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TOPONIUM-Z^0 INTERFERENCE*

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ABSTRACT

A study of interference of the Z^0 boson and toponium states is presented. The simple case of the Z^0 mixing with one $t\bar{t}$ state is discussed in detail. Effects of mixing with the full $t\bar{t}$ spectrum, of the smearing due to beam spread, and of different potentials, are then shown.

1. INTRODUCTION

Why do we expect toponium-Z^0 mixing to be of interest? From the absence of flavor-changing neutral currents in B decay, we are confident that the bottom quark must have an as-yet-unobserved partner. Experimentally, $m_t < 23$ GeV is excluded, while UA1 data suggests a top quark of mass between 30 and 50 GeV. It appears quite possible that $t\bar{t}$ bound states will have masses near that of the Z^0 (93 GeV), and thus vector ($J^{PC} = 1^{--}$) $t\bar{t}$ states (henceforth V) could be nearly degenerate with the Z^0. We expect the effects of $V - Z$ mixing to be seen soon, at both SLC and LEP.

I first present a few ways of understanding the nearly complete destructive interference of the Z boson with one V state. Then, after a brief review of toponium spectroscopy, I discuss the mixing of the Z with the full spectrum of toponium states (when the Z and V are nearly degenerate); I show the effects of finite beam width on the cross-sections and asymmetries. I then display the striking effects that remain if the Z is relatively far away from the V ($10 - 20$ GeV), and conclude by contrasting the effects of the Richardson potential, the Cornell potential, and a non-standard Higgs sector.

This talk is based on work done with Fred Gilman and Gregory Athanasiu.[1–3]

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2. MIXING OF THE Z^0 WITH A SINGLE $t\bar{t}$ STATE

I begin with a qualitative argument to show that the interference is indeed destructive. To be specific we consider the process $e^+e^- \rightarrow \mu^+\mu^-$(other final states are discussed analogously; see Ref. 1). This process occurs predominantly as $e^+e^- \rightarrow Z_0 \rightarrow \mu^+\mu^-$, while another contribution is $e^+e^- \rightarrow Z_0 \rightarrow V_0 \rightarrow Z_0 \rightarrow \mu^+\mu^-$ (for now, we neglect the small contributions due to γ couplings). The first has an amplitude proportional to the propagator $1/(s - M_{Z_0}^2 + i\Gamma_{Z_0}M_{Z_0})$, and therefore to $1/i\Gamma_{Z_0}$ on the peak of the Z_0 resonance. If, for simplicity, I choose the Z_0 and V_0 resonances to be degenerate, the amplitude from the second contribution is similarly proportional to $1/(i\Gamma_{Z_0}i\Gamma_{V_0}i\Gamma_{Z_0})$. Thus we have a relative minus sign between these two amplitudes, i.e., destructive interference.

I can extend this argument by replacing the Z_0 propagator by the iterated series

$$\frac{1}{s-M_{Z_0}^2} + \frac{1}{s-M_{Z_0}^2}\cdot\left(a\cdot\frac{1}{s-M_{V_0}^2}\cdot a\cdot\frac{1}{s-M_{Z_0}^2}\right) \tag{1}$$

$$+\frac{1}{s-M_{Z_0}^2}\cdot\left(a\cdot\frac{1}{s-M_{V_0}^2}\cdot a\cdot\frac{1}{s-M_{Z_0}^2}\right)^2 + \cdots = \frac{s-M_{V_0}^2}{(s-M_{Z_0}^2)(s-M_{V_0}^2)-a^2}\ .$$

Here, and often in what follows, $M_{Z_0}^2$ is used as a shorthand for the full expression $M_{Z_0}^2 - i\Gamma_{Z_0}M_{Z_0}$, and a is the $Z - V$ coupling.

What does this expression tell us? For energies a few GeV away from a V_0 resonance, $(s - M_{Z_0}^2)(s - M_{V_0}^2)$ is large compared to a^2; as expected, we recover the Z_0 propagator. When we are sitting on the V_0 resonance we get zero for the amplitude—*complete destructive interference.*

Strictly speaking, the amplitude only vanishes if we make some simplifying assumptions:

(1) I have ignored the fact that $e^+e^- \rightarrow \mu^+\mu^-$ can also proceed via a virtual photon. This is a good approximation, since the photon, by definition, contributes an R-value of about† one, while the R-value on the Z_0 peak is 200. (I note here that on the Z_0 peak, the Z amplitude is imaginary while that of the photon is real, so that there is no $\gamma - Z$ interference. However, in general we must compute $Z\gamma V$ mixing. The effect of the photon is small enough to be negligible, except in the determination of the asymmetry parameters.)

(2) I have implicitly assumed that the width of the V_0 is zero. The expression $s - M_{V_0}^2$ really stands for $s - M_{V_0}^2 + iM_{V_0}^2\Gamma_{V_0}$ which can only be zero (for a physically allowed value of s) if $\Gamma_{V_0} = 0$. This is also a good approximation, since the expected width of a $t\bar{t}$ $1S$ state (here, and throughout this section, I use the Richardson potential to estimate $t\bar{t}$ properties) is $\approx$ 100 keV, compared to Γ_Z=2.7 GeV.

† since R-value is defined in terms of the QED cross-section at the electron mass scale.

(3) Finally, I have ignored the "direct" couplings of the V_0, that is, the V_0 coupling to fermions through the photon instead of through the Z_0. This approximation is analogous to, and comparable in magnitude with, the second one.

2.1 Mass-Mixing Approach

Now I would like to present another way of analyzing this problem. The pure states V_0 and Z_0 are nearly degenerate, with mass-squared matrix

$$\mathcal{M}_0^2 = \begin{pmatrix} M_{V_0}^2 - i\Gamma_{V_0}M_{V_0} & \delta m^2 \\ \delta m^2 & M_{Z_0}^2 - i\Gamma_{Z_0}M_{Z_0} \end{pmatrix}. \tag{2}$$

δm^2 is the $Z_0 - V_0$ coupling, given by

$$\delta m^2 = \frac{(g_V)_t}{\frac{2}{3}e}\left[2\sqrt{3}\sqrt{4\pi\alpha(M_Z)}\,\frac{2}{3}\sqrt{M_{V_0}}\,|\Psi(0)|\right], \tag{3}$$

where $\alpha(M_Z)$ is the fine structure constant evaluated at the relevant mass scale and $\Psi(0)$ is the wave function of the $t\bar{t}$ system at the origin. The quantity in brackets is the $\gamma - V$ coupling, which is multiplied by the ratio of the weak charge of toponium to its electromagnetic charge to get the $Z - V$ coupling.

There are two ways one can deal with this mass matrix. It can be diagonalized, yielding a matrix with physical masses on the diagonal. The amplitude for the process $e^+e^- \to \mu^+\mu^-$ is then just the sum of the amplitudes for this process occurring via each of the physical particles, separately. This can be written as:

$$\mathcal{A} = (g_Z \quad g_V)_I \left[s - \begin{pmatrix} M_Z^2 - iM_Z\Gamma_Z & 0 \\ 0 & M_V^2 - iM_V\Gamma_V \end{pmatrix}\right]^{-1} \begin{pmatrix} g_Z \\ g_V \end{pmatrix}_F. \tag{4}$$

Here g_V and g_Z are the rotated, or physical, couplings. Equivalently, we have

$$\mathcal{A} = (g_{Z_0} \quad g_{V_0})_I \left[s - \begin{pmatrix} M_{Z_0}^2 - iM_{Z_0}\Gamma_{Z_0} & \delta m^2 \\ \delta m^2 & M_{V_0}^2 - iM_{V_0}\Gamma_{V_0} \end{pmatrix}\right]^{-1} \begin{pmatrix} g_{Z_0} \\ g_{V_0} \end{pmatrix}_F \tag{5}$$

where we have rewritten $(\vec{g}_0\mathcal{U})(\mathcal{U}^{-1}(s-\mathcal{M})^{-1}\mathcal{U})(\mathcal{U}^{-1}\vec{g}_0)$, canceling out the unitary transformations $\mathcal{U}$ that rotate couplings and masses from one basis to the other.

In this non-diagonalized basis, if we set $g_{V_0} = 0$, $\Gamma_{V_0} = 0$, and $s = M_{V_0}^2$, we have

$$\mathcal{A} = (g_{Z_0} \quad 0)_I \begin{pmatrix} 0 & -\delta m^2 \\ -\delta m^2 & M_{V_0}^2 - M_{Z_0}^2 \end{pmatrix} \begin{pmatrix} g_{Z_0} \\ 0 \end{pmatrix}_F = 0. \tag{6}$$

So in this formalism also, it is easy to see the complete destructive interference.

If we diagonalize the mass matrices, we find that the physical masses and widths are shifted away from their original values. This is not an important effect for the M_Z and M_V, which get shifted equally and oppositely by at most 4 MeV. Similarly, Γ_Z is $\Gamma_{Z_0} - \Delta\Gamma$ and Γ_V is $\Gamma_{V_0} + \Delta\Gamma$; $\Delta\Gamma$ can be as big as 20 MeV—irrelevant for the Z, but very important for the V, which has an unmixed width of 100 keV. This maximal $\Delta\Gamma$ is achieved when the Z and V are degenerate; when they are, e.g., 2 GeV apart, $\Delta\Gamma$ drops to 5 MeV (these $\Delta\Gamma$ are for the 1S state).

We have then, for the R-value, the following expression:

$$R = .1365 \frac{\alpha^2(M_Z)}{\alpha^2(m_e)} s^2 \left(\frac{s - M_{V_0}^2}{(s - M_{V_0}^2)(s - M_{Z_0}^2) - \delta m^2} \right)^2 \tag{7}$$

where .1365 comes from combinations of θ_W.* In Fig. 1 I show the results of our calculation: the solid line is exact (we include the V_0's direct couplings and width, and the photon term; we deal with the cross-sections for various helicity combinations separately); the dashed line is the result of ignoring the above parenthetical effects; the effect of the Z_0 alone is shown for comparison (dotted line). The two graphs differ only in scale.

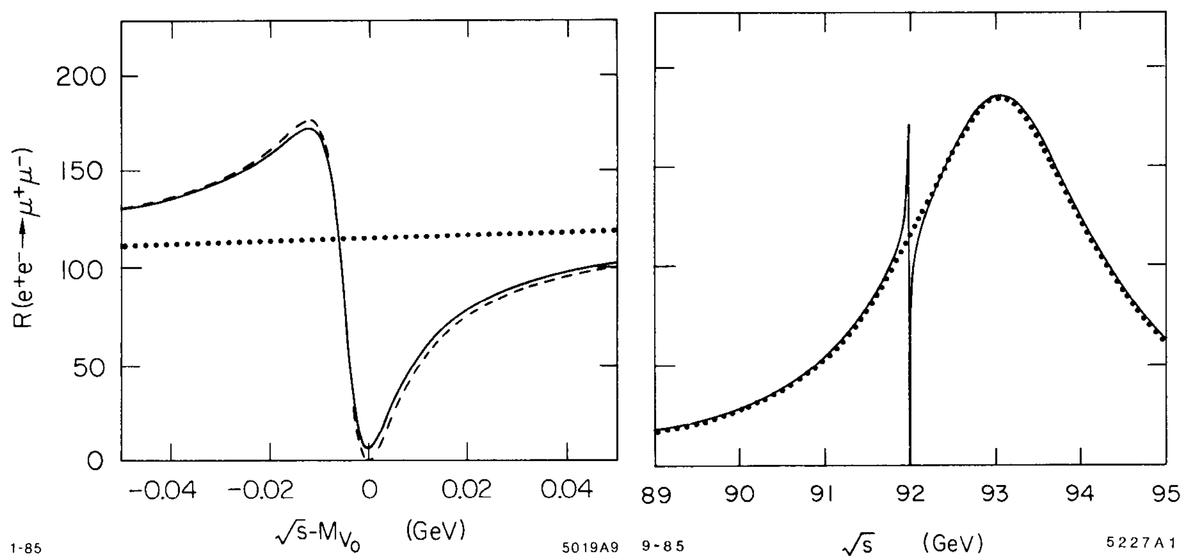

Figure 1. $R(e^+e^- \to \mu^+\mu^-)$ for one toponium state mixing with the Z_0.

3. WHAT WE WILL SEE: MANY STATES, SMEARING, AND ALL THAT

I begin with a brief review of heavy quarkonia, with particular reference to toponium. These systems are well-described by non-relativistic potential models; for the c and b quark systems, a wide range of successful forms have been proposed.

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* The ratio of α^2's in Eq. (7) is about 1.15. See previous footnote.

That they all work is not too surprising, as they approximately coincide in the range $.1 < R < 1$ fm, where the RMS radii of the observed $c\bar{c}$ and $b\bar{b}$ states lie. However, $t\bar{t}$, due to its large mass, will have a smaller radius, and test a region where the potentials differ. As examples, I choose:

1. Cornell:[4]

$$V(r) = \frac{-.48}{r} + \frac{r}{5.4756(\text{GeV})^{-2}} \tag{8}$$

—a combination of Coulomb at short range and linear confinement—and

2. Richardson:[5] the single dressed gluon exchange amplitude (in momentum space)

$$\widetilde{V}(q^2) = \frac{3\alpha_s(q^2)}{3q^2} \tag{9}$$

interpolated with a linear potential (in coordinate space) at large distances.

These two potentials give rather different level spectra (shown in Fig. 2) and wavefunctions; for example, $\psi(0)_{1S}$ is three times larger for Cornell than for Richardson (the Cornell potential, unlike Richardson's, does not incorporate asymptotic freedom, and is more singular at short distances). Since both the bare V widths and those acquired from mixing go as $|\psi(0)|^2$, these numbers increase by a factor of ten for Cornell—resulting in, for example, a maximal width from mixing of .2 GeV.

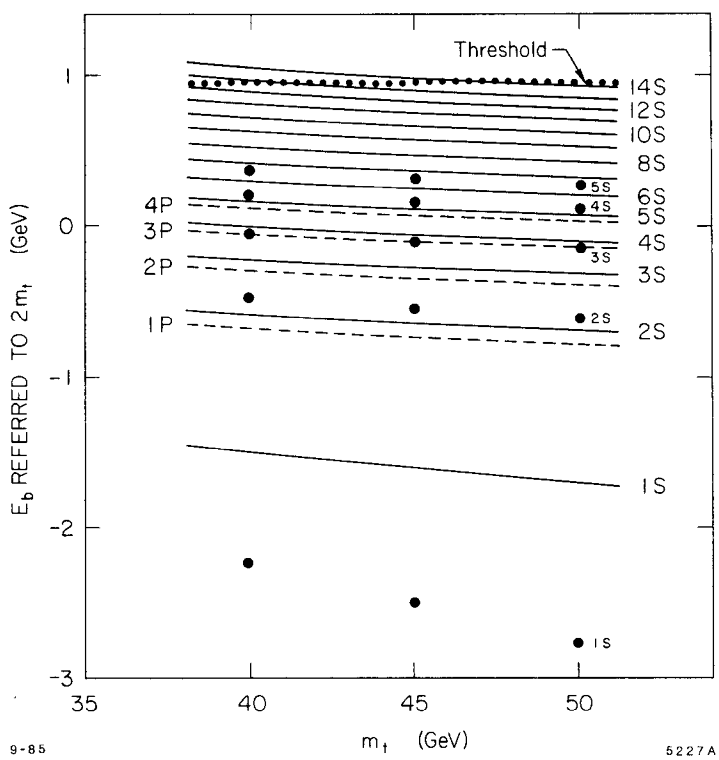

Figure 2. Binding energies of toponium, versus top mass, for Richardson (solid lines, S states; dashed, P states) and Cornell (dots–S states only) potentials. The threshold for open top production, calculated using the Richardson potential, is also shown (dotted line).

It is then a simple extension of the formalism developed in Sec. 2 to deal with the mixing of the Z_0 with the (about) thirteen toponium states we expect below the open top threshold. We obtain cross-sections such as the one shown in Fig. 3. The height of the "spikes" are given by

$$C * s^2 \left| \frac{1}{s - M_{Z_0}^2 + iM_{Z_0}\Gamma_{Z_0} - \frac{\delta m^2}{s - M_{V_0}^2}} \right|^2 . \qquad (10)$$

The peak of a given V_0 resonance occurs when the real part of the denominator vanishes, at a value of s very close to $M_{V_0}^2$. The height of the peak is thus $C * s^2/(m_{Z_0}\Gamma_{Z_0})^2$. The height of the Z_0 peak can also be gotten from Eq. (10), by dropping the V_0 mixing term; maximizing, we obtain the exact expression found for the spike (s equals the relevant mass squared). This explains why all the peaks, including that of the Z_0, are on the same gently rising curve. (In Fig. 3 the P-states are ignored; they cause similar spikes, but are unobservably narrow, as their coupling, and hence acquired width, is suppressed relative to the S-states.)

Of course, real machines, such as SLC and LEP, will not resolve these very narrow spikes; we must convolute the curves with a Gaussian (with width related to the beam spread) in order to approximate what will be measured. In Fig. 4, I show R for the Z alone, and Fig. 3 convoluted with Gaussians appropriate to σ_{beam}= 40 MeV and 100 MeV. LEP is expected to run (without wigglers) at the former beam width; SLC is expected to achieve the latter, and perhaps with special effort, the former.

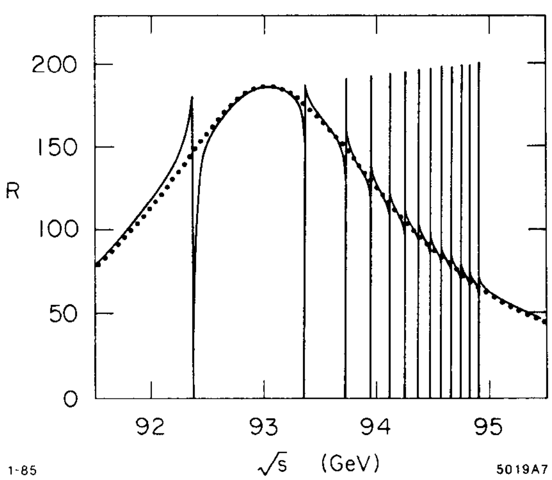

Figure 3. $R(e^+e^- \to \mu^+\mu^-)$ for several toponium states mixing with the Z (Richardson potential, $m_t = 47$ GeV). The dotted line is the Z_0 alone.

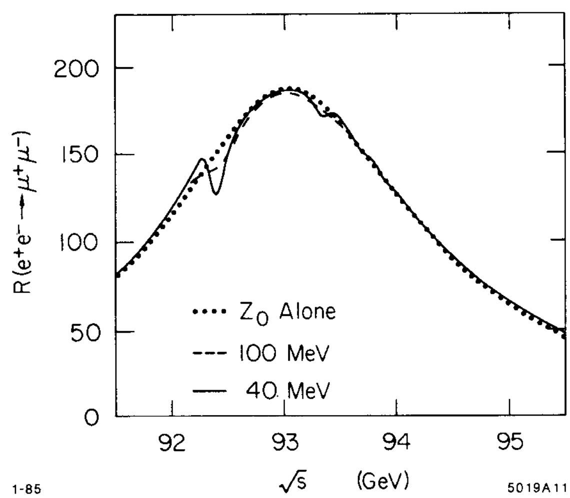

Figure 4. $R(e^+e^- \to \mu^+\mu^-)$, smeared, for various expected beam widths.

I next remark that even for a V relatively far away from the Z, the enhancement due to mixing should be quite noticeable. The height of the peak does not decrease, though its width does. The smeared height is therefore greatly reduced, but should be compared to the also much reduced background due to the Z.

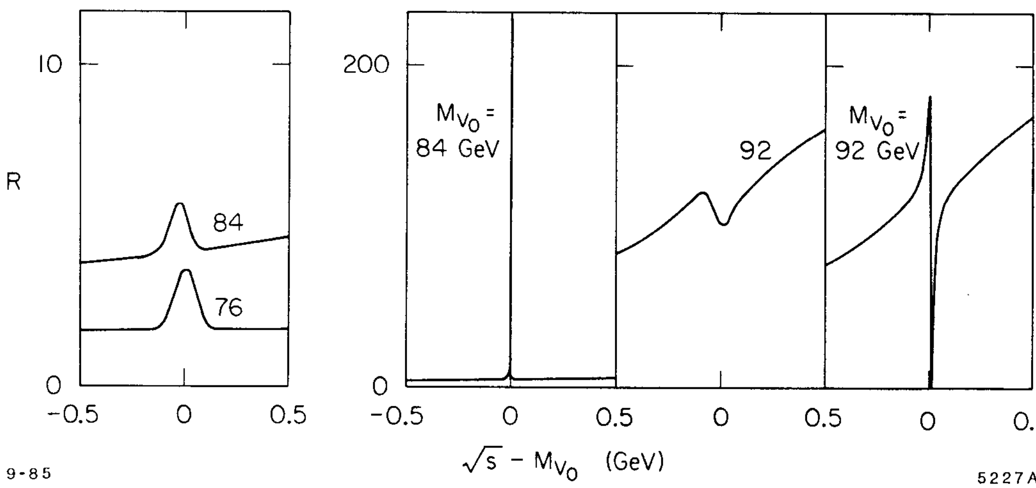

Figure 5. $R(e^+e^- \to \mu^+\mu^-)$ smeared and not, for $M_{V_0} = 76$, 84 and 92 GeV.

I now present smeared polarization and forward-backward asymmetries for various values of M_{V_0}. These are found by calculating the cross-sections (for each individual helicity configuration), smearing them, and then taking the appropriate differences and ratios. Since the asymmetries also crucially depend on the $ZV\gamma$ interference, the results do not seem to have a simple qualitative explanation. In Fig. 6 I show the asymmetries; the effects are in fact more striking for V moderately far away from Z.

All the results I have shown so far used the Richardson potential. I shall briefly show the effects of using the Cornell potential, and the Richardson potential combined with a non-standard Higgs sector. Consider the 2-Higgs model of Glashow, Weinberg and Paschos,[6] where one Higgs couples to up-type quarks, and one to down-type. There is a neutral-Higgs (H_0) exchange contribution to the toponium potential, where the H_0 coupling is enhanced by the vacuum-expectation-value ratio ξ/η (ξ being the VEV of the Higgs coupling to down type quarks and η to up-type). The extra contribution is an attractive Yukawa, in momentum space

$$-\left(\frac{\xi}{\eta}\frac{gm_t}{2M_W}\right)^2 \frac{1}{m_H^2+q^2} \qquad \text{or} \qquad -\left(\frac{\xi}{\eta}\frac{gm_t}{2M_W}\right)^2 \frac{e^{-rm_H}}{4\pi r} \tag{11}$$

in coordinate space. This addition has the effects of increasing the wavefunctions at the origin, since it pulls in the wavefunctions, and of lowering states (increasing binding energies); it changes the level spacings, since it affects the lowest lying states the most. Finally, if the Higgs term is strong enough* it has a very curious

* that is, ξ/η equals about 5, if we are using the Cornell potential, or 10, for Richardson.

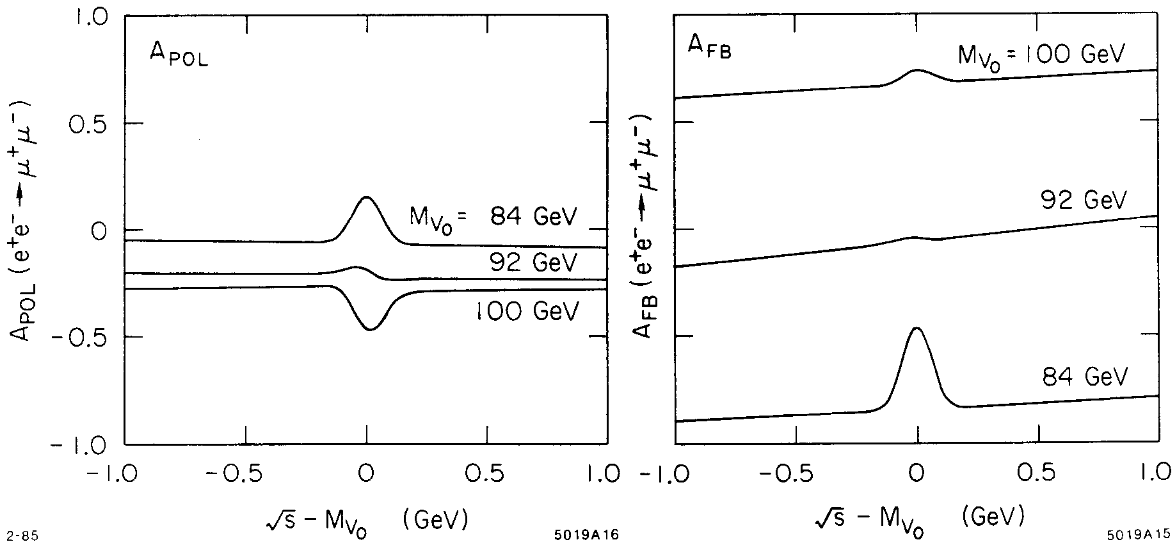

Figure 6. A_{pol} and A_{fb} for three different values of M_{V_0}.

effect—it causes the 2S state to lie below the 1P. This effect does not happen for any standard quarkonium potential, and is related[7] to the fact that $\Delta V(r) < 0$ for the Higgs potential and not so for any standard quarkonium potential. In Fig. 7, I show R($e^+e^- \to \mu^+\mu^-$), smeared (σ_{beam} = 40 MeV), for Richardson alone, Cornell alone, and Richardson with Higgs† . Note the qualitative similarity between the second and third figure.

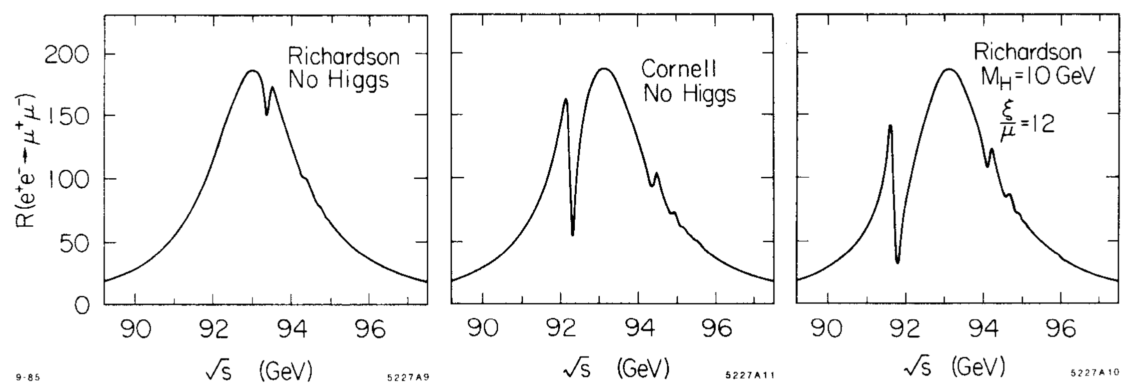

Figure 7. Effects of varying quarkonium potential.

In summary, we have seen that toponium and the Z_0 almost completely destructively interfere. Toponium states pick up a large width from mixing—the 1S state, with a bare width of 100 keV, can acquire a width of as much as 20 MeV

† the parameters have been chosen to be dramatic; they are all but excluded by $B\bar{B}$ mixing[3]

(using the Richardson potential). While the beam widths of machines such as SLC and LEP will greatly blur the sharp spikes that we find, effects will be visible as wiggles in cross-sections and asymmetry parameters. The exact potential for toponium (and thus exactly what we will see) is not very well known. The Higgs (in a 2-Higgs model) can have noticeable effects, but it may be hard to distinguish these effects from those of different potentials; the 2S-1P level inversion is a possible qualitative difference, if the Higgs couplings are rather large.

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