mu\nu}-H_{\mu\nu}$ and the mass matrix $\tilde{M}$ is given by:

$$\tilde{M}_{11} = \tilde{M}_{12} = \tilde{M}_{21} = \left(\frac{1}{6}\lambda G^2 - 32\lambda^2 - \frac{4}{3}G^2\lambda^2\gamma\right)\left(12\lambda + \frac{1}{3}\lambda G^2\gamma - \frac{G^2}{24}\right)^{-1} \qquad (4.11a)$$

$$\tilde{M}_{22} = \tilde{M}_{11} + 12\lambda + \frac{1}{3}\lambda G^2\gamma - \frac{G^2}{24} \qquad (4.11b$$

$\bar{D}_\lambda$ is the covariant derivative with respect to the background. We diagonalize $\tilde{M}$ by a transformation which leaves invariant the form of the kinetic part of (4.10). The resulting values of the masses are:

$$\tilde{M}^{(1)} = \mp 4\lambda \quad ; \quad \tilde{M}^{(2)} = \pm\left(8\lambda + \frac{1}{3}\lambda G^2\gamma - \frac{G^2}{24}\right) \qquad (4.12)$$

and the signs correspond to $\mathrm{Tr}\tilde{M} \lessgtr 0$.

We see immediately that flat space ($\lambda=0$) has a neutral stability. For $\lambda>0$ we must repeat the analysis of ref [13] for spin two fields. The lagrangian (4.10), after diagonalization, becomes the sum of two terms of the form:

$$\mathcal{L} = \pm \frac{1}{2}\sqrt{-\bar{g}}\left\{ \left(\bar{D}_\lambda G_{\mu\nu}\right)^2 + \alpha \frac{\bar{R}}{12} G_{\mu\nu}{}^2 \right\} \tag{4.13}$$

where $G_{\mu\nu}$ stands for the two linear combinations of $h'_{\mu\nu}$ and $H_{\mu\nu}$ which diagonalize $\tilde{M}$, $\bar{R} = 24\lambda v^2$ and

$$\alpha = \begin{cases} 2 \\ 4 + \frac{1}{6}G^2\gamma - \frac{1}{48}\frac{G^2}{\lambda} \end{cases} \tag{4.14}$$

The condition for instability, for scalar fields, was found in ref [13] to be $\alpha > 9/4$. In our case, for spin two fields, we find that the anti-de Sitter background is unstable if

$$\alpha > 2 + \frac{9}{4} \tag{4.15}$$

which gives

$$\lambda > \frac{G^2}{8G^2\gamma - 12} \tag{4.16}$$

We summarize the results of this section: in conformally invariant gravity the only non-trivial background which is stable against small fluctuations is an anti-de Sitter space ($\lambda>0$) with the coupling constants of the theory satisfying the conditions:

$$G^2\gamma > \frac{3}{2} \tag{4.17a}$$

$$0 < \lambda < \frac{G^2}{8G^2\gamma - 12} \tag{4.17b}$$

These conditions can be further sharpened by studying, through the renormalization group equations, the evolution of the various coupling constants. One can check whether they represent a real restriction on the admissible values of the couplings or whether there exists an energy scale for which they are always satisfied.

Acknowledgements

Two of us, I. Antoniadis and J. Iliopoulos would like to thank the physics department of the Rockefeller University for the hospitality extended to them when this work was initiated.

[f1]: In our notation the curvature tensor $R^{\mu}{}_{\nu\rho\sigma}$ is given by:

$$R^{\mu}{}_{\nu\rho\sigma} = \partial_{\rho}\Gamma^{\mu}{}_{\sigma\nu} - \partial_{\sigma}\Gamma^{\mu}{}_{\rho\nu} + \Gamma^{\mu}{}_{\rho\alpha}\Gamma^{\alpha}{}_{\sigma\nu} - \Gamma^{\mu}{}_{\sigma\alpha}\Gamma^{\alpha}{}_{\rho\nu}$$

and the Christoffel symbol by

$$\Gamma^{\mu}{}_{\nu\rho} = \frac{1}{2} g^{\mu\sigma}\left(\partial_{\nu} g_{\rho\sigma} + \partial_{\rho} g_{\nu\sigma} - \partial_{\sigma} g_{\nu\rho}\right)$$

[f2] We are ignoring the surface (topological) term one has on the right-hand-side of this equation. This is equivalent to the statement that we are only concerned with "asymptotically-flat" fluctuations which do not change the Euler characteristic of the background metric we start with, or to the choice zero for the analogue of θ_{QCD}, the coupling in front of such a term in the action.

[f3] With Φ_{μ} and Φ linear functions of the fields, Γ too generates the 1PI vertices of the theory, with a difference only in the 2-point functions of the fields that appear in Φ_{μ} and Φ.

References:

1. See, for example, C.W. Misner, K.S. Thorne and J.A. Wheeler; Gravitation (Freeman, San Francisco, 1973) p. 410.
2. L.F. Abbott and S. Deser; Nucl. Phys. B195 (1982) 76.
3. I. Antoniadis and N. Tsamis; Phys. Lett. 144B (1984) 55. I. Antoniadis: SLAC-PUB 3297, March 1984.
4. H. Weyl; Raum, Zeit, Materie, Vierte erweiterte Auflage, Berlin 1921.
5. C. Becchi A. Rouet and R. Stora; Commun. Math Phys. 42 (1975) 127.
6. K. S. Stelle; Phys. Rev. D16 (1977) 953.
7. E.T. Tomboulis; Phys. Rev. Lett. 52 (1984) 1173.
8. M. Kaku, Phys. Rev. D27 (1983) 2819.
9. E.T. Tomboulis, Phys. Lett. 70B (1977) 361; Phys. Lett. 97B (1980) 77; see also E.T. Tomboulis in "Quantum Theory of Gravity", St. M. Christensen, Editor, Adam Hilger 1984.
10. T.D. Lee and G.C. Wick, Nucl. Phys. B9 (1969) 209; ibid. B10 (1969) 1; R.E. Cutkosky, P.V. Landshoff, D.I. Olive and J.C. Polkinghorne, Nucl. Phys. B12 (1969) 281.
11. F. Englert, C. Truffin and KR. Gastmans, Nucl. Phys. B117 (1976) 407.
12. See, for example, S. Weinberg, Gravitation and Cosmology, (John Wiley and Sons Inc. 1972) Chapter 13.
13. P. Breitenlohner and D.Z. Freedman, Phys. Lett. 115B (1982) 197.
14. D.G. Boulware and D.J. Gross, Nucl. Phys. B233 (1984) 1.