# EXCLUSIVE HEAVY MESON PRODUCTION IN $Z^{0}-$ DECAY* 

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## ABSTRACT

The exclusive two body decay of the $Z^{0}$ to heavy mesons is analyzed in the framework of perturbative QCD. We present a general formalism for calculating the decay widths of vector vector, vector pseudoscalar, and pseudoscalar pseudoscalar mesons with arbitrary constituent masses. Numerical estimates of the branching ratios for different exclusive decay modes of the $Z^{0}$ are presented.

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## 1. INTRODUCTION

With the expectation for copious production of $Z^{0}$ s at SLC and LEP in the next few years, studying the various decay modes of $Z^{0}$ becomes of prime importance. The interest in the $Z^{0}$-decay processes is due to the fact that the purely leptonic branching ratios of $Z^{0}$ are smaller than hadronic branching ratios. ${ }^{1}$ Therefore, one would expect that the hadronic decay channels play a dominant role in the decay of $Z^{0}$. In this paper, we address an exclusive hadronic decay of $Z^{0}$ in which the final state mesons contain heavy quarks $c, b$, or $t$. From the decay of $Z^{0} \rightarrow q \bar{q}$, we expect that the final state hadrons are produced through a non-perturbative mechanism similar to final state hadronization in $e^{+} e^{-}$reactions. However, there are certain types of reactions in which a perturbative QCD analysis is available. These are the well known high $Q^{2}$ reactions in which the running coupling constant $\alpha_{s}\left(Q^{2}\right)$ is small. The exclusive processes involving large momentum transfer have been extensively studied in the framework of perturbative QCD, and their relevance to the type of processes we are considering has been demonstrated. ${ }^{2}$ A key result is that such amplitudes factorize at large momentum transfer into the convolution of a hard scattering amplitude $T_{H}$, which can be computed perturbatively from quark- gluon subprocesses, and process-independent "distribution amplitudes" $\phi_{M}$, which contain all of the bound-state non- perturbative dynamics of each of the interacting hadrons. ${ }^{3}$ In fact the amplitude for $Z^{0}$-decay can be obtained by the generic form

$$
\begin{equation*}
M=\int T_{H}\left(x_{i}, Q\right) \phi_{M}\left(x_{i}, Q\right) \phi_{M}\left(x_{i}, Q\right)[d x] \tag{1.1}
\end{equation*}
$$

where $T_{H}$ is the subprocess scattering amplitude for quarks with fractional momentum $0<x_{i}<1, \phi_{M}$ is the probability amplitude to find quarks which are
collinear up to the scale $Q$ in a mesonic bound state, and

$$
\begin{equation*}
[d x]=\prod_{i} d x_{i} \delta\left(1-\sum_{i} x_{i}\right) \tag{1.2}
\end{equation*}
$$

The derivation of this factorization of perturbative and non-perturbative dynamics is by the use of a Fock basis $\psi_{M}\left(x_{i}, \vec{k}_{\perp i}, \lambda_{i}\right)$ defined at equal $\tau=t+z / c$ on the light-cone to represent relativistic color singlet bound states. ${ }^{4}$ Here $\lambda_{i}$ is helicity, and $x_{i}=\left(k_{i}^{0}+k_{i}^{3}\right) /\left(p^{0}+p^{3}\right)$ and $\vec{k}_{\perp i}$ are the momentum coordinates which satisfy

$$
\sum_{i} x_{i}=1 ; \quad \sum_{i} \vec{k}_{\perp i}=0
$$

Thus the meson state is represented as column vectors like $\psi_{q \bar{q}}, \psi_{q \bar{q} g}, \psi_{q \bar{q} q \bar{q}}$, etc. In the light-cone gauge $A^{+}=A^{0}+A^{3}=0$, only the minimal "valence" Fock state needs to be considered at large momentum transfer, since any additional quark or gluon forced to absorb large momentum transfer yields a contribution to the hadronic amplitude which is suppressed by a power law. ${ }^{3,5}$ In Eq. (1.1) at large $Q^{2}, T_{H}$ is computed from $Z^{0} \rightarrow(q \bar{q})(q \bar{q})$ tree graph amplitudes and $\phi_{M}$ is related to $\psi_{M}$ by

$$
\begin{equation*}
\phi_{M}\left(x_{i}, Q\right)=\int\left[d^{2} \vec{k}_{\perp}\right] \psi_{M}\left(x_{i}, k_{\perp i}\right) \Theta\left(\vec{k}_{\perp i}^{2}<Q^{2}\right) \tag{1.3}
\end{equation*}
$$

which actually represents the valence quark and antiquark wave function evaluated at quark impact separation $b_{\perp} \sim O\left(Q^{-1}\right)$. In Eq. (1.3) we have not written $\lambda_{i}$ explicitly because the helicity is fixed to the quark helicity $\left|\lambda_{i}\right|=1 / 2$. Our calculations are mostly concerned with exclusive heavy meson decay products like $B_{c}, T_{b}, T_{c}$, etc. in which the binding energy is very small as compared to mass of the constituent quarks. In these processes one can invoke an approximate and
simple meson two-body wavefunction ${ }^{6}$

$$
\begin{equation*}
\psi_{M}\left(x_{i}, \vec{k}_{\perp i}\right)=\frac{128 \pi b^{3}\left(m_{1}+m_{2}\right)}{x_{1}^{2} x_{2}^{2}\left[M^{2}-\frac{m_{1}^{2}+\vec{k}_{\perp 1}^{2}}{x_{1}}-\frac{m_{2}^{2}+\vec{k}_{\perp 2}^{2}}{x_{2}}\right]^{2}} \tag{1.4}
\end{equation*}
$$

where $b$ is the binding energy of the two-body bound state and $M$ is the meson mass. Both in the equal mass case $m_{1}=m_{2}$, and the unequal mass case $m_{1} \gg m_{2}$, one can show the above wave function is the Schrödinger equation with a Coulomb potential which is the nonrelativisitic limit of the Bethe-Salpeter equation with the QCD kernel. ${ }^{2,6}$

The decay mode of $Z_{0} \rightarrow V \bar{V}$, where $V$ is a $(t \bar{u})$ vector meson, was considered in Ref.7, in which they studied the decay width for the case of heavy quark mass/meson mass $\approx 1$. However, in this paper, we give a more general formulation for the decay of $Z^{0}$ to different combinations of heavy vector and pseudoscalar mesons with arbitrary constituent masses in the final states.

The plan of our paper is as follows. In Section 2 we present the general framework of our calculations. In Section 3, we specifically consider the different spin configurations of final state mesons. The ratios of production rates turn out to be somewhat sensitive to the spins of the final state mesons because of spin correlations. In the numerical estimates of different widths and branching ratios presented in Section 4 we find that the width depends strongly on the constituent masses. Finally, in Section 5 we present a summary of our results and conclusions.

## 2. The $Z^{0}$ Decay Amplitudes

In the framework of perturbative $Q C D$, the leading contributions to the $Z^{0}$ decay processes with two final state mesons are shown in Fig. 1. As we discussed in the Introduction, by QCD factorization, the perturbative and non-perturbative parts of this process can in written in the following form

$$
\begin{equation*}
\mathcal{M}=-4 \pi C_{F} \alpha_{s} \frac{1}{\sqrt{2}}\left(\frac{G_{F} M_{z}^{2}}{\sqrt{2}}\right)^{1 / 2} \int d x d y A(q, p, x, y) \phi_{M}(x, q) \phi_{M}(y, q) \tag{2.1}
\end{equation*}
$$

where $C_{F}=4 / 3$ is the color factor, $\alpha_{B}$ and $G_{F}$ are the strong and Fermi coupling constants respectively, and $M_{z}$ is the $Z^{0}$ mass. The form of $A(q, p, x, y)$ for each case of the final state mesons will be given in the following section. $\phi_{M}\left(x_{i}, q\right)$ is given by Eqs. (1.3) and (1.4) and at large $q^{2}$ is

$$
\begin{align*}
\phi\left(x, q^{2}\right) & =\frac{\left(128 \pi b^{5} M\right)^{1 / 2}}{16 \pi^{2}\left[x(1-x) M^{2}-(1-x) m_{1}^{2}-x m_{2}^{2}\right]}  \tag{2.2}\\
& \simeq \frac{f_{M}}{2 \sqrt{3}} \delta\left(x-\frac{m_{1}}{m_{1}+m_{2}}\right)
\end{align*}
$$

where $f_{M}=\left(6 b^{3} / \pi M\right)^{1 / 2}$ is the meson decay constant. Since $\phi_{M}$ depends only logarithmically on $q^{2}$, the approximate form (2.2) does not change significantly with varying $q^{2}$. Therefore, by carrying out the integration in Eq. (2.1) we obtain

$$
\begin{equation*}
\mathcal{M}=4 \pi C_{F} \alpha_{s} \frac{1}{\sqrt{2}}\left(\frac{G_{F} M_{z}^{2}}{\sqrt{2}}\right)^{1 / 2}\left(\frac{f_{M}}{2 \sqrt{3}}\right)^{2} A(q, p, x) \tag{2.3}
\end{equation*}
$$

where now $x=y=m_{1} /\left(m_{1}+m_{2}\right)$ with $m_{1} \geq m_{2}$. In the following section we will utilize Eq. (2.3) to exhibit the expressions for the decay rates of the various decay products.

## 3. Decay Rates For Various Spin Combinations

(a) Vector Vector Case:

In the small $\overrightarrow{\boldsymbol{k}}_{\perp}$ approximation a vector meson with unequal constituent masses has a contribution to the vertex factor given by ${ }^{8}$

$$
\begin{equation*}
\sum_{s p i n} v(x p) \bar{u}((1-x) p)=\sqrt{\frac{x(1-x)}{2}} k_{1}(-i p+M) \tag{3.1}
\end{equation*}
$$

where $\epsilon_{1}$ is the polarization of the final state meson with momentum $p$. This form was obtained in an analogous way to that of a vector meson with equal mass constituents. ${ }^{9}$ Then the amplitude for the processes shown in Fig. 1(a) and (b) is given by Eq. (2.3) where $A(q, p, x)$ is

$$
\begin{align*}
A(q, p, x)= & \frac{1}{2 M_{z}^{4}(1-x)^{3}} \operatorname{Tr}\left[\notin \Gamma_{q} \not k_{2}\{-i(\phi-p p)+M\} \gamma_{\alpha} k_{1}(-i p p+M)\right. \\
& \times \gamma_{\alpha}[-i\{(1-x) \phi+x p\}+x M]  \tag{3.2}\\
& +\notin \Gamma_{q}\{-i(-\phi+x p p)+x M\} \gamma_{\alpha} k_{2}\{-i(\phi-p p)+M\} \\
& \left.\times \gamma_{\alpha} k_{1}(-i \not p+M)\right]
\end{align*}
$$

The factor $1 / M_{z}^{4}(1-x)^{3}$ is coming from quark and gluon propagators in Fig. 1 (a) and (b), $\epsilon, \epsilon_{1}$ and $\epsilon_{2}$ are the polarization vectors for the $Z^{0}$ and the two vector mesons respectively. $\Gamma_{q}$ is the $Z^{0}$ weak coupling with primary quark pairs (heavy quark pairs in this case) given by

$$
\begin{equation*}
\Gamma_{q}=R_{q}\left(1+\gamma_{5}\right)+L_{q}\left(1-\gamma_{5}\right) \tag{3.3}
\end{equation*}
$$

with

$$
\begin{equation*}
R_{Q}=-2 Q_{q} \sin ^{2} \theta_{w} \quad L_{q}=\tau_{3}-2 Q_{q} \sin ^{2} \theta_{w} \tag{3.4}
\end{equation*}
$$

where $\tau_{3}=2 I_{3}$ refers to the third component of the weak isospin and $Q_{q}$ is the magnitude of the charge of the quarks coupled to $Z^{0}$. The contributions from the diagrams obtained by exchanging primary and secondary quark pairs can be derived from Eq. (3.2) by letting $x \rightarrow 1-x$, where $\Gamma_{q}$ now refers to the coupling of $Z^{0}$ to light quark pairs. The expression for the decay rate now becomes

$$
\begin{align*}
\Gamma_{V \bar{V}}= & \frac{256}{243} \frac{\pi^{2} \alpha \alpha_{s}^{2}}{\sin ^{2} 2 \theta_{w}} \frac{f_{M}^{4} M^{2}}{M_{z}^{5}}\left(1-\frac{4 M^{2}}{M_{z}^{2}}\right)^{3 / 2} \\
& \times\left[\frac { 1 } { ( 1 - x ) ^ { 6 } } \left[\left(I_{3}^{h}-2 Q_{q}^{h} \sin ^{2} \theta_{w}\right)^{2}\right.\right. \\
& \left.\times\left\{\frac{(1-x)^{2} M_{z}^{2}}{4 M^{2}}-\left(x^{2}-x-1\right)+3 x^{2} \frac{M^{2}}{M_{z}^{2}}\right\}+\left(I_{3}^{h}\right)^{2} x^{2}\left(1-\frac{4 M^{2}}{M_{z}^{2}}\right)\right] \\
& +\frac{2}{x^{3}(1-x)^{3}}\left[\left(I_{3}^{h}-2 Q_{q}^{h} \sin ^{2} \theta_{w}\right)\left(I_{3}^{\ell}-2 Q_{q}^{\ell} \sin ^{2} \theta_{w}\right)\right. \\
& \left.\times\left\{\frac{3}{2}+x(1-x)\left(3 \frac{M^{2}}{M_{z}^{2}}-1+\frac{M_{z}^{2}}{4 M^{2}}\right)\right\}+I_{3}^{h} I_{3}^{\ell} x(1-x)\left(1-\frac{4 M^{2}}{M_{z}^{2}}\right)\right] \\
& +\frac{1}{x^{6}}\left[\left(I_{3}^{\ell}-2 Q_{q}^{\ell} \sin ^{2} \theta_{w}\right)^{2}\left\{\frac{x^{2} M_{z}^{2}}{4 M^{2}}-\left(x^{2}-x-1\right)+3(1-x)^{2} \frac{M^{2}}{M_{z}^{2}}\right\}\right. \\
& \left.\left.+\left(I_{3}^{\ell}\right)^{2}(1-x)^{2}\left(1-\frac{4 M^{2}}{M_{z}^{2}}\right)\right]\right] \tag{3.5}
\end{align*}
$$

An approximation to this form has been obtained by Horgan, et al. ${ }^{7}$ who considered the case $m_{1} \gg m_{2}$, so that $m_{1} \approx M$. Our result reduces to theirs in the
limit of $x \approx 1$.
(b) Two Pseudoscalar Mesons:

Using the same approximation as in Section 2(a) we find a vertex factor for a pseudoscalar meson with unequal quark masses to be given by ${ }^{10}$

$$
\begin{equation*}
\sum_{\mathrm{spin}} v(x p) \bar{u}[(1-x) p]=\sqrt{x(1-x)} \gamma_{5}(-i p+M) \tag{3.6}
\end{equation*}
$$

Similarly $A(q, p, x)$ for the diagrams of Fig. $1(a)$ and (b) is

$$
\begin{align*}
A(q, p, x)= & \frac{1}{M_{z}^{4}(1-x)^{3}} \operatorname{Tr}\left[k \Gamma_{q}\{(i(\phi-\not p)+M\}\right. \\
& \times \gamma_{5}\{-i(\not q-\not p)+M\} \gamma_{\alpha} \gamma_{5}(-i p p+M) \gamma_{\alpha} \\
& \times\{-i[(1-x) \phi+x \not p]+x M\}  \tag{3.7}\\
& +\notin \Gamma_{q}\{-i(-\not q+x p p)+x M\} \gamma_{\alpha} \gamma_{5} \\
& \left.\times\{-i(p-p p)+M\} \gamma_{\alpha} \gamma_{5}(-i p+M)\right]
\end{align*}
$$

Where all the symbols are as defined in Section 2(a). The corresponding decay rate is

$$
\begin{align*}
\Gamma_{P S P S}= & \frac{256 \pi^{2}}{243} \frac{\alpha \alpha_{s}^{2}}{\sin ^{2} 2 \theta_{w}} \frac{f_{M}^{4} M^{2}}{M_{z}^{5}}\left(1-\frac{4 M^{2}}{M_{z}^{2}}\right)^{3 / 2} \\
& \times\left[\frac{1}{(1-x)^{6}}\left(I_{3}^{h}-2 Q_{q}^{h} \sin ^{2} \theta_{w}\right)^{2}\left\{(1-x) \frac{M_{z}}{M}-2 x \frac{M}{M_{z}}\right\}^{2}\right. \\
& +\frac{2}{x^{3}(1-x)^{3}}\left(I_{3}^{h}-2 Q_{q}^{h} \sin ^{2} \theta_{w}\right)\left(I_{3}^{\ell}-2 Q_{q}^{\ell} \sin ^{2} \theta_{w}\right)  \tag{3.8}\\
& \times\left\{4 \frac{M^{2}}{M_{z}^{2}} x(1-x)-2\left(2 x^{2}-2 x+1\right)+\frac{M_{z}^{2}}{M^{2}} x(1-x)\right\} \\
& \left.+\frac{1}{x^{6}}\left(I_{3}^{\ell}-2 Q_{q}^{\ell} \sin ^{2} \theta_{w}\right)^{2}\left\{x \frac{M_{z}}{M}-2(1-x) \frac{M}{M_{z}}\right\}^{2}\right]
\end{align*}
$$

(c) Vector Pseudoscalar Mesons:

Using the vertex factors given by Eqs. (3.1) and (3.6), we determine $A(q, p, x)$ in this case to be

$$
\begin{align*}
A(q, p, x)= & \frac{1}{\sqrt{2} M_{z}^{4}(1-x)^{3}} \operatorname{Tr}\left[k \Gamma_{q} k_{2}\{-i(\phi-p p)+M\} \gamma_{\alpha} \gamma_{5}(-i p p+M)\right. \\
& \times \gamma_{\alpha}\{-i[(1-x) \phi+x p]+x M\}+\notin \Gamma_{q}\{-i[-\phi+x p]+x M\}  \tag{3.9}\\
& \left.\times \gamma_{\alpha} k_{2}\{-i[\phi-p]+M\} \gamma_{\alpha} \gamma_{5}(-i \not p+M)\right]
\end{align*}
$$

Consequently, the decay rate for vector pseudoscalar final states turns out to be

$$
\begin{align*}
\Gamma_{V P S}= & \frac{256}{243} \frac{\pi^{2} \alpha \alpha_{s}^{2}}{\sin ^{2} 2 \theta_{w}} \frac{f_{M}^{4} M^{2}}{M_{z}^{5}}\left(1-\frac{4 M^{2}}{M_{z}^{2}}\right)^{3 / 2} \\
& \times\left[\frac { 1 } { ( 1 - x ) ^ { 6 } } \left[\left(I_{3}^{h}-2 Q_{q}^{h} \sin ^{2} \theta_{w}\right)^{2}+\left(I_{3}^{h}\right)^{2}\right.\right. \\
& \left.\times\left\{2-x^{2}\left(1+\frac{4 M^{2}}{M_{z}^{2}}\right)+(1-x)^{2} \frac{M_{z}^{2}}{2 M^{2}} \frac{1-\left(\frac{4 M^{2}}{M_{z}^{2}}\right)^{3}}{\left(1-\frac{4 M^{2}}{M_{z}^{2}}\right)^{2}}\right\}\right] \\
& +\frac{2}{x^{3}(1-x)^{3}}\left[\left(I_{3}^{h}-2 Q_{q}^{h} \sin ^{2} \theta_{w}\right)\left(I_{3}^{\ell}-2 Q_{q}^{\ell} \sin ^{2} \theta_{w}\right)+I_{3}^{h} I_{3}^{\ell}\right.  \tag{3.10}\\
& \left.\times\left\{2-x(1-x)\left(3+\frac{4 M^{2}}{M_{z}^{2}}-\frac{M_{z}^{2}}{2 M^{2}}\right)+\frac{6 x(1-x)}{\left(1-\frac{4 M^{2}}{M_{z}^{2}}\right)}\right\}\right] \\
& +\frac{1}{x^{6}}\left[\left(I_{3}^{\ell}-2 Q_{q}^{\ell} \sin ^{2} \theta_{w}\right)^{2}+\left(I_{3}^{\ell}\right)^{2}\right. \\
& \left.\left.\times\left\{2-(1-x)^{2}\left(1+\frac{4 M^{2}}{M_{z}^{2}}\right)+x^{2} \frac{M_{z}^{2}}{2 M^{2}} \frac{1-\left(\frac{4 M^{2}}{M_{z}^{2}}\right)^{3}}{\left(1-\frac{4 M^{2}}{M_{z}^{2}}\right)^{2}}\right\}\right]\right]
\end{align*}
$$

The three expressions given by Eqs. (3.5), (3.8) and (3.10) will be used to determine the rates and ratios of the decay widths given in the next section.

## 4. NUMERICAL RESULTS

In this part we present some estimates of the absolute decay widths and branching ratios for different decay modes we studied in section III. As can be seen from Eqs. (3.5), (3.8) and (3.10) there are several factors for which we do not have accurate data. For example, the meson decay constant $f_{M}$ is not known for all the cases we are considering. For the case of B-mesons, we used the recent calculation from QCD sum rules, ${ }^{11}$ which is $f_{B}=190 \mathrm{Mev}$. However, it is expected that this value will vary depending on spin of the $B$-mesons. In fact by Eq. (3.5), $\Gamma \propto f_{M}^{4}$, therefore even a five percent variation amounts to about a twenty percent effect on our estimates. In the calculation of widths we used the following numerical values:

$$
\begin{array}{lll}
\alpha_{s}=0.2 & f_{B}=190 \mathrm{MeV} & f_{T}=200 \mathrm{MeV} \\
m_{t}=40 \mathrm{GeV} & m_{b}=1.5 \mathrm{GeV} & m_{u}=0.30 \mathrm{GeV} \\
M_{z}=93 \mathrm{GeV} & \Gamma_{z}=2.9 \mathrm{GeV} &
\end{array}
$$

The results for absolute widths and branching ratios are given in Table I. From Eq. (3.5), we observe that the contributions from the diagrams in which the secondary quark pairs are heavier than the primary quark pairs are negligible as compared to the opposite case. This can be seen by examining the factors $1 / x^{6}$, $1 / x^{3}(1-x)^{3}$, and $1 /(1-x)^{6}$. Since $x$ is defined by the ratio of the heavy quark mass to the meson mass, $1 / x^{6}$ and $1 / x^{3}(1-x)^{3}$ are much smaller than $1 /(1-x)^{6}$. This means that the main contributions to Eqs. (3.5), (3.8), and (3.10) come from the case in which the primary quark pairs are heavier than the secondary quark pairs. If the primary quark pairs are light as compared to $M_{Z}$ (e.g. (b, c, s, etc), then we expect the produced meson mass also will be light, and multiparticle
production will be the dominant decay model. Consequently, in this case, very small rates are expected for the exclusive two body decay mode. On the other hand, because the $t$-quark mass is close to $M_{z} / 2$, the decay mode $Z^{0} \rightarrow t \bar{t}$ picks up significant contributions from the two body exclusive decay. Of course for cases like $T_{b} \bar{T}_{b}$ mesons, the rate is down again due to the quark mass suppression (b-quark in this case) in the production of secondary quark pairs. However, for $Z^{0} \rightarrow T_{u} \bar{T}_{u}$, the mass effect does not suppress the pair production of $u \bar{u}$ quarks from the emitted gluon, [see Fig.(1)]. Consequently, the decay rate is expected to be much higher than the previous cases. It turns out that for this particular decay mode our calculations give a width of about 100 Mev which amounts to a branching ratio of about 3 percent. This is consistent with the results of the free quark calculation of $Z^{0} \rightarrow t \bar{t}^{1}$. Of course, the perturbative calculations as presented here may be less believable in this case. Nevertheless these results give an indication of an interesting decay mode to be searched for in the future.

Another interesting result, as given in Table $I$ is the ratio of the decay rates for different spin configurations among the final state mesons. The naive expectation based on spin considerations is $\Gamma_{v v} / \Gamma_{p s p s} \approx 3$. Our calculations give values between 2.4 and 5 which of course needs to be taken cautiously due to the uncertainties associated with the meson decay constant $f_{M}$ as discussed before. In processes like $B_{c} \bar{B}_{c}$ production, this naive expectation is not realized because the mass of the secondary quark pairs ( $c$ and $\bar{c}$ ) is not negligible as compared to the primary pairs ( $b$ and $\bar{b}$ ).

Generally the production of the vector-pseudoscalar spin combination is expected to dominate. This becomes more evident when we consider the production of mesons with masses comparable to $M_{Z} / 2$. In this case, the fact that the final
state consisting of vector and pseudoscalar mesons can exist in a state of relative angular momentum $l=0$ makes it the most favored decay mode among all possible final states.

## 5. CONCLUSIONS

In this paper we analyzed the decay of $Z^{0} \rightarrow M \bar{M}$, in the framework of perturbative QCD. The calculation was extended to different final state spin combinations and mesons with different constituent quark masses. In the limit of $x \rightarrow 1$, we found our results for vector the meson production case to be consistent with the results of Ref.7. In summary we found that:
(i) The decay widths, though small, are strongly correlated with different final state spin combinations of the produced heavy mesons. In fact, the combination of vector and pseudoscalar mesons is usually the dominant decay mode. This effect is much more evident when we consider mesons with masses comparable to that of $M_{Z} / 2$.
(ii) The exclusive decay rates and branching ratios for heavy mesons with masses much smaller than the mass of $Z^{0}$ are very small and in some cases almost negligible. Nevertheless, because $m_{t} \sim M_{z} / 2$, we found indications that the rate for production of two mesons like ( $T_{u} \bar{T}_{u}$ ) might be more accessible to experimental measurements and probably can be measured at both SLC and LEP. We would like to emphasize that the rates rather strongly depend on the binding energy of the mesons [see Eq. (1)]. This means that even though the order of magnitude of the rates does not change significantly with fluctuations of the value of $f_{M}$, the factors given in Table I can change. In these calculations we used the best estimates available from QCD sum rules.

Finally, in this paper our primary interest was focused on the exclusive two body decay of $Z^{0}$. However, we observed that multiparticle production probably accounts for a large fraction of the decay width of the $Z^{0}$. It is also expected that inclusive processes like $e^{+} e^{-} \rightarrow Z^{0} \rightarrow$ Meson +X , will yield higher rates than we calculated. Calculations of these inclusive cross sections is in progress by the authors.

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## FIGURE CAPTION

1. Decay of $Z^{0}$-boson into two mesons. Each meson can be either a vector or a pseudoscalar. Two more diagrams can be obtained from (a) and (b) by exchanging primary and secondary quark pairs.


Fig. 1

Table I

| Meson | $B_{u}$ |  |  | $B_{c}$ |  |  | $T_{c}$ |  |  | $T_{b}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Spin | $\Gamma_{1}=$ | $\Gamma_{2}=$ | $\Gamma_{3}=$ | $\Gamma_{1}=$ | $\Gamma_{2}=$ | $\Gamma_{3}=$ | $\Gamma_{1}=$ | $\Gamma_{2}=$ | $\Gamma_{3}=$ | $\Gamma_{1}=$ | $\Gamma_{2}=$ | $\Gamma_{3}=$ |
| Combination | $V \bar{V}$ | $P S \overline{P S}$ | $V \overline{P S}$ | $V \bar{V}$ | $P S \overline{P S}$ | $V \overline{P S}$ | $v \bar{V}$ | $P S \overline{P S}$ | $V \overline{P S}$ | $V \bar{V}$ | $P S \overline{P S}$ | $V \overline{P S}$ |
| Width ( GeV ) | $1.2 \times 10^{-7}$ | $0.5 \times 10^{-7}$ | $2.5 \times 10^{-7}$ | $3 \times 10^{-8}$ | $6.2 \times 10^{-8}$ | $9.5 \times 10^{-8}$ | $3.1 \times 10^{-6}$ | $6.2 \times 10^{-7}$ | $3.7 \times 10^{-6}$ | $7.5 \times 10^{-10}$ | $1.92 \times 10^{-10}$ | $7.52 \times 10^{-9}$ |
| $\Gamma_{i j}=\Gamma_{i} / \Gamma_{j}$ | $\Gamma_{12}=2.4$ | $\Gamma_{13}=0.5$ | $\Gamma_{32}=4.8$ | $\Gamma_{12}=0.47$ | $\Gamma_{13}=0.3$ | $\Gamma_{32}=1.6$ | $\mathrm{r}_{12}=5$ | $\mathrm{r}_{13}=0.8$ | $\Gamma_{32}=6.25$ | $\Gamma_{12}=3.8$ | $\Gamma_{13}=0.1$ | $\Gamma_{32}=38$ |
| Branching Ratio | $4.3 \times 10^{-8}$ | $1.7 \times 10^{-8}$ | $8.7 \times 10^{-8}$ | $1.0 \times 10^{-8}$ | $2.1 \times 10^{-8}$ | $3.3 \times 10^{-8}$ | $1.02 \times 10^{-6}$ | $2.1 \times 10^{-7}$ | $1.3 \times 10^{-6}$ | $2.5 \times 10^{-10}$ | $6.6 \times 10^{-11}$ | $2.5 \times 10^{-9}$ |


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