SLAC – PUB – 3711
June 1985
T/AS

Goldstone Bosons in String Models of Galaxy Formation*

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ABSTRACT

One reason why cosmic strings are interesting is because they may provide the primordial density fluctuations that began the process of galaxy formation. For the scenario in which galaxies condense around oscillating closed loops it is necessary that gravitational radiation be the dominant energy loss mechanism. It is shown that loops of strings from a broken exact global symmetry decay too quickly to serve this purpose. Loops of strings from a broken gauge symmetry may have Goldstone boson couplings as well. It is shown that the decay rate of these strings due to Goldstone boson emmission is strongly suppressed. This supports the conjecture that gauge strings may seed galaxy formation.

Submitted to *Physical Review D*

⋆ Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.

1. Introduction

Topological defects arise in a GUT which undergoes a sequence of phase transitions in the early expanding universe.[1,2] One type of defect is the cosmic string: a filament of a primordial vacuum topologically trapped in the present vacuum. Strings are particularly interesting because of recent suggestions that they may explain the density fluctuations which began the process of galaxy formation.[3–6] In the scenario of Vilenkin,[4] galaxies and clusters condense around oscillating closed loops which gradually decay in $\sim 10^4$ Hubble times from the instant of their creation. This requires that for large loops the dominant energy loss mechanism is gravitational radiation. To support this, it has been shown that both electromagnetic radiation and the radiation of massive particles fail to be significant factors for the energy loss of the large loops envisioned for galaxy formation.[3–7]

In this paper we consider the effects of massless Goldstone bosons. We find that for theories in which strings come from a broken global symmetry the lifetime of a large oscillating loop is much shorter than in the cases mentioned above, and consequently the scenario in which these loops seed galaxy formation does not work.

It is possible that strings from a broken local symmetry have couplings to massless Goldstone bosons. We show that the emission of these particles is suppressed compared to gravitational radiation, and consequently such strings are permitted for galaxy formation theories.

This paper is organized as follows: In the first section we review the properties of the simplest string. In the second we use a geometrical argument to derive the radiation rate of an oscillating global string. In the third we discuss small oscillations of infinite gauge strings and estimate the power loss from Goldstone boson emmission for a simple model. We conclude with a discussion of string models of galaxy formation and Goldstone bosons.

2. String Review

The simplest string is a classical solution to a spontaneously broken U(1) gauge theory, which is cylindrically symmetric and has finite energy per unit length. We take a complex scalar field ϕ with lagrangian

$$\mathcal{L} = |(\partial_\mu - ieA_\mu)\phi|^2 - V(\phi) - \frac{1}{4}\mathcal{F}^2$$

$$V(\phi) = \frac{\lambda}{4}(\phi^*\phi - v^2)^2$$

and as long as $v^2 > 0$ we can make the following classification of field configurations which minimize the action.

Vacuum sector:

$$\langle A_\mu \rangle = 0 \qquad \langle \phi \rangle = ve^{i\delta} \qquad \delta = arbitrary \quad phase$$

String sector:

$$e\,\langle A_\mu \rangle = n\delta^\theta_\mu \qquad \langle \phi \rangle = ve^{in\theta} \qquad \theta = azimuthal \quad angle$$

Where $\delta^\theta_x = \frac{-\sin\theta}{r}$, $\delta^\theta_y = \frac{\cos\theta}{r}$ and n is an integer. With these vacuum expectation values (VEVs) as the classical values of ϕ and A_μ, all the terms in the Lagrangian are zero.Furthermore, any other configurations which minimize $\mathcal{L}$ will be gauge equivalent to one of these, for some value of n. In the following n will allways be taken to be 1.

The string solutions have the peculiarity that $\langle \phi \rangle$ cannot lie in one of the degenerate vacua, where $|\langle \phi \rangle| = v$, at all points in space because single-valuedness requires that $\langle\phi\rangle \to 0$ as r $\to$ 0. Similarly $\langle A_\mu\rangle \to 0$ as r $\to$ 0. Thus we write

$$\langle\phi\rangle = vf(r)e^{i\theta} \tag{2.1}$$

$$e\langle A_\mu\rangle = g(r)\delta^\theta_\mu$$

where

$$f(r){\to}0, \quad g(r){\to}0 \quad as \quad r{\to}0.$$

The functions f and g may be obtained from the effective lagrangian

$$\mathcal{L}_{g,f} = -v^2(\partial_r f)^2 - v^2 f^2\left(\frac{1-g}{r}\right)^2 - \frac{\lambda v^4}{4}(f^2-1)^2 - \frac{1}{4e^2}\left[\frac{1}{r}\partial_r g\right]^2.$$

and they have the following limiting values:

$$f \sim r \quad for \quad r \ll \frac{1}{v\sqrt{\lambda}} \qquad\qquad g \sim r^2 \quad for \quad r \ll \frac{1}{ve}$$

$$f \sim 1 \quad for \quad r \gg \frac{1}{v\sqrt{\lambda}} \qquad\qquad g \sim 1 \quad for \quad r \gg \frac{1}{ve}.$$

Thus, assuming that $e \sim \sqrt{\lambda}$, the thickness of the string is roughly the Compton wavelength of the massive higgs or, equivalently, of the massive gauge boson.The magnetic field of the string is that of a very thin solenoid: the field is zero everywhere outside, but since around the string

$$\oint A_\mu dx^\mu = \frac{2\pi}{e}$$

there is a parallel magnetic field trapped in the core.

String solutions similar to the one above are a general feature of spontaneously broken gauge theories in which a discrete symmetry survives the breaking.[1]

3. Goldstone boson radiation from global strings

Global strings are classical solutions to a spontaneously broken global U(1) theory obtained by setting $A_\mu = 0$, or equivalently g(r)=0 in the previous example. In this case a single straight string has a logarithmically divergent energy per unit length.[4] One might think that such strings should be ignored as being unphysical. However, following a cosmological phase transition they form a random random network and may be in loops of finite total energy or in open strings with finite energy per horizon. In any case, the radiation problem is simpler for global strings so we consider this case first.

We start with the static string solution, eq. (2.1),

$$\langle\phi\rangle = vf(r)e^{i\theta}$$

and look for low energy excitations. We define fields $\widetilde{\rho}$ and $\widetilde{\alpha}$ and their shifted counterparts ρ and α, where

$$\phi \equiv \widetilde{\rho}e^{i\widetilde{\alpha}} \equiv (\rho + f(r)v)e^{i(\theta+\frac{\alpha}{v})} \tag{3.1}$$

$$\langle\widetilde{\rho}\rangle = f(r)v \quad \langle\widetilde{\alpha}\rangle = \theta \quad \langle\rho\rangle = 0 \quad \langle\alpha\rangle = 0.$$

The Lagrangian is

$$\mathcal{L} = (\partial_\mu\widetilde{\rho})^2 + \widetilde{\rho}^2(\partial_\mu\widetilde{\alpha})^2 - V(\widetilde{\rho})$$

$$= [\partial_\mu(\rho + vf)]^2 + (\rho + vf)^2\left(\delta_\mu^\theta + \frac{1}{v}\partial_\mu\alpha\right)^2 - V(\rho + vf). \tag{3.2}$$

Spontaneous symmetry breaking requires that ρ have mass $\sim v\sqrt{\lambda}$ and α be a massless goldstone boson. More generally, the string will not be straight. It will be curved, possibly in a loop, and moving under the influence of external forces and its own tension. As long as its typical inverse radii of curvature and frequencies of oscillation are small compared to v, the very massive ρ field will not be excited, so the low energy physics of the string will involve only the massless α field.

Now suppose that a long straight string oscillating back and forth with amplitude d and frequency ω. The solution for the radiation field of α is readily found under the condition d $\ll \frac{1}{\omega}$. This is because, for values of r such that $r \ll \frac{1}{\omega}$, the field will be carried rigidly along with the string as it moves. That is, at a particular time t and for $r \ll \frac{1}{\omega}$, the $\tilde{\alpha}$ field will appear to instantaneaously establish itself in the new string vacuum centered at $\vec{r} + \vec{d}(t)$. If the condition $d \ll r$ holds true as well, a small angle approximation can be used to obtain

$$\tilde{\alpha}(t) = \theta - \frac{d(t)}{r}\sin\theta = \theta - \frac{d\cos\omega t\sin\theta}{r}. \qquad (3.3)$$

Thus we have a boundary condition wich we can match to the solution for $\tilde{\alpha}$ when $\frac{1}{\omega} \ll r$.

For r$\gg$d and r$\gg$ $\frac{1}{v}$ we have $\tilde{\rho} = v = constant$,so $\tilde{\alpha}$ satisfies the massless wave equation

$$\partial^{\mu}\partial_{\mu}\tilde{\alpha} = 0.$$

We define the excitation field α as in eq.(3.1). This must satisfy

$$\partial^{\mu}\partial_{\mu}\alpha = 0 \qquad \frac{1}{\omega} \ll r$$

with boundary condition from eq.(3.3)

$$\alpha = \frac{-vd\cos\omega t\sin\theta}{r} \qquad d \ll r \ll \frac{1}{\omega}.$$

The solution which involves outgoing radiation is

$$\alpha = \frac{\pi v d\omega}{2}[N_1(\omega r)\cos\omega t - J_1(\omega r)\sin\omega t]\sin\theta$$

where N_1 and J_1 are Bessell functions. As $r \to \infty$ it has limiting behavior

$$\alpha \to vd\sin\theta\sqrt{\frac{\pi\omega}{2r}}\sin(\omega t - \omega r - \frac{3\pi}{2})$$

and the power radiated per unit length is

$$\frac{P}{L} = \frac{\omega}{2\pi} \int\limits_0^{\frac{2\pi}{\omega}} dt \int\limits_0^{2\pi} \partial_0 \alpha \partial_r \alpha (r d\theta) = \frac{\pi}{4} v^2 d^2 \omega^3 \tag{3.4}$$

Note that this result does not depend on the internal structure of the string, nor on any assumption that the amplitude d is small compared to the string thickness. We need only $\omega d \ll 1$.[8] To this rather unphysical example of a straight,rigidly oscillating string may be added dynamics in the string direction without changing the basic form of our result.

4. Goldstone boson radiation from gauge strings

4.1 Transverse Waves on Strings

First we show that there are small amplitude transverse waves travelling along the string at the speed of light. This is shown for gauge strings, though a similar result can be obtained for global strings by setting $g = \widetilde{g} = 0$ everywhere. In either case, it is these transverse modes that give off Goldstone boson radiation. The example of the last section was equivalent to having one very long wavelength mode excited.

We start with the static string solution of Sec.2 and look for low energy excitations. We suspect that such excitations must, at a given time and at a given point along the string, look like a translation of the unperturbed string from its unexcited position because any deformation of the cross sectional shape of the string will cost a large amount of energy. Therefore,we define the spacelike vector $\vec{d}(z,t) = d(z,t)\hat{x}$ and for the excited fields we try

$$\widetilde{\rho}(\vec{x},t) = vf(\vec{x} - \vec{d}) = vf(r^\star)$$

$$\widetilde{\alpha}(\vec{x},t) = \theta^\star(\vec{x} - \vec{d})$$

$$e\widetilde{A}_\mu(\vec{x},t) = g(\vec{x}-\vec{d})\frac{\partial[x^\nu - d^\nu]}{\partial x^\mu}\delta_\nu^{\theta^*} = \widetilde{g}[\delta_\mu^\nu - \partial_\mu d^\nu]\delta_\nu^{\theta^*}$$

where $r^\star$ and $\theta^\star$ are the cylindrical coordinates measured from the instantaneous center of the string, $\delta_x^{\theta^*} = \frac{-\sin\theta^*}{r^*}$ and $\delta_y^{\theta^*} = \frac{\cos\theta^*}{r^*}$. With these fields the lagrangian is

$$\widetilde{\mathcal{L}} = (\partial_\mu\widetilde{\rho})^2 + \widetilde{\rho}^2(\partial_\mu\widetilde{\alpha} - e\widetilde{A}_\mu)^2 - V(\widetilde{\rho}) - \frac{1}{4}(\partial\mu\widetilde{A}_\nu - \partial_\nu\widetilde{A}_\mu)^2$$

Let a=x,y and i=z,t. Since z and t appear only through the function d(z,t) we may derive the following:

$$(\partial_i\widetilde{\rho})^2 = \left(\frac{\partial\widetilde{\rho}}{\partial d}\right)^2(\partial_i d)^2 \tag{4.1}$$

$$(\partial_i\widetilde{\alpha} - e\widetilde{A}_i)^2 = \left(\frac{\partial}{\partial d}\theta^\star + \frac{\widetilde{g}\sin\theta^\star}{r^\star}\right)^2(\partial_i d)^2 \tag{4.2}$$

$$\widetilde{F}_{ia}\widetilde{F}^{ia} = \left[\frac{\partial}{\partial d}(\widetilde{g}\delta_a^{\theta^*}) + \partial_a\left(\frac{\widetilde{g}\sin\theta^\star}{r}\right)\right]^2(\partial_i d)^2 \tag{4.3}$$

$$\widetilde{F}_{ij} = 0$$

Thus we may write

$$\widetilde{\mathcal{L}} = (\partial_a\widetilde{\rho})^2 + \widetilde{\rho}^2(\partial_a\widetilde{\alpha} - e\widetilde{A}_a)^2 - V(\widetilde{\rho}) - \frac{1}{4}(\widetilde{F}_{ab})^2 + \Pi(x,y,d(z,t))(\partial_i d)^2$$

where Π comes from (4.1-3). Since at any fixed t and z the string configuration is just an undeformed translation from the static d=0 solution , we can integrate this lagrangian over the transverse dimensions x and y, and use translation invariance, to obtain

$$\int\widetilde{\mathcal{L}}dxdy = \int\mathcal{L}dxdy + (\partial_i d)^2\int\Pi dxdy$$

Finally, because translation invariance also requires that $\int\Pi dxdy \equiv K$ be independent of d, we find that the effective two- dimensional lagrangian for d(x,t) is

$$\mathcal{L}_{eff}[d] = K(\partial_i d)^2. \tag{4.4}$$

Hence the low energy excitations of the static string solution are described by a two-dimensional massless free field theory, ie., transverse waves travelling at the speed of light.

4.2 A MODEL

To calculate the rate of Goldstone boson emission from such an excited string we must choose a model in which the fields that make up the gauge string carry a charge for a spontaneously broken global symmetry. In the following we use a simple $U(1)_{gauge} \times U(1)_{global}$ model. There are three complex scalar fields $\phi_i = \rho_i e^{i\alpha_i} \quad i = 1, 2, 3$ with the following charges:

	$U(1)_{gauge}$	$U(1)_{global}$
ϕ_1	e	1
ϕ_2	$-e$	1
ϕ_3	0	2

and the scalar lagrangian is

$$\mathcal{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 + |\partial_\mu \phi_3|^2 + C[\phi_1 \phi_2 \phi_3^{*2} + h.c.] - \sum_i \frac{\lambda_i}{4}(\phi_i^2 - v_i^2)^2$$

Since both U(1)'s are broken there must be two Goldstone bosons, one of which becomes the longitudinal component of the massive gauge field, and three massive scalar particles. The Goldstone bosons are the linear combinations of the $\alpha_i's$ which transform only under $U(1)_{gauge}$ or $U(1)_{global}$, but not under both. The third linear combination transforms under neither. Accordingly we define

$$\beta = \frac{\alpha_1 + \alpha_2}{2} \quad \gamma = \frac{\alpha_1 - \alpha_2}{2} \quad \chi = \frac{\alpha_1 + \alpha_2 - 2\alpha_3}{2}.$$

The Lagrangian is now

$$\mathcal{L} = \sum_i \{(\partial_\mu \rho_i)^2 - \frac{\lambda}{4}(\rho_i^2 - v_i^2)^2\} + 2C\rho_1\rho_2\rho_3^2 \cos\chi$$

$$+\rho_1^2(\partial_\mu\beta + [\partial_\mu\gamma - eA_\mu])^2 + \rho_2^2(\partial_\mu\beta - [\partial_\mu\gamma - eA_\mu])^2 + \rho_3^2(\partial_\mu\beta - \partial_\mu\chi)^2$$

with minimal string solution

$$\langle\rho_i\rangle = v_i f_i(r) \quad \langle\gamma\rangle = \theta \quad e\,\langle A_\mu\rangle = g(r)\delta_\mu^\theta$$

$$\langle\beta\rangle = \langle\chi\rangle = 0.$$

We see that outside of the string β will remain massless, γ will be eaten, and χ will aquire mass from symmetry breaking.

As defined β is a dimensionless field. The physical field corresponding to it is

$$\hat{\beta} = \bar{v}\beta$$

with classical equation of motion

$$\partial^\mu W(r)\partial_\mu\hat{\beta} + \partial^\mu K_\mu = \partial^\mu W(r)\partial_\mu\hat{\beta} = 0 \tag{4.5}$$

where we have made the following additional definitions:

$$\bar{v} \equiv \sqrt{\Sigma_i v_i^2}$$

$$W \equiv \frac{\Sigma(v_i f_i)^2}{\bar{v}^2} \tag{4.6}$$

$$K_\mu \equiv \frac{1}{\bar{v}}[(v_1 f_1)^2 - (v_2 f_2)^2][\partial_\mu \theta - g\delta_\mu^\theta].$$

For the case of a static string the source term, $\partial \cdot K$, is zero and W is a function of r which goes to zero as $\sim r^2$ inside the string and is equal to 1 for $r > \frac{1}{\bar{v}}$, so β satisfies the massless wave equation outside the string. The time independent ground state for β is $\beta = 0$.

4.3 Exciting the Gauge String

Now we let the string wiggle. Following the method of Sec. 4.1, we shift the center of the string, using $\vec{d}(z,t) = \hat{x} d \cos\omega(z-t)$. As long as $\omega \ll v_i$ the string oscillations will not have enough energy to produce any of the massive particles of the theory, so we need only consider the interaction with the massless β field. The equation of motion for β becomes

$$\partial^\mu \widetilde{W} \partial_\mu \beta + \partial^\mu \widetilde{K}_\mu = 0. \tag{4.7}$$

In this case the source term is not zero. By translation invariance $\partial^a \widetilde{K}_a = \partial^a K_a = 0$, so

$$\partial^\mu \widetilde{K}_\mu = \partial^i \widetilde{K}_i \qquad i = t, z$$

From Sec. 4.1

$$\widetilde{K}_i = \frac{1}{\bar{v}}[(v_1 \widetilde{f}_1)^2 - (v_2 \widetilde{f}_2)^2]\left[\frac{\partial}{\partial d}\theta^\star + \frac{\widetilde{g}\sin\theta^\star}{r^\star}\right]\partial_i d$$

and therefore

$$\partial^i \widetilde{K}_i = \frac{1}{\bar{v}}\frac{\partial}{\partial d}\left\{[(v_1 \widetilde{f}_1)^2 - (v_2 \widetilde{f}_2)^2]\left[\frac{\partial}{\partial d}\theta^\star + \frac{\widetilde{g}\sin\theta^\star}{r^\star}\right]\right\}(\partial_i d)^2. \tag{4.8}$$

To estimate the effect of this source term we assume that our results will depend only slightly on the detailed profile of the string. Accordingly, we use

$\tilde{f}_i = v_i r^\star$ and $\tilde{g} = \bar{v} r^\star$ for $r^\star < \frac{1}{\bar{v}}$, and $\tilde{f}_i = \tilde{g} = 0$ for $r^\star > \frac{1}{\bar{v}}$. Furthermore, we exclude the case where $v_1 = v_2$ and $\lambda_1 = \lambda_2$ because it would imply an extra symmetry that prevents any coupling of the Goldstone boson to the string. Our purpose here is to investigate the case in which the Goldston boson *is* coupled to the string. Since all the VEVs are expected to be very large we take $v_1 - v_2 \sim \bar{v}$. Under these assumptions eq. 4.7 becomes

$$\begin{aligned} \partial^i \tilde{K}_i &\approx \left\{(v_1)^4 - (v_2)^4\right\}\left[\frac{(x-d)y}{\sqrt{(x-d)^2+y^2}}\right](\partial_i d)^2 \\ &\approx \bar{v}^4(\omega d)^2 \sin^2\omega(t-z)\frac{[x - d\cos\omega(t-z)]y}{\sqrt{[x-d\cos\omega(t-z)]^2+y^2}} \end{aligned} \tag{4.9}$$

in which we have gone to Cartesian coordinates and explicitly indicated the dependence on z and t, in this case due to a transverse wave moving in the +z direction.

Integrating the term over xy gives zero, so the net 'charge' vanishes and there will no static $\sim \ln r$ component of the field of the oscillating string. In fact, this term has the form of a 2-dimensional quadrupole with moment

$$Q_{xy} \sim \bar{v}^4(\omega d)^2 \int_{-\frac{1}{\bar{v}}}^{\frac{1}{\bar{v}}} \frac{x^2y^2}{\sqrt{x^2+y^2}} dxdy = \frac{(\omega d)^2}{16\bar{v}} \tag{4.10}$$

Looking back at eq.(4.9) we see that the source of radiation comes from two effects, the intrinsic oscillation of the quadrupole moment because of the $\sin^2\omega(t-z)$ factor, and the effect of this quadrupole oscillating back and forth with large amplitude d, which should to produce octupole radiation. We could straightforwardly calculate a radiation field and total power emmitted by using eq.(4.10). Though it would give the right order of magnitude, it turns out that this is not the correct path to follow because of the non-standard W term in eq.(4.7). In the appendix we discuss the generalization of 2-dimensional electrostatics to fields in a string background.

Just as in sec.2, we can find $\hat{\beta}$ for all $r \gg d$ by matching outgoing Bessell waves to the following boundary condition: that for $d \ll r \ll \frac{1}{\omega}$ the solution must look like the static field produced by the source (4.9) with z and t fixed. Using eq.(A6) this boundary condition is

$$\left[\frac{3\sqrt{5}-7}{4}\right] \sin^2 \omega(t-z) \frac{(\omega d)^2}{\bar{v}} \frac{(x - d\cos\omega(t-z))y}{[(x-d\cos\omega(t-z))^2 + y^2]^2} \qquad r \ll \frac{1}{\omega}$$

and if $r \gg d$ this becomes

$$\frac{(\omega d)^2}{\bar{v}} \left[\frac{3\sqrt{5}-7}{4}\right] \left[\frac{\cos\theta \sin\theta \cos^2 \omega(t-z)}{r^2} - \frac{d \sin 3\theta \sin\omega(t-z) \cos^2\omega(t-z)}{r^3}\right]$$

Which has both quadrupole and an octupole parts.We drop the octupole because it will be relatively suppressed by a factor of ωd. The solution for $\hat{\beta}$ which involves outgoing radiation is

$$\hat{\beta} = \left[\frac{3\sqrt{5}-7}{4}\right] \frac{(\omega d)^2}{\bar{v}} \left\{ \frac{-\pi\omega^2 \sin 2\theta}{2} [\cos 2\omega(t-z) N_2(2\omega r) - \sin 2\omega(t-z) J_2(2\omega r)] \right\}$$

and its limiting behavior for $r \gg \frac{1}{\omega}$ is

$$\hat{\beta} \rightarrow -\left[\frac{3\sqrt{5}-7}{4}\right] \frac{(\omega d)^2}{\bar{v}} \frac{\pi^{\frac{1}{2}} \omega^{\frac{3}{2}}}{2} \sqrt{\frac{1}{\pi \omega r}} \sin(2\omega t - 2\omega z - \omega r - \frac{5\pi}{4}).$$

The total power radiated per unit length is

$$\frac{P}{L} = \left[\frac{3\sqrt{5}-7}{4}\right]^2 \frac{(\omega d)^2}{4\bar{v}^2} \pi \omega^4$$

The reader will see in the folowing section that this result will be extrapolated up to values $\omega d \sim 1$, in which case the the octupole component of the power cannot be neglected. But since the two components of the power have the same order of magnitude for $\omega d \sim 1$ we will reach the same conclusions regardless of which we use.

5. Galaxy Formation and Goldstone bosons

Strings can be formed following a GUT phase transition in the early universe as long as the unbroken subgroup has a discrete factor.[1] Though not a property of the minimal $SU(5)$, such strings are copiously produced in non-minimal versions of $SU(5)$,[9] $SO(10)$,[10,11] $SO(18)$[12] and even in the $E_8 \times E_8$ and $SO(32)$ superstring theories.[13]

The initial configuration is a random tangled web throughout the universe, the strings traversing a Brownian walk of step length $\sim \xi_G$, the Ginzberg correlation length,[1] at the GUT time $\sim 10^{-37}s$. ξ_G depends crucially on the nature of the phase transition, though it is constrained by causality to be less than the horizon size at the time of symmetry breaking. After t $\sim 10^{-32}s$, long before the Weinberg-Salam transition at $\sim 10^{-11}s$, frictional effects become negligible and the strings oscillate[14] on scales less than the horizon. Vilenkin[4] has shown that these oscillations diminish and the strings tend to straighten out as far as the horizon, though on larger scales the system remains Brownian.

The sub-horizon straightening comes about through cosmological stretching and the formation of loops as the strings cross each other (intercommute) at relativistic speeds. These loops will then oscillate and shrink as they radiate away their energy,[15] eventually turning into elementary particles.[16] As the universe expands the loops continue to be created, to start to oscillate and eventually dissapear, and the scale of the string network continues to increase with the horizon.

Any new loops that appear within the horizon at time t will have a size roughly equal to t. Once a loop is in the horizon its lifetime is bounded by the decay time due to gravitational radiation. This is

$$\tau \sim .01R\frac{m_{planck}}{m_{GUT}} \sim 10^{-4}R \tag{5.1}$$

where R is the size of the loop and we have taken $m_{GUT} \sim 10^{16}$GEV.[6] Of course, the existence of a more efficient mechanism for energy loss will result in a shorter lifetime. Assuming that the dominant mechanism for energy loss is gravitational radiation, Vilenkin has shown that loop production generates energy density fluctuations which gives rise to structure in the universe that is in good agreement with observation.[6] [4]

To support the conjecture that gravitational radiation dominates, the authors of ref [7] have calculated the energy loss due to EM radiation and massive particles, finding that for R > $\frac{m_{planck}}{v^2}$ EM becomes negligible and for R > $\frac{1}{m_{particle}}$ the effect of massive particles becomes negligible.

Now we turn to examine the case of massless Goldstone bosons. Since an oscillating loop is an exceedingly complicated source we can only estimate the order of magnitude of the radiated power. Following [7], we do this by extrapolating the result for an infinite string, derived under the condition $\omega d \ll 1$, up to the typical value for loops, $\omega d \sim 1$. Taking $\omega \sim \frac{1}{R}$, where R is the length of the loop, and using (3.1), we obtain for global strings

$$P \sim v^2$$

Since the mass of a string of length R is $\sim v^2$R we find that the lifetime of an oscillating loop is roughly equal to its size

$$\tau \sim R.$$

This is much shorter than that for gravitational radiation (5.1), and consequently models with global strings cannot supply us with the persisting loops in the early universe which we need for a plausible galaxy formation scenario. In particular, this result rules out the global strings produced in variations of the inflationary $SU(5)$ model of ref.[8] as seeds for galaxy formation.

The case for gauge strings with Goldstone boson couplings is different. Setting $d \sim \frac{1}{\omega} \sim R$, where R is the size of a loop, we obtain from (4.9)

$$P \sim \frac{1}{vR^3}$$

$$\tau \sim R^4 v^3.$$

For loops much larger than the GUT scale this is enormous compared to the lifetime for gravitational radiation (5.1). We conclude that the radiation of massless Goldstone bosons from these strings does not pose a problem for galaxy formation.

APPENDIX

We engage here in a short discussion of static multipole fields in the background of a static string of the type in Sec. 4.2. To the Lagrangian may be added an explicit source term for $\hat{\beta}$ so that eq. (4.5) becomes

$$\partial^\mu W(r)\partial_\mu \hat{\beta}(r,\theta) = \rho(r,\theta). \tag{A1}$$

in which the ρ is taken to be independent of z and t. In the following these two dimensions are suppressed. Because of W, eq.(5.1) is only rotationally invariant about r=0, so only if the moments are calculated from r=0 will eq.(5.1) relate $\hat{\beta}$ and ρ term by term in their multipole expansions. Defining

$$\hat{\beta} = \sum \beta_n e^{in\theta}$$

$$\rho = \sum \rho_n e^{in\theta}$$

we find

$$\left[\frac{1}{r}\partial_r W r \partial_r - \frac{n^2 W}{r^2}\right]\beta_n = \rho_n \tag{A2}$$

Multiplying eq. (5.2) by an undetermined function f, and performing some manipulations, we get

$$\begin{aligned} f\rho_n &= \left[\frac{f}{r}\partial_r W r \partial_r - \frac{f n^2 W}{r}\right]\beta_n \\ &= \frac{1}{r^2}\partial_r\left\{\left[f W r \partial_r - f' W r\right]\beta_n\right\} + \frac{1}{r}\left\{(f' W r)' - \frac{f W n^2}{r}\right\}. \end{aligned} \tag{A3}$$

Now suppose that f(r) satisfies

$$(f' W r)' - \frac{f W n^2}{r} = 0 \tag{A4}$$

then we can integrate eq.(A3), finding

$$\mathcal{Q}_n \equiv \int_0^r f \rho_n r' dr' - \{\lim_{r\to 0} r W (f \beta_n' - f' \beta_n)\} = r W (f \beta_n' - f' \beta_n). \tag{A5}$$

Assuming that the source is localized, this tells us that the values of the field β_n on the surface of a large cylinder depends on ρ only through the constant $\mathcal{Q}_n$, which we call the generalized multipole charge. Note that if W=1 everywhere, we would just have 2-dimensional electrostatics and eq.(A5) would give the 2-dimensional Coulomb law.

For $r \gg \frac{1}{v}$ W=1. From eq.(A4) we find $f = r^n$ or $f = r^{-n}$; we use the first because it is the one that corresponds β_n as a *decreasing* function of r. The solution is then

$$\beta_n = -\frac{\mathcal{Q}_n}{2 n r^n}$$

which has the same functional form as we would get from an n-pole in 2-dimensional electrostatics. However, the generalized n-pole charge $\mathcal{Q}_n$ is defined by an integral over a region where $W \neq 1$, so its numerical value will be different.It turns out that eq.(A5) is an impractical formula for calculating $\mathcal{Q}_n$ because f is in general not defined at r=0, and so a delicate limiting procedure must be followed to obtain a finite $\mathcal{Q}_n$.

For a particular case such as that of Sec. 4.3 it is easier to use eq.(A2) to find the solution for β_n directly. We make the approximations $W = \bar{v}^2 r^2$, $\rho = \frac{1}{2}\bar{v}^4(\omega d)^2 \sin^2\omega(t-z)\sin 2\theta \equiv \rho_0 r \sin 2\theta$ inside the string , and $W = 1$ and $\rho = 0$ outside. The regular solution for β_2 is then

$$\begin{aligned}\beta_2 &= -\frac{\rho_0}{\bar{v}^2}r + Ar^{\sqrt{5}-1} \qquad r < \frac{1}{\bar{v}}\\ &= Br^{-2} \qquad\qquad r > \frac{1}{\bar{v}}\end{aligned}$$

where A and B are constants are determined by the requirement that β_2 and β_2' be continuous at $r = \frac{1}{\bar{v}}$. The result is

$$\begin{aligned}\beta_2 &= -\frac{\rho_0}{\bar{v}^2}r + \left[\frac{3}{\sqrt{5}+1}\right]\frac{\rho_0}{\bar{v}^{4-\sqrt{5}}}r^{\sqrt{5}-1} \qquad r < \frac{1}{\bar{v}}\\ &= \left[\frac{3\sqrt{5}-7}{4}\right]\frac{\rho_0}{\bar{v}^5}r^{-2} \qquad\qquad r > \frac{1}{\bar{v}}\end{aligned}$$

ACKNOWLEDGEMENTS

I am grateful to Savas Dimopoulos and Helen Quinn for insightful discussions.

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