SLAC – PUB – 3733
July 1985
T/E

PROBING THE DESERT WITH FERMION MASSES[*]

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We use the SU(3) × SU(2) × U(1) renormalization group equations to place *upper* limits on the scale where new physics beyond the standard model must emerge. Our bounds rely solely on the structure of the renormalization group equations and on the magnitudes of heavy quark masses. For example, if $\sum {m_q}^2 \geq (450\ \mathrm{GeV})^2$, new physics must be found below 10^5 GeV.

Submitted to *Physical Review Letters*

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[*] This work was supported by the Department of Energy, contract DE-AC03-76SF00515, and by the National Science Foundation, contract NSF-PHY-83-10654.

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The standard SU(3) × SU(2) × U(1) model provides a very successful description of the strong, weak and electromagnetic interactions. It is, however, in no sense a fundamental theory. The SU(3) × SU(2) × U(1) model contains at least 18 parameters, each of which must be adjusted by hand to account for low-energy phenomenology.

What lies beyond the standard model? A variety of extensions have been discussed in recent years. Most of these proposals predict new physics above a characteristic scale M_N. This threshold ranges from a few hundred GeV, in technicolor scenarios, to about 10^{10} GeV, in axion models, all the way up to 10^{15} GeV, in typical grand unified theories.

It is very important to constrain the scale M_N. Experiments place lower limits on M_N that are typically of order a few hundred GeV. The precise bounds depend on the model under consideration and on the values of various free parameters. In this letter we describe a new *upper* bound of the scale M_N. We shall show that heavy quarks provide a *model-independent* probe of the SU(3) × SU(2) × U(1) desert.

Our idea is based on the following two observations:

(1) Heavy fermions have large Yukawa couplings at the weak scale M_W. The structure of the SU(3) × SU(2) × U(1) renormalization group equations implies that such couplings blow up at a relatively low energy M_B.

(2) At the scale M_B, where the Yukawa couplings diverge, new physics should be found. For example, large Yukawas induce strong couplings in the Higgs sector. Such a strongly-coupled Higgs sector should give rise to a new spectrum of bound states and to new effective interactions. Of course, it is always possible that the gauge group might change at an energy $M_N < M_B$. In this case, new gauge bosons of mass M_N should then be found. No matter what happens, new physics must emerge before the Yukawa couplings diverge.

The idea that diverging couplings imply new physics is not new. In quantum electrodynamics, it is associated with the presence of "Landau ghosts." In QED, the electromagnetic coupling blows up at an astronomically high energy, about $M_B \gtrsim e^{137}$ GeV. If QED were a fundamental theory, it would – at the very least – become strongly coupled at the scale M_B. New physics would arise, and quantum electrodynamics as we know it would break down. Of course, we can never see this new physics because QED is only an effective theory. At the scale $M_N \simeq 80$ GeV, QED loses its separate identity, and becomes absorbed into the standard electroweak theory.[1]

In what follows we apply these ideas to the standard SU(3) × SU(2) × U(1) model. For ease of presentation, we first examine a three-family model with a heavy top quark (and a single Higgs doublet). We denote the top Yukawa coupling by g_t. Its evolution with energy is given by[2]

$$\frac{dg_t}{d\tau} = \frac{g_t}{2}\left(9g_t^2 - 2G\right), \tag{1}$$

where $\tau = \log(E/M_W)/16\pi^2$, $M_W = 80$ GeV, and $G = 8{g_3}^2 + 2.25{g_2}^2$. Here g_3 and g_2 are the strong and weak gauge couplings, and we neglect small hypercharge effects. As can be seen in equation (1), the evolution of g_t has two types of contributions. The first comes from Higgs boson exchanges and tends to increase the Yukawa at high energies. The second comes from the gauge bosons; it tends to decrease the Yukawa. For large values of g_t, the first term dominates, and the Yukawa coupling grows exponentially with energy.

In Figure 1 we show the evolution of g_t with energy for a variety of initial conditions ${g_t}^0$ (at the weak scale M_W). For ${g_t}^0 > 2G/9$, the coupling g_t increases with energy, and diverges at an energy M_B.[3] Because of equation (1), it diverges faster and faster for larger and larger initial conditions.

In Figure 2 we plot the scale M_B as a function of the top mass m_t (evaluated at the weak scale M_W). With our conventions, $m_t = {g_t}^0 \langle\phi\rangle$, where $\langle\phi\rangle$ is normalized to 175 GeV. For large initial conditions, the scale M_B is quite small. A top mass of 250 GeV implies new physics below $M_B = 10^{11}$ GeV. For $m_t = 300$ GeV, this tightens to just 10^6 GeV. If m_t is as large as 400 GeV, new physics should appear below 10 TeV! The top mass provides a very sensitive probe of the SU(3) × SU(2) × U(1) desert.

The value of M_B depends sensitively on the value of g_3 at the weak scale M_W. The larger the value of g_3, the larger the scale M_B. Since we are bounding M_N from above, we wish to choose the maximum value of g_3 consistent with experiment. Therefore we take g_3 to be given by $\Lambda_{\overline{MS}} = 250$ MeV, in the four-flavor regime.

More general bounds on M_N can be found by examining the renormalization of an arbitrary number of heavy families. The appropriate generalization of the top Yukawa is the so-called quark "radius" ${T_Q}^2 = \sum {g_q}^2$. Here the g_q denote the (real) eigenvalues of the quark Yukawa matrices, and the sum runs over all up- and down-type quarks. The renormalization of T_Q has been investigated in Reference [4]. For large Yukawas, the quark radius renormalizes even more quickly than the individual couplings themselves.

As with the top Yukawa, the evolution of T_Q can be used to bound the scale of new physics. Instead of m_t, we now plot $[\sum {m_q}^2]^{1/2}$, where the index q runs over all up- and down-type quarks. The results are shown in Figure 3. When $\sum {m_q}^2 > (355 \text{ GeV})^2$, $M_N < 10^{15}$GeV. The bound tightens very quickly. For $\sum {m_q}^2 = (450 \text{ GeV})^2$, the scale M_N lowers to just 10^5 GeV. Because of the renormalization group, the heavy quark spectrum provides a model-independent probe of the SU(3) × SU(2) × U(1) desert. If the sum $\sum {m_q}^2$ is large, the desert must bloom.

In Figure 3, we have evolved g_2 and g_3 with the beta functions appropriate for $N_F = 8$, 9, 10 and 11 families. The limits for $N_F < 8$ are not shown because they are even tighter. At any given energy, the bound on M_N is given by the curve with the greatest value of N_F. In Figure 3, the graphs for $N_F > 8$ are terminated when the color coupling blows up.

Certain extensions of the standard model have two Higgs doublets at low energies. These models usually have one Higgs doublet coupled to up-type quarks, and the other coupled to their down-type partners. This type of coupling naturally suppresses flavor-changing neutral currents. Our analysis is easily extended to include these two-Higgs models. The evolution of T_Q differs from the one-Higgs case, but it is not hard to see that the bounds on M_N remain the same.[5][6] Therefore the limits in Figure 3 are also valid in two-Higgs extensions of the standard SU(3) × SU(2) × U(1) model.

Finally, we consider the supersymmetric extension of the standard model. We shall see that if supersymmetry is found below the weak scale M_W, the bounds on M_N can be tightened still further. This follows from the fact that the superpartners of the quarks, leptons and gauge bosons all contribute to the SU(3) × SU(2) × U(1) beta functions.[7] The final results are shown in Figure 4, with the assumption that the superpartners are all lighter that 80 GeV. As before, we take $\Lambda_{\overline{MS}} = 250$ GeV. Because of the effects of the superpartners, the color beta function blows up very quickly, so Figure 4 includes only the four cases $N_F = 4$, 5, 6 and 7.

In this letter we have demonstrated that heavy quarks provide a model-independent probe of the desert. They extend the range of TeV accelerators to well beyond their center-of-mass energies. The discovery of new heavy quarks would indicate that new physics waits to be found in the SU(3) × SU(2) × U(1) desert.

This work was supported by the Department of Energy, contract DE-AC03-76SF00515, and by the National Science Foundation, contract NSF-PHY-83-10654.

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FIGURE CAPTIONS

1. The evolution of the top Yukawa coupling g_t as a function of the energy E. We have evolved the color gauge coupling using $\Lambda_{\overline{MS}} = 250$ MeV.
2. The upper limit on the scale of new physics, as a function of the top mass m_t in a three-family model. As in Figure 1, we take $\Lambda_{\overline{MS}} = 250$ MeV. Smaller values of $\Lambda_{\overline{MS}}$ lead to tighter bounds.
3. The upper limit on the scale of new physics, as a function of the quark radius $[\sum {m_q}^2]^{1/2}$. The bound is given by the graph with the greatest number of families in a given energy region. We have included results for $N_F = 8, 9, 10$ and 11. The graphs terminate when the color coupling diverges. As before, we take $\Lambda_{\overline{MS}} = 250$ MeV.
4. The same as in Figure 3, for the supersymmetric version of the standard model. We include the limits for $N_F = 4, 5, 6$ and 7.

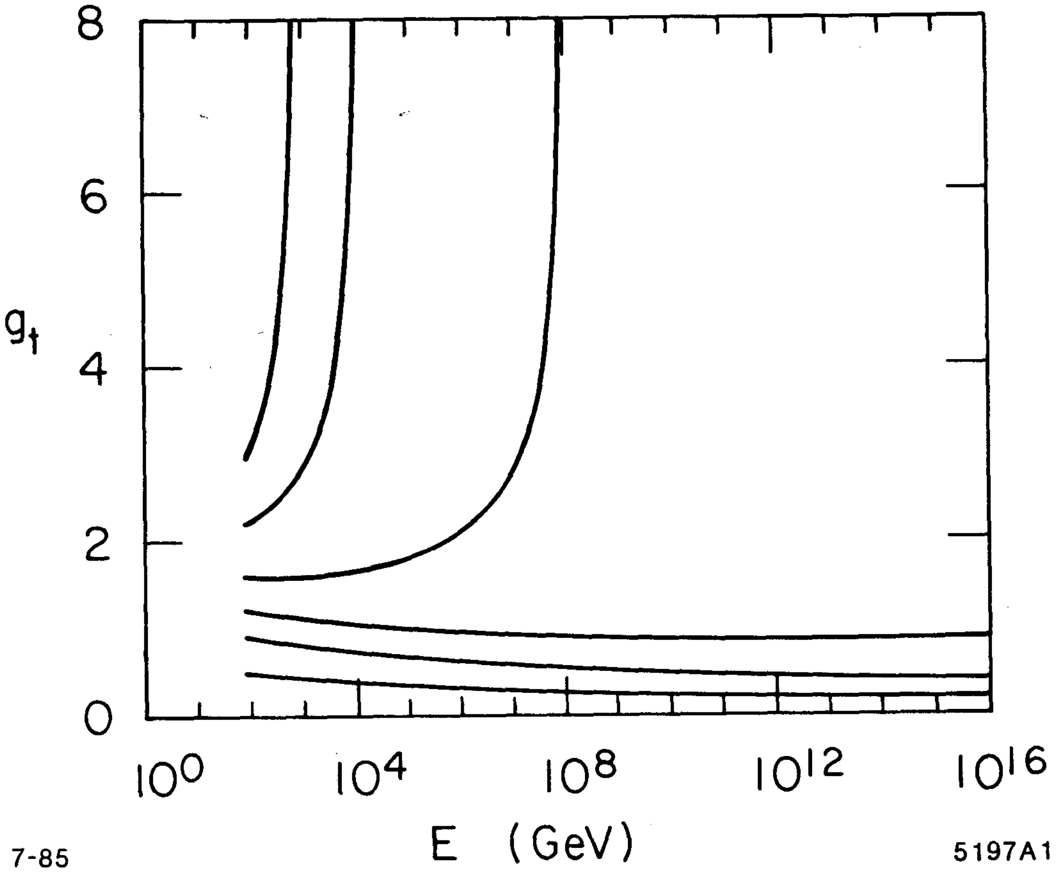

Fig. 1

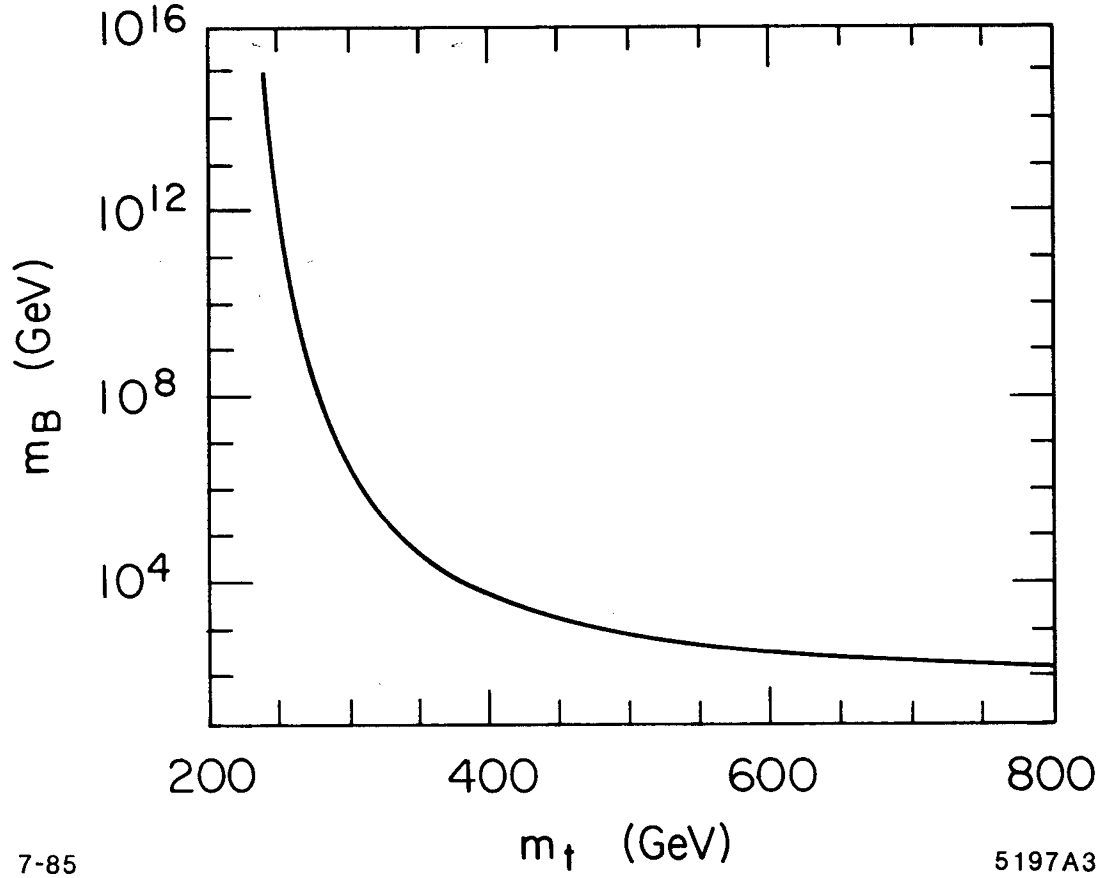

Fig. 2

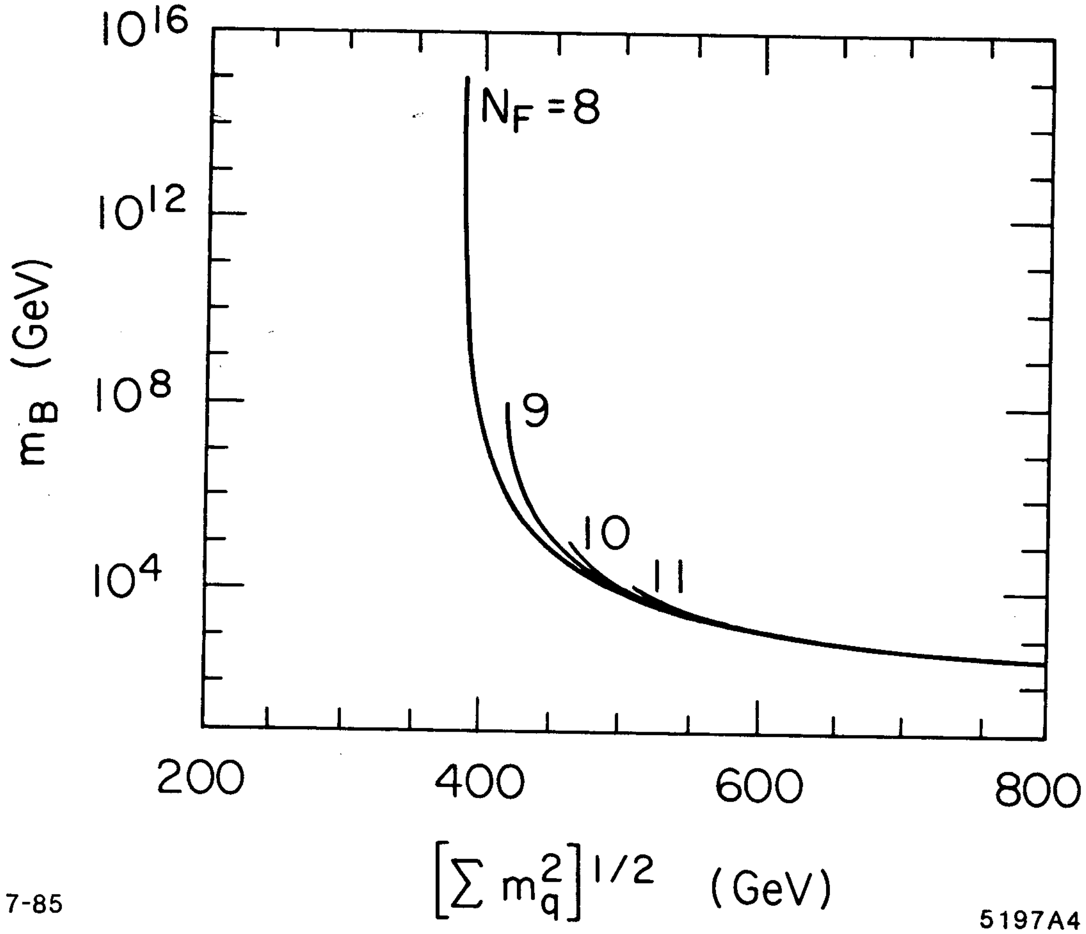
10^{16}
10^{12}
10^{8}
10^{4}
m_B (GeV)
$N_F = 8$
9
10
11
200
400
600
800
$\left[\sum m_q^2\right]^{1/2}$ (GeV)
7-85
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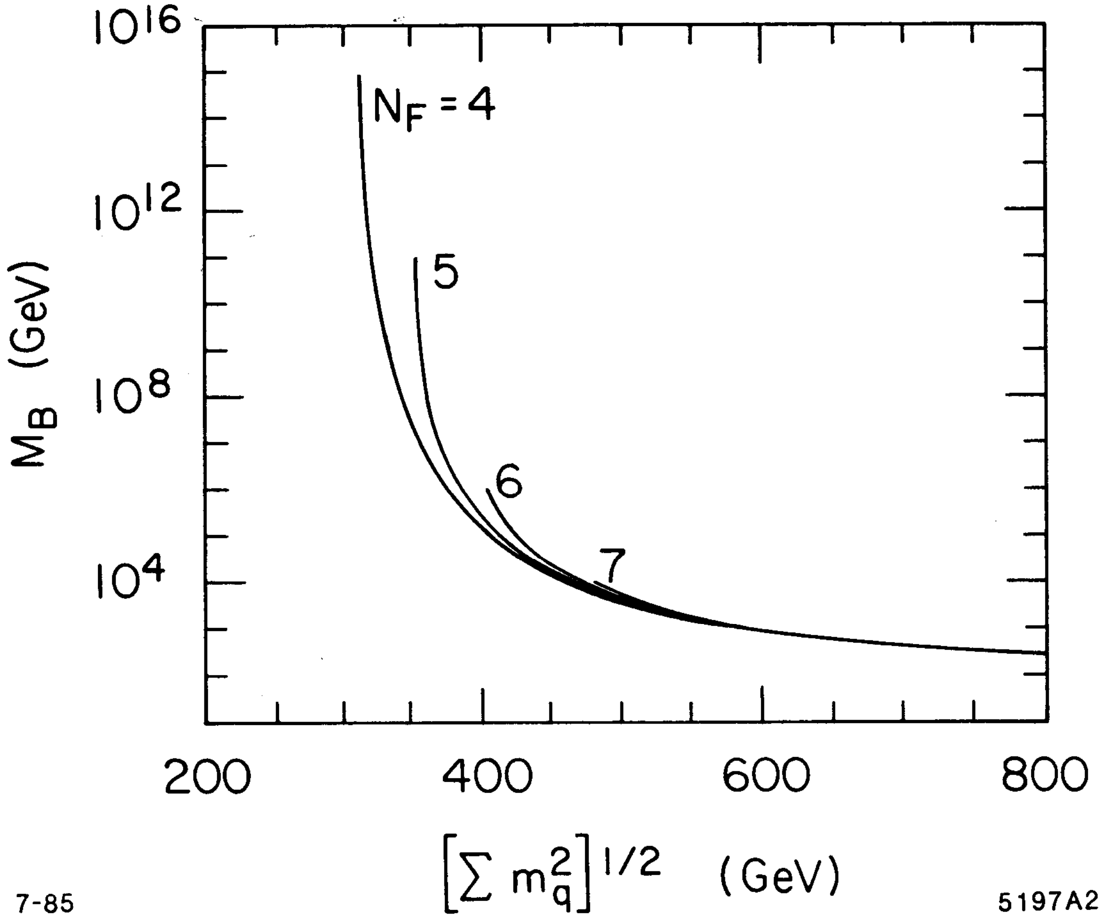
Fig. 3



10^{16}
10^{12}
10^{8}
10^{4}
M_B (GeV)
$N_F = 4$
5
6
7
200
400
600
800
$\left[\sum m_q^2\right]^{1/2}$ (GeV)
7-85
5197A2

Fig. 4