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# A NEW FORMULATION FOR LINEAR ACCELERATOR DESIGN*

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## ABSTRACT

We define three basic, calculable and measurable parameters. With these parameters we derive expressions for the radio frequency induced, beam induced, and loaded section voltages and average section gradients. Unlike the present well known expressions these alternate expressions are valid for continuous wave, pulsed, and single bunch beams, for both lossy and lossless sections and for both standing wave and travelling wave sections. We use these alternate expressions to maximize the efficiency and the gradient for a given peak power. TW sections with linearly variable group velocity are also considered.

## INTRODUCTION

Three local parameters characterize a traveling wave accelerator section: the elastance per unit length $s$, the group velocity $v_g$, and the unloaded (internal) time constant $T_o$. They are defined as follows:

$$s = \frac{E^2}{w} \tag{1}$$

$$v_g = \frac{P}{w} \tag{2}$$

$$T_o = \frac{2w}{p_d} \tag{3}$$

where $E$ is the accelerating gradient, $w$ is the energy stored per unit length, $P$ is the power transmitted and $p_d$ is the power dissipated per unit length. As the linear dimensions vary as the frequency $f$, we infer from the above definitions that for the same group velocity and same mode

$$s \propto f^2 \qquad \text{and} \qquad T_o \propto f^{-3/2} \tag{4}$$

We can express the rf induced gradient, the beam induced gradient and the difference between the rf and the beam induced section gradients in terms of the three parameters. With these parameters, and given the rf peak power and pulse energy into the section we obtain the beam voltage and total energy transferred to the beam. The ratio of energy in each beam pulse to the energy in each rf pulse is the conversion efficiency, or simply the efficiency.

## RF ENERGY TO BEAM ENERGY CONVERSION EFFICIENCY

We will derive expressions for the efficiency for two special cases: long beam pulse and single bunch.

The rf induced accelerating gradient, averaged over the section, is derived in the Appendix and is

$$\bar{E}_a = \frac{V_o}{L} = \sqrt{\frac{\eta_s s P_o T_f}{L}} \tag{5}$$

where $\eta_s$ is the section efficiency, $s$ is the elastance per unit length (M$\Omega$ $\mu s^{-1} m^{-1}$), $P_o$ is the section power input (MW), $T_f$ is the section fill time ($\mu$s), $L$ is the section length (m).

The section efficiency $\eta_s$ is the energy required for a given voltage divided by the energy required for the same voltage by an identical but lossless section.

### Long Beam Pulse

From the Appendix the expression for the beam induced gradient averaged over the section is

$$\bar{E}_b = \frac{\eta_i s I_o T_f}{4} \tag{6}$$

$\eta_i$ is the beam to section voltage conversion efficiency, defined as the beam induced voltage divided by the beam induced voltage in an identical but lossless section.

The loaded gradient is

$$\bar{E} = \bar{E}_a - \bar{E}_b = \bar{E}_a - \frac{\eta_i I_o T_f}{4} \tag{7}$$

Assuming a beam pulse of length $T_b$ is turned on as soon as the section is full the rf energy to beam energy conversion efficiency is

$$\eta = \frac{\bar{E} I_o T_b L}{P_o(T_f + T_b)} = \frac{\eta_s s I_o T_b T_f}{\bar{E}_a(T_f + T_b)}\left[1 - \frac{\eta_i s I_o T_f}{4\bar{E}_a}\right] \tag{8}$$

### Single Bunch

If we inject a single bunch of charge $q$ at $T_f$, the self-induced effective field acting on the bunch is:[1] $\bar{E}_b = sq/4$. In Ref. 1, $s/4$ is called the loss parameter $k_1$. The efficiency of transforming rf energy to bunch energy is

$$\eta = \frac{(\bar{E}_a - \bar{E}_b)Lq}{P_o T_f} = \frac{\eta_s s(\bar{E}_a - \bar{E}_b)q}{\bar{E}_a^2} = \frac{\eta_s sq}{\bar{E}_a}\left[1 - \frac{sq}{4\bar{E}_a}\right] \tag{9}$$

In the single bunch mode, when the group velocity is close to the particle velocity, assumed to be $c$, then we do not have to fill the section before we inject the beam pulse and the effective fill time is diminished by the factor $f_v = 1 - (v_g/c)$. The required klystron pulse width is reduced by this factor and $\eta$ in Eq. (7) is improved by the inverse of this factor. In both cases for light beam loading the rf energy to beam energy conversion efficiency varies as the elastance.

## SECTION DESIGN: CHOICE OF GROUP VELOCITY AND FREQUENCY

Define the section elastance $s_s$, and the section reactance $x_s$:

$$s_s = \frac{\bar{E}_a^2}{w_o} \quad , \quad x_s = \frac{\bar{E}_a^2}{p_o} \tag{10}$$

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Here $p_o$ and $w_o$ are respectively the section input peak power per meter and pulse energy per meter. Using (5) and (10) we obtain

$$s_s = \eta_s s f_v \quad , \quad x_s = \eta_s s T_f = \eta_s s \frac{L}{v_g} \tag{11}$$

The pulse energy and the peak input power to a section is

$$W_o = \frac{E_a^2 L}{s_s} = \frac{E_a^2 L}{\eta_s s f_v} \quad , \quad P_o = \frac{E_a^2 L}{x_s} = \frac{E_a^2 v_g}{\eta_s s} \tag{12}$$

The section elastance determines the energy per unit length and the section reactance determines the peak power per unit length needed to attain a given gradient. Figure 1 shows a plot of $x_s$ and $s_s$ as a function of fill time for a 2 cm aperture, 1.44 $\mu$s internal time constant, disk-loaded waveguide (DLWG) operating at 2856 MHz. Its length is 3 m and its fill time is 0.82 $\mu$s and hence $\tau = 0.57$ and $\eta_s = 0.581$. The elastance is 76.4 M$\Omega$/m$\mu$s and the group velocity is 3.66 m/$\mu$s. The fill time is a compromise: we accept a lower efficiency in order to reduce the peak power requirement. The local and section parameters are listed in the first line of Table 1.

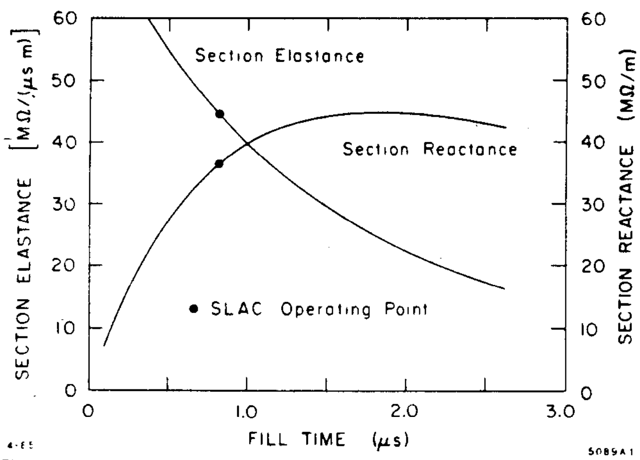


Fig. 1. TW section elastance and reactance vs fill time.

Table 1. Local and section parameters for a 2 cm aperture DLWG

| freq MHz | diam cm | $s$ M$\Omega$/$m\mu s$ | $v_g$ $m/\mu s$ | $T_o$ $\mu s$ | $T_f$ $\mu s$ | $L$ m | $s_s$ M$\Omega$/$m\mu s$ | $x_s$ M$\Omega$/m |
|---|---|---|---|---|---|---|---|---|
| 2856 | 10.5 | 76.4 | 3.66 | 1.44 | 0.82 | 3 | 44.4 | 36.4 |
| 11400 | 2.8 | 267 | 96 | .585 | 0.33 | 32 | 226 | 51.2 |

For a given aperture there is a frequency that will maximize the elastance. As we shall see in the following example, the elastance can be increased considerably over the value at 2856 MHz.

We now increase the aperture to 8 cm and increase the frequency so that the aperture returns to 2 cm as illustrated in Fig. 2. With our assumed light loading the design is the same for both a single bunch and long beam pulse. But when operating in a single bunch mode the velocity factor increases the section elastance by a factor of 1.46. The resulting parameters with maximized elastance are listed in the second line of Table 1. The section attenuation remains 0.57 nepers. We do pay for the 5 fold increase in elastance with a high peak power requirement, but this can be remedied with efficient peak power multiplication or by increasing the number of sources.

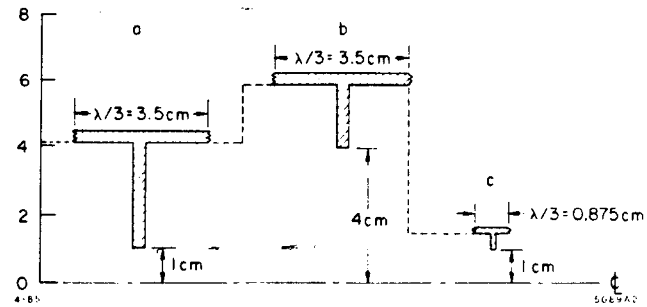


Fig. 2. Evolution of fixed aperture design from 2856 MHz to 11400 MHz.

High group velocity and high frequency combination afford a mechanical advantage: the accelerator becomes a long thin tube which can be shaped into a zig-zag structure.

The peak power and pulse energy requirements to obtain the 21 MV/m gradient required for the SLAC Linear Collider are listed in Table 2.

Table 2. Peak Power and Pulse Energy to Obtain 21 MV/m

| freq | $w_o$ | $p_o$ | $W_o$ | $P_o$ |
|---|---|---|---|---|
| MHz | joule/m | MW/m | joule | MW |
| 2856 | 9.93 | 12.1 | 30.3 | 36.9 |
| 11400 | 1.95 | 8.61 | 63.4 | 276 |

We see that although the input peak power is much higher at 11400 MHz, the peak power per unit length is actually five times lower.

## STANDING WAVE SECTION DESIGN

Define the internal and the external standing wave (SW) section time constants as

$$T_o = \frac{2W}{P_d} \quad , \quad T_e = \frac{2W}{P_e} \tag{13}$$

$W$ is the energy stored in the cavity, $P_d$ is the power dissipated in the cavity, $P_e$ is the power emitted by the cavity. The stored energy at the end of the input pulse $T_p$ is[2]

$$W = \frac{2\gamma^2 T_e (1 - e^{-T_p/\gamma T_e})^2}{T_p} P_o T_p \tag{14}$$

$$\gamma = \frac{1}{1 + T_e/T_o} \quad , \quad T_L = \gamma T_e \tag{15}$$

$\gamma$ is half the steady state normalized emitted field and $T_L$ is the loaded time constant.

Define
$$s = \frac{V^2}{WL} = \frac{E_a^2}{W/L} \tag{16}$$

where $V$ is the accelerating voltage. From (14) and (16) the rf induced accelerating gradient is

$$E_a = \sqrt{\frac{\eta_s s P_o T_p}{L}} \quad , \quad \eta_s = \frac{2\gamma^2 T_e (1 - e^{-T_p/\gamma T_e})^2}{T_p} \tag{17}$$

Figure 3 shows a plot of $T_e/T_o$ that maximizes $\eta_s$ for a given $T_p/T_o$. The $T_e$ obtainable from this plot matches a source with a given pulse length to a cavity with a given $T_o$. If $T_p \gg T_o$ then we have the steady state match condition of $T_e = T_o$. If $T_o \gg T_p$ then $\gamma = 1$ and the section efficiency has a maximum with respect to $T_e$ of 0.815 at $T_p = 1.257T_e$.

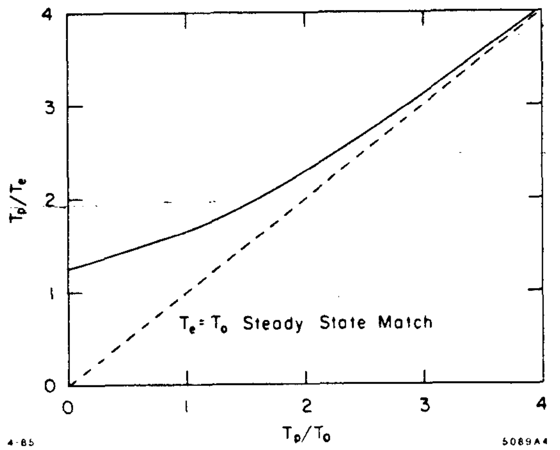


Fig. 3. Pulse matching.

If the pulse length is much greater than the external time constant then

$$\eta_s = \frac{2\gamma^2 T_e}{T_p} \qquad \text{and} \qquad E_a = \gamma\sqrt{\frac{2sP_oT_p}{L}} \tag{18}$$

The constants that characterize a SW section are the same as the ones that characterize a traveling wave section with the following exceptions. The lossless efficiency is 0.815 rather then unity as for a TW section, the group velocity is replaced by the external time constant, the elastance per unit length and the internal time constant is that of the whole section rather then obtained from per unit length quantities. The elastance and internal time constant can be obtained from computer codes such as SUPERFISH.

Figure 4 shows a plot of $x_s$ and $s_s$ as a function of the fill time for a SW section with the same elastance, internal time constant and length as a SLAC section. Just as with a TW section the section elastance approaches an asymptotic maximum as the fill time gets much smaller than the internal time constant. Unlike in a TW section in a SW section there is a slight increase in stored energy if the source pulse width increases beyond the fill time.

At each point we choose the external time constant that maximizes the section elastance and therefore we may consider the rf input pulse length as the fill time. For the same peak power per unit length a SW section can yield higher gradients than a TW section because it acts as its owm pulse compressor, and because its $T_e$, unlike the group velocity of a TW section, is not determined by the size of the aperture thru which the beam passes but by the input port coupling.

The SW section beam induced voltage and output power are respectively

$$V_b = \frac{\gamma S I_o T_e(1 - e^{-t/\gamma T_e})}{2} \quad , \quad P_b = \gamma V_b I_o \tag{19}$$

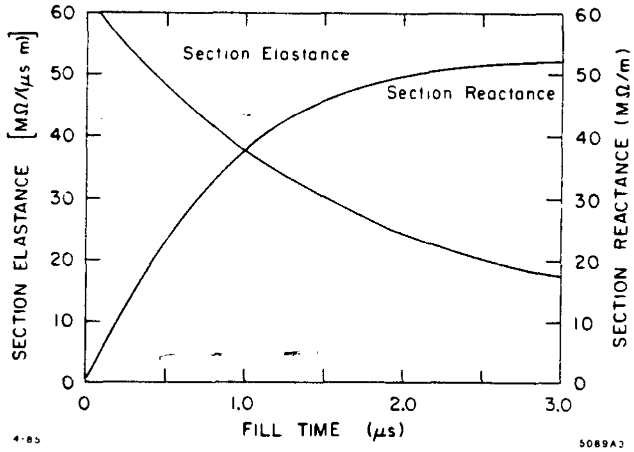


Fig. 4. SW section elastance and reactance vs fill time.

## A PROPOSAL

We propose the use of the parameters $s$, $v_g$, $T_o$ and $T_e$ for accelerator design. As we have shown, they are convenient and sufficient to design a TW or a SW accelerator section, as well as a superconducting section.[3] These parameters are clearly definable, have names, are directly measurable and are useful: that is they give information. We believe they simplify the present symbol soup.

From these parameters we can obtain the familiar parameters: the quality factor $Q$, the shunt resistance per unit length $r$, $r/Q$, $\omega r/Q$, and the loss parameter $k_1$:

$$Q = \frac{\omega T_o}{2}\ , \quad r = \frac{sT_o}{2}\ , \quad r/Q = \frac{s}{\omega}\ , \quad 4k_1 = \frac{\omega r}{Q} = s \tag{20}$$

With our parameters the familiar expressions for the no-load and beam induced voltages can be converted to expressions that do not become indeterminate as the resistivity of the section material approaches zero. indeterminate as the resistivity of the section material approaches zero. Also, many combination of parameters that are independent of loss can be written in terms of parameters that are in themselves independent of loss. For example in reference 1 frequently occurring products when expressed in terms of the new parameters do not become indeterminate as the section losses approach zero:

$$r\tau = \frac{sT_f}{2} \quad , \quad \alpha r = \frac{s}{2v_g} \tag{21}$$

Finally we note that the lumped parameter parallel circuit model is also applicable to elastance. It is easy to show that the 'shunt' elastance $1/C$ fits the definition $S = V^2/W$:

$$S = \frac{V_{rms}^2}{W} = \frac{V^2/2}{CV^2/2} = \frac{1}{C}. \tag{22}$$

Frequently the elastance is denoted by $\omega R/Q$. There is no basic objection to this, but why define a parameter that does not depend on loss, by three parameters two of which do depend on loss. The familiar $r/Q$ is the shunt reactance $x$ of the capacitor. We suggest that, analogous to shunt resistance, $r/Q = s/\omega$ be called the shunt reactance and given the symbol $x$.

## APPENDIX A: UNLOADED VOLTAGE

From (2) and (3) we have

$$p_d = \frac{-2P}{v_g T_o} \qquad (A1)$$

Approximating $p_d$ by $dP/dz$ Eq. (A1) can be solved to obtain

$$P(z) = P_o e^{-2z/v_g T_o} \qquad (A2)$$

Here $P_o$ is the section input power. From (1), (2) and (A2)

$$E = \sqrt{\frac{sP}{v_g}} = E_o e^{-z/v_g T_o} \quad , \quad E_o = \sqrt{\frac{sP_o}{v_g}} \qquad (A3)$$

Here $E$ and $E_o$ are the accelerating gradient as a function of $z$ and at the section input respectively. We integrate Eq. (A3) and obtain

$$V_o = \int_0^L E e^{-z/v_g T_o} = \left(\frac{1 - e^{-T_f/T_o}}{T_f/T_o}\right) \sqrt{sP_o T_f L} = \sqrt{\eta_s s P_o T_f L} \qquad (A4)$$

$$\eta_s = \frac{(1-e^{-\tau})^2}{\tau^2} \qquad (A5)$$

$$\tau = \frac{T_f}{T_o} \quad , \quad T_f = \frac{L}{v_g} \qquad (A6)$$

$\tau$ is the section attenuation in nepers.

For $\tau << 1$ $\qquad \eta_s = 1 - \tau \qquad (A7)$

## APPENDIX B: BEAM INDUCED VOLTAGE

From consideration of conservation of energy and using $P = (v_g/s)E^2$

$$\frac{dP}{dz} = I_o \sqrt{\frac{s}{v_g} P} - \frac{2P}{v_g T_o} \quad . \qquad (B1)$$

$$P = \frac{I_o^2 T_o^2 s v_g}{4}(1 - e^{-z/v_g T_o})^2 \qquad (B2)$$

$$E = \frac{I_o T_o s}{2}(1 - e^{-z/v_g T_o}) \qquad (B3)$$

$$V_b = \frac{I_o T_o s}{2}[z - v_g T_o(1 - e^{-z/v_g T_o})] \qquad (B4)$$

$$V_b = 2\left[\frac{1}{\tau} - \frac{1-e^{-\tau}}{\tau^2}\right] \frac{I_o T_f s L}{4} \qquad (B5)$$

$$\eta_i = 2\left[\frac{1}{\tau} - \frac{1-e^{-\tau}}{\tau^2}\right] \qquad (B6)$$

For $\tau << 1$ $\qquad \eta_s = 1 - \frac{\tau}{3} \qquad (B7)$

## APPENDIX C: VARIABLE GROUP VELOCITY RF INDUCED VOLTAGE

In equation (A1) substitute for the group velocity

$$v_g = v_{go}(1 + mz) = v_{go} + v_g' z \quad \text{where} \quad v_g' \equiv \frac{dv_g}{dz} \equiv m v_{go} \qquad (C1)$$

and obtain

$$P(z) = P_o \, e^{-\frac{2\ln(1+mz)}{T_o v_g'}} \qquad (C2)$$

$$E(z) = E_o(1 + mz)^{-.5 - 1/v_g' T_o}; \quad E_o = \sqrt{\frac{sP_o}{v_{go}}} \qquad (C3)$$

Let $x_{11} = .5 - \frac{1}{v_g' T_o}$ and $g \equiv mL$. Then

$$V_o = E_o L \frac{(1+g)^{x_{11}} - 1}{x_{11} g} \qquad (C4)$$

Hence

$$\eta_s = \frac{[(1+g)^{x_1} - 1]^2}{g \ln(1+g) x_1^2} \quad . \qquad (C5)$$

$$T_f = \frac{L}{v_{go}} \frac{\ln(1+g)}{g} \qquad (C6)$$

For a constant gradient section: $v_g' T_o = -2$ and

$$\eta_s = \frac{1 - e^{-2\tau}}{2\tau} \quad . \qquad (C7)$$

## APPENDIX D: VARIABLE GROUP VELOCITY BEAM INDUCED VOLTAGE

$$E = \frac{sI_o T_o}{(2 + v_g' T_o)} \left[1 - (1+mz)^{-(1 + 1/v_g' T_o)}\right] \quad . \qquad (D1)$$

$$V_b = \frac{sI_o T_o L}{2 + v_g' T_o} \left[1 - \frac{(1+g)^{x_2} - 1}{g x_2}\right] \quad . \qquad (D2)$$

$$x_2 = 0.5 - \frac{1}{v_g' T_o} \quad , \quad g = mL$$

$$\eta_i = \frac{4T_o}{T_f(2 + v_g' T_o)} \left[1 - \frac{(1+g)^{x_2} - 1}{g x_2}\right] \qquad (D3)$$

For a constant gradient section: $v_g' T_o = -2$ and

$$\eta_i = \frac{1}{\tau} - \frac{2e^{-2\tau}}{1 - e^{-2\tau}} \quad . \qquad (D4)$$

## ACKNOWLEDGEMENT

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