SLAC – PUB – 3644
April 1985
(T)

LONG DISTANCE DOMINANCE AND K^+ DECAYS*

H. GALIĆ

Stanford Linear Accelerator Center

Stanford University, Stanford, California, 94305

ABSTRACT

In an attempt to understand exclusive K decays within a long distance framework, a simple and operative model for the K^+ effective vertex is constructed and applied in the analysis of a few decay modes.

Submitted for Publication

⋆ Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.

1. Introduction

The QCD corrected effective Hamiltonian was for a long time the basic tool in analyses of nonleptonic decays. (For a review of successes and problems in such "standard" scenario see *e.g.* Refs. [1,2]). The popularity of that approach is in fact not completely understandable, when its rather poor record in description of exclusive decays is taken into account. Although in the first works on $K \to \pi\pi$ the old current algebra (CA) result was changed in right direction, an explanation of $\Delta I = 1/2$ wasn't achieved. In a similar way the method originally failed in analyses of D-decays, CP-violating parameters, Ω-decays, etc. It is true that one can *post factum* adopt a procedure and parameters in such a way that the observed values are reproduced, but that could hardly teach us anything. Even more than the lack of predictive power, the physical picture — in which the low energy decays of "large" systems[♮1] are assumed to be properly described by a short distance expansion — casts shadows on the entire procedure.

There has been therefore many attempts [3 - 6] to improve the method by systematic inclusion of long distance corrections to the existing framework. It is also possible to imagine another, completely different and to some extent more natural path. Indeed, various authors [7 - 14] have tried to describe exclusive weak decays of light hadrons relying only on long distance dynamics and neglecting altogether short distance contributions. One of the proposed schemes [13,14] was based on an effective field theory with a direct coupling of mesons to constituent quarks. The only role of QCD was to produce meson-quarks effective vertices, while no further gluons were allowed in diagrams.

♮1 The size is basically determined by the inverse of the reduced mass of bound states.

Here the mentioned program is illustrated by a simple example. A toy model compatible with the requirements of the new scheme is constructed and applied in the study of $K^+ \to \pi^+\pi^0$ and some other K^+ decays. (The same model will be used in more interesting and more challenging decays of neutral kaons in the subsequent paper.) Note that the method is designed only for the description of weak exclusive decays of *mesons*. If the long distance dynamics really plays an important role, then the inclusive decays, and the exclusive decays of baryons might need a completely different approach. Throughout the work the four-flavour version of the standard model is used, but a generalization to more flavours presents no problem.

2. Model

As mentioned previously, the simplified world will be considered in which mesons are constructed of valence quarks only, and gluons fulfill their role by producing effective vertices. To the order $1/M_W^2$, the decay $K^+ \to \pi^+\pi^0$ is then represented by four diagrams in Fig. 1. In Refs. [13,14], these diagrams were treated in a general context. Here a simple model for effective vertices will be constructed and — by making use of the Feynman rules (with minus signs and traces for closed loops of fermions, etc.) — the amplitude of the process will be calculated. Note that there will be no need for the soft pion technique and CA reductions, since diagrams can be treated exactly. (Still, results will be similar in form to the results of the CA analyses).

Whatever we choose for a vertex function, the Lorentz and colour structure of the function should respect the pseudoscalar nature of π and K mesons, and the fact that these mesons are colour singlets. In the construction of the effective vertices I have also made use of the following constraints:

(*a*) Diagram in Fig. 2a should give finite and correct weak decay constants f_π, f_K, etc. (Since a pointlike vertex necessarily leads to divergent integrals, it is clear that the vertex function therefore must be nonlocal);

(*b*) Diagram in Fig. 2b in the $SU(3)$ limit, with $M \to M'$, and $Q^2 \to 0$ (M denoting here the mass of a meson) should give properly normalized form factors, $F_+(0) = 1$ and $F_-(0) = 0$;

(*c*) The vertex should be such that the decay of a meson to a quark-antiquark pair is forbidden when quarks are on mass-shell. (With this requirement the "confinement" is introduced into the scheme).

Although the constraints restrict a class of acceptable functions, a certain freedom still exists. Another criterion, the simplicity, was then decisive for the choice proposed in this work. With the notation explained in Fig. 3, the "regular" effective vertex function[♮2] is chosen to be

$$\Gamma_{\mathcal{M}}(k, \pm P) = \pm\delta^{ij}(\not{k} \pm \frac{\not{P}}{2} - m_q)\, H_{\mathcal{M}}(k, P)(\not{k} \mp \frac{\not{P}}{2} - m_{q'}) \ , \tag{1}$$

with

$$H_{\mathcal{M}}(k, P) = \frac{\beta_{\mathcal{M}}(M_{\mathcal{M}})^{2n-3}\,\not{P}\gamma_5}{(k^2 - \alpha_{\mathcal{M}} P^2)^n} \ . \tag{2}$$

In (2), n is an integer not smaller than three ($n \geq 3$), while α and β are parameters of the model. It is easily seen that requirements (a) - (c) are satisfied

♮2 The "anomalous" vertices will be briefly discussed in Section 4.

with

$$\alpha_{\mathcal{M}} = (-)^{n+1}\pi(f_{\mathcal{M}}/M_{\mathcal{M}})\left[\frac{(n-1)(n-2)^2}{6(2n-1)}\right]^{1/2} ,$$
$$\beta_{\mathcal{M}} = 4\pi\,\alpha_{\mathcal{M}}^{n-1}\left[(2n-1)(2n-2)/3\right]^{1/2} . \tag{3}$$

It will be shown later that the larger n produces the better description of the $K^+ \to \pi^+\pi^0$ decay. However, the goal of the model is to provide a qualitative rather than exact quantitative picture. Therefore I will concentrate mostly on $n = 3$ case. For $n = 3$, the expressions (3) are reduced to

$$\alpha_{\mathcal{M}} = \pi(f_{\mathcal{M}}/M_{\mathcal{M}})/\sqrt{15} , \quad \beta_{\mathcal{M}} = 8\pi\,\alpha_{\mathcal{M}}^2\sqrt{15}/3 . \tag{4}$$

Note also that Clebsch-Gordan coefficients $\pm 1/\sqrt{2}$ should be added to vertices in which neutral mesons π^0, K_L and K_S appear.[♮3]

3. Amplitudes

Having defined the vertices, we can start with the analysis of diagrams in Fig. 1. Let the momenta of K^+, π^0 and π^+ be denoted as P, P_0, and P_+ respectively, and momentum of the strange (s) quark as $k - P/2$. The amplitude

♮3 More precisely, $+1/\sqrt{2}$ for $\pi^0(\bar{u}u)$, $K_L(\bar{d}s)$, $K_L(\bar{s}d)$ and $K_S(\bar{d}s)$ vertices, and $-1/\sqrt{2}$ for $\pi^0(\bar{d}d)$ and $K_S(\bar{s}d)$ vertices.

corresponding to the diagram (a) in Fig. 1 is then[♯4]

$$A(P_0, P_+) = -\frac{h^2}{8} \sin\vartheta \cos\vartheta \, (-3) \int \frac{d^4k}{(2\pi)^4} \frac{1}{P_+^2 - M_W^2}$$
$$\times \mathrm{Tr}\Big\{ -\frac{1}{\sqrt{2}} H_\pi(k + P_+/2, P_0) \, [\not k + \frac{\not P}{2} - m] \, H_K(k, P)\gamma^\mu(1-\gamma_5)\Big\}$$
$$\times (-3) \int \frac{d^4\ell}{(2\pi)^4} \, \mathrm{Tr} \, \{-H_\pi(\ell, P_+/2)\gamma_\mu(1-\gamma_5)\} \ . \tag{5}$$

In (5), ϑ denotes Cabibbo angle, and $h^2/(8M_W^2) = G_F/\sqrt{2}$ is the Fermi coupling constant. Masses of u and d quarks are assumed to be equal, and denoted by m.

In a similar way one can express amplitudes $B(P_0, P_+)$, $C(P_0, P_+)$ and $D(P_0, P_+)$, corresponding to diagrams 1b, 1c and 1d. It is easily seen (on the basis of a symmetry of integrands) that amplitudes C and D cancel each other. The remaining two amplitudes can be expanded in series of powers of $1/M_W^2$. Let $A_0(P_0, P_+)$ and $B_0(P_0, P_+)$ denote the first terms in these expansions. One obtains

$$A_0(P_0, P_+) = \frac{G_F}{\sqrt{2}} \sin\vartheta \cos\vartheta \;\; 9(2\pi)^{-8}\beta_K\beta_\pi^2(M_K M_\pi^2)^{2n-3}$$
$$\times \frac{1}{\sqrt{2}} \int d^4k \, \frac{\mathrm{Tr}\,\{\not P_0(\not k + \not P/2)\not P\gamma^\mu\}}{[(k+P_+/2)^2 - \alpha_\pi M_\pi^2]^n[k^2 - \alpha_K M_K^2]^n} \tag{6}$$
$$\times \int d^4\ell \, \frac{\mathrm{Tr}\,\{\gamma_\mu \not P_+\}}{[\ell^2 - \alpha_\pi M_\pi^2]^n} \ .$$

At the same time, as a consequence of a generalized Fiertz transformation (see

♯4 An early attempt of such "diagrammatic" approach to K decays is described in Ref. [15]. The authors however have used pointlike vertices, and applied some additional assumptions to eliminate a class of diagrams.

Ref. [14]), one finds

$$B_0(P_0, P_+) = A_0(P_+, P_0)/3 \ . \tag{7}$$

To simplify expression (6), I shall use the definitions for weak decay constants and form factors (compare to Fig. 2):

$$i\, f_{\mathcal{M}} P_\mu = (-3) \int \frac{d^4\ell}{(2\pi)^4} \operatorname{Tr}\left\{ H_{\mathcal{M}}(\ell, P) \gamma_\mu (1-\gamma_5) \right\} \ , \tag{8}$$

and (with $Q = P_A - P_B$),

$$\begin{aligned} & i\, F_+^{AB}(Q^2)[P_A + P_B]^\mu + i\, F_-^{AB}(Q^2)[P_A - P_B]^\mu \\ & = 3 \int \frac{d^4k}{(2\pi)^4} \operatorname{Tr}\left\{ H_B(k + Q/2, P_B)[\not k + \frac{\not P_A}{2} - m]\, H_A(k, P_A) \gamma^\mu (1-\gamma_5) \right\} \ . \end{aligned} \tag{9}$$

Expression (6) now can be written as

$$A_0(P_0, P_+) = \frac{G_F}{\sqrt{2}} \sin\vartheta \cos\vartheta \ \frac{1}{\sqrt{2}} f_\pi \ \{ \ F_+^{K\pi}(M_\pi^2) \ [M_K^2 - M_\pi^2] + F_-^{K\pi}(M_\pi^2)\, M_\pi^2 \ \} \ . \tag{10}$$

A consequence of (10) is that $A_0(P_0, P_+) = A_0(P_+, P_0)$, and the amplitude $\mathcal{A}$ for $K^+ \to \pi^+\pi^0$ decay becomes (to the lowest order in $1/M_W^2$)

$$\mathcal{A} = A_0(P_0, P_+) + A_0(P_+, P_0)/3 = 4A_0(P_0, P_+)/3 \ . \tag{11}$$

In order to find this amplitude, one must know the values of form factors in (10). From the definition (9), after some rearrangement, one obtains

$$\begin{aligned} F_\pm^{K\pi}(Q^2) &= (M_\pi/M_K)^{2n-3} (\alpha_\pi/\alpha_K)^{n-1} \int_0^1 dx\, g_n(x) \\ & \times \frac{[(1-x) M_\pi^2/M_K^2 \pm x]}{[(1-x)\alpha_\pi M_\pi^2/(\alpha_K M_K^2) + x - x(1-x)Q^2/(4\alpha_K M_K^2)]^{2n-2}} \ . \end{aligned} \tag{12}$$

In (12), constants β were re-expressed in terms of α's. The function $g_n(x)$ is the result of Feynman parametrization; if e.g., $n = 3$, $g(x) = 30\,x^2(1-x)^2$. Expression (12) has a suitable form for a numerical integration. The obtained values are presented in Table 1. , and will be discussed in the next section.

Note that for positive α_π and α_K, form factors $F_\pm(Q^2)$ reach a singularity when

$$\sqrt{Q^2} \equiv M^* = 2(M_K\sqrt{\alpha_K} + M_\pi\sqrt{\alpha_\pi}\ ) \ . \tag{13}$$

For $n = 3$, this point is at $M^* = 0.795$ GeV. Graph in Fig. 4 presents $F_\pm$ as functions of Q^2. A good fit for $F_+^{K\pi}(Q^2)$ is a function

$$F_+^{K\pi}(0)\left[\frac{(M^*)^2}{(M^*)^2 - Q^2}\right]^{3.5} \quad . \tag{14}$$

M^* in expression (13) thus reminds somehow of the pole K^* in the dispersion relation analysis of form factors (see *e.g.* Ref.[16]). For $n = 3$, the ratio $F_-^{K\pi}(Q^2)/F_+^{K\pi}(Q^2)$ comes out to be very slowly changing function which for any practical purpose can be considered as a constant. The well known soft pion result [16],

$$F_+(Q^2 = M_K{}^2) + F_-(Q^2 = M_K{}^2) = f_K/f_\pi \quad , \tag{15}$$

is quite well satisfied with form factors as defined by (12). (The l.h.s. in Eq. (15) has *e.g.* for $n = 3$ the value 1.37).

4. Discussion and Conclusion

Table I summarizes the parameters and the results of the analysis for several values of n. The inputs were $f_\pi = 0.9\, M_\pi$ and $f_K = 0.4\, M_K$. For the value $n = 3$, the amplitude comes out to be by a factor of two larger than the experimental value, $|\mathcal{A}_{\rm exp}(K^+ \to \pi^+\pi^0)| = 18.3\, eV$. However, taking into account the roughness of the used approximation, this is quite a good result. For larger values of n, the agreement is even better.

Having the expressions for form factors, one can in addition numerically calculate decay rates $\Gamma(K_{\ell 3})$, and they also agree with the observed values. For $n = 3$, one obtains

$$\Gamma(K_{e3}) = 3.46 \times 10^6\, s^{-1} \quad , \quad \Gamma(K_{\mu 3})/\Gamma(K_{e3}) = 0.68 \quad , \tag{16}$$

while the experimental values for these quantities are $3.90 \times 10^6\, s^{-1}$ and 0.66 respectively.

Besides the form (1), I have tested a few more complicated wave functions, but the results are similar: the constraints (a) - (c) in Section 2, seem to be strong enough to ensure the right order of magnitude for $K^+ \to \pi^+\pi^0$ and other K^+ decays.

One might now try to apply the same method to nonleptonic decays of neutral kaons. Taking into account only regular vertices (1), one easily obtains *e.g.*,

$$\begin{aligned} \mathcal{A}(K_L \to \pi^+\pi^-) &= \mathcal{A}(K_L \to \pi^0\pi^0) = 0\ , \\ \mathcal{A}(K_S \to \pi^+\pi^-) &= -3\mathcal{A}(K_S \to \pi^0\pi^0) = \frac{3}{2}\,\mathcal{A}(K^+ \to \pi^+\pi^0)\ . \end{aligned} \tag{17}$$

The first line in (17) is simply a consequence of a symmetry of the four-flavor model with no CP-violation. The second line however incorrectly describes K_S

decays. The experimental values are $|\mathcal{A}_S^{+-}| = 389\,eV$ and $|\mathcal{A}_S^{00}| = 372\,eV$, while (17) gives more than one order of magnitude smaller values. In fact, this disagreement between the theory and experiment is no surprise: the regular vertices alone cannot completely describe K-decays. As shown in Ref. [13], a new type of "anomalous" vertices is a natural part of the scheme. Such anomalous vertices, presented in Fig. 5, originate when the $s \to d$ transition (via the W-boson, in self-energy type exchanges) happens within the confinement radius. Unlike in the "penguin" graphs [1,2], here the dominant mechanism is assumed to be the nonperturbative long-distance dynamics.[♯5] The situation is symbolically presented in Fig. 6. A mechanism which might lead to a big contribution of diagrams with anomalous vertices (and thus to an explanation of the "octet" rule) will be presented in more detail in the subsequent paper. Note that anomalous vertices, carrying $\Delta I = 1/2$ isospin change, cannot affect the analysis of $K^+ \to \pi^+\pi^0$ decays.

In conclusion, the simple semiphenomenological model for the (regular) meson wave-functions is presented. This work is a step in the attempt of a description of nonleptonic decays of mesons in a new framework. The unavoidable arbitrariness and not too convincing physical picture of the so-called standard procedure urge indeed for radical improvements. Opposing the main assumption of the standard procedure, this work is exploring the possibility that properties of K-decays are mostly hidden in the wave functions, and that short distance QCD corrections are only marginaly important. It is shown that K^+ decays, in which only the regular vertices appear, are fairly described by rather general wave

♯5 There is an interesting (although not direct) analogy between anomalous vertices and "tadpoles" advocated in Refs. [8,12].

function.[♮6] One of advantages of the scheme is that amplitudes are calculated directly from diagrams, and therefore some ambiguites related to the transition from real pions to "soft" pions are not present. The proposed model in addition may be useful in the more thorough study of the behaviour of form factors in π, K, and D systems. It remains to be seen whether a combination of the regular vertices and long-distance parts of $s \to d$ transitions (Fig. 6) can bring naturally to the required enhancement of anomalous wave functions. If that is the case, we would gain a new insight into the $\Delta I = 1/2$ rule, and at the same time have a simple and elegant description of CP conserving and nonconserving decays of K meson and other light mesons.

NOTE ADDED: A different type of effective meson – quarks vertices was proposed recently by A. S. Bagdasaryan, S. V. Esaǐbegyan, N. L. Ter–Isaakyan: Yad. Fiz. 38, 402 (1983) [Sov. J. Nucl. Phys. 38, 240 (1983)]. A detailed study of $K - \pi$ form factors was presented by J. Gasser, H. Leutwyler: Nucl. Phys. B250, 517 (1985). In both of these works models are designed for the analyses of $Q^2 \approx 0$ region, and in form factors no poles (compare tó expression (13)) appear.

ACKNOWLEDGEMENTS

Kind hospitality in the SLAC Theory Group is appreciated.

♮6 The same method, with regular vertices only, should work in Cabibbo allowed D-decays. However, the presence of poles in form factors makes the analyses of nonleptonic D-decays much more dependent on the choice of vertices.

REFERENCES

1. R. Rückl: "Weak decays of heavy flavours", Geneve preprint, October 1983
2. J. F. Donoghue in "Phenomenology of Unified Theories", eds. H. Galić, B. Guberina, D. Tadić , pp. 1-24. Singapore: World Scientific 1984
3. H. Fritzsch, P. Minkowski: Phys. Lett. 90B, 455 (1980)
4. V. L. Chernyak, A. R. Zhitnitsky: Nucl. Phys. B201, 492 (1982)
5. N. Bilić, B. Guberina: Phys. Lett. 150B, 311 (1985); Z. Physik C – Particles and Fields 27, 399 (1985)
6. J. O. Eeg: Phys. Lett. 155B, 115 (1985)
7. G. Nardulli, G. Preparata: Phys. Lett. 104B, 399 (1981); G. Nardulli, G. Preparata, D. Rotondi: Phys. Rev. D27, 557 (1983)
8. B. H. J. McKellar, M. D. Scadron: Phys. Rev. D27, 157 (1983)
9. N. F. Nasrallah, N. A. Papadopoulos, K. Schilcher: Phys. Lett. 134B, 355 (1984)
10. K. Terasaki, S. Oneda, T. Tanuma: Phys. Rev. D29, 456 (1984)
11. T. N. Pham, D. G. Sutherland: Phys. Lett. 135B, 209 (1984)
12. M. D. Scadron: Universal nonperturbative tadpole dynamics, Arizona preprint, June 1984; G. Eilam, M. D. Scadron: Phys. Rev. D31, 2263 (1985)
13. H. Galić: Phys. Rev. D31, 2363 (1985); for more details see Ref. [14]
14. H. Galić: SLAC Report PUB–3383, July 1984 (unpublished)
15. P. Pascual, R. Tarrach: Phys. Lett. 87B, 64 (1979)
16. R. E. Marshak, Riazuddin, C. P. Ryan: "Theory of Weak Interactions in Particle Physics". New York: Wiley 1969

Table I

Parameters of the model, the values of the form factors, and $K^+ \to \pi^+\pi^0$ amplitude for various choices of n. In the SU(3) limit, $F_+(0) = 1$ and $F_-(0) = 0$.

	$n = 3$	$n = 5$	$n = 9$
α_π	0.73	2.31	5.54
α_K	0.32	1.03	2.46
$F_+^{K\pi}(0)$	0.75	0.57	0.47
$F_+^{K\pi}(M_\pi^2)$	0.83	0.61	0.49
$F_-^{K\pi}(0)$	-0.49	-0.32	-0.23
$F_-^{K\pi}(M_\pi^2)$	-0.54	-0.34	-0.24
$\mathcal{A}(K^+ \to \pi^+\pi^0)$	37.3 eV	27.5 eV	22.6 eV

FIGURE CAPTIONS

1. Diagrams for $K^+ \to \pi^+\pi^0$ decay. Heavy dots denote regular effective vertices.

2. Diagrams which define (a) weak decay constants f_M, and (b) weak form factors $F_\pm(Q^2)$.

3. Feynman rules for regular vertices. An ingoing meson is described by $\Gamma(k,+P)$, and an outgoing meson by $\Gamma(k,-P)$. The function Γ is defined in Eq. (1). Colour indices are denoted by i and j.

4. Form factors $F_+^{K\pi}$ and $F_-^{K\pi}$ as functions of Q^2, with $n = 3$. The position of the pole is denoted.

5. Typical anomalous vertices, with the "wrong" flavour structure.

6. The long-distance (L-D) contribution of diagrams on r.h.s. is assumed to produce anomalous vertices of order $\sim (m_c/M_W)^2$. Dashed lines correspond to gluons.

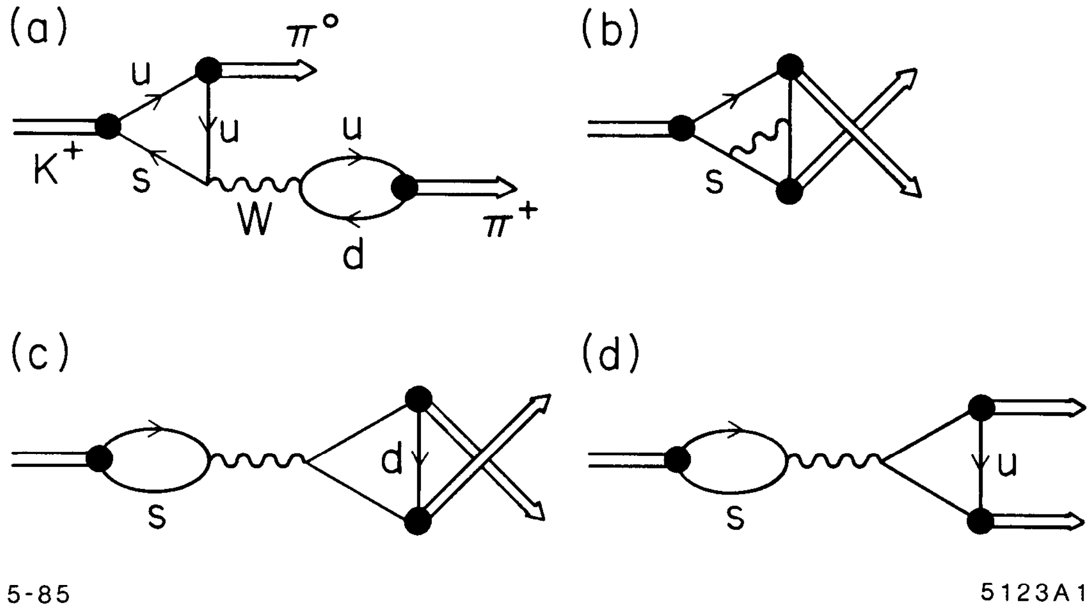


Fig. 1

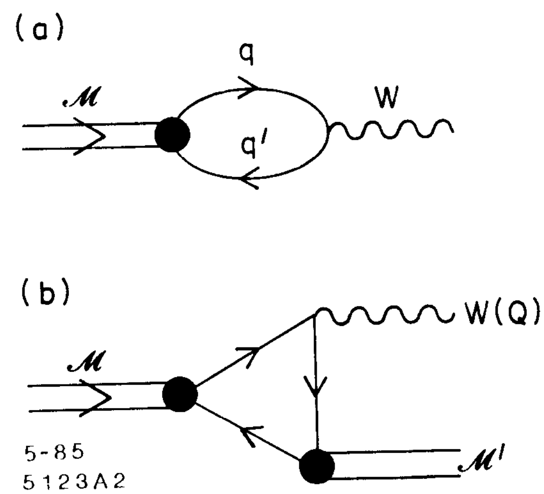
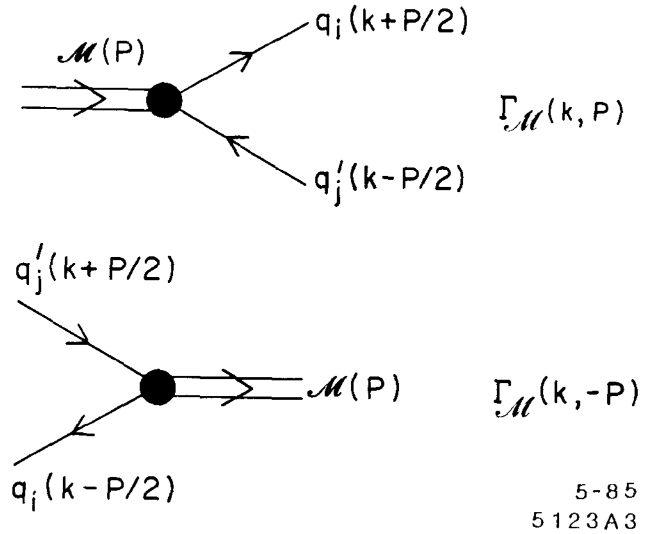


Fig. 2



$\mathcal{M}(P)$
$q_i(k+P/2)$
$q'_j(k-P/2)$
$\Gamma_{\mathcal{M}}(k,P)$
$q'_j(k+P/2)$
$\mathcal{M}(P)$
$q_i(k-P/2)$
$\Gamma_{\mathcal{M}}(k,-P)$
5-85
5123A3

Fig. 3

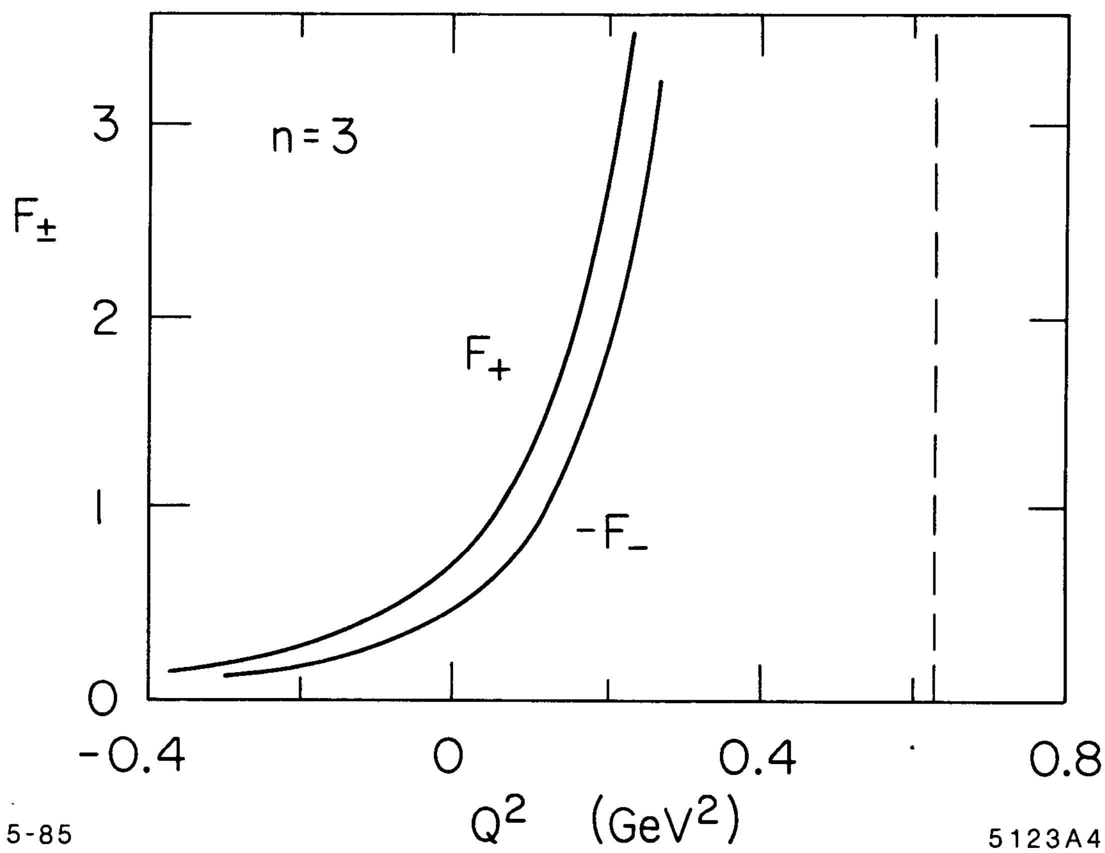


Fig. 4

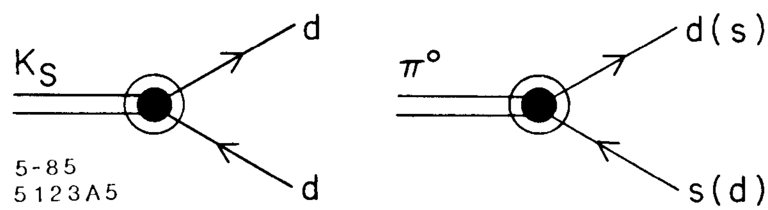


Fig. 5

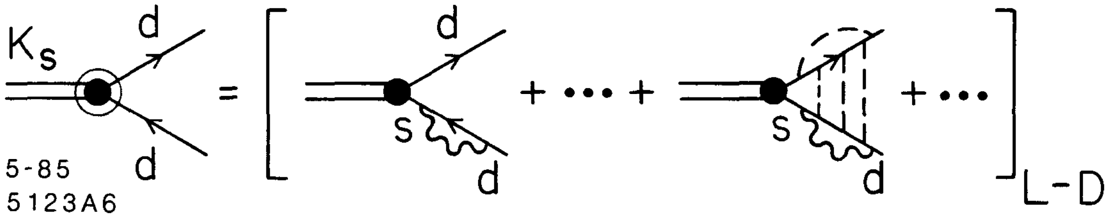


Fig. 6