# A DIAGNOSTIC FOR DYNAMIC APERTURE* 

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## INTRODUCTION

In large accelerators and low beta colliding beam storage rings, the strong sextupoles, which are required to correct the chromatic effects, produce strong nonlinear forces which act on particles in the beam. In addition in large hadron storage rings the superconducting magnets have significant nonlinear fields. To understand the effects of these nonlinearities on the particle motion there is currently a large theoretical effort using both analytic techniques and computer tracking. This effort is focused on the determination of the 'dynamic aperture' (the stable acceptance) of both present and future accelerators and storage rings. A great deal of progress has been made in understanding nonlinear particle motion, but very little experimental verification of the theoretical results is available. In this paper we describe 'dynamic tracking', a method being studied at the SPEAR storage ring, which can be used to obtain experimental results which are in a convenient form to be compared with the theoretical predictions.

## DESCRIPTION OF DYNAMIC TRACKING

In SPEAR it has been observed that for circulating currents greater than 2 ma the free betatron oscillation of a single bunch of electrons remains coherent. ${ }^{1}$ A coherent signal proportional to the transverse displacement of the electron bunch can be obtained by processing the signal from the beam position monitor electrodes. The method of dynamic tracking consists of exciting a free transverse betatron oscillation and then observing the transverse displacement at two different azimuths in the storage ring. By choosing the two pickup stations to have a $\pi / 2$ beiatron phase difference, the beam position at the second station is proportional to the beam angle at the first station. Hence, a plot of the beam position at the second station against the beam position at the first station (on a turn by turn basis) is equivalent to a phase plot of the particle motion at the first station. The two pickup stations used for dynamic tracking at SPEAR are 16 S 17 and $15 S 16$ respectively which are very nearly $\pi / 2$ apart in betatron phase. Since the bipolar signals from beam position monitors are too short to be read directly by a transient digitizer it is necessary to process the signals in such a way that we can obtain a reading proportional to the particles' transverse position on a turn-by-turn basis.

## SIGNAL PROCESSING

Two identical detectors constitute the interface between the pick-up electrodes and the transient digitizer. These detectors are similar to the circuits developed for the Transverse Feedback System of PEP and have been analyzed elsewhere in more detail. ${ }^{2}$ The basic approach for this detection consists of using a single pulse train as obtained from the sum of two adjacent electrodes, here 2 inward buttons, and to process the pulses' crests only.

[^0]The two buttons are in a 45 deg configuration; summing their signals is simply for the purpose of detecting the horizontal beam motion only. The power adder used to accomplish this does not modify in any way the sensitivity of the pick-up system which is

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\Delta x=\frac{\Delta V}{V} \frac{R}{\sqrt{2}} \quad(\Delta x<R)
$$

where $R=100 \mathrm{~mm}$ is the radius of the vacuum chamber and $\Delta V / V$ is the percentage of amplitude modulation observed on the pulse train. For a one millimeter bunch oscillation we get a $1.4 \%$ modulation.

The block diagram of Fig. 1 depicts the wide band processing of the beam pulses modulation. By wideband we mean that no filtering of any kind has been done, up to at least the tenth harmonic of the revolution frequency; this insures that the detector introduces no phase shifts for the significant sidebands of these harmonics.

The purpose of the automatic gain control loop (AGC) is to develop a pulse train having a constant average (therefore independent of both beam current variations and the residual DC orbit distortions), and to sample the peak of the pulse after shifting its baseline by a constant voltage (the AGC reference voltage). The trigger for this sampling is conveniently derived from the beam pulse train itself, since, except for the modulation riding on its crest, it has a constant amplitude. The final output of each detector resembles a staircase going up and down at the fractional betatron frequency, each step having a time duration of 790 nsec , the machine period.

## MEASUREMENTS

A single bunch of electrons is excited horizontally by pulsing one of the injection kickers. The coherent betatron oscillation amplitude is a linear function of the kicker voltage, while the damping rate is linearly related to both the current in the bunch and the chromaticity of the ring. In practice the coherent amplitude of the beam oscillation is displayed on an oscilloscope and the horizontal chromaticity adjusted to obtain damping times from less than 1000 turns to damping times greater than 10,000 turns. Presently the maximum number of turns that can be sampled is 2048. After the kicker is fired, a train of clock pulses at the SPEAR revolution frequency is used to sample the staircase output signals from the two beam pickup stations, once per revolution. These sampled signals are digitized and stored on the floppy disk of a Nicolet digital scope; the results are later transferred to a computer for analysis. It is important that the negative slope of the clock pulse occur during the flat portion of both of the staircase signals from the two position monitors. Figure 2 shows a display of the actual staircase output from the processor and the clock pulse train. Figure 3 displays the sampled signal as a function of revolution number and clearly shows the excitation of the coherent signal by the kicker and the subsequent decay.

A pseudo-phase plot can be obtained by plotting the coherent signal at position monitor $15 S 16$ against the coherent


Fig. 1. Signal processor for the beam position monitor pulses.


Fig. 2. (a) The output from the processor and (b) the clock pulse train.


Fig. 3. Position $x_{1}$ versus time.
signal at position monitor $16 S 17$ on a turn by turn basis (on the same turn the electron arrives at monitor 16 S 17 before monitor 15 S 17 ). The fact that the betatron functions are both
maximum and equal at the two position monitors means that the pseudo-phase plots of linear motion are circles when the phase difference is exactly $\pi / 2$. There are two ways to vary the betatron oscillation amplitude used in the pseudo-phase plots. The first is to vary the strength of the exciting kicker. The second is to vary the position of the time window where the motion is studied and allow the damping to reduce the oscillation amplitude to the desired value. Both methods have been used and give consistent results.


Fig. 4. Pseudo-phase space for 50 successive turns at 3 different amplitudes, $\nu_{z}=5.284, \nu_{y}=5.185$.

The horizontal pseudo-phase motion at three different oscillation amplitudes for tunes of $\nu_{x}=5.284$ and $\nu_{y}=5.185$ is shown in Fig. 4. The maximum amplitude shown in the figure is limited by the fact that the signal processing is not adequate at larger amplitudes. The motion appears to be nearly linear for all three ampiltudes shown in Fig 4. The slight departure of the motion from a perfect circle is due to the fact that the phase shift between the two position monitors was measured to be $0.44 \pi$ instead of $0.5 \pi$. In order to see a departure of the motion from linearity, studies were done at tunes of $\nu_{z}=$ 5.312 and $\nu_{y}=5.187$ (the third order resonance $\nu_{z}=16 / 3$
is an intrinsic resonance driven by the sextupole configuration which has an even periodicity in SPEAR). The pseudo-phase motion at three different oscillation amplitudes for the tunes of $\nu_{z}=5.312$ and $\nu_{y}=5.187$ is shown in Fig. 5. Note the appearance of the triangular shape phase motion at the large amplitude while the phase motion is fairly linear at the small amplitudes. This is indicative of phase motion near a third order resonance. In Fig. 6 all of the phase space points for the large amplitude oscillation are plotted to show the triangular shape more clearly.

The discrete Fourier transforms of the large amplitude oscillations displayed in Figs. 4 and 5 are shown in Figs. 7 and 8 respectively. Note the appearance of the additional frequencies, $2 \nu_{z}, 3 \nu_{x}$ and $4 \nu_{x}$ (aliased to less than 0.5 ) for the motion at the tune $\nu_{x_{2}}=5.284$, and the appearance of the additional frequencies $2 \nu_{x}, \ldots, 5 \nu_{x}$ at the tune of $\nu_{x}=5.312$. The frequency near 0.04 , in both figures, is due to a small amount of coherent synchrotron motion of the beam.


Fig. 5. Pseudo-phase space for $\mathbf{5 0}$ successive turns at 3 different amplitudes, $\nu_{z}=5.312, \nu_{y}=5.187$.


Fig. 6. Pseudo-phase space including 1790 turns for the large amplitude oscillation in Fig. 5, $\nu_{x}=5.312$, $\nu_{y}=5.187$.

Preliminary computer tracking studies using two different tracking codes have been done for lattice configurations close to the one discussed above. They show phase motion linear out


Fig. 7. Fourier transform of the large amplitude oscillation in Fig. 4.


Fig. 8. Fourier transform of the large amplitude oscillation in Fig. 5.
to amplitudes of about 4 cm , much larger than those shown in Figs. 4 to 8. A more detailed comparison between computer tracking and experiment is planned for the near future.

## ELECTRONICS LINEARITY AND DYNAMIC RANGE

Our experiment has been constrained, so far, by the inability of the electronics to process large beam oscillations (larger than 5 mm ). We have evidence that distortions occur in the processing of large beam kicks. We have attempted to simulate beam pulses having an initial modulation of large amplitude, decreasing exponentially, for the purpose of testing the linearity of the detectors. This test failed due to the lack of a linear modulator (a modulator that does not create new frequencies) which could generate a calibration signal with the mentioned wave shape. In spite of the great convenience offered by the AGC loop, it does not seem well suited to handling large oscillations. Thus we are contemplating a modification of this circuit to guarantee its linear operation over a larger dynamic range.

## REFERENCES

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