SLAC – PUB – 3688
May 1985
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SUPER DISRUPTION
(AND ITS USE IN LINEAR COLLIDERS)*

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Submitted for Publication

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.
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David Leith has suggested that the enhancement of luminosity due to disruption could be increased if the bunch consisted of two discrete short pulses. The first pulse would act as a lens to focus the second. The approach may be particularly helpful for a super high-energy collider for which the interacting bunch must be very short to avoid excessive quantum beamstrahlung.

In this analysis I will assume bunches with uniform charge density up to a radius a. I will, however, follow Tom Himel's convention of referring to the bunch size as σ where $\sigma = a/2$. With this convention the luminosity relation remains as with a Gaussian bunch. Clearly some refinements will be needed to get correct values for the Gaussian case. This analysis may, however, be useful to give the orders of magnitudes.

Define

$$
\begin{aligned}
N_1 = N &= \text{particles in bunch \#2} \\
N_2 = kN &= \text{particles in bunch \#1} \\
\sigma_1 = \sigma &= \text{half radius of bunch \#2} \\
\sigma_2 = b\sigma &= \text{half radius of bunch \#1} \\
s &= \underline{\text{half}} \text{ bunch spacing} \\
f &= \text{focal length of bunch \#1} \\
D &= \text{disruption parameter defined by:}
\end{aligned}
$$

$$D = \frac{s}{f} = \frac{r\,kN\,s}{2\gamma\,b^2\,\sigma^2}$$

where r = classical electron radius = $2.8\,10^{-15}$ m.

Note that the 2 arises from the uniform current case. It is not present in the Gaussian focus.

Consider the sequence of events as the two bunches pass through one another. See Fig. 1. I will assume for the moment that $\sigma_1 \gg \sigma_2$, i.e., $b \gg 1$. In this case bunch #2 "sees" a perfect spherical lens. Only chromatic abberation will be present. I will also assume that the bunch length $\ll s$.

After the two #1 bunches have passed through each other, they are converging to a point at a distance f from the center (Fig. 1b). After a further distance s they meet the #2 bunches. At that time the #1 bunches have decreased in size by a factor:

$$\sigma_1' = \sigma_1 \left(\frac{f-s}{f} \right)$$

and their focal lengths as seen by the #2 bunches have decreased:

$$f' = f \left(\frac{f-s}{f} \right)^2$$

What we require is that this new focal length is s so that the two second bunches collide at their respective foci, i.e.,

$$s = f \left(\frac{f-s}{f} \right)^2$$

from which we get

$$f = \frac{3+\sqrt{5}}{2} \, s \approx 2.62 \, s$$

and

$$D = .38 = \frac{r \, N_1 \, s}{2\gamma \, \sigma_1^2} = \frac{r \, kN \, s}{2\gamma \, b^2 \, \sigma} \tag{1}$$

We can note now that to avoid bunch #2 being larger than the lens mode from bunch #1, we require

$$b = \frac{\sigma_1}{\sigma_2} \geq \left(\frac{f}{f - s}\right) \approx \sqrt{2.62} \approx 1.62$$

For more Gaussian distributions we will need $b > 1.62$, but I will take $b = 1.62$ for the moment. From (1) we now get

$$s \approx 7.1\,10^{14} \left(\frac{\epsilon\beta}{kN}\right) \tag{2}$$

The half angle θ as the second bunches approach one another is

$$\theta = \frac{\sigma_2}{s}$$

and thus the new β^*, which I will call β' is given by

$$\beta' = \frac{\epsilon}{\gamma\theta^2} = \frac{s^2}{\beta}$$

and the enhancement factor for the second bunch collision is

$$\begin{aligned} H_2 = \frac{\beta}{\beta'} &= \left(\frac{r}{.76\,b^2}\right)^2 \frac{k^2\,N^2}{\epsilon^2} \\ &\approx 2\,10^{-30}\,\frac{k^2 N^2}{\epsilon^2} \end{aligned} \tag{3}$$

The overall enhancement must include the other collisions: 1 on 1, two 2 on 1 and 2 on 2:

$$H_t = \frac{(k/b)^2 + 2k + H_2}{(1+k)^2}$$

but provided $H_2 \gg 1$

$$
\begin{aligned}
H_t &\approx \frac{H_2}{(1+k)^2} = \left(\frac{r}{.76\,b^2}\right)^2 \frac{N^2}{\epsilon^2} \cdot \left(\frac{k}{1+k}\right)^2 \\
&\approx 2\,10^{-30}\,\frac{N^2}{\epsilon^2}\left(\frac{k}{1+k}\right)^2 \qquad (mks)
\end{aligned}
\tag{4}
$$

For SLC $\epsilon \approx 3\,10^{-5}$, $(1+k)N = 5\,10^{10}$. Take $k = 1$ then

$$H_2 \approx .35$$

which is <u>not</u> useful unfortunately. However, for future colliders we usually assume that far smaller emittances will be available. In these future colliders it is in general true that N will be limited by beamstrahlung and that in turn is dependent on how small the final spot is. We need, therefore, to solve for the correct N for a given beamstrahlung parameter δ.

From Himel and Siegrist SLAC-PUB-3572 we get:

$$
\begin{aligned}
\delta &= \left(\frac{r^5\,16(mc)^4}{3^{7/2}\,\hbar^4}\right)^{1/3}\left(\frac{\sigma_z N^2}{\epsilon\,\beta}\right)^{1/3} \\
&\approx 1.4\,10^{-8}\left(\frac{\sigma_z N^2}{\epsilon\,\beta}\right)^{1/3} \qquad (mks)
\end{aligned}
$$

from which

$$N \approx \delta^{3/2}\sqrt{\frac{\epsilon\,\beta'}{\sigma_z}} \times 5.9\,10^{11}$$

substitute for β' from Eq. (3)

$$
\begin{aligned}
N^2 &\approx (5.9\,10^{11})^2\,\delta^3\,\frac{\epsilon}{\sigma_2} \times \frac{\beta}{2\,10^{-30}}\,\frac{\epsilon^2}{k^2N^2} \\
N &\approx 2\,10^{13}\,\frac{\epsilon^{3/4}\beta^{1/4}\delta^{3/4}}{k^{1/2}\sigma_z^{1/4}}
\end{aligned}
\tag{5}
$$

and putting this back into Eq. (4)

$$H_2 = .8\,10^{-3}\,\delta^{3/2}\left(\frac{\beta}{\epsilon\,\sigma_2}\right)^{1/2}\,k^{3/2} \tag{6}$$

Now we can see how much we have reduced the total beam power for a given luminosity. For round beams

$$\begin{aligned} L &\approx \frac{1}{4\pi}\frac{N^2 f}{\sigma^2}H_2 \\ P &= (mc^2)\,N\,f\,\gamma(1+k) \qquad \text{per beam} \\ P &\approx 1.02\,10^{-12}\,L\frac{\epsilon\,\beta}{N\,H_2}(1+k) \qquad (mks) \end{aligned} \tag{7}$$

Which with Leith's super enhancement gives:

$$P = 6.4\,10^{-19}\frac{\epsilon^{3/4}\beta^{1/2}\sigma_z^{3/4}}{\delta^{9/4}} \times L \times \left(\frac{1+k}{k}\right) \tag{8}$$

where all units are mks except L is in $\text{cm}^{-2}\text{sec}^{-1}$. For examples I take $L = 10^{34}\ \text{cm}^{-2}\text{sec}^{-1}$, $\beta = 1$ cm, $k = 1$, $\sigma_z = .1\,\mu$, $\delta = .3$

Invariant ϵ m	10^{-6}	10^{-7}	10^{-8}	
N	$4.6\,10^9$	$.81\,10^9$	$1.5\,10^8$	$\epsilon^{3/4}$
H_2	41	130	410	$\epsilon^{-(1/2)}$
β'	240 μ	76 μ	24 μ	$\epsilon^{1/2}$
σ' (for 5 TeV)	49 A°	27 A°	15 A°	$\epsilon^{1/4}$
P (one beam)	10.8 MW	1.9 MW	.3 MW	$\epsilon^{3/4}$
s	1.5mm	.8 mm	.5 mm	$\epsilon^{1/4}$

cf without Leith double bunch

$$p \approx 1.7\,10^{-24}\,L\,(\epsilon\,\beta\,\sigma_z)^{1/2}\;\delta^{3/2}$$

N	$1.7\,10^{10}$	$.5\,10^{10}$	$1.7\,10^{9}$	$\epsilon^{1/2}$
P (one beam)	60 MW	20 MW	6 MW	$\epsilon^{-(1/2)}$
Power reduction	6	10	20	$\epsilon^{1/4}$

One notes that even greater power reduction results from accepting a larger δ, e.g., for $\delta = .6$ the beam power for $\epsilon = 10^{-8}$ drops to only 63 kW! This is significant since it allows us to think (at least allows Tom and I to think) of 50 TeV $*$ 50 TeV and $L = 10^{36}$!

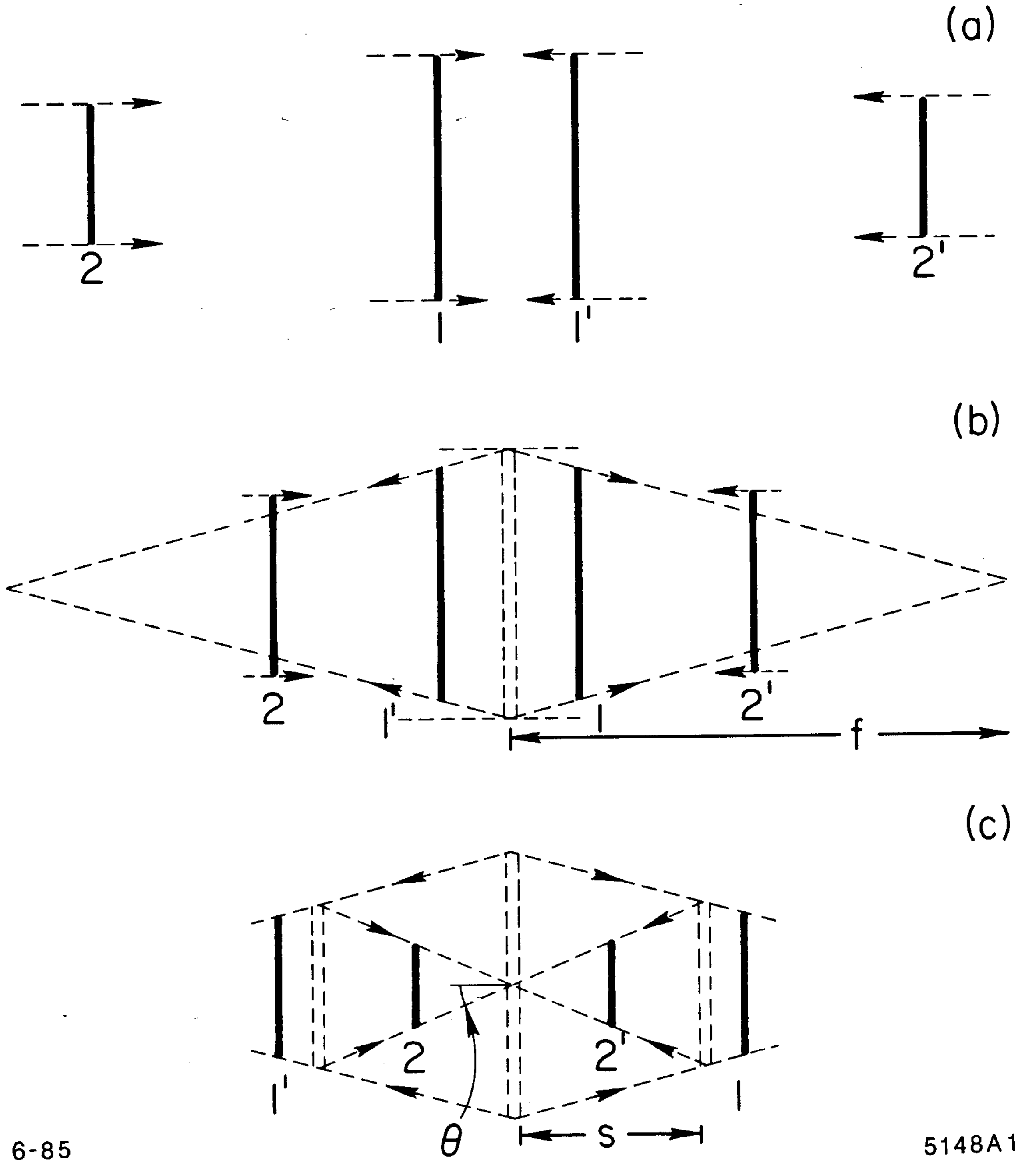

Fig. 1