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THEORETICAL UNCERTAINTIES IN THE PHOTON STRUCTURE FUNCTION*

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ABSTRACT

We study the theoretical uncertainties associated with higher order singularities and hadronic contributions in the photon structure function. We find that they give negligible contributions for $x \gtrsim 0.15$. Therefore, the second order QCD analysis still provides a reliable prediction in $\gamma\gamma$-scattering.

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The analysis of deep-inelastic photon-photon scattering has, in the past few years, generated a great deal of enthusiasm and hope as a most incisive test of QCD predictions.[1,2] The important feature here is that QCD predicts not only the Q^2-evolution, as in most other processes, but also the normalization and shape of the photon structure function[3] up to second order in the strong coupling constant $\alpha_s(Q)$,[4] in terms only of the QCD scale parameter Λ. However, the naive treatment of next-to-leading order corrections gives a negative structure function at small x, even at large values of Q.[4,5] This negative piece comes a simple pole singularity in the "point-like" contribution at the $n = 2$ moment leading to a $-1/x$ behavior in the structure function. This singularity is cancelled[6,7] by corresponding term in the perturbatively noncalculable "hadronic" contribution which becomes important only in the second moment.

It is possible to write a simple parametrization of the real photon structure function, free of the $1/x$ singularity, using an explicit separation of the singular terms.[8] The method consists of a proper regularization of the singularity without loosing the predictive power of perturbative QCD. It introduces an additional parameter λ (besides $\Lambda_{\overline{MS}}$) associated to the noncalculable constant term in the second moment. For reasonable values of λ, the structure function remains positive and the perturbative expansion is well behaved. Furthermore, the structure function turns out to be sensitive to the values of this parameter only in the region of small x's ($x \lesssim .2$).

The experimental data appear to be nicely compatible with such a picture.[2] The analysis gives $\Lambda_{\overline{MS}} = 230 \pm 70$ MeV (including systematical errors) which is in good agreement with results from other processes. The experimental accuracy in the determination of $\Lambda_{\overline{MS}}$ compares favorably with other measurements which suggest that deep-inelastic $\gamma\gamma$-scattering is best suited for an accurate measurement of the QCD scale parameter. The corresponding value of λ turns out to be around 10.

However, the success may be premature since there are theoretical uncertainties that seem to be even larger than experimental errors. On one hand, higher order singularities are expected to appear as poles at the $n > 2$ moments.[8,9] Although, in principle, similar regularization techniques could be used, such singularities might give important contributions[9] since they lead to more singular x-behavior, i.e. $1/x^p$ with $p > 1$. On the other hand, beyond next-to-leading order, the shape of the structure function is no more predictable as the "hadronic" part becomes significant. A conservative approach consists in pretending that only the Q^2-evolution of the structure function can be successfully predicted by QCD but then all sensitivity to Λ is lost.[10]

In this work, we do a systematic study of the aforementioned theoretical uncertainties and we show that, under very reasonable assumptions, these uncertainties are constrained to a region of small x ($x \lesssim .15$) giving negligible contribution for large values of x. We thus find the "regularized" second order QCD analysis is sufficient to describe accurately the theoretical feature of QCD in $\gamma\gamma$-scattering and can be used for comparison with experiment.

ANALYSIS OF HIGHER ORDER SINGULARITIES

Using operator product expansion and renormalization group methods, the moments of the photon structure function $F_2^\gamma(x, Q^2)$ can be written as:[3]

$$
\begin{aligned}
\frac{4\pi}{\alpha}\, M_n^\gamma(Q^2) \equiv \frac{4\pi}{\alpha} \int_0^1 dx\; x^{n-2}\; F_2^\gamma(x, Q^2) \\
= \frac{4\pi}{\beta_0}\, \frac{a_n}{\alpha_s(Q)} + b_n + c_n\, \frac{\alpha_s(Q)}{4\pi} + \dots \qquad (1) \\
+ \sum_{i=+,-,NS} A_n^i\, [\alpha_s(Q)]^{d_i^n} \left(1 + r_n^i\, \frac{\alpha_s(Q)}{4\pi} + \dots\right)
\end{aligned}
$$

where α is the e.m. coupling constant, $a_n, b_n, c_n, \dots$ and $r_n^i, \dots$ are calculable quantities while A_n^i are hadronic coefficients and finally, $d_i^n = \gamma_i^n / 2\beta_0$ $(i =$

$+, -, NS)$ with γ_i^n being the eigenvalues of the one-loop anomalous dimension matrix and β_0, the one-loop coefficient of the β-function.

The term b_n is known[4] to have a singularity at $n = 2$, i.e. $b_n \sim -\frac{1}{n-2}$, which leads to a negative structure function[4,5] in the region of small x

$$b(x) \sim -\frac{1}{x}$$

Fortunately, this singularity is cancelled[6] by a corresponding term in A_n^-.[7] The remaining piece is finite and of order $O(1)$ in $\alpha_s(Q)$. However, since only b_n is calculable, it is best to separate the singular terms prior to regularization. We use the following procedure:[8] we expand around $n = 2$

$$b_n = \frac{b}{n-2} + b_n^r \tag{2a}$$

$$A_n^- = -\frac{b}{n-2}\,\lambda^{d_-^n} + A_n^r \tag{2b}$$

$$d_-^n = d(n-2) + O\left[(n-2)^2\right] \tag{2c}$$

where b_n^r is regular and λ is a hadronic parameter defined such that $A_n^r|_{n=2} = 0$. Thus, the moments of F_2^γ take the form:

$$\frac{4\pi}{\alpha}\,M_n^\gamma(Q^2) = \frac{4\pi}{\beta_0}\,\frac{a_n}{\alpha_s(Q)} + b_n^r + \frac{b}{n-2}\left\{1 - [\lambda\alpha_s(Q)]^{d_-^n}\right\} + O\left[(\alpha_s(Q))^{\delta_n}\right] \tag{3}$$

where $\delta_n > 0$ for $n \geq 2$. Notice that the above expression is now well behaved at $n = 2$:

$$\frac{4\pi}{\alpha}\,M_2^\gamma(Q^2) = \frac{4\pi}{\beta_0}\,\frac{a_2}{\alpha_s(Q)} + b_2^r - bd\ln\left[\lambda\alpha_s(Q)\right] \;. \tag{4}$$

When λ is zero, one recovers the singularity. However, for λ not too small M_2^γ as well as $F_2^\gamma(x, Q^2)$ are positive.

Similar singularities appear at higher orders in $\alpha_s(Q)$.[8,9] They are generated by the mixing between the hadronic and photon components and arise in the solution of the renormalization group equations:

$$
\begin{aligned}
&\overline{\alpha}_s^{\,d_i^n} \int_{\overline{\alpha}_s}^{\alpha_s} d\alpha'_s \; {\alpha'_s}^{-d_i^n-2} \left(q_0^n + q_1^n\, \alpha'_s + q_2^n\, {\alpha'_s}^2 + \ldots\right) \\
&= \sum_{\ell\geq 0} (\overline{\alpha}_s)^{\ell-1} \; \frac{q_\ell^n}{d_i^n+1-\ell} \left\{ 1 - \left(\frac{\overline{\alpha}_s}{\alpha_s}\right)^{d_i^n+1-\ell} \right\} , \qquad \overline{\alpha}_s \equiv \alpha_s(Q)
\end{aligned}
\tag{5}
$$

Each of the two terms in the curly bracket has precise meaning: they are the point-like and hadronic parts respectively. Although the total expression is perfectly regular, both contributions become singular when the denominator $d_i^n+1-\ell$ is zero. We see that for each value of $\ell = 0, 1, \ldots$ the simple pole singularity in the point-like part appearing at the moment n_ℓ^i (such that $d_i^n|_{n=n_\ell} = \ell-1$) gives a contribution to the structure function at least of order $[\alpha_s(Q)]^{\ell-1}$. This singular term is cancelled by a corresponding term in the hadronic part and leads to an expression proportional to $\ln[\alpha_s(Q)]$ besides powers.

Adopting the same regularization technique as in the $n = 2$ case, we expand

$$
b_n^\ell = \frac{\tilde{b}_\ell^i}{n - n_\ell^i} + \left(b_n^\ell\right)^r \tag{6a}
$$

$$
A_n^i = -\frac{\tilde{b}_\ell^i}{n - n_\ell^i} \; \left(\lambda_\ell^i\right)^{d_i^n+1-\ell} + \left(A_{n,i}^\ell\right)^r \tag{6b}
$$

$$
d_i^n = \ell - 1 + \tilde{d}_\ell^i \left(n - n_\ell^i\right) + O\left[\left(n - n_\ell^i\right)^2\right] \tag{6c}
$$

where b_n^ℓ, $\ell = 0, 1, 2, \ldots$, stands for $a_n, b_n, c_n, \ldots$ [$\tilde{b}_1^- = b$, $\lambda_1^- = \lambda$ and $\tilde{d}_\ell^- = d$ defined in eq. (2)]. In table I, we present the numerical values of n_ℓ^i, $i = +, -, NS$, for $\ell = 0, 1, 2, 3$ together with the coefficients $\tilde{d}_\ell^i$.

In principle, there is no reason why λ_ℓ^i's should not depend on n. However, for the purpose of studying the effect of the singularities no loss of generality results if we take them as constants. The contribution of the regular parts of the hadronic piece in (6b) will be discussed in the next section. Moreover, $(b_{n,i}^\ell)^r$ for $\ell \geq 2$ are the regular parts of $c_n^i, \ldots$ which give contributions at least of order $O[(\alpha_s(Q))]$. They represent small corrections to (3) and will not be discussed here. Instead, we consider the regularization of the most singular pieces which take the form [see eq. (5)]:

$$\Delta_{n,i}^\ell\left(\lambda_\ell^i\right) = \frac{\tilde{b}_\ell^i}{n - n_\ell^i}\left\{1 - \left[\lambda_\ell^i\,\alpha_s(Q)\right]^{d_i^n+1-\ell}\right\}\left[\frac{\alpha_s(Q)}{4\pi}\right]^{\ell-1} \tag{7}$$

It is instructive to look at this expression by approximating d_i^n by its form near $n = n_\ell^i$. Then

$$\Delta_{n,i}^\ell\left(\lambda_\ell^i\right) = \frac{\tilde{b}_\ell^i}{n - n_\ell^i}\left\{1 - \left[\lambda_\ell^i\,\alpha_s(Q)\right]^{\tilde{d}_\ell^i(n-n_\ell^i)}\right\}\left[\frac{\alpha_s(Q)}{4\pi}\right]^{\ell-1} \tag{8}$$

In that form the moments $\Delta_{n,i}^\ell$ are analytically invertible and the contribution to $F_2^\gamma(x, Q^2)$ is

$$\Delta_i^\ell\left(x, \lambda_\ell^i\right) = \frac{\tilde{b}_\ell^i}{x^{n_\ell^i-1}}\,\theta\left\{x - \left[\lambda_\ell^i\,\alpha_s(Q)\right]^{\tilde{d}_\ell^i}\right\}\left[\frac{\alpha_s(Q)}{4\pi}\right]^{\ell-1} \tag{9}$$

When we do not take into account the θ-function, this expression blows up as x goes to zero and, as ℓ increases, the singular behavior becomes more severe and affects the photon structure function from zero to larger and larger values of x, despite the presence of higher powers of $\alpha_s(Q)$.[9] However, the θ-function acts as a cutoff procedure and Δ_i^ℓ vanishes in the interval of x running from zero to $[\lambda_\ell^i\,\alpha_s(Q)]^{\tilde{d}_\ell^i}$. Moreover, considering larger values of ℓ (or n_ℓ^i), $\tilde{d}_\ell^i$ decrease significantly (see fig. 1) and the θ-function cuts off contributions for larger and larger values of x. Assuming that the hadronic contributions are all of the same order of magnitude, it is reasonable to consider the parameters λ_ℓ^i as independent of i $(= +, -, NS)$ and ℓ $(= 0, 1, 2, \ldots)$. Then, as n_ℓ^i become very large, $\tilde{d}_\ell^i \rightarrow 0$ and $[\lambda \alpha_s(Q)]^{\tilde{d}_\ell^i}$ tends to one, more or less rapidly depending on λ and $\alpha_s(Q)$.

In general however, Δ_i^ℓ is not given by eq. (9) but rather by a "smooth θ-function". We take advantage of this fact in the parametrization we need to assume to invert the complete $\Delta_{n,i}^\ell$ in (7) since it is otherwise impossible to perform analytically; we choose the form

$$\Delta_i^\ell(x,\lambda) = \tilde{b}_\ell^i \left[\frac{\alpha_s(Q)}{4\pi}\right]^{\ell-1} \left\{1 - (1-x)^{-d_i' \ln[\lambda_\ell^i \alpha_s(Q)]} \sum_{k=0}^{5} \delta_k \; x^k \right\} \tag{10}$$

where d_i' is defined by

$$\lim_{n\rightarrow\infty} d_i^n \;\sim\; d_i' \;\ln n \tag{11}$$

and for 4 flavors

$$d_i' = \begin{cases} 16/25 & i = -, NS \\ 36/25 & i = + \end{cases} \quad . \tag{12}$$

In (10), the behavior at $x \rightarrow 1$, which corresponds to the large n limit, is explicitly factored out. The coefficients δ_k are fitted using first six moments given by eq. (7). The constants $\tilde{b}_\ell^i$ are calculable in perturbation theory, but require higher-loop calculations of b_n^ℓ. We assume, they are of the order $O(1)$.

To determine the numerical importance of $\Delta_i^\ell(x,\lambda)$, we use the regularized second order QCD results, $F_2^{\gamma,r}$, as a basis for comparison and compute

$$\frac{F_2^\gamma(x,Q^2)}{\alpha} = \frac{F_2^{\gamma,r}(x,Q^2)}{\alpha} + \frac{\Delta_i^\ell(x,\lambda)}{4\pi} \tag{13}$$

where $F_2^{\gamma,r}$ is given by the inverse Mellin transform of the first two terms of eq. (3). In the results we will present below, we use $\alpha_s(Q) = 0.2$; then, $\lambda = 5$ corresponds to $\Delta_i^\ell(x,\lambda) = 0$ while for $\lambda = 0$ we recover the singularity of the point-like part.

We begin by examining the case $\ell = 1$ since we are most familiar with the singularity appearing in $\Delta^1_{n,-}$ at $n = 2$. The results are shown in fig. 2 where $F_2^\gamma(x, Q^2)/\alpha$ is plotted for $\lambda = 5$, 1 and 0. Then, for $\ell = 2$, three new singularities appear in the c_n coefficient of eq. (1), two of which occur very near each other at $n \simeq 5.3$ and correspond to the (NS) and $(-)$ components of c_n and a third one which arises around $n = 2.4$ in c_n^+ (see table I). Since the first two singularities are very similar in all respect, we show only the results related to Δ^2_-, for $\lambda = 5$, 0.1, 0.01 and 0 (fig. 3). The contributions of Δ^2_- are very small compared to those of Δ^1_- for corresponding values of λ. Even for $\lambda = .1$ the result is hardly distinguishable from $F_2^{\gamma,r}(x, Q^2)$. We have to consider very small values of λ ($\lambda \lesssim .01$) in order to get appreciable deviations. For completeness, we also present the results associated with Δ^2_+ for $\lambda = 5$, 0.1 and 0 (fig. 4). Finally, we give an example for $\ell = 3$, for the singularity at $n \simeq 4.4$ in the $(+)$ component (see table I). Figure 5 shows the results for $\lambda = 5$, 0.1, 0.01 and 0.

Two facts are worth noticing: First, when ℓ increases, for fixed i and λ, the contributions from $\Delta_i^\ell(x, \lambda)$ decreases significantly. This is of course due to the additional powers of $\alpha_s(Q)$ which suppress higher order contributions but, more importantly, to the fact that $\tilde{d}_\ell^i$ decreases in the argument of the smooth θ-function [see eq. (9) and the discussion below]. Furthermore, the contribution from $\Delta^1_-(x, \lambda)$, i.e. from the regularization of the lowest pole, is clearly the largest for a given value of λ.

HADRONIC CONTRIBUTIONS

In the previous analysis, we did not consider the nonsingular hadronic contributions of the form

$$\left(A^\ell_{n,i}\right)^r [\alpha_s(Q)]^{d_i^n} \left(1 + r_n^i \ \frac{\alpha_s(Q)}{4\pi} + \ldots\right) . \tag{14}$$

In the following treatment we shall assume that the regular hadronic piece, $(A^\ell_{n,i})^r$, has no additional singular behavior. Then we see that expression (14) becomes important compared with the second order QCD prediction of (3) and (4) when $d_i^n < 1$. It even dominates the point-like part for n very small $n < 1.6$ where $d_-^n < -1$. But these values of n correspond to very small x's, or the Regge region, where perturbative QCD does not apply. We concern ourselves to the region of x controlled by moments larger than $n = 2$.

In fig. 6, d_i^n $(i = +, -, NS)$ are plotted as functions of n, where we have analytically continued for noninteger n's. We see that for $n \gg 2$ the contribution of type (14) can be discarded. However, for $n \lesssim 4$, d_i^n are smaller than one and the contributions might not be negligible. We will now examine how such contributions may affect the theoretical accuracy of (3) and (4).

We proceed as follows: The magnitude of (14) decreases fast as n increases assuming $(A_{n,i}^{\ell})^r$ are smooth functions of n. To investigate the region of x where such terms may have a significant effect, we replace them by a smooth θ-function,

$$\left(A_{n,i}^{\ell}\right)^r [\alpha_s(Q)]^{d_i^n} \;\rightarrow\; A_n\, \tilde{\theta}\,(n_0 - n) \tag{15}$$

and invert to get the x-behavior; here, n_0 is indicative of the value of n at which the damping $\alpha_s^{d_i^n}$ becomes significant ($n_0 \sim 4$).

We choose an appropriate function $\tilde{\theta}$ which is analytically invertible:

$$\tilde{\theta}_k(n_0 - n) = \frac{1 - \tanh k(n - n_0)}{2} \quad . \tag{16}$$

In the limit $k \rightarrow \infty$, $\tilde{\theta}_k$ reduces to $\theta(n_0 - n)$. Its inverse Mellin transform is

$$\tilde{\theta}_k(x) = \frac{1}{x^{n_0 - 1}} \sum_{m \geq 1} (-)^{m+1}\, \delta\left(\ln\frac{1}{x} - 2mk\right) \;. \tag{17}$$

Using the convolution property

$$\int_0^1 dx\; x^{n-2} \int_x^1 \frac{dy}{y}\; \alpha(y)\; \beta\left(\frac{x}{y}\right) = \alpha_n\; \beta_n \tag{18}$$

one finds that the inverse Mellin tranform of the product $A_n\tilde{\theta}(n_0 - n)$ [see eq.

(15)], $f(x)$, is

$$
\begin{aligned}
f(x) &= \int\limits_0^1 \frac{dy}{y}\, A\left(\frac{x}{y}\right) \frac{1}{y^{n_0-1}} \sum_{m\geq 1} (-)^{m+1}\, \delta\left(\ln\frac{1}{y} - 2mk\right) \\
&= \sum_{m=1}^{\left[\frac{1}{2k}\ln\frac{1}{x}\right]} (-)^{m+1}\, e^{2mk(n_0-1)}\, A\left(xe^{-2mk}\right)
\end{aligned}
\qquad (19)
$$

where $[t]$ stands for the integer part of t.

Clearly, no contribution arises from (19) for $x > e^{-2k}$. In our attempt to use a smooth θ-function, $\tilde{\theta}_k$, we introduce the parameter k which indicates how fast the contributions of higher moments are suppressed near $n = n_0$. It is determined by, among other things, $\alpha_s(Q)$ and the rate of growth of d_i^n. Clearly at very high energy, only contributions associated to $d_i^n \simeq 0$ will be important and k will be relatively large. But even for $k = 1$, the smooth θ-function leads to nonzero effects only in a region where $x \lesssim .14$. A closer look to expression (19) reveals that $f(x)$ starts to oscillate rapidly as x approaches zero, due to the alternating signs which appear in the sum, and although the amplitude of these oscillations grows exponentially with k, they contained to smaller and smaller x's where in any case the perturbative treatment is no longer valid.

We end with a brief discussion of contributions that can be attributed to vector dominant processes. From jet analysis, there is evidence that such contributions are negligible for $Q^2 > 10$ GeV^2.[2] We argue that at least part of the VDM contribution to the structure function is included through the hadronic parameter λ and, on theoretical grounds alone, no significant VDM contribution for $x \gtrsim 0.15$ and large Q can be found. This is substantiated by an experimental analysis, which consists of fitting simultaneously $\Lambda_{\overline{MS}}$, λ as well as a "VDM coupling constant," that finds the VDM part to be consistent with zero.[2]

In conclusion, we find that the theoretical uncertainties associated with higher order singularities and hadronic contributions can be estimated and are shown to be important only in a restricted region of x, $x \lesssim .15$. Therefore, the "regularized" second order QCD analysis provides a reliable prediction that can be used for comparison with experiment and as an accurate tool to determine the scale parameter Λ.

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REFERENCES

1. e.g. P. M. Zerwas, XV Symposium on Multiparticle Dynamics, Lund, June 1984; W. A. Bardeen, VIth International Workshop on Photon-Photon Collisions, Lake Tahoe, September 1984.

2. W. Wagner, VIth International Workshop on Photon-Photon Collisions, Lake Tahoe, September 1984.

3. E. Witten, Nucl. Phys. B120 (1977) 189.

4. W. A. Bardeen and A. J. Buras, Phys. Rev. D22 (1979) 166; Erratum, D21 (1980) 2041.

5. D. W. Duke and J. F. Owens, Phys. Rev. D22 (1980) 2280.

6. W. A. Bardeen, Proceedings of the 1981 International Symposium on Lepton and Photon Interactions, ed. by W. Pfiel, Bonn (1981).

7. T. Uematsu and T. F. Walsh, Phys. Lett. 101B (1981) 263; Nucl. Phys. B199 (1982) 93; C. Peterson, P. M. Zerwas and T. F. Walsh, Nucl. Phys. B229 (1983) 301.

8. I. Antoniadis and G. Grunberg, Nucl. Phys. B213 (1983) 445.

9. G. Rossi, Phys. Lett. 130B (1983) 105; Phys. Rev. D29 (1984) 852.

10. M. Glück and E. Reya, Phys. Rev. D28 (1983) 2749; M. Glück, K. Grassie and E. Reya, Phys. Rev. D30 (1984) 1447; M. Drees, M. Glück, K. Grassie and E. Reya, Dortmund University preprint DO-TH84/12.

Table I

Numerical values for n_ℓ^i and $\tilde{d}_\ell^i$ for $i = +, -, NS$ and $\ell = 0, 1, 2, 3$

ℓ	n_ℓ^+	n_ℓ^-	n_ℓ^{NS}	$\tilde{d}_\ell^+$	$\tilde{d}_\ell^-$	$\tilde{d}_\ell^{NS}$
0		1.596	0.310			
1		2.000	1.000		1.4143	
2	2.386	5.326	5.251	0.6801	0.1176	0.1144
3	4.403	26.579	26.572	0.3467	0.0237	0.0237

FIGURE CAPTIONS

1. $\tilde{d}^i_\ell$ (analytically continued by $\frac{d}{dn} d^n_i$) in the region $2 \leq n_\ell \leq 20$, for $i = +, -, NS$ (dotted, solid and dot-dashed line respectively).

2. $F_2^\gamma(x, Q^2)/\alpha$ corresponding to $\Delta^1_-(x, \lambda)$ for $\lambda = 5$ (solid), 1 (dot-dashes) and 0 (dots) [see eq. (13)].

3. $F_2^\gamma(x, Q^2)/\alpha$ corresponding to $\Delta^2_-(x, \lambda)$ for $\lambda = 5$ (solid), 0.1 (dot-dashes), 0.01 (dashes) and 0 (dots).

4. $F_2^\gamma(x, Q^2)/\alpha$ corresponding to $\Delta^2_+(x, \lambda)$ for $\lambda = 5$ (solid), 0.1 (dashes) and 0 (dots).

5. $F_2^\gamma(x, Q^2)/\alpha$ corresponding to $\Delta^3_+(x, \lambda)$ for $\lambda = 5$ (solid), 0.1 (dot-dashes), 0.01 (dashes) and 0 (dots).

6. d^n_i for $i = +$ (dots), $-$ (solid) and NS (dot-dashes).

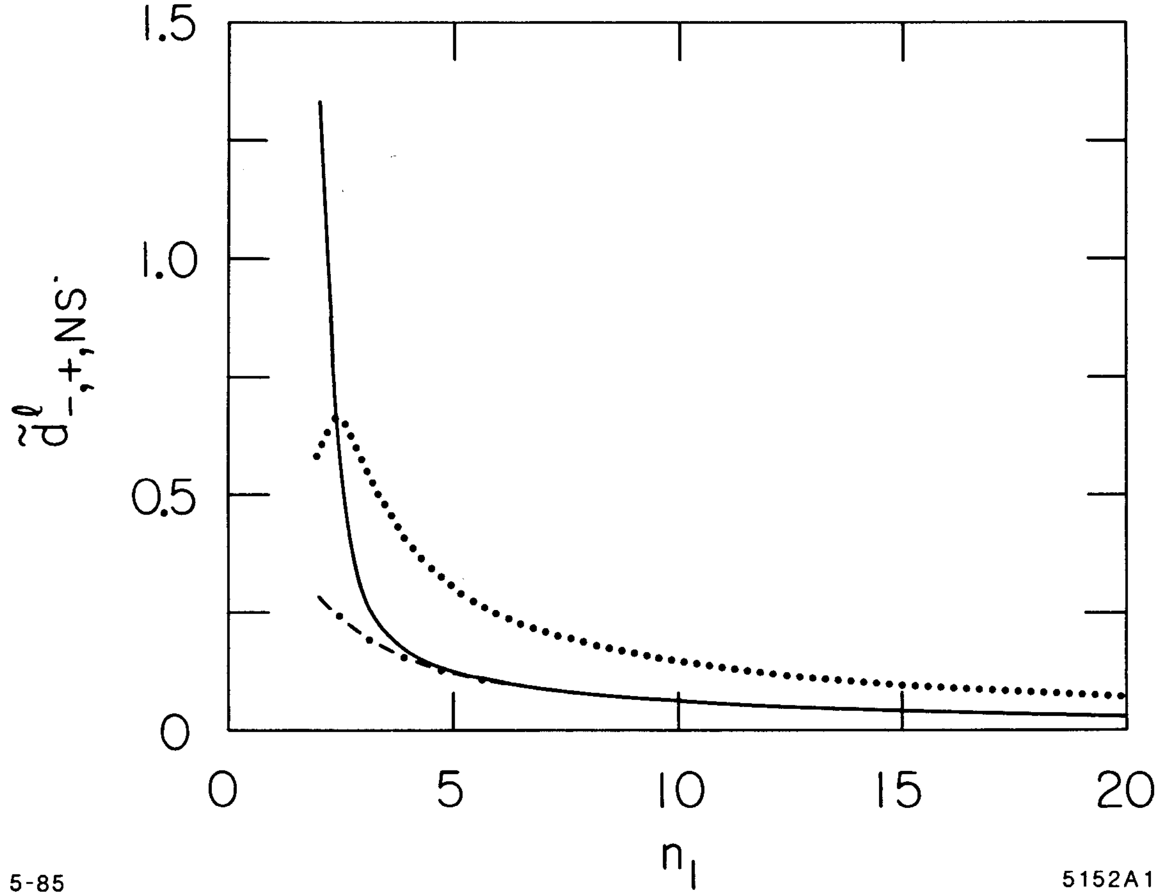

Fig. 1

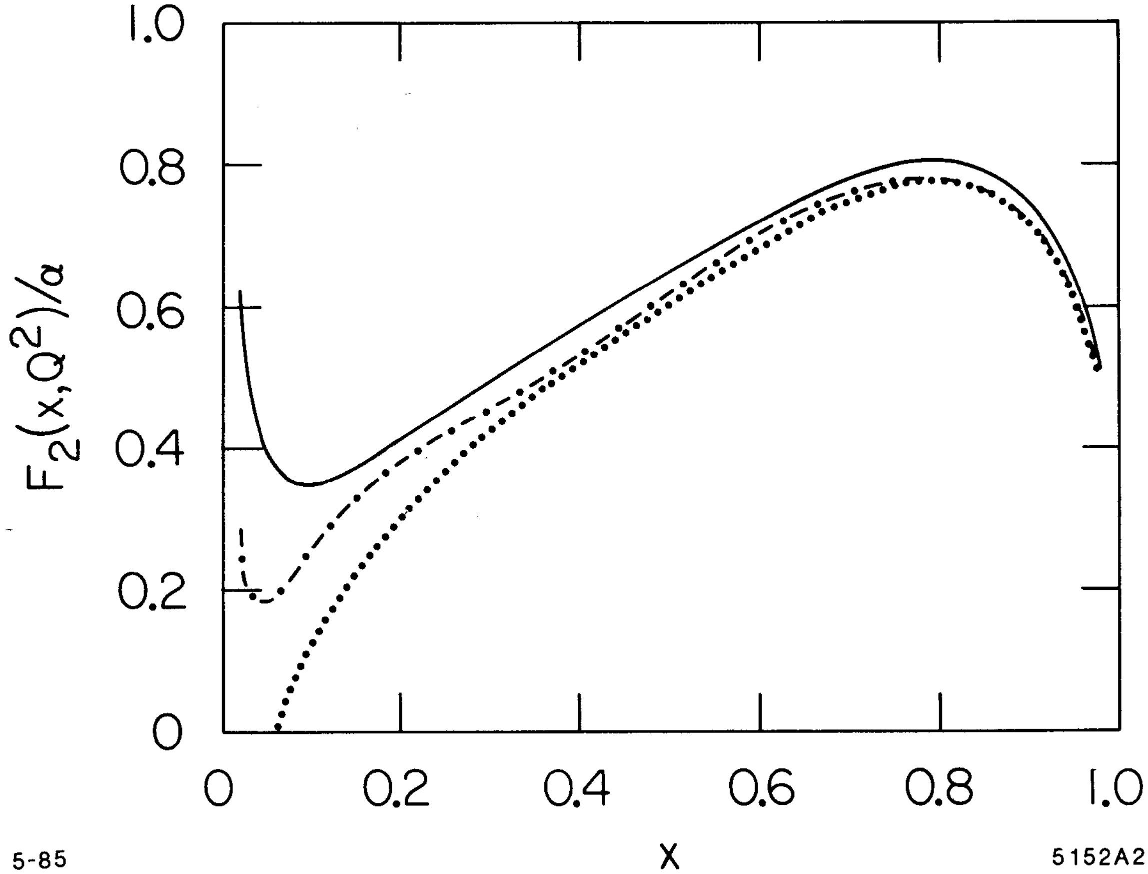

Fig. 2

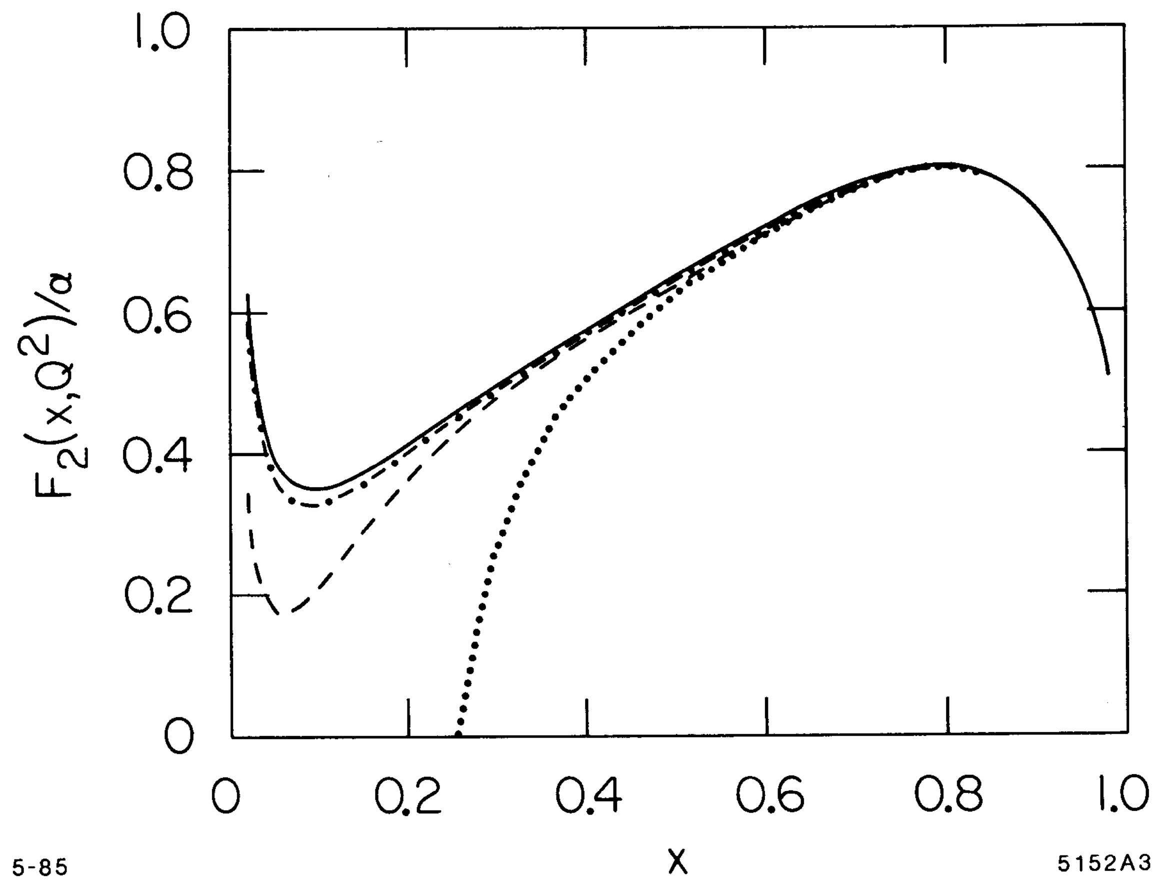

Fig. 3

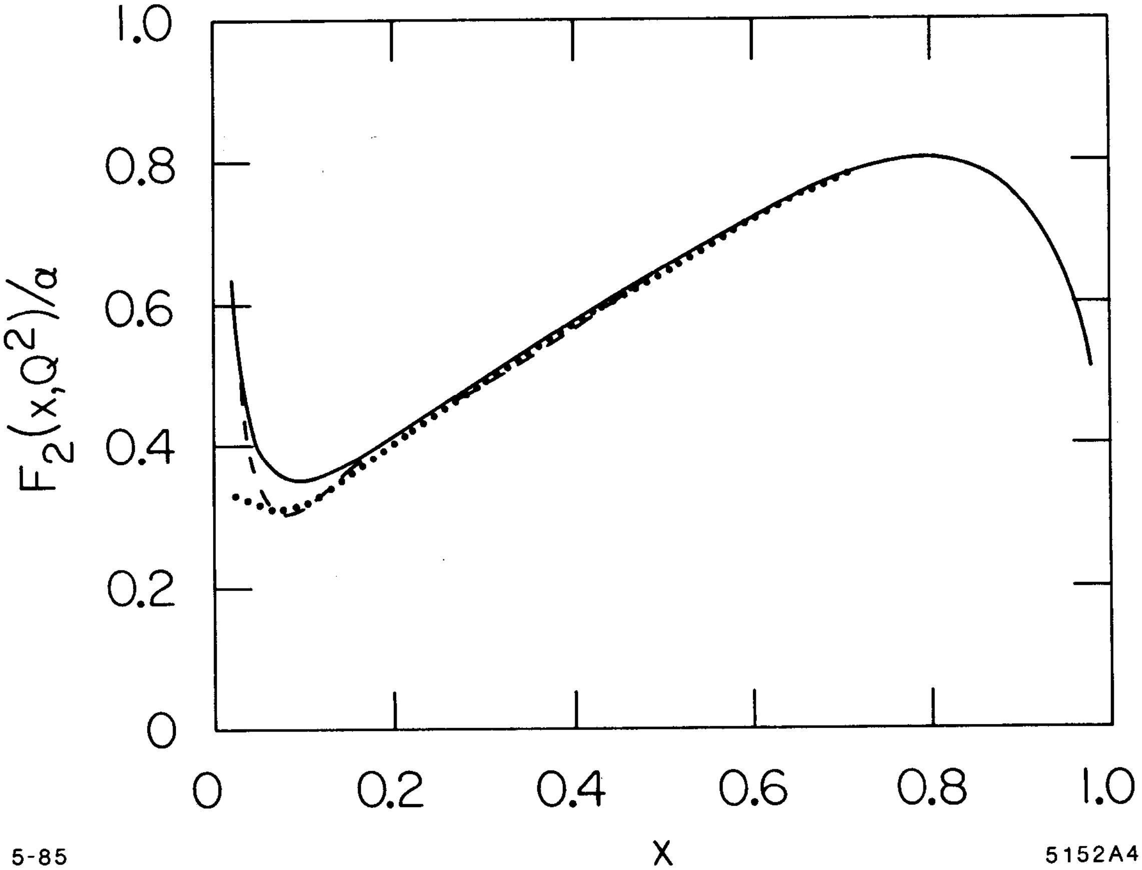

Fig. 4

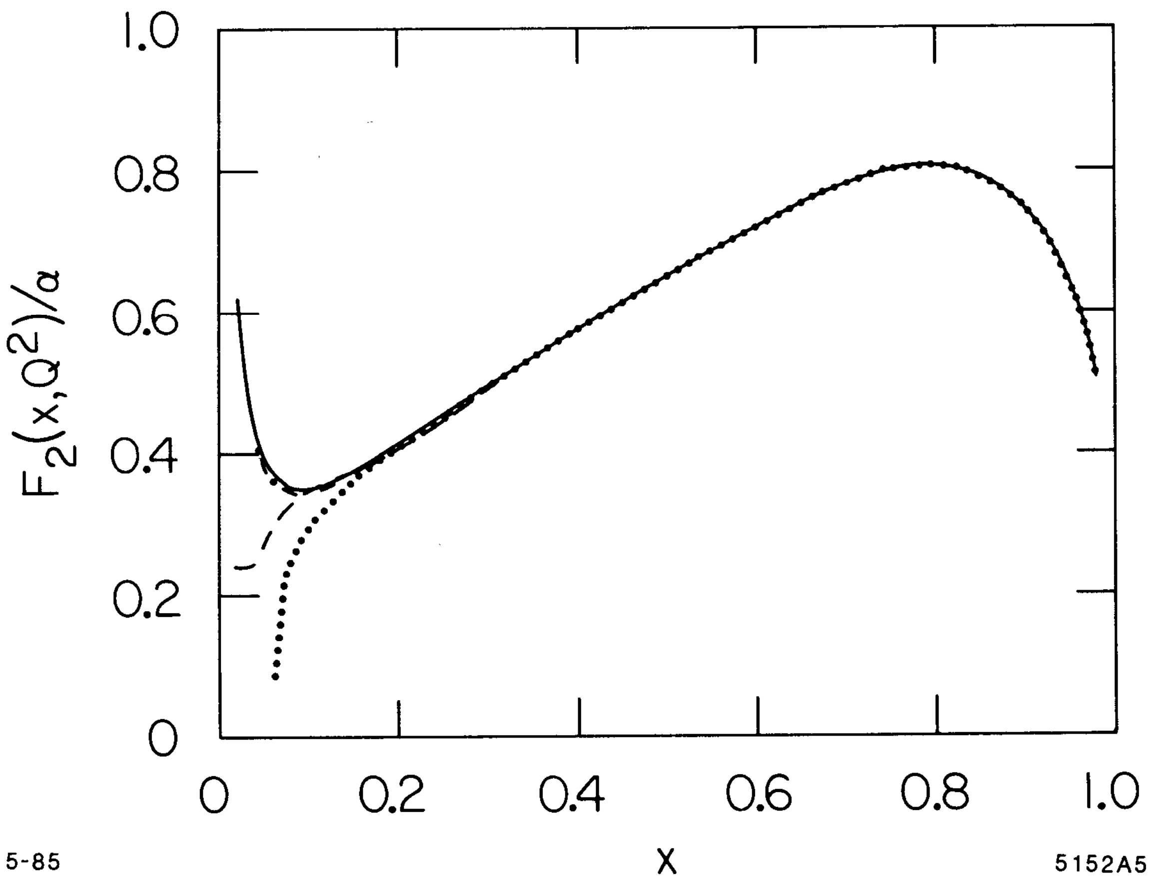

Fig. 5

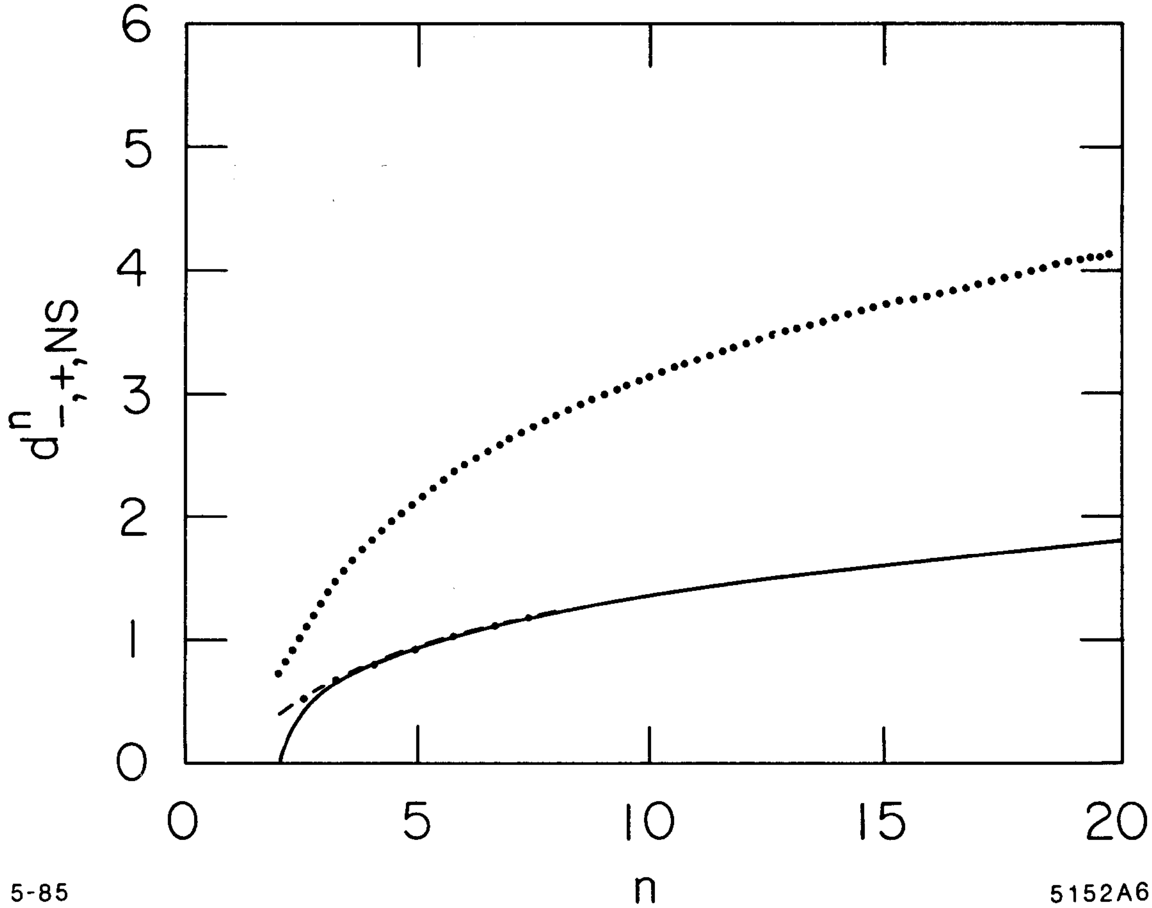

Fig. 6