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VACUUM STABILITY BOUNDS ON ENHANCED COUPLINGS OF LIGHT HIGGS FIELDS*

MIRJAM CVETIČ AND CHRISTIAN R. PREITSCHOPF

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

and

MARC SHER

Division of Natural Sciences II
University of California, Santa Cruz, CA 95064

ABSTRACT

In the standard model with more than one Higgs doublet, Yukawa couplings can be enhanced by ratios of vacuum values. Georgi, Manohar and Moore argued that an enhanced t-quark coupling to a light Higgs could destabilize the effective potential of this Higgs; they used this argument to rule out an enhanced t-quark coupling to the $\varsigma(8.3)$. We extend their argument to Higgs fields of any mass, critically examine the sensitivity of the result to higher order corrections and to effects of the additional scalars, and we examine the validity of the effective field theory approximation.

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One of the interpretations of the now-defunct $\varsigma(8.3)$ was that it was a Higgs boson whose coupling to the t-quark was enhanced by a factor of 10. Such an enhancement can occur in two-Higgs models in which the ratio of vacuum expectation values, v_2/v_1, is about 10. A crucial factor in determining the properties of such a particle is knowing whether the couplings to the charge-2/3 quarks are also enhanced. Shortly after the discovery of the $\varsigma(8.3)$, Georgi, Manohar and Moore[1] (GMM) argued that the requirement of vacuum stability eliminated this possibility. They argued that if v_2/v_1 was large, then one scalar (the $\varsigma(8.3)$) would get a mass of $0(v_1)$ and the others would get a mass of $0(v_2)$. One can then integrate out the heavy degrees of freedom and consider the effective field theory (EFT) below v_2. By considering the self-coupling, λ, of the light scalar field, they showed that λ, while positive at $q^2 = v_1^2$, is driven negative at $q^2 < v_2^2$ by contribution of the enhanced coupling of the t-quark to the renormalization group equation (RGE). Thus the scalar potential for the light field is unstable if the coupling to the top quark is enhanced.

It is easy to see where this instability comes from. In the standard, single Higgs model, a fermion will contribute with a negative sign to the one-loop term in the effective potential. Thus a sufficiently heavy fermion will cause the potential to be unbounded from below. This was first used to bound fermion masses in the standard model in Refs. [2-3], and a more detailed and accurate renormalization group analysis was done in Refs. [4-5]. In the EFT for $q^2 < v_2^2$ mentioned above, the scale of the potential (corresponding to 250 GeV in the standard model) is v_1, which is of order 25 GeV. If one simply scales all masses of this effective theory up by a factor of 10, then one has a theory whose potential looks just like the standard model (except for gauge boson loops), with an 83 GeV Higgs *and* a 400 GeV top quark. Such a potential, as found in Refs. [2-5], is unstable due to the one-loop effect is of the heavy fermion.

The $\varsigma(8.3)$ is no longer with us, but the GMM argument will still apply to any situation in which there is a light Higgs and an enhanced t-quark coupling. In this letter, we calculate the upper bound to the enhancement of this coupling for any Higgs mass. It is shown that, for a light Higgs, this bound is much more severe than any previous bounds; in the EFT approach $v_2/v_1 \lesssim 2$. We then examine the approximations used by GMM more carefully; first estimating the higher order effects by considering the renormalization scheme dependence of the calculation, then including the effects of the fields with masses of $O(v_2)$ on the RGE's and examining the validity of the EFT approximation (which is certainly suspect if $v_2/v_1 \sim 2$).

In the two-Higgs model consistent with the discrete symmetry $\Phi_2 \to -\Phi_2$, the potential is

$$
\begin{aligned}
V = & -\tfrac{1}{2} m_1^2 \Phi_1^\dagger \Phi_1 - \tfrac{1}{2} m_2^2 \Phi_2^\dagger \Phi_2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\
& + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \tfrac{1}{2} \lambda_5 \left[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right] .
\end{aligned} \tag{1}
$$

The masses of the χ^+ (charged Higgs), χ^0 (pseudoscalar Higgs) and ϕ and η (scalar Higgs) are

$$
\begin{aligned}
m_{\chi^+}^2 &= -(\lambda_4 + \lambda_5)\,(v_1^2 + v_2^2) \\
m_{\chi^0}^2 &= -2\lambda_5 (v_1^2 + v_2^2) \\
m_{\phi,\eta}^2 &= +2\lambda_2 v_2^2 + 2\lambda_1 v_1^2 \pm \sqrt{(2\lambda_2 v_2^2 - 2\lambda_1 v_1^2)^2 + 4(\lambda_3 + \lambda_4 + \lambda_5)^2 v_1^2 v_2^2}
\end{aligned} \tag{2}
$$

where v_1 and v_2 are vacuum expectation values of Φ_1 and Φ_2, respectively. Constraints for the minimum of the Higgs potential are the following:

$$
\left\{ \lambda_1, \lambda_2, -\lambda_5, (-\lambda_4 - \lambda_5), 4\lambda_1\lambda_2 - (\lambda_3 + \lambda_4 + \lambda_5)^2 \right\} > 0 . \tag{3}
$$

In the limit $v_1/v_2 \to 0$, the neutral scalar masses become

$$
\begin{aligned}
m_\eta^2 &= 4\lambda_2 v_2^2 \\
M^2 \equiv m_\phi^2 &= \left[4\lambda_1 - (\lambda_3 + \lambda_4 + \lambda_5)^2/\lambda_2 \right] v_1^2
\end{aligned} \tag{4}
$$

and we have one light scalar and four heavy scalars. Integrating out the heavy degrees of freedom and defining $\lambda \equiv \lambda_1 - (\lambda_3 + \lambda_4 + \lambda_5)^2/4\lambda_2$, the potential for

ϕ is $V(\phi) = \frac{1}{4}\lambda(\phi^2 - 2v_1^2)^2$. As discussed in Ref. [5], one-loop corrections to this potential can be found by replacing λ by $\lambda(q)$ where q is the energy scale. Strictly speaking, this neglects the anomalous dimension of ϕ and the running of v_1 which, as shown in the renormalization group analysis of Ref. [5] give very small effects. The RGE for λ is[F1]

$$\frac{d\lambda}{dt} = 24\lambda^2 + (12h^2 - 9g^2 - 3g'^2)\lambda - 6h^4 + \tfrac{9}{8}g^4 + \tfrac{3}{4}g^2g'^2 + \tfrac{3}{8}g'^4 \qquad (5)$$

where $h = m_t/v_1$, $t = \frac{1}{16\pi^2}\ell n\,\frac{q}{v_2}$ and we are assuming $m_t = 40$ GeV. By requiring that $\lambda(q = v_2)$ be positive (above v_2 there is no effective theory), one gets a lower bound on $\lambda(v_1)$ which translates into a lower bound on $M \equiv m_\phi$. In the RGE, we follow GMM and only include the effects of the Yukawa coupling for $q^2 > m_t^2$ and of the gauge coupling for $q^2 > M_W^2$. The results are shown on the solid line in Fig. 1. We see that the bound on the enhancement is quite strong ($\lesssim 2$ for $m_\phi < 10$ GeV), much stronger than any previous bounds[6-8]. The $\zeta(8.3)$ is clearly excluded, as is the $\chi(2.2)$, with an enhancement of ~ 4. The solid line in Fig. 1 does turn over for $v_2/v_1 \sim 12$, going back towards $m_\phi = 0$. However, perturbation theory breaks down for $v_2/v_1 \sim 25$, so, this turnaround may not be meaningful.

We now examine the approximations used in the calculation more carefully.

In GMM, the contribution of the Yukawa coupling to the RGE was neglected for $q^2 \leq q_t^2 = m_t^2$. An alternative possibility would be to neglect this contribution for $q^2 \leq q_t^2 = 4m_t^2$, as is done in considering the contribution of a heavy fermion to the QCD beta-function[9]. The third possibility is to *never* neglect the Yukawa coupling contribution; this mass-independent scheme is precisely that used by Coleman and Weinberg[10] in their pioneering paper. These three possibilities are all reasonable renormalization procedures. A renormalization scheme should be chosen in such a way as to minimize higher-order corrections; without doing a higher-order calculation in a similar problem, we have no way of knowing which is preferable. We have therefore repeated the calculation using all three schemes;

the discrepancy is a measure of the uncertainty due to higher-order corrections. The dotted line in Fig. 1 corresponds to the scheme in which the Yukawa coupling contribution to Eq. (5) is dropped for $q^2 \leq q_t^2 = 4m_t^2$, and the dashed line is for the mass-independent scheme in which it is never dropped. We see that the uncertainty is extremely small for a light Higgs mass, but increases somewhat for larger ($\gtrsim$ 20 GeV) Higgs masses. In Fig. 2, we have examined the sensitivity of the bound to m_t, varying m_t from 30 to 50 GeV.

In the EFT approximation, all fields with masses of $0(v_2)$ are integrated out. It is possible, however, that due to small couplings, some of the fields may have masses which are small enough to contribute to the RGE's at values of q^2 well below v_2^2. We can improve the EFT approximation by keeping the contribution to the RGE's of these fields for values of q^2 greater than their masses. This was done above for gauge bosons by keeping the gauge boson contribution to Eq. (5) for $q^2 > M_W^2$. We have also found that if one includes this contribution for all q^2 instead, then the change in the results is negligible due to the small value of gauge coupling constant g and g' compared to the Yukawa coupling h.

The nature of the curves presented in Figs. 1-2 can also be analyzed by using the analytic solution for $\lambda(q)$ which in turn determines v_2/v_1 as a function of m_ϕ. For the case $q_t^2 = 0$ one obtains the following analytic solution:

$$\left(\frac{v_2}{v_1}\right)^{(1+(v_2/v_1)^2)} = \left[\frac{(\sqrt{5}-1)m_\phi^2 + 4m_t^2}{(-\sqrt{5}-1)m_\phi^2 + 4m_t^2}\right]^{(4\pi^2 v^2/3\sqrt{5}\,m_t^2)} \tag{6}$$

where $v^2 \equiv v_1^2 + v_2^2 = (173\ GeV)^2$. This solution is in agreement with the numerical result presented in Fig. 1 for the dashed line. From Eq. (6) one can easily see that when $\left[(v_2/v_1)^2 + 1\right]\ \ell n\ v_2/v_1 \gg \left(4\pi^2 v^2/3\sqrt{5}\,m_t^2\right)$, m_ϕ approaches its fixed point value $m_\phi^{f.p.} = (\sqrt{5}-1)^{1/2}\ m_t = 1.11\,m_t$. For the case with $m_t = 40$ GeV the fixed point $m_\phi^{f.p.} = 44.4$ GeV is reached to within a few percent when $v_2/v_1 > 10$.

On the other hand when $q_t^2 = 4m_t^2$ or $q_t^2 = m_t^2$ the analytic solution has a complicated form. In this case one can see that the fixed point for m_ϕ is *not* reached. Namely, when $v_2/v_1 \geq \sqrt{(v/q_t)^2 - 1}$, the Yukawa coupling h does not contribute to the RGE (5) for a range of q between v_1 and q_t. This results in a departure from the dashed curve in Fig. 1 in such a way that m_ϕ increases with a smaller slope as v_2/v_1 is increased. Eventually, long before the fixed point for m_ϕ is reached, the turnover takes place, so that m_ϕ starts decreasing with increasing v_2/v_1. For the case with $m_t = 40$ GeV this departure from the dashed line occurs when $v_2/v_1 \geq 1.9$ and $v_2/v_1 \geq 4.2$ for $q_t^2 = 4m_t^2$ and $q_t^2 = m_t^2$, respectively. These values for v_2/v_1 are significantly smaller than the value $v_2/v_1 > 10$, which is near the fixed point.

We now turn to the effects of additional scalars. To include these effects, it is necessary to use the full RGE's:

$$\frac{d\lambda_1}{dt} = 24\lambda_1^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_5^2 + \lambda_4^2 + 3(4h^2 - 3g^2 - g'^2)\lambda_1 \\ + \tfrac{9}{8}g^4 + \tfrac{3}{4}g^2g'^2 + \tfrac{3}{8}g'^4 - 6h^4 \tag{7a}$$

$$\frac{d\lambda_2}{dt} = 24\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_5^2 + \lambda_4^2 - 9g^2\lambda_2 - 3g'^2\lambda_2 \\ + \tfrac{9}{8}g^4 + \tfrac{3}{8}g'^2g^2 + \tfrac{3}{8}g'^4 \tag{7b}$$

$$\frac{d\lambda_3}{dt} = 4(\lambda_1 + \lambda_2)(3\lambda_3 + \lambda_4) + 4\lambda_3^2 + 2\lambda_5^2 + 2\lambda_4^2 + 3(2h^2 - 3g^2 - g'^2)\lambda_3 \\ + \tfrac{9}{4}g^4 - \tfrac{3}{2}g^2g'^2 + \tfrac{3}{4}g'^4 \tag{7c}$$

$$\frac{d\lambda_4}{dt} = 4(\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4)\lambda_4 + 8\lambda_5^2 + 3(2h^2 - 3g^2 - g'^2)\lambda_4 + 3g^2g'^2 \tag{7d}$$

$$\frac{d\lambda_5}{dt} = \lambda_5[4\lambda_1 + 4\lambda_2 + 8\lambda_3 + 12\lambda_4 - 9g^2 - 3g'^2 + 6h^2]\ . \tag{7e}$$

Equation (5) is obtained from Eqs. (7) by ignoring the contribution from λ_2 λ_3, λ_4 and λ_5. One can now include the effects of other scalars by integrating the other RGE's. Consider, for example, the effects of λ_5. If λ_5 is large, then $m^2_{\chi^0}$ is large and thus λ_5 will not contribute to the RGE's since λ_5 only contributes for $q^2 > m^2_{\chi^0}$. If it is small, then $m^2_{\chi^0}$ is small and λ_5 will contribute to the RGE's. However, the effect will be small simply because λ_5 is small. We have found the value of λ_5 which produces the largest effect on our results, and found the size of the effect to be negligible. The same is true for λ_2 and λ_4.[F2]

A different result occurs when considering the scalar coupling, λ_3.[F3] The reason is simple; as can be seen from Eq. (2), it is possible to have λ_3 large and *all* scalar masses small as long as the minimum constraints (3) are satisfied. This necessitates some fine-tuning, of course, since λ_1 must be very near $\lambda_3^2/4\lambda_2$ in order for the light Higgs to remain light. Thus, λ_3 can contribute to the RGE's for all values of q^2, even if it is large. If $\lambda_3 > 0$, we find that bound in Fig. 3a, where we have plotted the bound for $\lambda_3 = 0, 1, 2$ and 3. The bounds in this case are weakened somewhat, but not enough to salvage the $\chi(2.2)$ or $\varsigma(8.3)$. When $\lambda_3 < 0$, the entire picture is different. When λ_3 is positive or small, as in all previous cases, the smallest Higgs mass would occur when $\lambda(v_2) = 0$, and the requirement that $\lambda(v_2) > 0$ ensured that $\lambda > 0$ between v_1 and v_2. In the case of λ_3 negative and large, however, $\left.\frac{d\lambda}{dt}\right|_{v_2} < 0$, thus the vacuum stability requirement (that $\lambda > 0$ between v_1 and v_2) means that $\lambda(v_2)$ must be large. We have plotted the resulting bound in Fig. 3b for $\lambda_3 = 0, -1, -2$ and -3. We see that the bounds are weakened considerably if λ_3 is large and negative. For how large a λ_3 should perturbation theory be considered valid? This has been a subject of much discussion; arguments[11] based on the triviality of $\lambda\phi^4$ theory generally give a bound of $\lambda \sim 1$. If one continues the RGE's to higher scales, one finds that if $|\lambda_3(v_2)| \gtrsim 1.5$ then λ_3 reaches a Landau pole for scales in the TeV range. In the absence of a clear upper limit to the perturbatively reliable value of λ_3, we have simply plotted the bound for various values of λ_3 and will let the reader decide whether the GMM bound can be significantly weakened by a large value of λ_3.

How is this potentially large effect of λ_3 compatible with the EFT approximation? In making the EFT approximation, one does *not* assume v_1^2/v_2^2 is small, but rather that $f(\lambda_i)v_1^2/v_2^2$ is small, where $f(\lambda_i)$ is a ratio of some combination of the λ_i. In the case where λ_3 is large, this function depends on $\lambda_3/\sqrt{4\lambda_1\lambda_2 - \lambda_3^2}$, and thus the function is large if we have a large λ_3 and light Higgs scalars (i.e. $4\lambda_1\lambda_2 - \lambda_3^2$ is small).

The new features of the bound which arise when the evolution of all the λ_i parameters has been taken into account have its origin in the structure of the RGE's (7). Namely, one observes that $d\lambda_i/dt$'s are in general quadratic functions of λ_i's. Thus, RGE's have in general two sets of fixed points (one attractive and one repulsive). The attractive fixed points are:

$$\lambda_1^{f.p.} = \frac{h^2}{4}\,(\sqrt{5}-1)\ , \qquad \lambda_{2,3,4,5}^{f.p.} = 0\ . \tag{8}$$

Depending on the initial choice of $\lambda_i(v_2)$'s one can be in a region of either attractive fixed points or repulsive ones.[F4] Results are especially sensitive to the initial choice of λ_3. E.g., for $\lambda_3(v_2) \ll 0$, one is in a region of repulsive fixed points. In this case the evolution of λ_i's yields significantly different results from the one which arises from the evolution of effective λ determined by Eq. (5).

Finally, we can consider (in the case where λ_3 is not large and negative) the validity of the EFT approximation. By considering the effects of all of the scalars and gauge bosons, we have included many of the higher-order (in v_1^2/v_2^2) effects. Another source of corrections involve $0(v_1^2/v_2^2)$ corrections to the mass of the light Higgs. The mass given in Eq. (3), including first order corrections, gets multiplied by $1-(\lambda_3+\lambda_4+\lambda_5)^2v_1^2/4\lambda_2^2v_2^2$. Including this correction changes our figures very slightly (the change in v_1/v_2 for a given mass is $\lesssim 0.3$), primarily because when v_2/v_1 is relatively small, the curves are fairly flat and so a small change in the Higgs mass is unimportant.

Another possible source of corrections will come from mixing between the two neutral scalars, which is $0(v_1^2/v_2^2)$. The only significant effect of this mixing will be in the Yukawa coupling. The coupling to the weak eigenstate is m_t/v_1, but the coupling to the mass eigenstate is $m_t \cos\theta/v_1$. Since $\cos\theta < 1$, it is easy to see that, if one uses $(v_2/v_1)\cos\theta$ on the ordinate of Figs. 1-3, then the bounds are strengthened.

It thus appears that corrections to the EFT approximation will not significantly change our results. A complete analysis of the full potential without the EFT approximation would necessitate a full renormalization group analysis on a potential of more than one field. Except in certain special cases, such as the EFT approximation or Coleman-Weinberg symmetry breaking,[12] such an analysis has not been done, is extremely complicated, and beyond the scope of this letter.

We have examined different approaches in determining the upper bound of the ratio of vacuum expectation values v_2/v_1 as a function of the light neutral Higgs mass m_ϕ. Using the EFT approach we determined the bound with different prescriptions for the t-quark contribution to the one-loop RGE (Fig. 1), and different choices for m_t (Fig. 2). In all cases the nature of the bound is the same and it is in agreement with the GMM result[1] for light Higgs masses, thus excluding an enhanced t-quark coupling of $\varsigma(8.3)$ and $\chi(2.2)$. If one includes the effects of all the λ_i parameters the results are not changed significantly except when λ_3 is large and negative (Fig. 3b), in which case bounds are weakened considerably.

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FIGURE CAPTIONS

1. The upper bound to v_2/v_1 as a function of the light Higgs mass $M \equiv m_\phi$, calculated in the effective field theory approximation with $m_t = 40$ GeV, cutting off the top quark contribution to the beta-function at m_t (solid line), $2m_t$ (dotted line) and 0 (dashed line).
2. The upper bound to v_2/v_1 as a function of the light Higgs mass $M \equiv m_\phi$, calculated in the effective field theory approximation with different top quark masses.
3. The upper bound to v_2/v_1 as a function of the light Higgs mass $M \equiv m_\phi$, calculated for various values of λ_3. In (a), $\lambda_3 > 0$ and in (b), $\lambda_3 < 0$.

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FOOTNOTES

$F1$ This disagrees slightly with the equation used in Ref. [1].

$F2$ We redefined $\overline{\lambda}$ to equal $\lambda_4 + \lambda_5$, then cut off $\overline{\lambda}$ at the charged Higgs mass $(-\overline{\lambda}(v_1^2 + v_2^2))$. We have assumed throughout that none of the λ_i's is extremely small since radiative corrections will generate effective λ_i's of order α^2 (except λ_5).

$F3$ For the sake of simplicity we put $\lambda_4 = \lambda_5 = 0$ in this analysis.

$F4$ Note that the initial choice for effective λ, i.e., $\lambda(v_2) = 0$, is always in the regime of the attractive fixed point $\lambda^{f.p.} = \frac{h^2}{4}\,(\sqrt{5} - 1)$ of RGE (5).

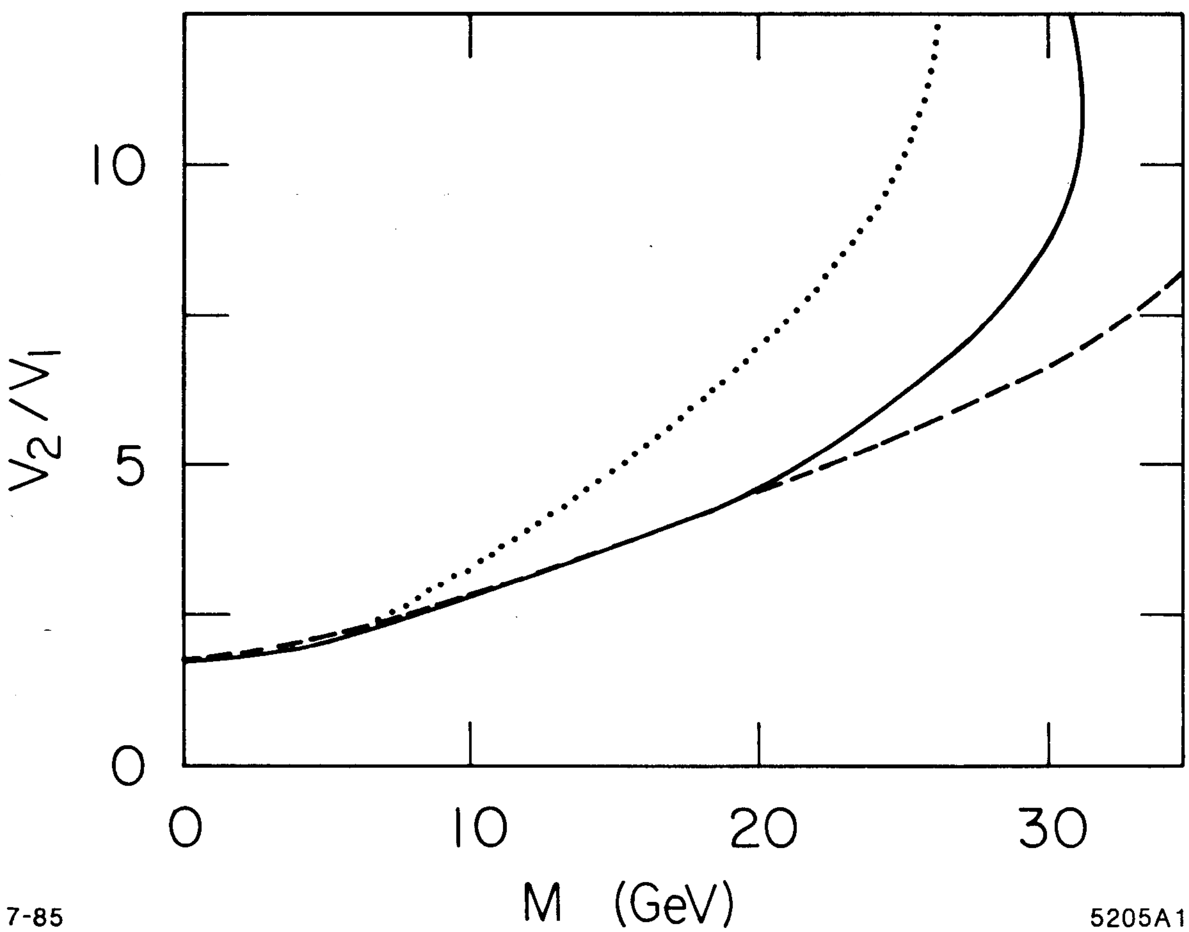

Fig. 1

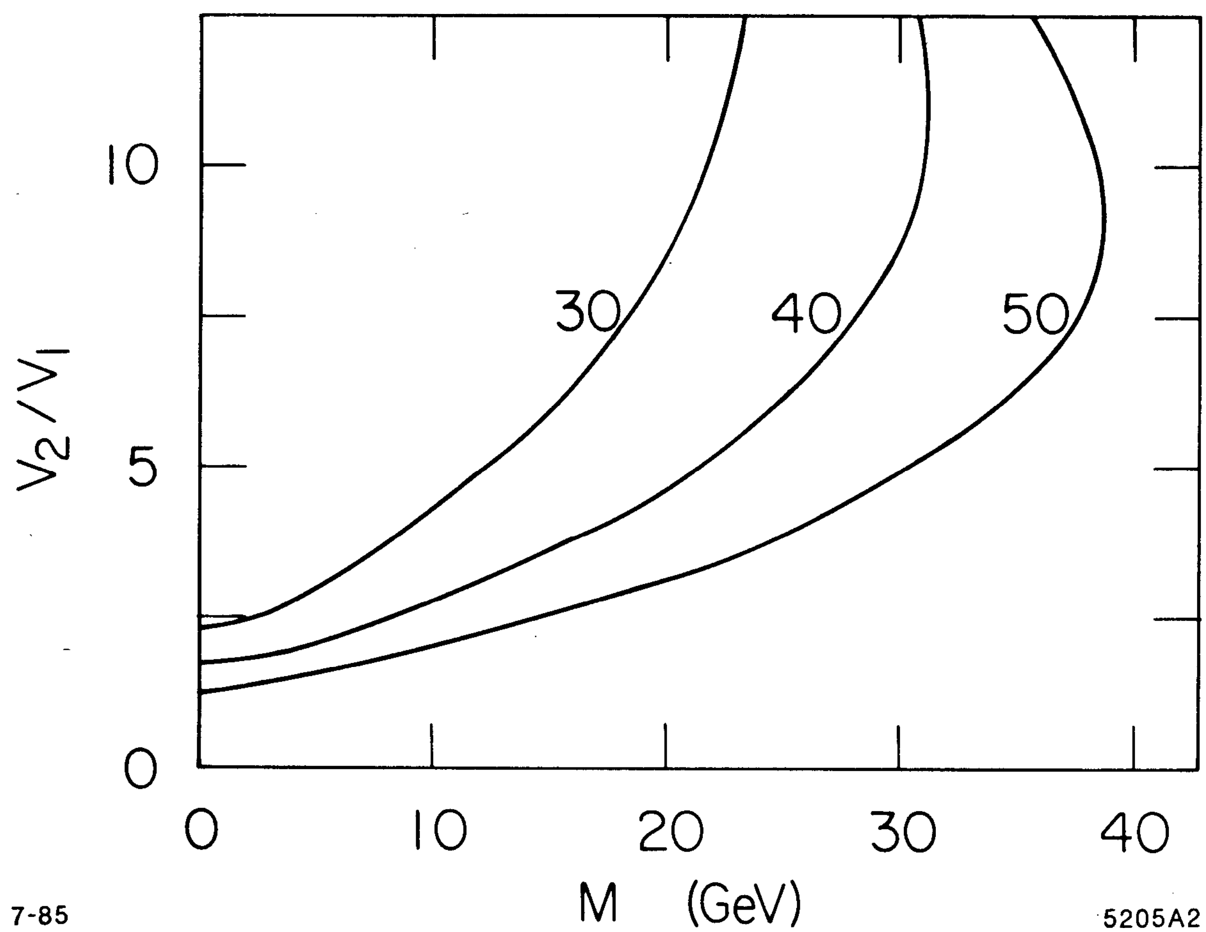

Fig. 2

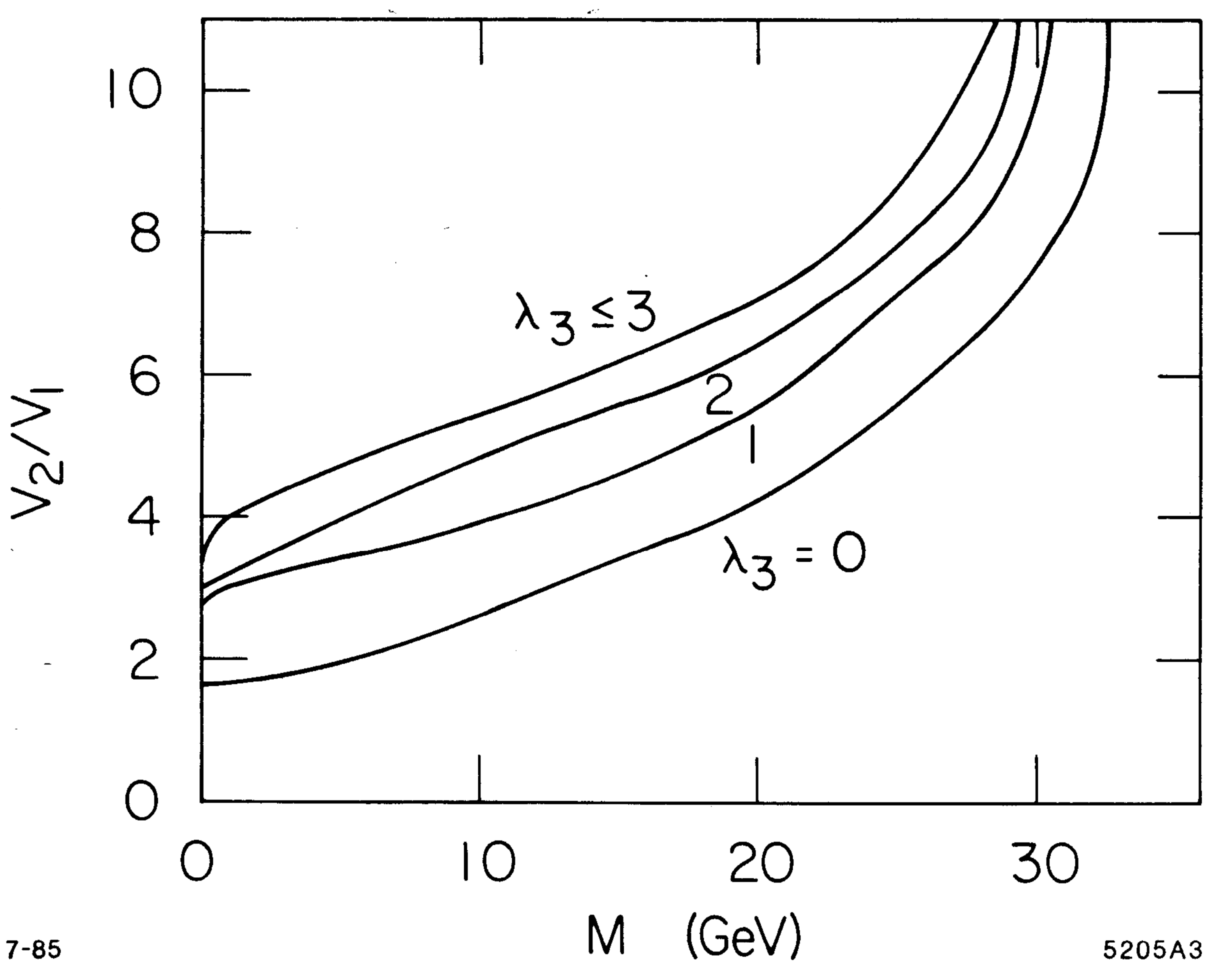

Fig. 3a

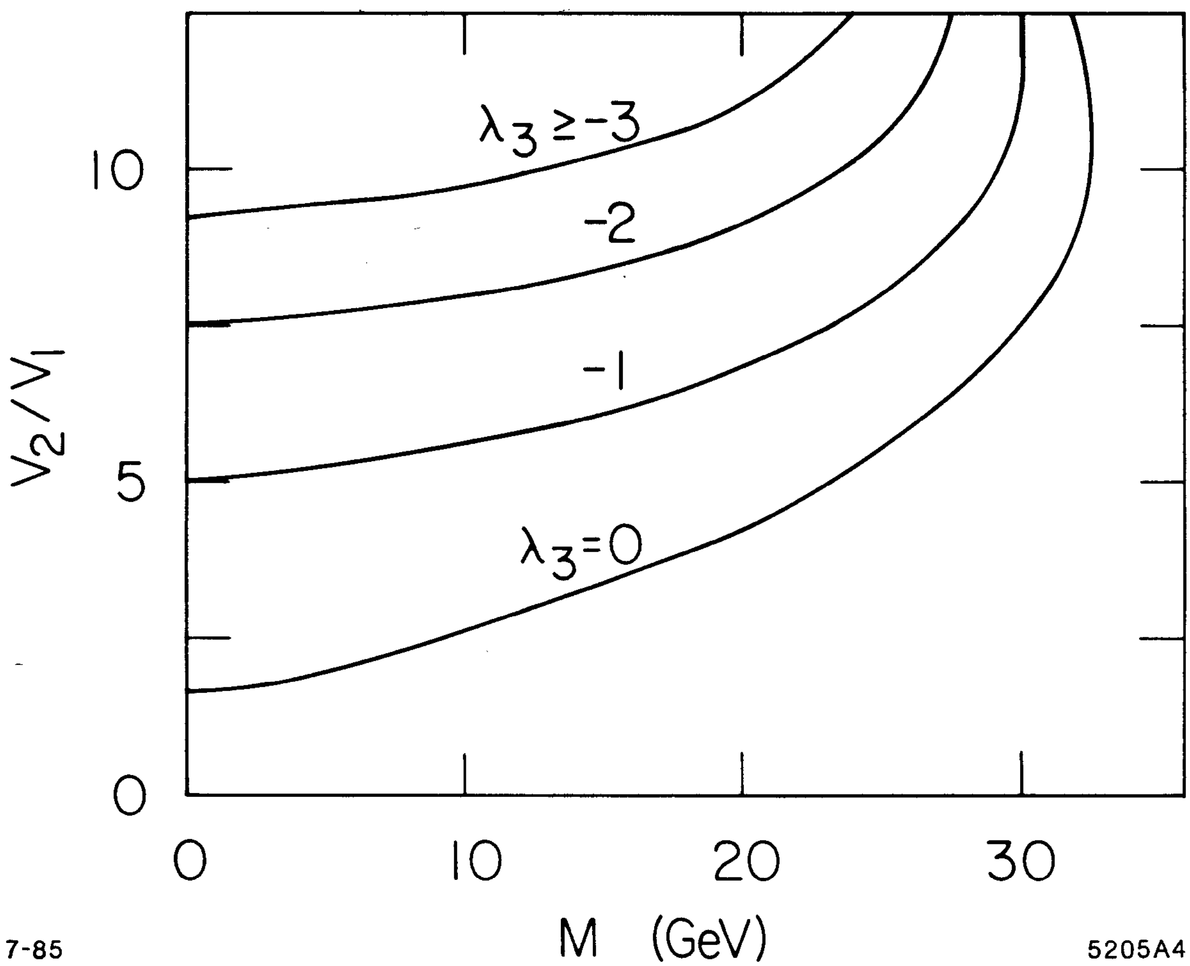

Fig. 3b