SLAC – PUB – 3757
August 1985
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EXACT N-DEPENDENCE OF MULTIQUARK OPERATORS*

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ABSTRACT

We develop a method which enables us to calculate *explicitly* matrix elements of multiquark operators between two nonstrange baryon states, thus obtaining the *exact* dependence of these operators on N-number of color degrees of quarks. The method employs permutation symmetry of the N-quark wave function and it is illustrated by evaluating g_A/g_V and the matrix elements of four-quark (current-current) operators which appear in the $\Delta S = 0$ effective weak Hamiltonian. Implications of these results for the low energy phenomenology and for the interpretation of the Skyrme model results are discussed.

Submitted to *Physical Review Letters*

⋆ Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.

‡ On leave of absence from Rudjer Bošković Institute, Zagreb-Croatia-Yugoslavia Work supported in part by the U.S. National Science Foundation Grants No. INT-8509369 and YOR 84/078 under the program U.S.-Yugoslavia joint board on scientific and technical cooperation.

There are many strong arguments to believe that the $SU(3)_C$ gauge theory of quarks and gluons, known as quantum chromodynamics (QCD), is the theory which properly describes the world of hadronic physics. Unfortunately, the complexity of the phenomena which this theory tries to explain makes it unsolvable exactly. However there exists an expansion parameter, i.e. the number of colors, $N = 3$, which gives the possibility of developing a good approximate theory. 't Hooft[1] originally proposed a generalization of QCD from three colors with the $SU(3)_C$ gauge group to N colors with $SU(N)_C$ gauge group. It was hoped that such a generalization, together with $1/N$ expansion, could provide an interesting insight into QCD problems.[2] Witten in his excellent paper[2] reviewed the existing results concerning mesons and gluon states in the $1/N$ expansion scheme and showed how to fit baryons into this picture. This approximation scheme does provide useful selection rules for strong meson decays and also provides us with a systematic way to calculate hadronic matrix elements involving weak currents. The $SU(N)_C$—quantum chromodynamics for strong interaction—and the Weinberg-Salam gauge theory for weak interaction have been applied to the nonleptonic weak decays of D and K mesons,[3] and give a reasonable description of $D \to K\pi$ and $K \to 2\pi$ decay amplitudes.

In the last few years chiral perturbation theory, and the Skyrme model in particular,[4,5] received revival and extensive study. It became clear that large N-properties of many physical quantities are very important.[6] An important finding is that in the $N \to \infty$ limit baryons appear to be solitons in the effective mesonic field theory.[2]

In this paper we shall develop the method which enables us to evaluate explicitly the matrix elements of multiquark operators, i.e., dimension four (current) or dimension six (current-current) operators, between baryonic states as a function of N. This will be done for the simplest case with 2 flavors only, i.e., $SU(2)_I$ case. The method is based on permutation symmetry[7] of the baryon wave function.[8]

The nucleon wave function with N quarks is written as a product of the single particle wave functions. It is composed of products of the wave functions in four subspaces: the spatial-(X), spin-$SU(2)_S$, isospin-$SU(2)_I$ and color-$SU(N)_C$ subspaces. The wave functions in each of the subspaces are characterized by[7] Young tableau $[f]$, unitary quantum number ρ and Yamanouchi symbol r which is the quantum number of permutation symmetry.[8] Quantum numbers $[f], \rho, r$ are extensively explained in Appendix 1C of A. Bohr and B. Mottelson, Ref. 7. Here we only briefly mention definitions of these quantum numbers. The Young tableau is defined as $[f] \equiv [f_1, f_2, \ldots, f_n]$, $f_1 + f_2 \ldots + f_n = N$, where f_i is the number of boxes in the i^{th} row. E.g., the nucleon wave function has $[f_X] = [N]$ (symmetric), $[f_S] = [f_I] = [\frac{N+1}{2}, \frac{N-1}{2}]$ and $[f_C] = [1, 1, \ldots, 1]$ (antisymmetric). The unitary quantum number ρ is determined by the single particle wave functions arranged in the Young tableau. E.g., ρ_I determines the third component of isospin T_3. Yamanouchi symbol r is determined by the ciphers from 1 to N arranged in the Young tableau in an increasing order in each row and each column. The number of different Yamanouchi symbols $h[f]$ (or equivalently the dimension of the Young tableau) can be cast in the following formula:[7]

$$h[f] = \frac{N! \prod_{i<j\le n}(f_i - f_j + j - i)!}{\prod_{i=1}^{n}(f_i + n - i)!} \,. \tag{1}$$

The explicit form for the wave function with good $[f]$, r and ρ is generally complicated.[9] However, in the standard, the so-called Yamanouchi representation, one ensures the orthonormality of the wave functions,[7] i.e.,

$\langle [f], r, \rho | [f'], r', \rho' \rangle = \delta_{[f][f']} \delta_{rr'} \delta_{\rho\rho'}$, as long as the single particle wave functions are orthonormal. Also, this representation ensures good permutation symmetry with respect to the first two particles.

Coupling of the wave functions with $[f_\alpha]$, r_α, ρ_α and $[f_\beta]$, r_β, ρ_β from two different subspaces, e.g., spin and isospin subspace, into the wave function with new $[f], r$ is determined by the Clebsch-Gordon coefficients for the permutation group $\langle [f_\alpha] r_\alpha, [f_\beta] r_\beta | [f] r \rangle$. They are difficult to evaluate for the number of quarks > 4, so that their evaluation with a computer is needed.[10] However, when the new wave function is symmetric, i.e., $[f] = [N]$, the Clebsch-Gordon coefficients have a simple form $\langle [f_\alpha] r_\alpha, [f_\beta] r_\beta | [N] \rangle = \delta_{[f_\alpha][f_\beta]} \delta_{r_\alpha r_\beta} / \sqrt{h[f_\alpha]}$.

Since the nucleon wave function has symmetric spatial and antisymmetric color parts, the spin-isospin part should be symmetric in order to ensure that the total wave function is antisymmetric. Using the above results, we can finally express the nucleon wave function in the following forms:

$$\psi_{\mathcal{N}} = |[N], \rho_X\rangle \, |[1,1,\ldots,1]\rho_C\rangle \; \frac{1}{\sqrt{h\left[\frac{N+1}{2}, \frac{N-1}{2}\right]}} \sum_{r=1}^{h\left[\frac{N+1}{2}, \frac{N-1}{2}\right]} \left| \left[\frac{N+1}{2}, \frac{N-1}{2}\right] r, \rho_S \right\rangle \left| \left[\frac{N+1}{2}, \frac{N-1}{2}\right] r, \rho_I \right\rangle , \tag{2}$$

where $\mathcal{N}$ denotes nucleon. Here $h\left[\frac{N+1}{2}, \frac{N-1}{2}\right] = 2N! / \left[\left(\frac{N+3}{2}\right)! \left(\frac{N-1}{2}\right)!\right]$ as obtained from formula (1). Note that by a similar procedure one can construct also Δ state and other higher spin baryonic states. Although we do not have explicit form for the wave functions $\left|\left[\frac{N+1}{2}\; \frac{N-1}{2}\right] r, \rho_{S,I}\right\rangle$, it is not necessary. We will use only the symmetry properties of the wave function for the evaluation of the matrix element.

For an illustration of the method, we shall first evaluate the ratio of the axial-vector and vector coupling constants, g_A/g_V. In the nonrelativistic limit with the assumption of contact-interaction for the spatial parts one obtains:

$$\frac{g_A}{g_V} = \frac{\left\langle \psi_N^{\uparrow} \right| \sum_{i=1}^{N} (\sigma_3)_i (\tau_3)_i \left| \psi_N^{\uparrow} \right\rangle}{\left\langle \psi_N^{\uparrow} \right| \sum_{i=1}^{N} (\tau_3)_i \left| \psi_N^{\uparrow} \right\rangle} . \tag{3}$$

Here $\psi_N^{\uparrow}$ is the nucleon wave function with the total third component of spin $\Sigma_3 = +1$, while $(\sigma_3)_i$ and $(\tau_3)_i$ denote the single particle operators for the third component of spin and isospin, respectively. Because ψ_N is totally antisymmetric one can use the identity $\left\langle \psi_N^{\uparrow} \right| \sum_{i=1}^{N} (\sigma_3)_i (\tau_3)_i \left| \psi_N^{\uparrow} \right\rangle = N \left\langle \psi_N^{\uparrow} \right| (\sigma_3)_N (\tau_3)_N \left| \psi_N^{\uparrow} \right\rangle$. Expectation value of operator $(\tau_3)_N ((\sigma_3)_N)$ is different for each of the two sets of the wave functions in the isospin (spin) subspace. These two sets are determined by the following Yamanouchi symbols:

$$r_0 \equiv \begin{array}{|c|c|c|c|c|} \hline & & \cdots & & \text{N} \\ \hline & & \cdots & \\ \cline{1-4} \end{array} , \qquad r_0' \equiv \begin{array}{|c|c|c|c|c|} \hline & & \cdots & & \\ \hline & & \cdots & \text{N} \\ \cline{1-4} \end{array} \tag{4}$$

One can write the isospin part (and similarly the spin part) of the proton wave function with r_0 and r_0' in the following way:

$$\psi_{r_0} \equiv \left| [\tfrac{N+1}{2}, \tfrac{N-1}{2}] r_0, \rho_I \right\rangle = \sum_{i_1, i_2, \ldots, i_{N-1}} C^{r_0}_{i_1, i_2, \ldots, i_{N-1}}$$
$$\times \prod_{k=1}^{N-2} \frac{1}{\sqrt{2}} \left[u(i_k)\, d(i_{k+1}) - d(i_k) u(i_{k+1}) \right] u(N) \tag{5a}$$

$$\psi_{r_0'} \equiv \left| [\tfrac{N+1}{2}, \tfrac{N-1}{2}] r_0', \rho_I \right\rangle = \sum_{i_1, i_2, \ldots, i_{N-1}} C^{r_0'}_{i_1, i_2, \ldots, i_{N-1}}$$
$$\times \prod_{k=1}^{N-4} \frac{1}{\sqrt{2}} \left[u(i_k)\, d(i_{k+1}) - d(i_k) u(i_{k+1}) \right] \tag{5b}$$

$$\times \frac{1}{\sqrt{6}} \left\{ [u(i_{N-2})d(i_{N-1}) + d(i_{N-2})u(i_{N-1})]u(N) - 2u(i_{N-2})u(i_{N-1})d(N) \right\} .$$

One can write the corresponding parts of the neutron wave function in the isospin and spin space in a similar way. Although we *do not know* the coefficients $C^{r_0 r_0'}_{i_1,i_2,\dots,i_{N-1}}$, the evaluation of $(\tau_3)_N$ can be finally performed using the orthonormality of the single particle wave functions. One finds:

$$\langle \psi_{r_0} | (\tau_3)_N | \psi_{r_0'} \rangle = 0 \ , \tag{6a}$$

$$\langle \psi_{r_0} | (\tau_3)_N | \psi_{r_0} \rangle = -3 \langle \psi_{r_0'} | (\tau_3)_N | \psi_{r_0'} \rangle = T_3 \ , \tag{6b}$$

where T_3, the total third component of the isospin, is +1 and -1 for proton and neutron, respectively. An equivalent result is obtained for $(\sigma_3)_N$ operator. Using Eqs. (2), (3) and (6) we arrive at the following expression for g_A/g_V

$$\frac{g_A}{g_V} = N \left[\frac{h\left[\frac{N-1}{2}, \frac{N-1}{2}\right]}{h\left[\frac{N+1}{2}, \frac{N-1}{2}\right]} + \frac{1}{9} \left(1 - \frac{h\left[\frac{N-1}{2}, \frac{N-1}{2}\right]}{h\left[\frac{N+1}{2}, \frac{N-1}{2}\right]} \right) \right] = \frac{N+2}{3} . \tag{7}$$

The first part of Eq. (7) is obtained by noting that the number of Yamanouchi symbols r_0 is equal to $h\left[\frac{N-1}{2}, \frac{N-1}{2}\right]$ and is equal to $(N-1)!/\left[\left(\frac{N+1}{2}\right)!\left(\frac{N-1}{2}\right)!\right]$ according to formula (1). This then leads to the final, well-known result, first derived by using the full explicit form of the N-quark nucleon wave function.[11]

In our approach result (9) is obtained in an almost trivial way as a function of the ratio of dimensions for two different Young tableaux. This method can be extended to a calculation of other current operators and to baryon states with higher spins, like Δ.

We shall now apply the same method to $SU(2)_I$ dimension six (four-quark) operators which appear in the $\Delta S = 0$ effective weak Hamiltonian and have been classified in Ref. 12 (formula (3.2)). For the purpose of this paper we are dealing only with the parity-conserving parts of each operator, because the parity-violating operator between two states of the same spin and parity ($\frac{1}{2}^+$ in our case) vanishes identically. Here we shall calculate the parity-violating (PV) $n \to p\pi^-$ weak amplitude. So we start with PV operators O_i^{PV}'s from Ref. 12 and then through the so-called equal time commutator $[F_I^5, O_j^{PV}]$ arrive at the isospin rotated parity-conserving (PC) operators $\widehat{O}_j^{PC}$. In the nonrelativistic limit our method gives:

$$\widehat{O}_1 = [(\overline{u}u\overline{d}d + \overline{d}d\overline{u}u) - (\overline{u}d\overline{d}u + \overline{d}u\overline{u}d)]_{VV+AA}$$

$$= \tfrac{1}{2}\, N(N-1)(1_1 1_2 - \vec{\sigma}_1\vec{\sigma}_2)(1_1 1_2 - \vec{\tau}_1\vec{\tau}_2) \tag{8a}$$

$$\widehat{O}_2 = [2(\overline{u}u\overline{u}u + \overline{d}d\overline{d}d) + \overline{u}u\overline{d}d + \overline{d}d\overline{u}u + \overline{u}d\overline{d}u + \overline{d}u\overline{u}d]_{VV+AA}$$

$$= \tfrac{1}{2}\, N(N-1)(1_1 1_2 - \vec{\sigma}_1\vec{\sigma}_2)(3\, 1_1 1_2 + \vec{\tau}_1\vec{\tau}_2) \tag{8b}$$

$$\widehat{O}_2' = [(\overline{u}u\overline{u}u + \overline{d}d\overline{d}d) - (\overline{u}u\overline{d}d + \overline{d}d\overline{u}u + \overline{u}d\overline{d}u + \overline{d}u\overline{u}d)]_{VV+AA}$$

$$= \tfrac{1}{2}\, N(N-1)(1_1 1_2 - \vec{\sigma}_1\vec{\sigma}_2)[3(\tau_3)_1(\tau_3)_2 - \vec{\tau}_1\vec{\tau}_2] \tag{8c}$$

$$\widehat{O}_2^1 = (ouu\overline{u}u + \overline{d}d\overline{d}d + \overline{u}u\overline{d}d + \overline{d}d\overline{u}u)_{VV+AA}$$

$$= N(N-1)(1_1 1_2 - \vec{\sigma}_1\vec{\sigma}_2) \tag{8d}$$

$$\widehat{O}_3^1 = (\overline{u}\vec{\lambda}u\overline{u}\vec{\lambda}u + \overline{d}\vec{\lambda}d\overline{d}\vec{\lambda}d + \overline{u}\vec{\lambda}u\overline{d}\vec{\lambda}d + \overline{d}\vec{\lambda}d\overline{u}\vec{\lambda}u)_{VV+AA}$$

$$= \vec{\lambda}_i\vec{\lambda}_j\widehat{O}_3^1 \tag{8e}$$

$$\widehat{O}_{\frac{1}{3}}^{15} = (\overline{u}u\overline{d}d - \overline{d}d\overline{u}u)_{VV+AA} \pm (\overline{u}u\overline{u}u - \overline{d}d\overline{d}d)_{VV-AA}$$

$$= \pm(N-4)\, T_3 \pm N\, \overrightarrow{\Sigma}\, \vec{\sigma}_N(\tau_3)_N \tag{8f}$$

$$\begin{aligned}\widehat{O}^{15}_{\underset{4}{2}} &= (\overline{u}\vec{\lambda}u\overline{d}\vec{\lambda}d - \overline{d}\vec{\lambda}d\overline{u}\vec{\lambda}u)_{VV+AA} \pm (\overline{u}\vec{\lambda}u\overline{u}\vec{\lambda}u + \overline{d}\vec{\lambda}d\overline{d}\vec{\lambda}d)_{VV-AA} \\ &= \vec{\lambda}_i\vec{\lambda}_j\widehat{O}^{15}_{\underset{3}{1}} \; . \end{aligned} \tag{8g}$$

Here $\vec{\lambda}_i$ denotes the single particle color operator and $\overrightarrow{\Sigma}$ is the total spin operator. We have used the complete permutation symmetry of the baryon wave function, i.e., the two body operator is replaced by the operator which acts only on the first two particles (Eqs. (8a,e)) and the single particle operator is replaced by the operator which acts on the N^{th} particle (Eqs. (8f,g)). In Eqs. (8e,g) the total antisymmetry of the color part of the baryon wave function gives the same expectation value of the operator $\vec{\lambda}_i\vec{\lambda}_j$ for any i^{th} and j^{th} particle. Further, we make use of the connection between the unitarity and permutation symmetry, i.e., the spin or isospin of the first two particles is one (zero) if the spin or isospin part of the wave function is symmetric (antisymmetric) with respect to the exchange of the first two particles. In this case:

$$1_1 1_2 - \vec{\sigma}_1\vec{\sigma}_2 = 2\left(1 - \widehat{P}^S_{12}\right) \; , \tag{9a}$$

$$\vec{\tau}_1\vec{\tau}_2 = -1 + 2\widehat{P}^I_{12} \; . \tag{9b}$$

Here $\widehat{P}^{S,I}_{12}$ are permutation operators between the first two particles acting on the spin and isospin part of the wave function, respectively. Also, for the wave function which is antisymmetric with respect to the first two particles, one has:

$$(\tau_3)_1(\tau_3)_2 = -1 \; , \tag{10a}$$

$$\vec{\lambda}_1\vec{\lambda}_2 = -2\left(1 + \frac{1}{N}\right) \; . \tag{10b}$$

Here the normalization condition $Tr\lambda^i\lambda^j = 2\delta_{ij}$ was used. Using formulae (9,10). we get the matrix elements of operators (8a-e) between the nucleon states:

$$\left\langle \hat{O}_1 \right\rangle = \tfrac{1}{2}\, N(N-1)\, 16\, \frac{h\left[\frac{N+1}{2}\,,\frac{N-3}{2}\right]}{h\left[\frac{N+1}{2}\,,\frac{N-1}{2}\right]} = 2(N^2+2N-3)\ , \tag{11a}$$

$$\left\langle \hat{O}_2 \right\rangle = \left\langle \hat{O}_2' \right\rangle = 0\ , \tag{11b}$$

$$\left\langle \hat{O}_3^1 \right\rangle = -2\left(1+\frac{1}{N}\right)\left\langle \hat{O}_2^1 \right\rangle = -\left(1+\frac{1}{N}\right)\left\langle \hat{O}_1 \right\rangle\ . \tag{11c}$$

Here $h\left[\frac{N-1}{2}\,,\frac{N-3}{2}\right] = 2(N-2)!/\left[\left(\frac{N+1}{2}\right)!\,\left(\frac{N-3}{2}\right)!\right]$ appears as the number of Yamanouchi symbols of the type $\begin{array}{|c|c|c|}\hline 1 & \cdots & \\ \hline 2 & \cdots \\ \cline{1-2}\end{array}$.

In order to evaluate matrix elements of operators $\hat{O}_i^{15}$ we rewrite $\vec{\Sigma}\,\vec{\sigma}_N$ as $\frac{1}{2}\left[\vec{\Sigma}^2 + \vec{\sigma}_N^2 - (\vec{\Sigma}-\vec{\sigma}_N)^2\right]$ where $(\vec{\Sigma}-\vec{\sigma}_N)^2 = 0,8$ for the wave functions with r_0, r_0' (see Eq. (7)), respectively. This finally yields:

$$\left\langle \hat{O}^{15}_{\substack{1\\3}} \right\rangle = \pm\left\{ N\left[\tfrac{1}{2}\,(3+3-0)\,\frac{h\left[\frac{N-1}{2}\,,\frac{N-1}{2}\right]}{h\left[\frac{N+1}{2}\,,\frac{N-1}{2}\right]} + \tfrac{1}{2}\,(3+3-8)\,\left(\tfrac{-1}{3}\right)\left(1-\frac{h\left[\frac{N-1}{2}\,,\frac{N-1}{2}\right]}{h\left[\frac{N+1}{2}\,,\frac{N-1}{2}\right]}\right)\right]\right.$$

$$\left. +\,(N-4)\right\} T_3 = \pm 2(N-1)\,T_3\ , \tag{12a}$$

$$\left\langle \hat{O}^{15}_{\substack{2\\4}} \right\rangle = -2\left(1+\frac{1}{N}\right)\left\langle \hat{O}^{15}_{\substack{1\\3}} \right\rangle = \mp 4\left(N-\frac{1}{N}\right)T_3\ . \tag{12b}$$

Note that matrix elements for operators $\hat{O}_i^{15}$'s have the opposite sign for the neutron and proton states.

Again the explicit N dependence of the matrix elements for operators $\hat{O}_i$ appears simply as a function of the ratio of dimensions for different Young tableaux. The generalization of the method to higher spin states, e.g., Δ, and for other four quark operators can again be done. However, an extension of this approach to baryon states with nonzero strangeness is not obvious because the flavor parts of the wave function cannot be written in a way similar to the one given by Eq. (5). We expect that the *leading* N dependence of the matrix elements will remain similar also in this case.

Matrix elements (11, 12) calculated in a quark model with explicit nucleon wave functions, e.g., the MIT-bag model or the nonrelativistic harmonic oscillators give of course exactly the same result for $N = 3$. Expressions (11, 12) should also be multiplied by an integral I over the quark radial wave function. In the MIT-bag model a typical value for I is 0.002 GeV^3.

We can now evaluate $\mathcal{N}\mathcal{N}\pi$ PV amplitude[13] $A(n_-^0 : n \to p\pi^-)$ which is studied in PV nuclear electromagnetic transition.[14] When $p_\pi \to 0$, application of the soft-pion theorem, PCAC and current algebra gives the nonzero contribution $A^{CA}(n_-^0)$ from the so-called nonseparable diagrams. With the help of Eqs. (11,12) $A^{CA}(n_-^0)$ assumes the following form:

$$\begin{aligned} A^{CA}(n_-^0) &\equiv \frac{1}{f_\pi}\ \left[\left\langle p|H_W^{eff}(PC)|p\right\rangle - \left\langle n|H_W^{eff}(PC)|n\right\rangle\right] \\ &= \frac{-\sqrt{2}\,G_F}{3f_\pi}\sin^2\theta_W \left[(C_1^{15} - C_3^{15}) - 2\left(1+\frac{1}{N}\right)(C_2^{15} - C_4^{15})\right]\ (N-1)\,I\ . \end{aligned} \tag{13}$$

Here f_π, G_F and θ_W are the pion coupling constant, Fermi constant and Weinberg angle, respectively, while C_i^{15}'s denote renormalized coefficients in front of O_i^{15}'s operators. E.g., for $N = 3$ one obtains $A^{CA}(n_-^0) = 5 \cdot 10^{-8}$ by using values for C_i^{15} from Ref. 12. We do not present the explicit N-dependence for f_π, G_F and C_i^{15}s; however, employing the presented technique this can be done.

The explicit N-dependence of multiquark operators is also crucial for a proper interpretation of the Skyrme model results which are related to the ones of $SU(N \to \infty)_C$ theory. However, one would really like to link these results with the real world of $SU(N=3)_C$ theory. For that purpose, in the Skyrme model, the experimental values of the nucleon ($\mathcal{N}$) and Δ masses are chosen as input parameters, thus determining f_π and $e-$ the coefficient in front of the Skyrme term (see Ref. 5). On the other hand, in the large N limit, $m_\mathcal{N} \propto N$, $m_\Delta \equiv m_\mathcal{N}[1+\mathcal{O}(1/N)]$ and correspondingly $f_\pi \propto \sqrt{N}$, $e \propto 1/\sqrt{N}$. This means that in the Skyrme model $m_{\mathcal{N},\Delta}$, which have singular N dependence in $SU(N \to \infty)_C$ theory, were adjusted to fit the real world with finite $N=3$. One therefore expects that quantities of the Skyrme model with nonzero leading N-dependence, e.g., g_A, $\mu_{n,p}$ etc., will disagree with the corresponding experimental values, while quantities which are N independent in the leading order of N, e.g., f_π^2/g_A, $|\mu_p/\mu_n|$, etc., will be in good agreement with the real world. This is really the case; e.g., values for f_π^2/g_A and μ_p/μ_n are in agreement within 2% with the experimental values, although those for f_π, g_A or $\mu_{n,p}$ are far away from their experimental values (see Ref. 5 for numerical results). Also, the $\mathcal{N}-\pi$ scattering phase shifts,[15] which do not have leading N-dependence, are in good agreement with experiment.

The validity of the Skyrme model results in connection with the real world should thus be reexamined by keeping in mind the explicit N-dependence of relevant operators; results are reliable only for those quantities which do not have leading N dependence and possibly have N-dependent corrections of order $1/N^2$.[16] Therefore, having explicit N-dependence of relevant quantities is very useful in order to understand which Skyrme model results have strong predictive power. Similarly, matrix elements of $\widehat{O}_i$'s (see Eqs.(8)) evaluated in the Skyrme model[17] should be interpreted with care in comparison with corresponding QCD calculations ($SU(3)_C$); only ratios of certain $\langle \widehat{O}_i \rangle$'s (see Eqs. (11,12)) which in $SU(N)_C$ do not have leading N-dependence, e.g., $\langle \widehat{O}_1 \rangle / \langle \widehat{O}_2^2 \rangle$ etc., should be in good agreement with QCD results.

We believe that the method for the evaluation of matrix elements for the multi-quark operators between any two non-strange baryon states is not only of academic importance, but it will significantly contribute to deeper understanding of the large N expansion, to proper interpretation of the Skyrme model results, as well as to a simplified analysis of the weak interaction phenomenology. There is no doubt that the method drastically simplifies and shortens previously long, hard and tedious quark model calculations. It is straightforward and extremely useful.

ACKNOWLEDGEMENTS

We would like to thank M. Peskin for useful comments. One of us (J.T.) has a great pleasure to acknowledge the hospitality of L.-L. Chau, M. Creutz and other members of BNL High Energy Theory Group, as well as F. Gilman and other members of SLAC Theory Group.

REFERENCES

1. G. 't Hooft, Nucl. Phys. B72, 461 (1974); B75, 461 (1974). G. Rossi and G. Veneziano, Nucl. Phys. B123, 507 (1977).
2. E. Witten, Nucl. Phys. B160, 57 (1979).
3. M. Fukugita, T. Inami, N. Sakai and S. Yazaki, Phys. Lett. 72B, 237 (1977); D. Tadić and J. Trampetić, Phys. Lett. 114B, 179 (1982); A. Buras and J. M. Gerard, MPI-PAE/PTh40/85, June 1985.
4. N. K. Pak and H. C. Tze, Ann. Phys. 117, 164 (1979); A. P. Balachandran, Proceedings of the XXII International Conference on High Energy Physics, Leipzig 1984.
5. G. Adkins, C. Nappi and E. Witten, Nucl. Phys. B228, 552 (1983).
6. M. Mattis and M. Peskin, Phys. Rev. D32, 58 (1985). M. Karliner, M. Mattis and M. Peskin, private communications.
7. M. Hamermesh, Group Theory and its Application to Physics Problems, Addison-Wesley, Reading, Mass. (1962); A. Bohr and B. Mottelson, Nuclear Structure, Vol. 1, W. A. Benjamin, New York (1969).
8. T. Yamanouchi, Proc. Phys.-Math. Soc. Japan 19, 436 (1937).
9. For baryon states with $SU(N=3)_C$ see for example Appendix 1C, p. 130, of A. Bohr and B. Mottelson, Ref. 7. For dibaryon states with $SU(N=3)_C$ see M. Cvetič, B. Golli, N. Mankoč-Borštnik and M. Rosina, Nucl. Phys. A395, 349 (1983).
10. S. Shindler and R. Mirman, J. Math. Phys. 18, 1678 (1977).
11. A. Kakuto and F. Toyoda, Prog. Theor. Phys. 66, 2307 (1980); M. Mattis, unpublished.

12. D. Palle, I. Picek, D. Tadić and J. Trampetić, Nucl. Phys. B166, 149 (1980).

13. D. Guberina, D. Tadić and J. Trampetić, Nucl. Phys. B152, 429 (1979).

14. J. F. Covaignac, B. Vignon and R. Wilson, Phys. Lett. 67B, 148 (1977).

15. M. Karliner and M. Mattis, Phys. Rev. D31, 2833 (1985).

16. For example, in $SU(N)_C$ one obtains (in the same way as one obtains g_A/g_V), $\mu_{n,p} = \mu[\mp(N+2)/3 + 1/(2N)]$ and $|\mu_p/\mu_n| = 1 + 6/[2N(N+2) - 1] = 1 + O(1/N^2)$.

17. J. Trampetić, Phys. Lett. 144B, 250 (1984).