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NONCOMPACT SYMMETRIES AND VANISHING OF THE COSMOLOGICAL CONSTANT*

I. ANTONIADIS†

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

and

C. KOUNNAS*

Physics Department, University of California
Berkeley, California 94720

and

D. V. NANOPOULOS

CERN, CH-1211 Geneva 23, Switzerland
and
University of California Santa Cruz, California 95064

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† On leave of absence from Centre de Physique Théorique, Ecole Polytechnique, 91128 Palaiseau, France.

∗ On leave of absence from Laboratoire de Physique Théorique de l'E.N.S., 24 Rue Lhomond, 75231 Paris, France.

ABSTRACT

The vanishing of the cosmological constant at the quantum level is achieved by considering it as the physically irrelevant scale of a spontaneously broken and anomaly free, global, noncompact U(1) symmetry. This symmetry is naturally contained in the SU(N,1) no-scale supergravity model, which may be interpreted as the effective four dimensional limit of superstring theories. A firm prediction of such a mechanism is the unavoidable existence of a physical massless Goldstone boson.

Astrophysical observations on the cosmological constant value indicate that the vacuum energy density in our universe is extremely small [1]. Actually it is about 120 orders of magnitude smaller than the gravitational scale ($M = (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV) raised to the appropriate power: $\Lambda \simeq V_0 \leq 10^{-120}\ \mathrm{M}^4 \simeq (3 \times 10^{-12}\ \mathrm{GeV})^4$.

Any observed massive particle has a mass hierarchically larger than $\Lambda^{1/4} \lesssim 3 \times 10^{-12}$ GeV and therefore a hard-to-understand scale problem appears in any theory with massive states. Indeed, any individual massive degree of freedom induces an infinite contribution to the vacuum energy density as well as a net finite one at least of order $(\mathrm{mass})^4$. In the absence of a relevant symmetry reason and even if the infinities are disregarded, it is hard to imagine how such an accurate cancellation among unrelated individual contributions could occur.

In the presence of supersymmetry the cosmological constant problem is in much better shape due to some miraculous cancellations between the bosonic and fermionic contributions. Although the cosmological constant vanishes automatically in the exact limit of global supersymmetry [2], this is not the case in any realistic model where the supersymmetry is spontaneously broken. When supersymmetry (either local or global) is broken, the vacuum energy density is in general different from zero and gets unacceptably large contributions proportional to the supersymmetry breaking scale. However, this is not a general feature in supergravity; there is an interesting class of $N = 1$ supergravity models [3-6] where the vanishing of the cosmological constant occurs naturally at the classical level of the theory, whether supersymmetry is spontaneously broken or not, and this is due to the flatness properties of the scalar potential. The main feature of these supergravity models is the nonminimality of the kinetic terms of the scalars which form a noncompact symmetric Kähler manifold; namely SU(1,1)/U(1) in the less symmetric case [3,4] and SU(N,1)/SU(N) $\otimes$ U(1) in the maximally symmetric case [5] (N being the number of complex scalars in chiral supermultiplets).

Recently, $N = 2$ spontaneously broken supergravities with flat potentials have been proposed in the literature [7]. In all these models, as in the former [SU(1,1) and SU(N,1)] $N = 1$ supergravity, the cosmological constant is zero and the scalar manifold always forms a noncompact symmetric structure. We found this as an evidence for an interplay between the vanishing of the cosmological constant and the underlying noncompact symmetries which is the unifying thread of these theories.

Realistic and physically relevant models based on SU(1,1) and SU(N,1) no scale supergravity have been constructed [8,4,5]: they are known in the literature as no-scale models. They were mainly proposed as a solution to the scale hierarchy problem, by means of the dynamical determination of the hierarchical ratio $M_W/M \simeq M_{SUSY}/M \simeq 10^{-16}$. It is worth noticing that the maximally symmetric no-scale models are found as a four dimensional limit [9] of the ten-dimensional $E_8 \times E_8$ superstring theory [10] which, in turn, is claimed to be a finite theory successfully unifying all known interactions, including gravity.

When supersymmetry is spontaneously broken, the underlying SU(1,1) symmetry of the no-scale models is not respected by the supersymmetry breaking terms, like for instance the gravitino mass term. In the absence of the SU(1,1) symmetry (which guaranteed the vanishing of the cosmological constant), one might expect a nonzero vacuum energy at the quantum level of the theory. However, there is a remaining $\mathrm{U}(1)_{\mathrm{NC}}$ noncompact [4] global symmetry which is spontaneously broken simultaneously with supersymmetry.

In the present work we will show that the cosmological constant remains zero at the quantum level of the theory due to the presence of this particular anomaly-free $\mathrm{U(1)_{NC}}$ symmetry. A firm prediction of such a mechanism is the unavoidable existence of a physical massless boson: "the plation," the Goldstone mode of the above symmetry.

The main points of our proof are the following:The SU(1,1) transformations are linearized in a simple way by introducing an unphysical chiral superfield (ϕ_0, χ_0); the scalar component ϕ_0 acts as an "unphysical dilaton" field and restores the Weyl invariance of the theory as well as a U(1)–local and S–supersymmetry [11]. We then show that the remaining noncompact $\mathrm{U(1)_{NC}}$ global symmetry implies the *spontaneous* breaking of the local Weyl invariance which, in turn, guarantees [12] the stability of Minkowski space-time at the quantum level of the theory. Of course, the whole mechanism makes sense in the context of a quantum theory of gravity whose existence is our basic assumption.

At this point, let us stress that this mechanism is more general and in principle can be applied to any theory which exhibits a suitable noncompact and anomaly-free global symmetry. The advantage of supersymmetry in the no-scale models is that it implies naturally the relevant $\mathrm{U(1)_{NC}}$ and gives $\Lambda \equiv 0$ at the classical level of the theory; also, the presence of supersymmetry in the effective theory stabilizes the scalar masses to hierarchically smaller values than the gravitational scale.

For the sake of simplicity we will present the proof of our mechanism in the simplest SU(1,1) model, when only one chiral multiplet is coupled to supergravity. The extension to more general cases is straightforward. In fact, we will show that the presence of $\mathrm{U(1)_{NC}} \subset \mathrm{SU(1,1)}$, leads to the vanishing of the cosmological constant as a consequence of the following identity:

$$V\left(\langle\phi_i\rangle\right) \;=\; \left\{ C_D\,\frac{\partial V}{\partial \phi_D} + C_P\,\frac{\partial^2 V}{\partial \phi_P \partial \phi_P} \right\}_{\substack{\text{at the}\\ \text{minimum}}} \;\equiv\; 0 \qquad (1)$$

where ϕ_D and ϕ_P are the scalar and pseudoscalar fields which form the SU(1,1)/U(1) manifold and C_D, C_P are constants related to the $\langle\phi_D\rangle$ vacuum expectation value. The pseudoscalar field ϕ_P is a massless physical state corresponding to the Goldstone $\mathrm{U(1)_{NC}}$ mode and we will refer to it as the "plation" field. The scalar ϕ_D is a physical "dilaton" field and couples naturally to the trace of the energy momentum tensor. The magnitude of the supersymmetry breaking scale is defined by the vacuum expectation value of ϕ_D. At the classical level, the potential is flat in both directions (ϕ_D and ϕ_P) and identity (1) is automatically satisfied ($V \equiv 0$). When supersymmetry is spontaneously broken, there is no symmetry to protect the flatness of the potential in the ϕ_D direction and therefore $\langle\phi_D\rangle$ is dynamically determined [8,4]. The relevant information which should be extracted from eq. (1) is that at the minimum of the potential ($\partial V/\partial\phi_D = 0$), the vacuum energy is zero because of the presence of the massless plation.

The SU(1,1) no-scale supergravity is defined [3,4] by the Kähler potential

$$G(Z, Z^\dagger) \;=\; -3\,\ell n\,(Z + Z^\dagger) + \ell n\,|F|^2 \tag{2}$$

where $F = c$ is a constant "superpotential." In what follows we will work in units of $M = 2.4 \times 10^{18}$ GeV. The tree level scalar potential in this model vanishes identically for all values of the scalar field Z

$$V \;=\; e^G \left[G_Z G^{-1}_{ZZ^\dagger} G_{Z^\dagger} - 3 \right] \;\equiv\; 0\,, \quad \forall\, Z \tag{3}$$

$G_Z \;\equiv\; (\partial/\partial Z)G$ and $G_{ZZ^\dagger} \;=\; (\partial/\partial Z)\,(\partial/\partial Z^\dagger)\,G$; when $c \neq 0$, the supersymmetry breaking scale (gravitino mass) does not vanish, but it is undetermined due to the flatness of the potential.

$$m_{3/2} \;=\; e^{G/2}\Big|_{\langle Z\rangle} \neq 0 \qquad \text{undetermined} \quad . \tag{4}$$

The scalar Kähler manifold is defined by the metric $G_{ZZ^\dagger}$ and one can easily see that it forms an Einstein space with a constant curvature $R_M = 2/3$.

$$
\begin{aligned}
R_{ZZ^\dagger} &= \partial_Z \partial_{Z^\dagger} \ell n\, G_{ZZ^\dagger} = \frac{2}{3}\, G_{ZZ^\dagger} \\
R_M &= \frac{R_{ZZ^\dagger}}{G_{ZZ^\dagger}} = \frac{2}{3}
\end{aligned}
\tag{5}
$$

The isometries of the space form a noncompact SU(1,1) group and leave the bosonic part of the Lagrangian invariant

$$
\mathcal{L}^{bosons}_{SU(1,1)} = -\frac{1}{2}\sqrt{-g}\, R + \sqrt{-g}\, g^{\mu\nu}\, G_{ZZ^\dagger}\, \partial_\mu Z\, \partial_\nu Z^\dagger \tag{6}
$$

where R is the space-time curvature scalar.

Furthermore the SU(1,1) Möbius transformations

$$
Z \longrightarrow \frac{\alpha Z + i\beta}{i\gamma Z + \delta} \qquad \text{with} \qquad \begin{array}{l} \alpha\delta + \beta\gamma = 1 \\ \alpha, \beta, \gamma, \delta \text{ real} \end{array} \tag{7}
$$

leave the whole Lagrangian, except the gravitino–goldstino mass terms, invariant after simultaneous chiral rotations on the fermionic fields. By setting the super potential $F = c = 0$, all SU(1,1) breaking terms disappear from the Lagrangian and supersymmetry remains unbroken ($m_{3/2} \equiv 0$). When $c \neq 0$, the supersymmetry is spontaneously broken and the SU(1,1) symmetry breaks down to a $\mathrm{U}(1)_{\mathrm{NC}}$ defined by the imaginary translation [4]

$$
Z \longrightarrow Z + i\beta \tag{8}
$$

obtained from eq. (7) with $\alpha = \delta = 1$ and $\gamma = 0$. The corresponding Goldstone boson (plation) of the spontaneously broken $\mathrm{U}(1)_{\mathrm{NC}}$ symmetry is identified with

$\text{Im}\, Z \sim \phi_P$ and appears in the Lagrangian only through its space time derivatives. In terms of Z and $Z\dagger$ fields, the two physical fields ϕ_P and ϕ_D are given by:

$$\begin{aligned} \text{Plation:} \qquad \phi_P &= \sqrt{\frac{3}{2}}\, i \left(Z - Z^\dagger\right) \\ \text{Dilaton:} \qquad \phi_D &= -\frac{3}{\sqrt{6}}\, \ell n \left(Z + Z^\dagger\right) \end{aligned} \tag{9}$$

In this unitary representation the scalar kinetic terms take the form

$$\mathcal{L}_{KT}^{bosons} = \sqrt{-g}\, g^{\mu\nu} \left\{ \frac{1}{2}\, \partial_\mu \phi_D\, \partial_\nu \phi_D + \frac{1}{2}\, e^{\sqrt{8/3}\,\phi_D}\, \partial_\mu\, \phi_P\, \partial_\nu\, \phi_P \right\} \tag{10}$$

while the supersymmetry breaking parameter, $m_{3/2}$, depends only on the physical dilaton vacuum expectation value

$$m_{3/2} = c \times exp \left\{ \sqrt{\frac{3}{2}}\, \langle \phi_D \rangle \right\} \quad . \tag{11}$$

To better examine the consequences of the U(1)$_{\text{NC}}$ symmetry of the model, it is convenient to use a field representation where the SU(1,1) approximate symmetry of the model is linearly realized. This can be easily done due to the fact that the SU(1,1) model with $R_M = 2/3$, accepts a very simple superconformal representation. Indeed, the $\mathcal{L}_{SU(1,1)}^{bosons}$ of eq. (10) can be given by the following superconformal structure

$$\begin{aligned} \frac{1}{\sqrt{-g}}\, \mathcal{L}_{SU(1,1)}^{bosons} = &- \frac{1}{6} \left(|\phi_0|^2 - |\phi_1|^2 \right) R \\ &- g^{\mu\nu} \left[(D_\mu \phi_0)^\dagger\, D_\nu \phi_0 - (D_\mu \phi_1)^\dagger\, D_\nu \phi_1 \right] \end{aligned} \tag{12}$$

with $D_\mu = \partial_\mu + i A_\mu$, where the analytic field redefinition

$$Z = \frac{1}{2}\, \frac{\sqrt{3} + \phi_1}{\sqrt{3} - \phi_1} \tag{13a}$$

and a suitable Weyl rescaling of the metric

$$g_{\mu\nu} \;\rightarrow\; g_{\mu\nu}\,\frac{3-\phi_1\phi_1^\dagger}{3} \tag{13b}$$

have been performed. We also introduced the unphysical compensator ϕ_0 and the auxiliary vector field A_μ of the graviton supermultiplet. In the presence of ϕ_0 and A_μ the Weyl invariance and the U(1)–local symmetries are restored while the SU(1,1) transformations become linear. More specifically:

α) <u>Weyl invariance:</u>

$$\begin{aligned} \delta_W\, g_{\mu\nu} &= 2\omega(x)\, g_{\mu\nu} \\ \delta_W \begin{pmatrix}\phi_0\\ \phi_1\end{pmatrix} &= -\omega(x)\begin{pmatrix}\phi_0\\ \phi_1\end{pmatrix} \end{aligned} \tag{15a}$$

β) <u>U(1)–local invariance:</u>

$$\begin{aligned} \delta_c \begin{pmatrix}\phi_0\\ \phi_1\end{pmatrix} &= i\theta_0(x)\begin{pmatrix}\phi_0\\ \phi_1\end{pmatrix} \\ \delta A_\mu &= -i\partial_\mu\,\theta_0(x) \end{aligned} \tag{15b}$$

γ) <u>SU(1,1) transformations:</u>

$$\delta_{SU(1,1)}\begin{pmatrix}\phi_0\\ \phi_1\end{pmatrix} = (\theta_1\sigma_1+\theta_2\sigma_2+i\sigma_3\theta_3)\begin{pmatrix}\phi_0\\ \phi_1\end{pmatrix} \tag{16}$$

where $\omega(x)$ and $\theta_0(x)$ are real local infinitesimal parameters and θ_i, $i = 1,2,3$ global ones; σ_i are the Pauli matrices:

$$\sigma_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{17}$$

$\theta_1\sigma_1$ and $\theta_2\sigma_2$ define the two noncompact SU(1,1) transformations and $i\sigma_3\theta_3$ the compact one. Notice that the SU(1,1) Lagrangian and particularly its bosonic part (12) is ***uniquely*** determined by the requirement of local superconformal invariance and the global SU(1,1) symmetry. Terms like $\lambda(|\phi_o|^2 - |\phi_1|^2)^2$ in the potential are forbidden by supersymmetry.

The relevant global $\mathrm{U(1)_{NC}}$ symmetry of the model leaves the supersymmetry breaking terms invariant. These terms, triggered by a nonzero value of the constant c are

$$
\begin{aligned}
\frac{1}{\sqrt{-g}}\,\mathcal{L}_{SUSY\,breaking} \;=\; & \frac{1}{2}\,c\,\Big\{(\phi_0-\phi_1)^3\,\psi_\mu\,\sigma^{\mu\nu}\,(1-\gamma_5)\,\psi_\nu \\
& +\;3\,\overline{\psi}\cdot\gamma\,(1-\gamma_5)\,(\chi_0-\chi_1)\,(\phi_0-\phi_1)^2 \\
& -\;6\left[\,\overline{\chi}_0\,(1-\gamma_5)\,\chi_0\;-\;\overline{\chi}_1\,(1-\gamma_5)\,\chi_1\,\right](\phi_0-\phi_1)\Big\}\;+\;\text{h.c.}
\end{aligned}
\tag{18}
$$

where ψ_μ is the spin-3/2 gravitino and χ_i are the spin-1/2 superpartners of ϕ_i. Simple inspection of eq. 18 indicates that the remaining symmetry is defined by a combination of the SU(1,1) transformations which leave the form $(\phi_0 - \phi_1)$ invariant: $\theta_1 = 0$, $\theta_2 = \theta_3 = \theta_{\mathrm{NC}}$ in eq. (16). Indeed, the whole supergravity Lagrangian remains invariant under the transformation:

$$
\delta_{\mathrm{NC}}\begin{pmatrix}\phi_0\\ \phi_1\end{pmatrix} = \theta_{\mathrm{NC}}\,Q_P\begin{pmatrix}\phi_0\\ \phi_1\end{pmatrix}\,,\quad \delta_{\mathrm{NC}}\begin{pmatrix}\chi_0\\ \chi_1\end{pmatrix} = \theta_{\mathrm{NC}}\,Q_P\begin{pmatrix}\chi_0\\ \chi_1\end{pmatrix}\,,
\tag{19}
$$

with

$$
Q_P \;=\; \sigma_2 + i\sigma_3 \;=\; i\begin{pmatrix}1 & -1\\ 1 & -1\end{pmatrix}\quad .
$$

Under $\mathrm{U(1)_{NC}}$ the fermions χ_0 and χ_1 transform nontrivially although this is not required by eq. 18). Terms like $\overline{\psi}_\mu\;(\phi_0\,\partial^\mu\chi_0 - \phi_1\,\partial^\mu\chi_1)$ in the rest of the Lagrangian imply the transformation (19). As we will see later, it is of crucial importance for the vanishing of the cosmological constant to keep $\mathrm{U(1)_{NC}}$ free of any possible anomaly. It is amazing that the $\mathrm{U(1)_{NC}}$ generator Q_P satisfies the anomaly cancellation condition

$$\mathrm{Tr}\, Q_P^n \;=\; 0 \;, \qquad n = 1, 2, 3, \ldots \quad . \tag{20}$$

The absence of $\mathrm{U(1)_{NC}}$ anomalies can be also understood by observing that χ_0 is unphysical and can be eliminated by fixing the S–supersymmetry gauge. On the other hand, χ_1 is the goldstino field and it is absorbed by the gravitino (superhiggs mechanism) [13]. In the superunitary gauge of the anomalous free Q–supersymmetry χ_0 and χ_1 do not appear in the Lagrangian. In that gauge, the $\mathrm{U(1)_{NC}}$ transforms only the scalar fields and therefore no anomaly is present.

When ϕ_0 and ϕ_1 take nonzero vacuum expectation values, $\mathrm{U(1)_{NC}}$ and the local supersymmetry will be broken spontaneously. $v_0 \;\equiv\; \langle\phi_0\rangle$ and $v_1 \;\equiv\; \langle\phi_1\rangle$ can be chosen real without loss of generality. Indeed, by performing a $\mathrm{U(1)_{NC}}$ transformation with θ_{NC} chosen as:

$$\theta^{\circ}_{\mathrm{NC}} \;=\; \frac{\mathrm{Im}\,\left(\langle\phi_0\rangle\,\langle\phi_1^{\dagger}\rangle\right)}{\left|\langle\phi_0\rangle - \langle\phi_1\rangle\right|^2} \tag{21}$$

one can identify the $\langle\phi_0\rangle$ and $\langle\phi_1\rangle$ phases. The remaining phase can be rotated away by a $\mathrm{U(1)}_{local}$ rotation.

We are now in a position to prove the identity (1). To do that it is sufficient to assume that the effective SU(1,1) supergravity model makes sense at the quantum level, by embedding it in a more fundamental theory where quantum gravity makes sense. Under those circumstances, the generating functional $W(J_i)$ of the theory exists and can be written as usual

$$exp\,\{iW(J_i)\} \;=\; \int\, [\mathcal{D}\,\varphi]\; exp\left\{i\int\, d^4x\, \left(\mathcal{L}\;+\;\sum_i\; J_i\,\varphi_i\right)\right\} \tag{22}$$

where J_i denote the sources of the quantum fields φ_i. Performing a standard Legendre transformation we define the generating functional of the 1–particle irreducible Green functions as

$$\Gamma(\varphi_i) = W(J_i) - \int d^4x \sum_i \varphi_i J_i \tag{23}$$

where now φ_i are the classical fields defined by

$$\varphi_i = \frac{\delta}{\delta J_i} W(J_i) \quad . \tag{24}$$

In terms of $\Gamma(\varphi_i)$ the relevant $\mathrm{U(1)_{NC}}$ Ward–identity ($I_{\rm NC}$) takes the form:

$$I_{\rm NC} \equiv \frac{\delta\Gamma}{\delta\phi_j^R}\,\delta_{\rm NC}\,\phi_j^R + \frac{\delta\Gamma}{\delta\phi_j^I}\,\delta_{\rm NC}\,\phi_j^I + \text{fermionic contribution} = 0 \tag{25}$$

where $\phi_j = \phi_j^R + i\phi_j^I\,, \qquad j = 0,1$. Using eq. (25) together with [see eq. (19)]

$$\begin{aligned} \delta_{\rm NC}\,\phi_0^R &= \delta_{\rm NC}\,\phi_1^R = -\left(\phi_0^I - \phi_1^I\right) \\ \delta_{\rm NC}\,\phi_0^I &= \delta_{\rm NC}\,\phi_1^I = \left(v_0 - v_1 + \phi_0^R - \phi_1^R\right) \end{aligned} \tag{26}$$

and taking derivatives with respect to ϕ_0^I and ϕ_1^I, we obtain:

$$0 = \left.\frac{\delta I_{\rm NC}}{\delta\phi_1^I}\right|_{F=0} = \frac{\delta\Gamma}{\delta\phi_0^R} + \frac{\delta\Gamma}{\delta\phi_1^R} + (v_0 - v_1)\left(\frac{\delta^2\Gamma}{\delta\phi_1^{I2}} + \frac{\delta^2\Gamma}{\delta\phi_1^I\delta\phi_0^I}\right)_{0\text{–momentum}} \tag{27a}$$

$$0 = \left.\frac{\delta I_{\rm NC}}{\delta\phi_0^I}\right|_{F=0} = -\left(\frac{\delta\Gamma}{\delta\phi_0^R} + \frac{\delta\Gamma}{\delta\phi_1^R}\right) + (v_0 - v_1)\left(\frac{\delta^2\Gamma}{\delta\phi_0^{I2}} + \frac{\delta^2\Gamma}{\delta\phi_1^I\delta\phi_0^I}\right)_{0\text{–momentum}} \tag{27b}$$

where F denotes collectively all classical fields of the theory. These identities, as they stand, are not useful. They relate some Green functions between physical and unphysical fields. However, using the $U(1)_{local}$ and Weyl invariance [see eqs. (15a,b)] we will be able to eliminate the unphysical degees of freedom.

The $U(1)_{local}$ Ward identities I_c are obtained using eq. (15b) in a similar way as before. One finds:

$$0 = \frac{\delta I_c}{\delta \phi_1^I}\Big|_{F=0} = -\frac{\delta \Gamma}{\delta \phi_1^R} + \left(v_0 \frac{\delta^2 \Gamma}{\delta \phi_0^I \delta \phi_1^I} + v_1 \frac{\delta^2 \Gamma}{\delta \phi_1^{I2}} \right)_{0\text{–momentum}} \tag{28a}$$

$$0 = \frac{\delta I_c}{\delta \phi_0^I}\Big|_{F=0} = -\frac{\delta \Gamma}{\delta \phi_0^R} + \left(v_1 \frac{\delta^2 \Gamma}{\delta \phi_0^I \delta \phi_1^I} + v_0 \frac{\delta^2 \Gamma}{\delta \phi_0^{I2}} \right)_{0\text{–momentum}} . \tag{28b}$$

Finally, the Ward–identities of the Weyl invariance [see eq. (15a)] give:

$$2\, \eta_{\mu\nu} \frac{\delta \Gamma}{\delta h_{\mu\nu}} = v_0 \frac{\delta \Gamma}{\delta \phi_0^R} + v_1 \frac{\delta \Gamma}{\delta \phi_1^R} \quad , \tag{29}$$

where $h_{\mu\nu}$ is the graviton field defined by $g_{\mu\nu} = n_{\mu\nu} + h_{\mu\nu}$. The above identity is derived in ref. [12] and takes this simple form when the gauge conditions $\partial^\nu h_{\nu\mu} = 0$ and $h_\mu^\mu = 0$ are used to fix the local coordinate and Weyl invariance respectively. Because of the $\phi_0 - \phi_1 - h_\mu^\mu$ mixing, it is convenient to define:

$$\begin{pmatrix} \phi_0' \\ \phi_1' \end{pmatrix} = \begin{pmatrix} ch\omega & sh\omega \\ sh\omega & ch\omega \end{pmatrix} \begin{pmatrix} \phi_0 \\ \phi_1 \end{pmatrix} \tag{30a}$$

where ϕ_0' is the unphysical field whose real part couples to the trace of the energy momentum tensor (unphysical dilaton) and $\phi_1' \equiv \phi_D + i\phi_P$ is the physical field (physical dilaton and plation). The "Lorentz rotation" is dictated by the relative sign difference of the kinetic energies of ϕ_0 and ϕ_1. The parameter ω is given by

$$sh\omega = \frac{-v_1}{\sqrt{v_0^2 - v_1^2}} \quad . \tag{30b}$$

Combining eqs. (27–30) and eliminating the unphysical ϕ_0' we obtain the promising identity (1)

$$-\eta_{\mu\nu}\,\frac{\delta\Gamma}{\delta h_{\mu\nu}} \;=\; \sqrt{v_0^2 - v_1^2}\;\frac{\delta\Gamma}{\delta\phi_D} + \frac{1}{2}\,\left(v_0^2 - v_1^2\right)\,\frac{\delta^2\Gamma}{\delta\phi_P^2}\bigg|_{\text{0–momentum}} \quad . \tag{31}$$

This equation shows clearly that the vanishing of the vacuum energy, $V(\langle\phi_i\rangle) \simeq n^{\mu\nu}\,\delta\Gamma/(\delta h_{\mu\nu})$, at the physical ground state, $\delta\Gamma/(\delta\phi_D) = -\delta V/(\delta\phi_D)$ $=$ 0, follows as a consequence of the existence of a massless plation, $[m_P^2 = -\delta^2\Gamma/(\delta\phi_P^2) \equiv 0]$, the Goldstone mode of the anomaly free $\mathrm{U(1)_{NC}}$ symmetry.

It is important to note that the above identities can be preserved by the regularization procedure in a consistent quantum theory of supergravity (if such a theory exists). In fact one needs a regularization which preserves the general coordinate and Weyl invariance as well as supersymmetry. In a nonsupersymmetric theory such a regularization exists and consists of constructing an n–dimensional conformally-invariant theory by replacing any scale μ by the unphysical dilaton field raised to an appropriate number of dimensions [14]. In supergravity theories this regularization can be easily extended using the compensating superfield instead of the unphysical dilaton [15]. In the present case, in order to respect the additional $\mathrm{U(1)_{NC}}$ symmetry, one has to use as a superconformal regulator the superfield combination $(\phi_0 - \phi_1)$. Let us note here that the minimization of the effective potential should lead to different VEV's for the fields ϕ_0 and ϕ_1 to avoid the conformally symmetric point $\langle\phi_0 - \phi_1\rangle = 0$. This condition must be satisfied in any realistic model. Using the above regularization one can easily verify the validity of the derived Ward identities order by order in the loop expansion [16]. In the effective theory, in the superunitary gauge, when no such regularization is used, the derived identity (1,31) *has to be respected* by the renormalization conditions. Otherwise spurious and ambiguous contributions to the vacuum energy will be introduced from the inconsistent regularization. Of course if the final theory is finite, the question of a consistent regularization becomes much simpler.

There is a U(1) compact analogue of the proposed mechanism, namely the Peccei–Quinn U(1) symmetry [17]. There, the corresponding Goldstone boson, "the axion," is used to rotate away the θ parameter ($\theta\,\widetilde{F}_{\mu\nu}\,F^{\mu\nu}$) and therefore to explain its almost vanishing value. In our case, the symmetry is noncompact (complex rotations) and the corresponding Goldstone boson, "the plation," eliminates the cosmological constant. The difference with the axion case is that our $\mathrm{U(1)_{NC}}$ is anomaly free symmetry and therefore without any explicit breaking. The plation is a real Goldstone boson. Note that the "plation" is not a "Brans–Dicke" type massless scalar field [see eq. (10)] and therefore there is not any cosmological problem following from its existence [18]. In the SU(1,1) model without matter fields, the proposed mechanism seems to be realized in a trivial way without any interesting minimum with $v_0 \neq v_1$.

In the more interesting cases where matter fields are present, the identity of eq. (31) remains valid provided their couplings respect the $\mathrm{U(1)_{NC}}$ symmetry. Also the absence of anomaly creating terms like $\phi_P\,\widetilde{F}_{\mu\nu}\,F^{\mu\nu}$ must be requested. In the presence of such terms $\mathrm{U(1)_{NC}}$ will be explicitly broken because of instanton effects [19].

The SU(N,1) no-scale model is based on the following Kähler potential G [5]:

$$G \;=\; -3\,\ell n\left(Z + Z^{\dagger} - \frac{\phi_i^{\dagger}\phi^i}{3}\right) + \ell n\left|F\left(\phi^i\right)\right|^2 \tag{32a}$$

where the superpotential F depends only on the gauge nonsinglet fields ϕ_i,

$$F \;=\; c + d_{ijk}\,\phi^i\,\phi^j\,\phi^k \quad . \tag{32b}$$

The gauge singlet field Z plays a similar role, as in the simple SU(1,1) case and the parameter c breaks supersymmetry spontaneously. The scalar fields form an Einstein–Kahler manifold with curvature $R_M = (N+1)/3$,

$$R^I_J \;\equiv\; \partial^I\,\partial_J\,\ell n\,\mathrm{Det}\,G^M_N \;=\; \frac{N+1}{3}\,G^I_J \quad . \tag{33}$$

In eq. (33), $G^I_J = \partial^I \partial_J G$ is the metric and R^I_J is the Kähler–Ricci tensor; $\phi_I \equiv (Z, \phi_i)$. The physical dilaton and the plation in that case are given by

$$\begin{aligned} \phi_D &= -\frac{3}{\sqrt{6}}\, \ell n \left(Z + Z^\dagger - \frac{\phi^\dagger_i \phi^i}{3} \right) \\ \phi_P &= \sqrt{\frac{3}{2}}\, i \left(Z - Z^\dagger \right) \end{aligned} \tag{34}$$

and the scalar part of the Lagrangian in this unitary representation is

$$\begin{aligned} \frac{1}{\sqrt{-g}}\, \mathcal{L}_{scalars} = g^{\mu\nu} \Bigg\{ \frac{1}{2}\, \partial_\mu \phi_D\, \partial_\nu \phi_D + \frac{1}{2}\, e^{\sqrt{8/3}\, \phi_D}\, I_\mu I_\nu \\ + e^{\sqrt{2/3}\, \phi_D}\, \mathcal{D}_\mu \phi^\dagger_i\, \mathcal{D}_\nu \phi^i \Bigg\} - e^{\sqrt{8/3}\, \phi_D} \left\{ \sum_{i \neq D,P} \left| F_{\phi_i} \right|^2 + \frac{1}{2}\, g^2_{GUT}\, D^2 \right\} \end{aligned} \tag{35}$$

with $I_\mu = \partial_\mu \phi_P + (i/\sqrt{6})\, \phi^\dagger_i \overleftrightarrow{D}_\mu \phi^i$ and $D^a = \phi^\dagger_i\, (T^a)^i_j\, \phi^j$.

The scalar potential is positive semidefinite and flat in the ϕ_D and ϕ_P directions. The supersymmetry is spontaneously broken when $c \neq 0$ $\left[m_{3/2}\, (\langle \phi_D \rangle) = c\ exp\{ \sqrt{3/2}\ \langle \phi_D \rangle \} \right]$. Because of the specific value of the curvature $R_M = (N+1)/3$, the model has a simple superconformal representation:

$$\begin{aligned} \frac{1}{\sqrt{-g}}\, \mathcal{L}^{bosons}_{\mathrm{SU(N,1)}} = &- \frac{1}{6} \left(|\phi_o|^2 - |\phi_1|^2 - \sum_i |\phi_i|^2 \right) R \\ &- \left(|\mathcal{D}_\mu \phi_o|^2 - |\mathcal{D}_\mu \phi_1|^2 - \sum_i |\mathcal{D}_\mu \phi_i|^2 \right) \\ &- \frac{1}{4}\, F^a_{\mu\nu}\, F^{\mu\nu}_a - \left| F_{\phi_i} \right|^2 - \frac{1}{2}\, g^2_{GUT}\, D^2 \end{aligned} \tag{36}$$

and the introduction of the compensator ϕ_0 and the gravitation auxiliary field A_μ inside the covariant derivatives $\mathcal{D}_\mu$ linearizes the relevant SU(1,1) symmetry as previously.

The connection of the SU(N,1) model with the ten-dimensional $E_8 \times E_8$ superstring is manifest after the compactification of the extra 10–4 dimensions on a Ricci flat manifold [20]. The obtained effective potential is given by [9]

$$
\begin{aligned}
\mathcal{G} \; &= \; - \; 3\,\ell n \left(Z + Z^\dagger - \frac{\phi_i^\dagger \phi^i}{3} \right) \\
&\quad + \; \ell n \left| \left[W(S) + dijk\,\phi^i\,\phi^j\,\phi^k \right] \right|^2 \; - \; \ell n \left(S + S^\dagger \right)
\end{aligned}
\tag{37}
$$

where $W(S) \;=\; a \,+\, b\, e^{\gamma S}$ is an effective S–dependent superpotential of the "hidden" field S which is generated by the "hidden E_8" gaugino condensation, and a, b and γ are some condensation parameters. a, b are different from zero at energies smaller than the condensation scale Λ_c inducing a nonzero supersymmetry breaking parameter

$$
c^{eff} \;=\; W\left(\langle S \rangle\right) \qquad E < \Lambda_c \quad , \tag{38}
$$

where $\langle S \rangle$ is determined by the minimization condition

$$
\frac{\partial}{\partial S}\, \mathcal{G}\Big|_{\langle S \rangle} = 0 \quad . \tag{39}
$$

By integrating out the S–field one finds

$$
\mathcal{G}\Big|_{W(\langle S \rangle) \;\equiv\; c} \;\equiv\; G^{\mathrm{SU(N,1)}} \qquad \text{for} \quad E < \Lambda_c \quad , \tag{40}
$$

with E_6 as the grand unification group.

It is of great importance that in the effective ($E < \Lambda_c$) SU(N,1) model obtained from superstrings, dangerous terms like $\phi_P \widetilde{F}^a_{\mu\nu} F^{\mu\nu}_a$ are not allowed. It is also important to note that for energies larger than Λ_c there are two SU(1,1) symmetries corresponding to the Z and S fields. However, SU(1,1)$_S$ is explicitly broken from the condensation mechanism and the instanton effects, due to the presence in the Lagrangian of terms like $(S + S^\dagger) F^a_{\mu\nu} F^{\mu\nu}_a$ and also $i(S - S^\dagger) \widetilde{F}^a_{\mu\nu} F^{\mu\nu}_a$. No similar Z–field depending terms appear in the gauge kinetic terms and none will be generated by radiative corrections because the Z–field is gauge singlet; therefore the anomaly requirement is automatically satisfied.

Because of eq. (39) the gaugino mass terms are not present classically. $[m_{gauginos} \sim e^{\mathcal{G}/2} \mathcal{G}_s/[\mathcal{G}_{ss^\dagger}(S + S^\dagger)] = 0]$. The only supersymmetry breaking scale at the tree level is the undetermined gravitino mass $\big[m_{3/2} = W(\langle S\rangle) \times exp\{\sqrt{3/2}\,\langle\phi_D\rangle\}\big]$. However, through higher order corrections the supersymmetry breaking will be communicated to the gauge interacting sector of the theory (squarks, sleptons, gauginos, ...) creating an effective global SUSY breaking boson-fermion $(\mathrm{mass})^2$ splitting m^2_{susy} [21]. The latter will be determined dynamically and simultaneously with the electroweak scale M_W, as usually in the no-scale models [8, 4, 5]. The essential difference now is that the cosmological constant is identically equal to zero at the SU(2) $\times$ U(1) breaking minimum [16].

In the framework of superstring theories all requirements of the $\Lambda = 0$ mechanism are automatically fulfilled. However, as we have already stressed, the proposed mechanism for the vanishing of the cosmological constant is more general and can be implemented in any consistent theory of gravity assuming the existence of a noncompact and anomaly-free global symmetry.

The present experimental limit on $\Lambda \leq 10^{-120} M^4 \sim (M_W/M)^8 M^4$ gives us the possibility for a very small residual vacuum energy or equivalently, a very small plation mass, $m_p \leq 10^{-60}\ M \simeq 3 \times 10^{-42}$ GeV, which could be cosmologically interesting [22].

We end by noting that candidate plation fields are naturally present in $D > 4$, $N = 1$ supergravity theories. They correspond to some antisymmetric tensor fields which reduce, after compactification in four dimensions, to "axion" or "plation" type pseudoscalar fields.

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To Professor J. Prentki for his 65th birthday.

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