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CABIBBO-ANGLE FAVORED $D \to PP$ DECAYS: A DISPERSION APPROACH*

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ABSTRACT

$D \to PP$ Cabibbo-Angle favored decays are studied in a dispersion approach. It is pointed out that if the hadronic matrix element of the divergence of weak current satisfies a once subtracted dispersion relation, it is possible to generate a large flavor-annihilation amplitude. This lifts the color suppression of $D^o \to \bar{K}^o\pi^o$. Final state interactions are used to get a fit to $D \to K\pi$ data.

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1. INTRODUCTION

Extensive data on two-body Cabibbo-Angle favored decays now exist[1] for the channels $D \to PP$ and $D \to VP$. The data confirm that $D^o \to \bar{K}^o\pi^o$ is not color suppressed and that there is evidence of perhaps a larger degree of color suppression in the modes $D^o \to \bar{K}^o\rho^o$ and $D^o \to \bar{K}^{*o}\pi^o$. The relevant ratios are[1,2,3] (these are 1984 numbers; new MARK III numbers should be available soon):

$$\begin{aligned} R_{00} &\equiv \Gamma(D^o \to \bar{K}^o\pi^o)/\Gamma(D^o \to K^-\pi^+) = 0.35 \pm 0.07 \pm .07 \\ R_{0+} &\equiv \Gamma(D^o \to K^-\pi^+)/\Gamma(D^+ \to \bar{K}^o\pi^+) = 3.7 \pm 1.0 \pm 0.08 \end{aligned} \tag{1}$$

$$\begin{aligned} &\Gamma(D^o \to \bar{K}^o\rho^o)/\Gamma(D^o \to K^-\rho^+) = 0.16 \pm 0.06 \pm 0.04 \\ &\Gamma(D^o \to K^-\rho^+)/\Gamma(D^+ \to \bar{K}^o\rho^+) = 2.16 \pm 0.7 \pm 0.37 \end{aligned} \tag{2}$$

$$\begin{aligned} &\Gamma(D^o \to \bar{K}^{*o}\pi^o)/\Gamma(D^o \to \bar{K}^{*-}\pi^+) = 0.11 \pm 0.04 \pm .05 \\ &\Gamma(D^o \to K^{*-}\pi^+)/\Gamma(D^+ \to \bar{K}^{*o}\pi^+) = 6.66 \pm 4.3 \pm 3.4 \end{aligned} \tag{3}$$

We have used $\tau_{D^+}/\tau_{D^o} = 2.5 \pm 0.6$ (all statistical) in (1), (2) and (3). A model independent analysis[4] of the $D \to K\pi$ data has shown that (i) complex amplitudes are needed to fit the data and (ii) non-spectator processes play an important role. With the latter point in mind the problem of $D \to K\pi$ decay was investigated[5] in a vector pole model constrained by current algebra. A near-fit to the data was obtained with real amplitudes. After unitarization of the amplitudes through final state interactions the authors of Ref. 5 obtained a fit to the data[6]. In contrast the data on $D \to \rho K$ and $D \to K^*\pi$ are not yet accurate enough to require complex amplitudes. $D \to PV$ decays will be the subject of a future publication.

In the present paper we have studied the two-body Cabibbo-angle favored $D \to PP$ decays from a dispersion view point, including the flavor-annihilation

(W-exchange process) which is usually argued to be helicity suppressed. We investigate the role of these channels and the circumstances under which the annihilation channel could become important. The reader is referred to earlier investigations, similar in spirit, by Fakirov and Stech[7], Milosevic, Tadic and Trampetic[8] and Rückl[9].

Section 2 of this paper deals with $D \to K\pi$ decays. It is pointed out that if the hadronic matrix element of the divergence of the weak current satisfies a once subtracted dispersion relation then the flavor-annihilation amplitude could become large. This, then, lifts color-suppression of $D^o \to \bar{K}^o \pi^o$ giving a near-fit to the data with real amplitudes. Sections 3 and 4 deal with $D^o \to \bar{K}^o \eta$ and $D^o \to \bar{K}^o \eta'$ decays respectively. In Section 5 we unitarize $D \to K\pi$ decay amplitudes through final state interactions and obtain a fit to the data. The prescription for unitarization used here is the simplest one can think of. The conclusions follow in Section 6.

2. $D \to K\pi$

The hard gluon corrected Hamiltonian for the Cabibbo-angle favored charm decays is,

$$H_W = \frac{G_F}{\sqrt{2}} \cos^2\theta_c \left[\frac{1}{2} (C_+ + C_-) (\bar{u}d)(\bar{s}c) + \frac{1}{2} (C_+ - C_-) (\bar{u}c)(\bar{s}d) \right] \tag{4}$$

where $(\bar{u}d)$ etc. represents the left handed hadronic current, θ_c is the Cabibbo-angle an C_+ and C_- are taken to be [4]

$$C_+ = 0.69, \quad C_- = 2.09, \quad C_+^2 C_- = 1 \tag{5}$$

Sandwiching the Hamiltonian (4) between the initial and the final states and linking up the quark lines in all color-singlet combinations, one obtains the decay amplitudes in the factorization approximation (details are provided for $D^o \to K^- \pi^+$ channel only),

$$
\begin{aligned}
A\left(D^o \to K^-\pi^+\right) &= C_1 \langle \pi^+ | (\bar{u}d) | 0 \rangle \langle K^- | (\bar{s}c) | D^o \rangle \\
&\quad + C_2 \langle \pi^+ K^- | (\bar{s}d) | 0 \rangle \langle 0 | (\bar{u}c) | D^o \rangle
\end{aligned}
\tag{6}
$$

where

$$
\begin{aligned}
C_1 &= \frac{1}{3}\left(2C_+ + C_-\right) \\
C_2 &= \frac{1}{3}\left(2C_+ - C_-\right)
\end{aligned}
\tag{7}
$$

The first term in (6) is the usual spectator (and color-suppressed spectator) term while the second term is the flavor-annihilation term.

To proceed further we use

$$
\begin{aligned}
\langle 0 | (\bar{u}c) | D^o \rangle &= i\sqrt{2} f_D p_D^\mu \\
\langle \pi^+ | (\bar{u}d) | 0 \rangle &= -i\sqrt{2} f_\pi p_\pi^\mu
\end{aligned}
\tag{8}
$$

and

$$
\langle P_i | V_j^\mu | P_k \rangle = i f_{ijk} \left[(p_k + p_i)^\mu f_+(q^2) + (p_k - p_i)^\mu f_-(q^2) \right] \tag{9}
$$

where i, j, k are the SU(4) indices and $q^\mu = (p_k - p_i)^\mu$.

In evaluating the matrix elements in (6) one encounters the hadronic matrix element of the divergence of the vector current,

$$
\begin{aligned}
q_\mu \langle P_i | V_j^\mu | P_k \rangle &= i f_{ijk} \left[(m_k^2 - m_i^2) f_+(q^2) + q^2 f_-(q^2) \right] \\
&\equiv i f_{ijk} f_0(q^2)
\end{aligned}
\tag{10}
$$

The scalar form factor, $f_0(q^2)$, is normalized such that

$$
f_0(0) = (m_k^2 - m_i^2) f_+(0)
$$

$f_0(q^2)$ appearing in the first term in (6) gets contribution from a 0^+ state with flavor content $\bar{s}c$, i.e., F_s, while the second term gets contribution from a 0^+ state with flavor content $\bar{s}d$, i.e. κ (kappa)[10].

The final result for the $D \to K\pi$ decay amplitudes, up to an overall constant is[8],

$$A\left(D^o \to K^-\pi^+\right) = \left(C_1 f_\pi f_0^{F_s}(m_\pi^2) - C_2 f_D f_0^{\kappa}(m_D^2)\right)$$

$$A\left(D^o \to \bar{K}^o\pi^o\right) = \frac{C_2}{\sqrt{2}}\left(f_K f_0^{D_s}(m_K^2) + f_D f_0^{\kappa}(m_D^2)\right) \tag{12}$$

$$A\left(D^+ \to \bar{K}^o\pi^+\right) = \left(C_1 f_\pi f_0^{F_s}(m_\pi^2) + C_2 f_K f_0^{D_s}(m_K^2)\right)$$

Note that the $\Delta I = 1$ isospin sum rule

$$A\left(D^o \to K^-\pi^+\right) + \sqrt{2}A\left(D^o \to \bar{K}^o\pi^o\right) = A\left(D^+ \to \bar{K}^o\pi^+\right) \tag{13}$$

is identically satisfied.

The naive spectator model results are obtained from (12) in the limit $f_\pi = f_K$, $f_0^{F_s}(m_\pi^2) = f_0^{D_s}(m_K^2)$ and the neglect of terms proportional to $f_0^{\kappa}(m_D^2)$, the flavor-annihilation channel contribution.

Let us next assume that $f_0(q^2)$ satisfies an unsubtracted dispersion relation, which would require $f_-(q^2)$to decay faster than $1/q^2$ asymptotically, then

$$f_0^{F_s}(q^2) = \frac{f_+(0)(m_D^2 - m_K^2)}{1 - q^2/m_{F_s}^2} \tag{14}$$

and

$$f_0^{F_s}(m_\pi^2) \approx f_+(0)(m_D^2 - m_K^2) \tag{15}$$

Similarly,

$$\begin{aligned} f_0^{D_s}(m_K^2) &= \frac{f_+(0)(m_D^2 - m_\pi^2)}{1 - m_K^2/m_{D_s}^2} \\ &\approx f_+(0)(m_D^2 - m_\pi^2)\ . \end{aligned} \tag{16}$$

and,

$$f_0^{\kappa}(m_D^2) = \frac{f_+(0)(m_K^2 - m_\pi^2)}{1 - m_D^2/m_\kappa^2}$$
$$\simeq -1.1 f_+(0)(m_K^2 - m_\pi^2) \tag{17}$$

with[10] $m_\kappa = 1.35$ GeV.

The factor $(m_K^2 - m_\pi^2)$ in (17) signals helicity suppression. Clearly $f_0^{\kappa}(m_D^2)$ is helicity suppressed relative to $f_0^{F_s}(m_\pi^2)$ or $f_0^{D_s}(m_K^2)$. Hence, if we assume an unsubtracted dispersion relation for $f_0(q^2)$, and further assume that it is dominated by a single particle pole, then helicity suppression is not lifted and the non-spectator processes are not important.

It is clear from (10) that in an exact SU(4) limit, $f_-(q^2) = 0$, helicity suppression will always operate. However, SU(4) symmetry is broken and since $f_0^{\kappa}(q^2)$ is required at $q^2 = m_D^2$, it is quite possible that $f_+(m_D^2)$ and $f_-(m_D^2)$ are comparable[11]. Since $f_-(q^2)$ appears multiplied by m_D^2 in $f_0^{\kappa}(q^2)$, it is also quite possible that $m_D^2 f_-(m_D^2)$ would dominate over $(m_K^2 - m_\pi^2) f_+(m_D^2)$. We have just shown, however, that so long as $f_0(q^2)$ satisfies an unsubtracted dispersion relation helicity suppression is unlikely to be lifted.

Let us assume, next, that $f_-(q^2)$ decays no faster than $1/q^2$ asymptotically. $f_0(q^2)$ then satisfies a subtracted dispersion relation. Let us assume further, that $f_0(q^2)$ satisfies a once-subtracted dispersion relation such that,

$$f_0^{F_s}(q^2) = f_+(0)(m_D^2 - m_K^2) + \frac{\lambda^{F_s} q^2}{q^2 - m_{F_s}^2} \tag{18}$$

and

$$f_0^{F_s}(m_\pi^2) \simeq f_+(0)(m_D^2 - m_K^2) \tag{19}$$

unless, of course, λ^{F_s} is unusually large. Notice that (19) has not changed from the unsubtracted value (15).

Similarly,

$$f_0^{D_*}(q^2) = f_+(0)(m_D^2 - m_\pi^2) + \frac{\lambda^{D_*} q^2}{q^2 - m_{D_*}^2} \tag{20}$$

and

$$f_0^{D_*}(m_K^2) \approx f_+(0)(m_D^2 - m_\pi^2) \tag{21}$$

which is also the same as in (16).

However,

$$f_0^{\kappa}(q^2) = f_+(0)(m_K^2 - m_\pi^2) + \frac{\lambda q^2}{q^2 - m_\kappa^2} \tag{22}$$

and

$$f_0^{\kappa}(m_D^2) = f_+(0)(m_K^2 - m_\pi^2) + \frac{\lambda m_D^2}{m_D^2 - m_\kappa^2} \tag{23}$$

If we look for solutions with the condition (λ stands for any λ in (18), (20) or (22))

$$m_K^2 \ll \frac{|\lambda|}{f_+(0)} \ll \frac{m_D^2 m_{D_*}^2}{m_K^2} \tag{24}$$

then (19) and (21) will remain unaltered; however, for (23) we will obtain,

$$f_0^{\kappa}(m_D^2) \approx \frac{\lambda m_D^2}{m_D^2 - m_\kappa^2} \tag{25}$$

It is now possible to lift the helicity suppression of the annihilation process. The decay amplitudes are then,

$$\begin{aligned}
A(D^o \to K^-\pi^+) &= \left[C_1 f_\pi f_+(0)(m_D^2 - m_K^2) - C_2 f_D \frac{\lambda m_D^2}{m_D^2 - m_\kappa^2}\right] \\
A(D^o \to \bar{K}^o\pi^o) &= \frac{C_2}{\sqrt{2}}\left[f_K f_+(0)(m_D^2 - m_\pi^2) + f_D \frac{\lambda m_D^2}{m_D^2 - m_\kappa^2}\right] \\
A(D^+ \to \bar{K}^o\pi^+) &= \left[C_1 f_\pi f_+(0)(m_D^2 - m_K^2) + C_2 f_K f_+(0)(m_D^2 - m_\pi^2)\right]
\end{aligned} \tag{26}$$

Note that in the limit $f_\pi = f_K$ and $m_D^2 - m_\pi^2 \approx m_D^2 - m_K^2 \simeq m_D^2$ and the neglect of flavor-annihilation terms, signalled by the parameter λ, the usual color suppression occurs, $\Gamma(D^o \to \bar{K}^o\pi^o)/\Gamma(D^o \to K^-\pi^+) \approx 1/2C_2^2 \approx 1/50$. However, $\lambda > 0$ raises both $A(D^o \to \bar{K}^o\pi^o)$ and $A(D^o \to \bar{K}\pi^+)$. The percentage rise in $A(D^o \to \bar{K}^o\pi^o)$ is larger since the two terms appear with equal weights. In contrast, the annihilation term in $A(D^o \to K^-\pi^+)$ is proportional to C_2 while the spectator term is proportional to C_1. Since $C_2/C_1 \approx -1/5$, the percentage rise in $A(D^o \to K^-\pi^+)$ is smaller.

In Table I we have compiled the ratios R_{00} and R_{0+} as functions of f_D/f_π and $\lambda/f_+(0)$. Clearly, though one does not expect to fit both the ratios R_{00} and R_{0+} with real amplitudes, one comes close to fitting the data. The important point being that helicity-suppression of the flavor-annihilation process is lifted. In turn, this lifts the color-suppression of $D^o \to \bar{K}^o\pi^o$ decay.

3. $D^o \to \bar{K}^o\eta$

We assume that η is a pure SU(3) octet i.e. $\eta = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) = P_8$, in SU(4). In the factorization approximation,

$$A(D^o \to \bar{K}^o\eta) = C_2\left[\langle\bar{K}^o|(\bar{s}d)|0\rangle\langle\eta|(\bar{u}c)D^o\rangle + \langle\bar{K}^o\eta|(\bar{s}d)|0\rangle\langle 0|(\bar{u}c)|D^o\rangle\right] . \quad (27)$$

Relating $\langle\eta|(\bar{u}c)|D^o\rangle$ to $\langle\pi^o|(\bar{u}c)|D^o\rangle$ through the SU(4) rotation (9), one obtains, up to an overall constant,

$$A(D^o \to \bar{K}^o\eta) = \frac{C_2}{\sqrt{6}}\left[f_K f_0^{D_s}(m_K^2) - 3 f_D f_0^{\kappa}(m_D^2)\right] . \quad (28)$$

It is worth noting that the flavor-annihilation term is larger than in $D^o \to \bar{K}^o\pi^o$ by a factor of $\sqrt{3}$. This is due to the fact that η has a $s\bar{s}$ component while π^o does not.

However, the flavor-annihilation term may not be significant for two reasons. First, one expects that it is harder to excite a $s\bar{s}$-pair from the vacuum[12] and second, kappa does not appear to couple to $\bar{K}\eta$ channel[13]. If the latter statement is taken seriously then one does not expect the kappa structure in $f_0^{\kappa}(q^2)$ appearing in (28). $f_0^{\kappa}(q^2)$ will be essentially structure-free and approximated by it's value at $q^2 = 0$,

$$f_0^{\kappa}(q^2) \approx f_0^{\kappa}(0) = f_+(0)(m_K^2 - m_\eta^2) \tag{29}$$

Notice that due to the closeness of the K-mass to the η-mass the mass-suppression is rather strong.

This argument will apply regardless of whether $s\bar{s}$-pair is excited or not so long as kappa does not couple to $\bar{K}^o\eta$. *If the arguments made here apply* then flavor-annihilation will not contribute significantly to $D^o \to \bar{K}^o\eta$ and one will obtain,

$$A(D^o \to \bar{K}^o\eta) \simeq \frac{C_2}{\sqrt{6}} f_K f_+(0)(m_D^2 - m_\eta^2) \tag{30}$$

Since $A(D^+ \to \bar{K}^o\pi^+)$ does not depend on the flavor-annihilation amplitude either, one can calculate:

$$B(D^o \to \bar{K}^o\eta)/B(D^+ \to \bar{K}^o\pi^+) = 0.7 \times 10^{-2} \tag{31}$$

where we have used $\tau_{D^+}/\tau_{D^o} = 2.5$ and $f_K/f_\pi = 1.2$. In the naive spectator model one expects the ratio of (31) to be 2.2×10^{-2}. The difference is due to $f_\pi \neq f_K$ and the inclusion of the pseudo-scalar masses.

If the flavor-annihilation term in (28) is *not* suppressed, and κ couples to $\bar{K}^o\eta$ through SU(3), then the ratio in (31) could be larger by as much as two orders of magnitude. A measurement of $B(D^o \to \bar{K}\eta)$ will be quite a sensitive test of non-spectator contribution to $D^o \to \bar{K}^o\eta$. In Table II we have tabulated $B(D^o \to \bar{K}^o\eta)/B(D^+ \to \bar{K}^o\pi^+)$ as a function of f_D/f_π and $\lambda/f_+(0)$. In anticipation of the results of Section 5 we have allowed $\lambda/f_+(0)$ to be of order 10 GeV^2. We notice from this Table that $B(D^o \to \bar{K}^o\eta) \sim (1-2)\%$ signals a large flavor-annihilation contribution. The reader is reminded that $B(D^+ \to \bar{K}^o\pi^+)$ is (2 - 3)%.

4. $D^o \rightarrow \bar{K}^o\eta'$

We assume η' to be an SU(3) singlet, $\eta' = \frac{1}{\sqrt{3}}(u\bar{u}+d\bar{d}+s\bar{s}) = \frac{1}{2}(\sqrt{3}P_0+P_{15})$, in SU(4). We obtain, in a fashion analogous to the analysis of the last section,

$$
\begin{aligned}
A(D^o \to \bar{K}^o\eta') &= C_2\Big\{\langle\bar{K}^o|(\bar{s}d)|0\langle\eta'|(\bar{u}c)|D^o\rangle + \ \langle\bar{K}^o\eta'|(\bar{s}d)|0\rangle\langle 0|(\bar{u}c)|D^o\rangle\Big\} \\
&= \frac{C_2}{\sqrt{3}} f_K f_+(0)(m_D^2 - m_{\eta'}^2)
\end{aligned}
\tag{32}
$$

The flavor-annihilation term vanishes due to the vanishing of the SU(4) structure function f_{ijk}, as one would expect from the appearance of $d\bar{d}$ and $s\bar{s}$ with equal weights in the η'. However, the same reasons which allow us to argue away the flavor-annihilation terms in $D^o \to \bar{K}^o\eta$ conspire to resurrect the flavor-annihilation term in $D^o \to \bar{K}^o\eta'$. For example, if a $s\bar{s}$-pair could not be excited from the vacuum with the same probability as the $d\bar{d}$-pair then the cancellation of the flavor-annihilation term would not be complete.

In the approximation that the $s\bar{s}$-pair is *not* excited from the vacuum *and* kappa ***does not couple*** to the closed channel $\bar{K}^o\eta'$, such that $f_0^\kappa(q^2) \approx f_0^\kappa(0)$, one obtains,

$$
A(D^o \to \bar{K}^o\eta') = \frac{C_2}{\sqrt{3}}\left[f_K f_+(0)(m_D^2 - m_{\eta'}^2) + f_D f_+(0)(m_{\eta'}^2 - m_K^2)\right] . \tag{33}
$$

If the flavor-annihilation is absent in $A(D^o \to \bar{K}^o\eta')$, that is, the amplitude is given by (32), then one obtains (using $f_D \approx f_K$),

$$B(D^o \to \bar{K}^o \eta')/B(D^+ \to \bar{K}^o \pi^+) = 0.58 \times 10^{-2} \tag{34}$$

On the other hand using (33), with the flavor-annihilation terms present, one gets

$$B(D^o \to \bar{K}^o \eta')/B(D^+ \to \bar{K}^o \pi^+) = 0.93 \times 10^{-2} \ . \tag{35}$$

Thus this ratio is not a very sensitive test of the presence of annihilation term, unless it can be measured very accurately.

5. FINAL STATE INTERACTIONS

Re-scattering in the final state endows the weak decay amplitudes with phases. A number of authors[14] have studied the problem of final state interactions in $D \to K\pi$ decays.

Let us introduce the decay amplitudes, A_1 and A_3, for decays into $I = 1/2$ and $3/2$ states and their phases δ_1 and δ_3 as follows:

$$\begin{aligned} A(D^o \to K^- \pi^+) &= \frac{1}{\sqrt{3}}(A_3 e^{i\delta_3} - \sqrt{2} A_1 e^{i\delta_1}) \\ A(D^o \to \bar{K}^o \pi^o) &= \frac{1}{\sqrt{3}}(\sqrt{2} A_3 e^{i\delta_3} + A_1 e^{i\delta_1}) \\ A(D^+ \to \bar{K}^o \pi^+) &= \sqrt{3} A_3 e^{i\delta_3} \ . \end{aligned} \tag{36}$$

If the scattering in the final state is elastic then the phase of the weak decay amplitude is the scattering phase shift in the relevant two-body channel. The effect is to generate a complex amplitude ($i = 1, 3$),

$$A_i(s) e^{i\delta_i}(s) = A_i(s_0) \ \exp \left\{ \frac{(s - s_0)}{\pi} \int \frac{\delta_i(s') ds'}{(s' - s_0)(s' - s + i\epsilon)} \right\} \ , \tag{37}$$

through the solution of Muskhelishvili-Omnes integral equation[15]. s_0 is a normalization point and, in our case, we eventually set $s = m_D^2$.

If one parametrizes the partial wave scattering amplitude in the N/D form[16], then the Muskhelishvili-Omnes function, the exponential in (37), is proportional to $D^{-1}(s)$. A real decay amplitude, $A_i^{(o)}(s)$, may then unitarized by final state interactions through,

$$A_i(s)e^{i\delta_i(s)} = A_i^{(o)}(s)\frac{D_i(s_0)}{D_i(s)} \tag{38}$$

we choose s_0, the normalization point, to be the πK threshold $s_0 = (m_K + m_\pi)^2$, such that the phase vanishes at this point, as it indeed should, and $A_i(s_0) = A_i^{(o)}(s_0)$.

We begin by switching off the final state interactions and evaluate $A_i^{(o)}(s)$ using (12) and (36) with $\delta_i = 0$. We obtain (eventually we set $s = m_D^2$)

$$\begin{aligned} A_1^{(o)}(s) &= -\sqrt{\frac{3}{2}}\left(\frac{2}{3}C_1 f_\pi f_0^{F_s}(s,m_\pi^2) - \frac{1}{3}C_2 f_K f_0^{D_s}(s,m_K^2) - C_2 f_D f_0^{\kappa}(s,m_\kappa^2)\right) \\ A_3^{(o)}(s) &= \frac{1}{\sqrt{3}}\left(C_1 f_\pi f_0^{F_s}(s,m_\pi^2) + C_2 f_K f_0^{D_s}(s,m_K^2)\right) \end{aligned} \tag{39}$$

where

$$\begin{aligned} f_0^{F_s}(s,m_\pi^2) &= f_+(0)(s-m_K^2) \\ f_0^{D_s}(s,m_K^2) &= f_+(0)(s-m_\pi^2) \\ f_0^{\kappa}(s,m_\kappa^2) &= \frac{\lambda s}{s-m_\kappa^2} \end{aligned} \tag{40}$$

We make a simplifying assumption for $D_3(s)$. We assume that there is very little re-scattering in the non-resonant $I = 3/2$ channel. Thus $\delta_3(s) = 0$ and $D_3(s) = 1$. $I = 1/2$, 0^+ channel, on the other hand, resonates[13]. The *simplest* way to unitarize $A_1^{(o)}(s)$ would be by the prescription (38) where $D_1(s)$ is chosen to have a resonance structure. The unitarized form of (39) is then,

$$A_1(s)e^{i\delta_1(s)} = \frac{A_1^{(o)}(s)(s_0 - m_\kappa^2)}{(s - m_\kappa^2 + i\gamma k)} \tag{41}$$

$$A_3(s)e^{i\delta_3(s)} \approx A_3^{(o)}(s) \tag{42}$$

where γ is the reduced width of kappa and k, the 3-momentum in the πK center-of-mass. We have chosen the subtraction point to be the πK threshold, $s_0 = (m_K + m_\pi)^2$, and

$$D(s) = (s - m_\kappa^2 + i\gamma k) \tag{43}$$

R_{00} and R_{0+} are then calculated by using,

$$R_{00} = \frac{\left(1 + 2r^2 + 2\sqrt{2}\, r\cos\delta\right)}{\left(2 + r^2 - 2\sqrt{2}\, r\cos\delta\right)} \tag{44}$$

$$R_{0+} = \frac{1}{9}\left(1 + \frac{2}{r^2} - \frac{2\sqrt{2}}{r}\cos\delta\right) \tag{45}$$

where $\delta = \delta_1 - \delta_3 = \delta_1$ (in our case) and $r = A_3/A_1$.

The results are summarized in Table III. We chose rather a broad kappa, $\gamma = 1.4$ GeV, to generate $\delta_1 = 144^o$ at $s = m_D^2$. m_κ was chosen to be 1.35 GeV[10]. We get a fit to the data with $\lambda/f_+(0)$ in the vicinity of 10 GeV^2, well within the limits of (24).

A word about the effect of final state interaction of $D^o \to \bar{K}^o\eta$ and $D^o \to \bar{K}^o\eta'$ is in order. Since these decays involve only one isospin amplitude and thus only one overall phase, their rates are unaffected by final state interactions. Further, since $D \to \bar{K}^o\pi^+$ also depends on a single isospin amplitude, the ratios $B(D^o \to \bar{K}^o\eta)/B(D^+ \to \bar{K}^o\pi^+)$ and $B(D^o \to \bar{K}^o\eta')/B(D^+ \to \bar{K}^o\pi^+)$ as also $B(D^o \to \bar{K}^o\eta)/B(D^o \to \bar{K}^o\eta')$ are insensitive to the details of final state interaction.

6. CONCLUSIONS

In this paper we have paid attention to Cabibbo-angle favored $D \to PP$ decays only. The decay amplitudes were written down in terms of the matrix elements of the hadronic weak currents in a factorization approximation. Single-particle dominated dispersion relations were postulated for these hadronic matrix elements. The naive spectator model results are recovered from this analysis in the limit $f_\pi = f_K$ and $f_0^{F_s}(m_\pi^2) = f_0^{D^*}(m_K^2)$ *and* the neglect of the flavor-annihilation term proportional to $f_0^\kappa(m_D^2)$ in (12).

We showed that if $f_0^\kappa(q^2)$, which appears in the flavor-annihilation channel only, satisfies an unsubtracted dispersion relation then color-suppression of $D^o \to \bar{K}^o\pi^o$ is not lifted. On the other hand if $f_o^\kappa(q^2)$ satisfies a once-subtracted dispersion relation then it is possible to find a value of the new parameter, λ, introduced in (22) within the range required by (24), such that color-suppression of $D^o \to \bar{K}^o\pi^o$ is lifted. Another way to see this result is that in the flavor-annihilation channel $f_+(q^2)$, introduced in (10), appears multiplied by a mass-suppression factor of $(m_K^2 - m_\pi^2)$ while $f_-(q^2)$ appears with a large factor of m_D^2. Thus if $f_-(m_D^2) \approx f_+(m_D^2)$ then obviously $f_0^\kappa(m_D^2)$ will be large and helicity-suppression of flavor-annihilation process will be lifted. This, in turn, lifts the color-suppression of $D^o \to \bar{K}^o\pi^o$.

Experimentally, $f_-(q^2)$ is not accessible in $D \to \bar{K}\ell\nu$ due to the small charged lepton mass. However, theoretical model calculations[8,11] favor $f_-(m_D^2)$ comparable to $f_+(m_D^2)$. Thus the conjecture that $m_D^2 f_-(m_D^2)$ might dominate $(m_K^2 - m_\pi^2)f_+(m_D^2)$ is very likely to be true.

We have also studied $D^o \to \bar{K}^o\eta$ and $D^o \to \bar{K}^o\eta'$ decays. Theoretically these are, surprisingly, not very clean channels. For example, in $D^o \to \bar{K}^o\eta$ a straight forward SU(4) rotation applied to $D \to K\pi$ amplitudes generates a large annihilation term. This is due to the additional $s\bar{s}$ content of η. However, SU(3) breaking makes the excitation of a $s\bar{s}$-pair from the vacuum less likely[12] than, say, a $d\bar{d}$-pair. This alone would reduce the annihilation term by a factor of 3. Further, since kappa does not appear[13] to couple to $\bar{K}^0\eta$ channel, one expects $f_0^\kappa(q^2)$ not to have any structure. If so, then $f_0^\kappa(q^2) \approx f_0^\kappa(0) \simeq (m_K^2 - m_\eta^2)f_+(0)$, which is vanishingly small due to the closeness of K-meson and kappa masses. This uncertainty in handling the annihilation term can give rise to an uncertainty of two orders of magnitude in the rate for $(D^o \to \bar{K}^o\eta)$. The ratio $B(D^o \to \bar{K}^o\eta)/B(D^+ \to \bar{K}^o\pi^+)$ will test the presence of annihilation term in $A(D^o \to \bar{K}^o\eta)$ since $A(D^+ \to \bar{K}^o\pi^+)$ does not have an annihilation contribution. We have shown that with $\lambda/f_+(0)$ in the region of 10 GeV^2 one can generate $B(D^o \to \bar{K}^o\eta) \sim (1-2)\%$. If experiments would measure the branching ratio at this level, it would be an indication of a large flavor-annihilation contribution to $D^o \to \bar{K}^o\eta$.

Similar uncertainties apply to the flavor-annihilation term in $A(D^o \to \bar{K}^o\eta')$. However, in this channel one would be surprised if $B(D^o \to \bar{K}^o\eta')/B(D^+ \to \bar{K}^o\pi^+)$ turned out very different from $\approx 10^{-2}$.

Finally we unitarized $D \to K\pi$ decay amplitudes through final state interactions and showed that it is possible to fit the data with the assumption of a broad kappa meson in 0^+, $I = 1/2$ channel. The method of unitarization used here is the simplest one we can use (certainly not the last word on final state interactions) and shows that once a mechanism for lifting color suppression is found, it is possible to fit $D \to K\pi$ data with final state interactions.

The approach employed in this paper is complementary to that used in Ref. 5 but couched in different, and hopefully more familiar, language. The approach of Ref. 5 was largely algebraic where current algebra was used to constrain the decay amplitudes. The approach adopted in this paper is analytic in nature where dispersion relations are invoked for the hadronic matrix elements. The particles are always kept on mass-shell.

Lastly, we treated C_1/C_2 as parameter. The values of C_+ and C_- used in this paper imply $C_1/C_2 \simeq -5$. If C_-/C_+ is allowed to rise ('sextet dominance') then C_1/C_2 moves towards -1. The precise relationship between C_-/C_+ and

C_1/C_2 is

$$\frac{C_-}{C_+} = 2\,\frac{(C_1/C_2 - 1)}{(C_1/C_2 + 1)} \tag{46}$$

In Table IV we have listed R_{00} and R_{0+}, computed with final state interactions (kappa parameters as in Table III) and $f_D/f_\pi = 1.2$, as functions of C_1/C_2. It is evident that the effect of lowering the magnitude of C_1/C_2 (equivalent to raising the ratio C_-/C_+) is to simulate the flavor-annihilation term, since less and less of it is needed (λ decreases) to fit the data.

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Table I

R_{00} and R_{0+} without Final State Interactions

f_D/f_π	$\lambda/f_+(0)$ (in GeV^2)	R_{00}	R_{0+}
1	5	0.15	4.96
1	6	0.17	5.78
1.1	5	0.16	5.36
1.1	6	0.18	6.30
1.2	5	0.17	5.78
1.2	6	0.19	6.84
1.3	4	0.16	5.12
1.3	5	0.18	6.21
1.4	4	0.16	5.44
1.4	5	0.19	6.66

Table II

$B(D^o \to \bar{K}^o\eta)/B(D^+ \to \bar{K}^o\pi^+)$ as a function of f_D/f_π and $\lambda/f_+(0)$. Larger values of $\lambda/f_+(0)$ are used in anticipation of the results of Section 5.

f_D/f_π	$\lambda/f_+(0)$ (in GeV^2)	$B(D^o \to \bar{K}^o\eta)$ / $B(D^+ \to \bar{K}^o\pi^+)$
1	6	0.10
	8	0.20
	10	0.33
	12	0.51
1.2	6	0.15
	8	0.30
	10	0.51
	12	0.76
1.4	6	0.22
	8	0.43
	10	0.71
	12	1.06

Table III

R_{00} and R_{0+} with Final State Interactions. Parameters used:
$m_\kappa = 1.35$ GeV, $\gamma = 1.4$ GeV ($\delta_1 = 144°$).

f_D/f_π	$\lambda/f_+(0)$ in GeV^2	R_{00}	R_{0+}
1.0	7	0.18	3.38
	8	0.19	3.85
	9	0.20	4.34
	10	0.21	4.86
1.2	7	0.19	4.04
	8	0.21	4.65
	9	0.22	5.30
	10	0.23	6.00
1.4	7	0.21	4.75
	8	0.23	5.53
	9	0.24	6.36
	10	0.25	7.25

Table IV

R_{00} and R_{0+} as functions of C_1/C_2. Final State Interactions included. $f_D/f_\pi = 1.2$. Kappa Parameters as in Table III.

C_1/C_2	$\lambda/f_+(0)$ (in GeV^2)	R_{00}	R_{0+}
−5	7	0.19	4.04
	8	0.21	4.65
	9	0.22	5.30
	10	0.23	6.00
−4	5	0.20	4.30
	6	0.22	5.17
	7	0.24	6.13
−3	3	0.21	4.9
	4	0.24	6.4
−2	0.5	0.23	5.60
	0.6	0.23	5.96
	0.7	0.24	6.34

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