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CONFORMAL INVARIANCE AND PARASTATISTICS IN TWO DIMENSIONS*

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ABSTRACT

We show that the conformally invariant $\mathcal{O}(N)$ nonlinear σ–models with a Wess–Zumino term with coefficient k, have the same current and conformal algebras with a theory of N Majorana fermions with a hidden (gauged) $\mathcal{O}(k)$ quantum number. These latter are interpreted as N particles obeying the free Dirac equation but quantized with (nonlocal) parafermi statistics of order k, an interpretation that we verify by deriving the exact finite-temperature propagator, thereby explicitly relating the anomalous scaling dimensions to non-standard statistics.

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1. Introduction

Thanks mainly to the emergence of strings as potential cures to (practically) everything, a lot of attention has been recently drawn on conformally invariant two-dimensional field theories [1]. An interesting class of such theories consists of the nonlinear σ–models with action:

$$S = \frac{1}{4\lambda^2} \int Tr\, \partial^\mu g\, \partial^\mu g^{-1}\, d^2x + k\,\Gamma(g) \tag{1}$$

where g is an element of $\mathcal{O}(N)$ [or $\mathcal{U}(N)$]* and Γ is the Wess–Zumino term [2] with quantized coefficient ($k = 1, 2, \ldots ,$). Witten [3] has shown how at $\lambda^2 = 4\pi/k$ these theories become scale-invariant, and, for $k = 1$, equivalent to N free Majorana fermions. In this paper we will generalize Witten's result, by showing that for all k these theories have the same conformal and current algebras as N (Majorana) fermions with an internal (hidden or gauged) $\mathcal{O}(k)$ quantum number. This provides us with a fermionic representation of the infinite conformal (Virasoro) and current (Kac–Moody) algebras of the nonlinear σ–models (1). Besides its mathematical interest, this representation might prove useful in the study of candidate theories of strings. We should nevertheless point out that our result does not necessarily imply the quantum equivalence of these models, since the representation of the current and conformal algebras need not, in general, be unique.

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* We will limit ourselves to the group $\mathcal{O}(N)$, but our discussion is also applicable to $\mathcal{U}(N)$ if one replaces Majorana with Dirac fields in what follows.

Another possible interpretation of these models is in terms of N particles obeying the free Dirac equation but (nonlocal) parafermi statistics of order k [4,5]. We will in fact explicitly exhibit the relation between anomalous scaling dimensions and nonstandard statistics by deriving the exact finite-temperature propagator of any conformally covariant field. This interpretation is also made plausible by a formal transformation from the gauged-fermionic model.

In sect. 2, we prove the equivalence of the algebras of the σ–model (1) with those of the theory of gauged Majorana fermions. In sect. 3 this latter is formally transformed to a theory of Majorana particles quantized with anomalous (parafermi) statistics. In sect. 4 we show how to obtain the finite-temperature Green's functions of any scale-invariant theory, by means of a conformal mapping, and show how anomalous scaling dimensions are related to anomalous statistics. Finally, sect. 5 has some concluding remarks.

2. Equivalence with a Gauged Theory of Fermions

Let us begin by recalling that the $O(N)$ currents of the σ-model (1):

$$J_+^\alpha t^\alpha \equiv \frac{ik}{\pi} g^{-1} \, (\partial_+ g)$$

$$J_-^\alpha t^\alpha \equiv -\frac{ik}{\pi} \, (\partial_- g) \, g^{-1}$$

have the commutation relations [3]:

$$\left[J_+^\alpha(x_+), J_+^\beta(x_+')\right] = 2i\, f^{\alpha\beta\gamma}\, J_+^\gamma(x_+)\,\delta(x_+ - x_+') + \frac{ik}{2\pi}\,\delta^{\alpha\beta}\,\delta'(x_+ - x_+') \quad (2a)$$

$$\left[J_-^\alpha(x_-), J_-^\beta(x_-')\right] = 2i\, f^{\alpha\beta\gamma}\, J_-^\gamma(x_-)\,\delta(x_- - x_-') - \frac{ik}{2\pi}\,\delta^{\alpha\beta}\,\delta'(x_- - x_-') \quad (2b)$$

$$\left[J_+^\alpha(x_+), J_-^\beta(x_-')\right] = 0 \quad (2c)$$

where t^α and $f^{\alpha\beta\gamma}$ are the generators and structure constants of $\mathcal{O}(N)$, and we have used light-cone coordinates ($u_\pm = u_0 \pm u_1$ for any vector u). The energy-momentum tensor, on the other hand, is of the Sugawara type [6]:

$$T_{\substack{++ \\ (--)}} = \frac{1}{2(N+k-2)} \; : \; J^\alpha_{\substack{+ \\ (-}} J^\alpha_{\substack{+ \\ -)}} \; : \quad (3)$$

(where normal ordering is with respect to the current quanta), so that the conformal generators:

$$L_n \equiv \frac{1}{2\pi i} \oint x_+^{-n-1}\, T_{++}(x_+)$$

satisfy the algebra:

$$[L_n, L_m] = (n-m)\, L_{n+m} + \frac{c}{12}\, \left(n^3 - n\right)\, \delta_{n,-m} \quad (4a)$$

with

$$c = \frac{1}{2}\, \frac{N(N-1)k}{N+k-2} \quad (4b)$$

as shown by several authors [7-9]. The algebras Eqs. (2) and (4a) are known as Kac-Moody [10] and Virasoro [11] algebras, and are uniquely characterized by the coefficients of the anomalous terms, k and c respectively.

We will now show that these same current and conformal algebras can be derived from a theory of N Majorana fermions, with an internal gauged $\mathcal{O}(k)$ symmetry:

$$i\mathcal{L} = \frac{1}{2}\overline{\psi}^i_a \,\partial\!\!\!/\, \psi^i_a + \frac{1}{2}\,\overline{\psi}^i_a \,A\!\!\!/^I\, \lambda^I_{ij}\, \psi^j_a \quad . \tag{5}$$

Here and in what follows, $i, j = 1, \ldots, k$ are the $\mathcal{O}(k)$ ("color") indices, $a = 1, \ldots, N$ is the $\mathcal{O}(N)$ ("flavor") index, λ^I_{ij} are the antisymmetric generators of $\mathcal{O}(k)$, and summation over repeated indices is implied. A kinetic energy for the gauge fields A^I_μ has not been introduced, and will not be generated by renormalization in two dimensions.* Therefore, the A^I_μ are not real dynamical variables, but rather Lagrange multipliers whose equations of motion ensure the vanishing of all (normal-ordered) $\mathcal{O}(k)$-currents:

$$j^I_\mu \equiv \,:\, \overline{\psi}^i_a \lambda^I_{ij} \gamma_\mu \psi^j_a \,:\, = 0 \,. \tag{6}$$

In the language of constrained systems [13], these are second-class constraints that modify the canonical free-field commutation relations of any two operators according to:

$$[O_1, O_2] = [O_1, O_2]_f - \int d^2x\, d^2y\, [O_1, j^I_\mu(x)]_f M^{IJ,\mu\nu}(x,y) [j^J_\nu(y), O_2]_f \tag{7a}$$

* Bosonization of this model, with dynamical gauge fields, has been discussed in Ref. [12].

where $[\ ,\ ]_f$ stands for the canonical commutator calculated as if the fermions were free fields, and M is the inverse matrix of commutators of the constraints:

$$\int d^2y\, M^{IJ,\mu\nu}(x,y)\,[j^I_\nu(y), j^K_\rho(z)]_f = \delta^{IK}\delta^\mu{}_\rho\delta^{(2)}(x-z)\ . \qquad (7b)$$

Here the free commutators are defined with an $O(N)_L \times O(N)_R$ symmetric regularization, in which left and right fermion currents commute [14]. It should, furthermore, be pointed out that Eq. (7a) may, in principle, require modification when dealing with composite operators, due to operator ordering problems. Nevertheless, this won't be necessary for the purposes of our present discussion.

A couple of conclusions follow immediately from Eqs. (7a,b): firstly, the $O(k)$-currents j^I_μ commute with everything, as they should for the consistent implementation of the constraints (6). Secondly, if the operator O_1 is such that its *free-field* commutators with all $O(k)$-currents vanish, then for all O_2:

$$[O_1, O_2] = [O_1, O_2]_f\ .$$

The $O(N)$-currents:

$$J^\alpha_\mu =: \overline{\psi}^i_a t^\alpha_{ab}\gamma_\mu\psi^i_b\ :$$

in particular, satisfy $[J^\alpha_\mu, j^I_\nu]_f = 0$, so that their algebra can be calculated using free-fermion contractions. The result is easily seen to be the same as Eqs. (2a,b,c), since each color will make an equal independent contribution to the anomaly, whose strength will thus be trivially equal to k.

To calculate the algebra of the energy-momentum tensor we will use a trick. We consider the operator

$$T_{++} = \frac{1}{2(N+k-2)} \ : \ J_+^\alpha J_+^\alpha \ : + \frac{1}{2(N+k-2)} \ : \ j_+^I j_+^I \ : \ . \tag{8}$$

As can be easily shown, its *free-field* commutators with the $\mathcal{O}(N)$ and $\mathcal{O}(k)$ currents, as well as with itself, are identical to the corresponding free-field commutators of the energy-momentum tensor. In particular the central charge of the Virasoro algebra is

$$\frac{1}{2}\ \frac{N(N-1)k}{N+k-2} + \frac{1}{2}\ \frac{k(k-1)N}{N+k-2} = \frac{1}{2}\ Nk \ .$$

which is the correct anomaly for Nk free Majorana fermions. The first and second terms on the left-hand side correspond to the first and second terms in expression (8).

Since for calculating the actual (modified Dirac) algebra all one needs are these free-field commutators (see Eqs. (7)), we can legitimately use expression (8) in place of the real energy-momentum tensor of the theory.

We are now almost done. Indeed, as explained above, the $\mathcal{O}(k)$ currents j^I have vanishing Dirac commutators with everything,[⋆] while the $\mathcal{O}(N)$ currents J^α have identical Dirac and free-field commutation relations, and in particular satisfy the algebra (2a,b,c). Hence, the modified Virasoro algebra is identical to

⋆ The careful reader might want to verify that operator orderings do not destroy this result, that is that the composite operator $: j^I j^I :$ also has vanishing (modified) commutators with everything.

(4a,b) and in particular the central charge is $\frac{1}{2}N(N-1)k/(N+k-2)$, which completes our proof.

Let us note in passing that the fermionic Lagrangian [Eq. (5)] can be trivially supersymmetrized by adding "color"-singlet free fermions in the adjoint representation of $\mathcal{O}(N)$; the supersymmetry is then non-linearly realized, but this will not be discussed any further here. In the remainder of this paper we would like to give an alternative interpretation of the conformally invariant models [Eq. (5)] as theories of parastatistical particles.

3. Parastatistics

Parastatistics were introduced more than thirty years ago by Green [4,5], who noticed that canonical (anti)commutation relations were not the only ones consistent with the axioms of field theory. For instance, assuming a unique vacuum, the most general quantum field [15] satisfying the free Dirac equation can be written:

$$\Psi(x) \;=\; \sum_{i=1}^{k} \Psi_{(i)}(x)$$

where the auxiliary mathematical entities $\Psi_{(i)}$, called Green components, are free (Majorana) fields that obey normal equal-time anticommutation relations for the same Green index, while commuting for different Green indices:

$$\left\{\Psi_{(i)}(x),\, \Psi_{(i)}(y)\right\}\big|_{x_0\,=\,y_0} \;=\; -\left(\gamma_0 C\right)\,\delta\left(x_1-y_1\right) \tag{9a}$$

$$\left[\Psi_{(i)}(x),\, \Psi_{(j)}(y)\right]\big|_{x_0\,=\,y_0} \;=\; 0 \quad \text{if} \quad i\neq j \tag{9b}$$

Here C is the charge-conjugation matrix. Note that only $\Psi(x)$ has a direct physical meaning, and all observables should be built out of it; k is called the order or parastatistics and is the maximum number of particles that can simultaneously occupy a state. If the parafields also carry an $\mathcal{O}(N)$ "flavor" index, the anomalous commutation relations can be extended in an $\mathcal{O}(N)$–invariant way by modifying Eq. (9a) to:

$$\left\{ \Psi^a_{(i)}(x),\, \Psi^b_{(i)}(y) \right\} \Big|_{x_0=y_0} \;=\; -\,(\gamma_0\, C)\; \delta^{ab}\, \delta\,(x_1 - y_1) \qquad (9a')$$

while retaining Eq. (9b) for all possible "flavor" indices.

We should here clarify that although the field Ψ obeys the free Dirac equation, it really corresponds to a particular interacting theory; the interactions are manifested through the nonstandard statistics.

Now the $\mathcal{O}(k)$–gauged fermionic model [Eq. (5)] can be formally transformed into a theory of parafermions of order k. The transformation, which is nonlocal in the fields, but local in all gauge-invariant bilinear observables, can be constructed in two steps as follows. We first define new fields by attaching path ordered exponentials to the fermions:

$$\varphi^a_{+i}(x) \equiv P \exp \left\{ i \int_0^{x_-} A_- dx_- \right\}_{ij} \psi^a_{+j}(x) \qquad (10)$$

$$\varphi^a_{-i}(x) \;\equiv\; P\; exp \left\{ i \int_0^{x_+} A_+ dx_+ \right\}_{ij} \psi^a_{-j}(x)\;. \qquad (11)$$

where ψ_+, ψ_- are the left and right handed components of ψ respectively. The fields $\varphi^a_{\pm i}(x)$ obey the free Dirac equation, and have a global (space-time independent) gauge-transformation law, as can be easily seen. There now exists a well known Klein transformation [16,17], that converts such fields to free parafermions (with anomalous commutation relations):

$$\Psi^a_{(i)}(x) \;=\; K_i\,\varphi^a_i(x)$$

where

$$K_i \;=\; exp\left\{ i\pi \int dx_1 \sum_{j=i}^{k} \overline{\varphi}^b_j\,\varphi^b_j \right\} \quad .$$

To check the validity of this formal transformation, a more careful treatment would be required. We here only point out that this interpretation of the gauged fermionic model [Eq. (5)] is not very surprising, since in two dimensions the gauge symmetry has no dynamical content, but simply provides a hidden quantum number that modifies the particle statistics. A familiar analogy may help make this clear: suppose that the scale of real-world chromodynamics (Λ_{QCD}) is sent to infinity. Confinement would then be local and glueball states will disappear from the spectrum. One would be left with a theory of free flavor currents with modified statistics—the only souvenir of the unobservable color. It is in fact amusing to recall that color was historically first introduced as the hidden Green index of paraquarks, that explained the observed symmetry of baryon wavefunctions [18].

4. Scale Invariant Theories at Finite Temperature

One can also arrive at the above interpretation of the σ–models by what appears to be a completely different route, namely by considering these theories at finite temperature. Let us begin by recalling that in a scale-invariant Euclidean two-dimensional theory, a conformal transformation:

$$\phi'_\ell(z', \bar{z}') = \left(\frac{\partial z}{\partial z'}\right)^{\Delta_\ell} \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{\Delta}_\ell} \phi_\ell(z, \bar{z}) \tag{12}$$

leaves the field equations invariant, while in general changing the boundary conditions. Here $z = x_1 + ix_0$ and $\bar{z} = x_1 - ix_0$ are the imaginary-time light-cone coordinates, $z'(z)$ is holomorphic and $\Delta_\ell \pm \bar{\Delta}_\ell$ are the scaling dimension and spin of the field, respectively. Finite-temperature Euclidean Green's functions, on the other hand, obey the field equations with periodic (antiperiodic) boundary conditions in the time direction for integral (half-integral)-spin fields, respectively [19].

Consider now, in particular, the mapping:*

$$z' = \frac{1}{2\pi T} \ell n \left(2\pi T z + 1\right) \tag{13}$$

of the entire complex plane onto the strip $0 \leq x'_0 \leq T^{-1}$. It follows from Eq. (12) that, under this transformation, integral (half-integral)-spin fields become periodic (antiperiodic) in time with period T^{-1}. Zero-temperature Green's functions are thus automatically transformed into their finite-temperature counterparts. Consider, for instance, a free fermion; in the chiral representation the

* This mapping has also been used by Cardy [20], to explain the observed universality of finite-size scaling amplitudes in two dimensions.

upper and lower components have $\Delta = 1/2$, $\bar{\Delta} = 0$ and $\Delta = 0$, $\bar{\Delta} = 1/2$, respectively. The Dirac operator is:

$$\not{\partial} = \begin{pmatrix} 0 & \partial/\partial z \\ \partial/\partial \bar{z} & 0 \end{pmatrix}$$

and its inverse:

$$\not{\partial}^{-1} = \begin{pmatrix} 0 & (2\pi z)^{-1} \\ (2\pi \bar{z})^{-1} & 0 \end{pmatrix} \quad .$$

Applying transformation (13) yields:

$$D_{ferm,T} = \begin{pmatrix} 0 & T/\sin h\,(\pi T z) \\ T/\sin h\,(\pi T \bar{z}) & 0 \end{pmatrix}$$

which is indeed the finite-temperature free-fermionic propagator.

More generally, for any field ϕ_ℓ with nonvanishing† scaling dimension $\eta = \Delta_\ell + \bar{\Delta}_\ell$, the zero-temperature propagator is (up to a normalization constant):

$$\langle \phi_\ell\,(z, \bar{z})\ \phi_\ell(0) \rangle = z^{-2\Delta_\ell}\ \bar{z}^{-2\bar{\Delta}_\ell}$$

and the finite-temperature propagator becomes:

$$D_{\ell,T} = (\pi T)^{2\eta} \cdot \sin h^{-2\Delta_\ell}\ (\pi T z) \cdot \sin h^{-2\bar{\Delta}_\ell}\ (\pi T \bar{z}) \quad .$$

† Mapping Eq. (13) can actually also be used to obtain the finite-temperature free-boson propagator. The calculation is, however, in this case subtle and must be carried out in momentum space with an infinitesimal-mass-prescription that cures infrared divergences [21]. We only mention this in order to make an interesting comment: since the same mapping transforms both bosons and fermions to finite-temperature, supersymmetry is not explicitly broken by a heat bath [22], at least in the case of two-dimensional scale-invariant theories.

At large distances this falls off exponentially with an effective mass proportional to the scaling dimension:

$$m_{eff} = 2\pi T \eta \quad . \tag{14}$$

This is the energy required in order to create a zero-momentum excitation at finite temperature. It vanishes, as expected, for free bosons ($\eta = 0$), while for free fermions ($\eta = 1/2$), because of the exclusion principle, it equals the average energy of the nearest unoccupied level:

$$m_{eff} = \pi T \quad .$$

For parafermions of order k, one would of course expect:

$$m_{eff} = \frac{\pi T}{k} \tag{15}$$

since up to k particles can occupy the same state.

Now the critical exponents for the nonlinear σ–models have been computed by Knizhnik and Zamolodchikov [7]. In fact their calculation only makes use of the conformal and current algebras, and the transformation properties of fields, but not of the specific form of the Lagrangian. For fields transforming in the fundamental $\otimes$ trivial representation of $\mathcal{O}(N)_L \times \mathcal{O}(N)_R$, they find:

$$\Delta = \frac{N-1}{2(N+k-2)} \quad ; \qquad \bar{\Delta} = 0 \quad .$$

Putting $N = 2$, one indeed recovers the answer expected of a Dirac parafermion of order k, Eq. (15). For $N > 2$, on the other hand, the parastatistics of N different species of Majorana particles is considerable more complicated, as shown by the formula for the effective mass:

$$m_{eff} = \frac{\pi T(N-1)}{N+k-2} \quad .$$

Note that the fact that the critical exponent is neither integer nor half-integer, implies that the propagator has a cut, which is due to the nonlocal character of the parafermions.

5. Conclusions

In conclusion, we have argued for the equivalence of the scale-invariant nonlinear $\mathcal{O}(N)$ σ–models with theories of massless Majorana particles with anomalous statistics. The coefficient of the Wess-Zumino term was identified with the order of parastatistics. An alternative, equivalent, representation that can be trivially supersymmetrized is in terms of fermions with a gauged (hidden) $\mathcal{O}(k)$ quantum number. We have also derived the most general finite-temperature propagator of a conformally covariant field, and thereby explicitly related anomalous scaling dimensions to anomalous statistics.

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Note added in proof : After submission of this paper there appeared a preprint by Goddard, Nahm and Olive [23] where it is shown that *for some exceptional values* of the Wess–Zumino coefficient, the σ–models have the same algebraic structure as theories of *free* fermions. This should be contrasted to the present work where the Wess–Zumino coefficient is left arbitrary but the fermions are not necessarily free. We cannot however conclude that for these exceptional values of the Wess–Zumino coefficient the gauged fermionic model is quantum equivalent to a theory of free fermions, since as we already pointed out the representation of the current and conformal algebra, need not in general be unique.

References

[1] A. Polyakov, Pisma ZhETP 12 (1970) 538 (JETP Lett. 12, 381); A. Belavin, A. Polyakov, A Zamolodchikov, Nucl. Phys. B241 (1984) 333 ; D. Friedan, Z. Qiu, S. Shenker, Phys. Rev. Lett. 52 (1984) 1575.

[2] J. Wess, B. Zumino, Phys. Lett. 37B (1971) 95.

[3] E. Witten, Commun. Math. Phys. 92 (1984) 455.

[4] H. S. Green, Phys. Rev. 90 (1953) 270.

[5] Y. Ohnuki, S.Kamefuchi, Quantum Field Theory and Parastatistics (Univ. of Tokyo Press and Springer-Verlag, Tokyo, 1982), and references therein.

[6] H. Sugawara, Phys. Rev. 170 (1968) 1659.

[7] V. G. Knizhnik, A. B. Zamolodchikov, Nucl. Phys. B247 (1984) 83.

[8] D. Nemeschansky, S. Yankielowicz, Phys. Rev. Lett. 54 (1985) 620.

[9] P. Goddard, D. Olive, Nucl. Phys. B257 (1985) 226; S. Jain, R. Shankar, S. Wadia, TIFR/TH/85–3 preprint (1985).

[10] V. G. Kac, J. Funct. Anal. Appl. 8 (1974) 68; I. Frenkel, V. G. Kac, Invent. Math. 62 (1980) 23; G. Segal, Commun. Math. Phys. 80 (1981) 301.

[11] M. Virasoro, Phys. Rev. D1 (1970) 2933.

[12] D. Gonzales and A. N. Redlich, Phys. Lett. 147B (1984) 150; D. Gepner, Nucl. Phys. B252 (1985) 481.

[13] P.A.M. Dirac, Canad. J. Math. 2 (1950) 129; A. Hanson, T. Regge and C. Teitelboim in "Constrainted Hamiltonian Systems" ed. Academia Nazionale dei Lincei, Roma (1976) and references therein. For a generalization of Dirac's quantization to fermionic (anticommuting) fields see: C. Teitelboim, Lectures at Princeton University (1977), unpublished; R. Casalbuoni, Nuovo Cimento 33A (1976) 115, 389; F. A. Berezin and M. S. Marinov, Ann. Phys. 104 (1977) 336.

[14] P. di Vecchia and P. Rossi, Phys. Lett. 140B (1984) 344; P. di Vecchia, B. Durhuus and J. L. Petersen, Phys. Lett. 144B (1984) 245; Y. Frishman, Phys. Lett. 146B (1984) 204.

[15] O. Greenberg, A. Messiah, Phys. Rev. 138 (1965) 1155.

[16] O. Klein, J. Phys. Rad. (7), 9 (1938) 1; Y. Ohnuki, S. Kamefuchi, Prog. Theor. Phys. 50 (1973) 258.

[17] Y. Nambu, M. Y. Han, Phys. Rev. D10 (1974) 674.

[18] O. Greenberg, Phys. Rev. Lett. 13 (1964) 598.

[19] R. Feynman, A. Hibbs, Quantum Mechanics and Path Integrals (McGraw–Hill, 1965); C. Bernard, Phys. Rev. D9 (1974) 3312; L. Dolan, R. Jackiw, ibid., 3320; S. Weinberg, ibid., 3357.

[20] J. Cardy, Nucl. Phys. B240 [FS 12] (1984) 514; J. Phys. A: Math. Gen. 17 (1984) L385.

[21] I. Antoniadis, C. Bachas, unpublished.

[22] A. Das, M. Kaku, Phys. Rev. D18 (1978) 4540; L. Girardello, M. T. Grisaru, P. Solomonson, Nucl. Phys. B178 (1981) 331; L. Van Hove, Nucl. Phys. B207 (1982) 15; D. Boyanovsky, Phys. Rev. D29 (1984) 743; H. Aoyama, D. Boyanovsky, Phys. Rev. D30 (1984) 1356.

[23] P. Goddard, W. Nahm and D. Olive, Phys. Lett. 160B (1985) 111.