SLAC – PUB – 3710
June 1985
T/AS

Nucleosynthesis Problems for String Models of Galaxy Formation*

RICHARD LYNN DAVIS

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

ABSTRACT

The effect of having a large density of gravitational radiation at the time of nucleosynthesis is discussed. We derive a general constraint that relates the parameters in string models to the GUT scale.

Submitted to *Physics Letters B*

⋆ Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.

1. Introduction

Cosmic strings are produced in a cosmological phase transition in which the unbroken subgroup appears with a discrete factor[1,2]. Recent work has focused on the possibility that strings formed at the GUT scale could provide the densitiy fluctuations needed to begin galaxy formation[3-6]. These fluctuations depend on the string tension, $\mu \sim M_{gut}^2$, and three other parameters, α, β and γ, whose definitions will be given below.

Here we examine the possibility that gravitational radiation from such strings can disturb nucleosynthesis in the standard cosmological model. We find that, though at any given time the density of strings is very small compared to the background density of relativistic particles, there are two effects which enhance the gravitational radiation from these strings, allowing it to eventually overtake the background. That the string radiation should not become significant until after nucleosynthesis provides a tight constraint on the GUT scale and the parameters of the string models, a constraint which the commonly used values of these parameters do not satisfy. We conclude with a discussion of the seriousness of this problem and the need for more careful investigation into the actual values of the string model parameters.

2. The Standard String Model

A knot appears in the universe as it passes through the GUT phase transition at $t \sim 10^{-37}s$. Originally it is very complicated in its structure and highly damped by friction with the surrounding gas of matter and radiation. It has been estimated that frictional effects become negligible at[6]

$$t_\star \sim t_{planck}\left[\frac{M_{planck}}{M_{gut}}\right]^4 \sim 10^{-32} - 10^{-30}s \tag{2.1}$$

after which time the string system reaches the following configuration: at any cosmic time t it consists of strings executing Brownian steps with a continuously increasing step length t, the horizon distance, filling the 3-dimensional space†. For the step length to increase, length of string must be lost in some way. The scenario suggested by Vilenkin is that loops are formed by the crossing of moving strings[4]. The loops which appear at time t are roughly of size t and the rate of change of the number density of loops is

$$dn(t) = \beta \frac{dt}{t^4} \tag{2.2}$$

where β is an numerical constant, typically of order ~ 1.

Let us assume for the moment that loops created at time τ undergo stable oscillations and have size $R = \alpha\tau$. Since the number density of loops of size R is diluted by the expansion for times greater than $\sim \frac{R}{\alpha}$, the density of loops of all sizes will be

$$n(t) = \beta \int\limits_{t_*}^{t} (\tau/t)^{\frac{3}{2}} \frac{d\tau}{\tau^4} = \frac{2\beta}{3t^3}. \tag{2.3}$$

We can define $n_R(t)$ so that

$$n(t) = \int\limits_{R=\alpha t_*}^{R=\alpha t} n_R(t) dR. \tag{2.4}$$

and using eq.(2.2) and eq.(2.3) we find

$$n_R(t) = \frac{\beta\alpha^{\frac{3}{2}}}{R^{5/2}t^{3/2}}. \tag{2.5}$$

Note that all of these calculations are done in the radiation dominated epoch.

These formula must be corrected for the fact that these loops are not really stable. There are two ways that a loop can lose energy. One is by radiation and

† Cosmic time is defined by the metric $g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)d\vec{x}^2$.

the other is by fragmentation. It appears likely that a significant proportion of the loops will not fragment[7]. Thus oscillating loops are always shrinking due to radiation loss; the amazing thing is that the dominant mechanism is that of gravitational radiation[8,9]. The rate of power loss is[10]

$$P = \gamma G \mu^2 \tag{2.6}$$

where $\gamma \sim 100$, G is the gravitational constant and σ is the string tension $\sim M_{gut}^2$. This rate is independent of the size of the loop and results in a loop lifetime of

$$\tau = (\gamma G \mu)^{-1} R.$$

A loop of size R at time t had a larger size

$$R_0 = R + \gamma G \mu (t - t_0) \approx R + \gamma G \sigma t$$

at the instant $t_0 \sim \frac{R_0}{\alpha}$ of its creation. This allows us to find the corrected form of eq.(2.5), simply by making the replacement $R \leftrightarrow R_0$. Thus

$$n_R(t) = \frac{\beta \alpha^{\frac{3}{2}}}{(R + \gamma G \mu t)^{5/2} t^{3/2}}. \tag{2.7}$$

Using this and eq.(2.4), the corrected form of $n(t)$ is

$$n(t) = \frac{2 \beta \alpha^{\frac{3}{2}}}{3 (\gamma G \mu)^{\frac{3}{2}} t^3}.$$

The density in eq.(2.7) is defined for those values of R and t shown in fig.1. The loops start being produced at $t_\star$. Once formed, they will oscillate and decay by gravitational radiation until they reach a size $\sim 10^3 R_{gut}$[10], where R_{gut} is the GUT compton wavelength, at which point the loop disolves into particles other than gravitons. Vilenkin has shown that the density fluctuations caused by string loops give rise to structure in the universe that is in good agreement with observation[6].

3. The Radiation Density from Strings

Now we would like to know the accumulated gravitational radiation density at some time t. The contribution that was emmitted at time τ, where $t^\star < \tau < t$, is

$$d\rho_{gr} = (\frac{\tau}{t})^2 n(\tau)\gamma G\mu^2 d\tau.$$

The $(\tau/t)^2$ factor accounts for the cosmological redshift. The total radiation density is thus

$$\rho(t)_{gr} = \int_{t^*}^{t} \frac{\tau^2}{t^2} n(\tau)\gamma G\mu^2 d\tau = \left[\frac{\alpha^3\beta^2}{\gamma}\right]^{\frac{1}{2}} \frac{2\mu(G\mu)^{-\frac{1}{2}}}{3t^2} \ln(\frac{t}{t^\star}).$$

The physical meaning of the terms in this expression can be easily understood. The enhancement $(\gamma G\mu)^{-1/2}$ comes from the finite lifetime of the loops. The energy in a loop, all of which eventually becomes gravitational radiation, is not red-shifted as long as it remains stored in the loop, so the longer it takes for the loop to decay the greater the density of gravitational radiation will be. The $\ln(\frac{t}{t^\star})$ appears because energy is being continuously pumped into the universe as loops form and decay. Though the logarithm changes very slowly, the scale of time over which gravitiational radiation is being produced is enormous, and this term turns out to have important consequences. In fact, the presence of this term means that the energy density of gravitational radiation will slowly overtake that of the background gas of relativistic particles (fig.2).

4. String Radiation and Nucleosynthesis

The presence of a significant amount of gravitational radiation at the time of nucleosynthesis will, just like the presence of another light neutrino, increase the rate of expansion in that epoch and lead to an overadundance of helium. It has been estimated that an extra component to the radiation density must be less than 7% of the background.* This puts a constraint on string models

$$\rho_{gr} < \frac{1}{10}\rho = \frac{3}{320\pi G t^2}$$

or

$$\ln(\frac{t}{t_\star}) < \frac{9}{640\pi}\left[\frac{\alpha^3\beta^2}{\gamma}\right]^{-\frac{1}{2}}(G\mu)^{-\frac{1}{2}} \tag{4.1}$$

at $t_{nucsyn} = 1s$. Typical values of the various parameters are[6]

$$\alpha \sim 1 \quad \beta \sim 1 \quad \gamma \sim 100 \quad G\mu \sim 10^{-6} \quad t_\star \sim 10^{-32} - 10^{-30}s.$$

Inserting into eq.(4.1) we get

$$30 \lesssim 20 \tag{4.2}$$

so the constraint is not satisfied.

Is this a serious problem for string models? It is not at all clear. The problem (4.2) can be easily remedied by recalling that $\mu \propto M_{gut}^2$ and changing the GUT mass

$$M_{gut} \rightarrow \frac{2}{3}M_{gut}$$

hardly a major change in the scenario. On the other hand, the extreme sensitivity of this bound to the actual values of the parameters makes it essential to pin them down. This question will be discussed further in the next section.

$\star$ This corresponds to the presence of another species of light neutrino.

5. Conclusion

The constraint (4.1) can be written in a more manageable form by defining

$$s \equiv \frac{(G\mu)^{\frac{1}{2}}}{10^{-3}} \approx \frac{M_{gut}}{10^{16}Gev} \tag{5.1}$$

and

$$n \equiv log_{10}\left[\frac{t_{nucsyn}}{t_\star(s=1)}\right].$$

That is, n is the number of orders of magnitude between nucleosynthesis and the time at which frictional effects become unimportant, if $s = 1$. Then, using eqs.(2.1) and (4.1), we obtain

$$(1.7 \ln s + n) < \frac{2.0}{s}\left[\frac{\alpha^3\beta^2}{\gamma}\right]^{-\frac{1}{2}}.$$

As long as $s \ll e^{\frac{30}{1.7}}$ the logarithm can be neglected. Using n=30, squaring and rearranging, it becomes

$$(G\mu)\left[\frac{\alpha^3\beta^2}{\gamma}\right] \lesssim 4.2 \times 10^{-9}. \tag{5.2}$$

This result merits several comments:

1. It is insensitive to uncertainty in t_{nucsyn} and the considerable fuzziness of $t_\star$ in eq.(2.1).
2. The assumptions are remarkably few. It is a necessary consequence of the 'standard' string model of sec.2 and nucleosynthesis in the standard cosmological model.
3. Because $\mu \sim M_{gut}^2$, eq.(5.2) is a bound on the GUT scale in terms of things which in principle can be calculated. A value of $[\frac{\alpha^3\beta^2}{\gamma}] \gtrsim .42$, requiring that $G\mu \lesssim 10^{-8}$, could be be pushing us against the lower bound on the GUT

scale imposed by the decay rate of the proton. A detailed calculation of the string tension in terms of the GUT scale is needed to make this statement more precise.

The question remains as to how serious a problem we have. It has been shown that the string model, along with astrophysical observations of the scale at which galaxy distribution goes nonlinear, gives the following relationship among the various parameters[11,6]

$$\begin{aligned} Baryon-dominated\ universe: \ G\mu\alpha\beta^{\frac{2}{3}} &\approx 4\times10^{-5} \\ Neutrino-dominated\ universe: \ G\mu\alpha\beta^{\frac{1}{3}} &\approx 4\times10^{-6} \end{aligned} \tag{5.3}$$

Taking α=β=1 and $\gamma = 100$, neither of these give values of $G\mu$ which satisfy our inequality (5.2). We can eliminate $G\mu$ to obtain

$$Baryon-dominated\ universe: \ \frac{\alpha^2\beta^{\frac{4}{3}}}{\gamma} \lesssim 1.1\times10^{-4}$$

$$Neutrino-dominated\ universe: \ \frac{\alpha^2\beta^{\frac{5}{3}}}{\gamma} \lesssim 1.1\times10^{-3}$$

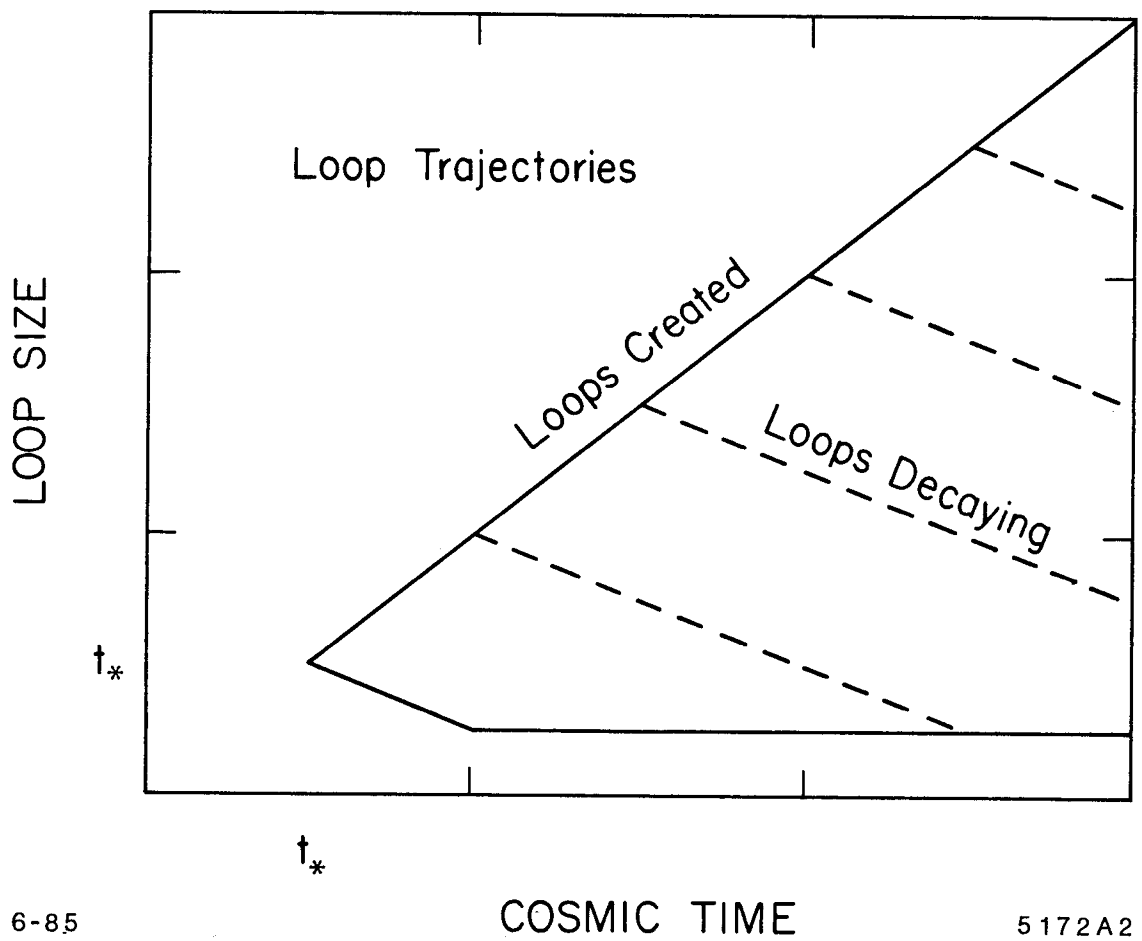
If these inequalities are strongly broken, then the galaxy formation scenario will have to be modified or discarded. On the other hand, if they are satisfied, then eq.(5.3) may push the GUT scale too high. Further investigation into the values of the string model parameters is necessary.

ACKNOWLEDGEMENTS

I would like to thank Dick Bond for several interesting discussions.

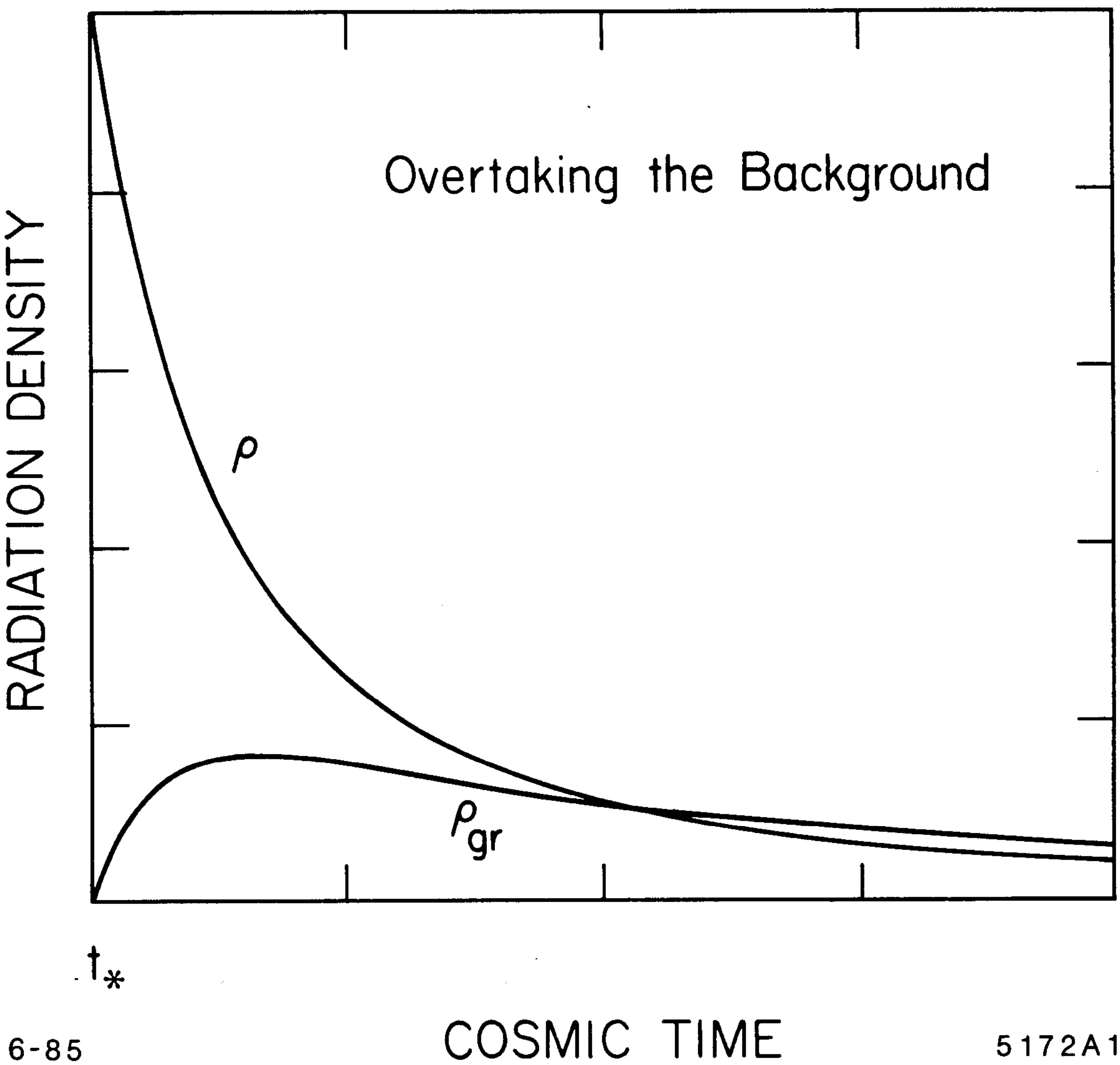
REFERENCES

1. T.W.B.Kibble, *Phys. Rep. 67* **No.1** (1980) 183-199
2. T.W.B Kibble, *J. Phys.* **A9** (1976) 1387
3. Ya. B. Zeldovich, *Mon. Not. R. Astr. Soc.* **192** (1980) 663
4. A. Vilenkin, *Phys Rev Lett* **46** (1981) 1169
5. A. Vilenkin and Q. Shafi, *Phys Rev Lett* **51** (1983) 1716
6. A thourough review and bibliography may be found in: Vilenkin, *Tufts-84-084* (1984)
7. Kibble and Turok, *Phys Lett* **116B** (1982) 141
8. Vachaspati,Everett and Vilenkin, *Tufts-84-0557* (1984)
9. R.L. Davis, *SLAC PUB 3011* (1985)
10. Vachaspati, Vilenkin, *HUTP* **-84/A065** (1984)
11. Shafi and Vilenkin, *Phys Rev Lett* **51** (1984) 1716



6-85 5172A2

Fig. 1



Overtaking the Background
RADIATION DENSITY
ρ
ρ_{gr}
t_*
COSMIC TIME
6-85
5172A1

Fig. 2