

A COMPARISON OF THE PLASMA BEAT WAVE ACCELERATOR AND THE PLASMA WAKE FIELD ACCELERATOR*

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ABSTRACT

In this paper we compare the Plasma Beat Wave Accelerator and the Plasma Wake Field Accelerator. We show that the electric fields in the plasma for both schemes are very similar, and thus the dynamics of the driven beams are very similar. The differences appear in the parameters associated with the driving beams. In particular to obtain a given accelerating gradient, the Plasma Wake Field Accelerator has a higher efficiency and a lower total energy for the driving beam.

INTRODUCTION

Recently there have been two similar types of plasma accelerator schemes proposed. The Plasma Beat Wave Accelerator (PBWA)^{1,2} employs two laser beams beating at the plasma frequency to drive the plasma while the Plasma Wake Field Accelerator (PWFA)^{3,4} replaces the laser beams by a bunched relativistic electron beam. Since the two schemes make use of different sources, the corresponding mechanisms that drive the plasma waves are different. In the PBWA, it is the ponderomotive force which comes from the beating lasers that drives the plasma, whereas in the PWFA the driving bunch is decelerated by the plasma and thus transfers energy to the plasma wave. Other than this difference, however, the two schemes are very similar. In both cases large longitudinal electric fields are generated in the plasma which oscillates at the fundamental plasma frequency ω_p . These fields are then used to accelerate an electron beam. It is interesting to ask how these two schemes compare to each other in detail. To make a fair comparison, in this paper we emphasize self consistency among the various accelerator parameters common to the schemes. We will follow Refs. 2 and 4 in most of the calculations; however, we will include transverse effects in the PBWA to calculate and compare focusing effects.

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FIELDS IN A PLASMA WAVE OF FINITE EXTENT

To find the electric fields in the plasma waves for both schemes, we start with the linearized, nonrelativistic fluid equations,

$$\frac{\partial n_1}{\partial t} + n_0(\nabla \cdot \vec{v}_1) = 0 \quad (1)$$

$$\frac{\partial \vec{v}_1}{\partial t} = \frac{e\vec{\mathcal{E}}_1}{m} + \frac{\vec{F}_{ext}}{m},$$

and solve for the perturbed plasma density n_1 . $\vec{\mathcal{E}}_1$ is the electric field due to n_1 and \vec{F}_{ext} is the external force due to either a driving beam or a beating laser. In the case of the PBWA the force is most easily calculated from a Hamiltonian which has been averaged over the fast oscillation of the laser frequency. This leaves only the beating effect at a frequency ω_p . The averaged Hamiltonian is given by

$$H = \frac{\vec{p}^2}{2m} + e\phi_1 + \frac{e^2}{4m\omega^2} E_0^2(r) \cos(k_p z - \omega_p t) \quad (2)$$

where ω and ω_p are the laser and the plasma frequency respectively and k_p is the plasma wave number. The last term is simply the ponderomotive potential due to a beating laser with a finite cross section. For the sake of a comparison with the PWFA later in this paper, we will assume a radial dependence of the ponderomotive potential given by

$$E_0^2(r) = 2E_0^2 \begin{cases} K_2(k_p a) I_0(k_p r) + \frac{1}{2} - \frac{2}{(k_p a)^2} - \frac{r^2}{2a^2} & r < a \\ I_2(k_p a) K_0(k_p r) & r > a \end{cases} \quad (3)$$

where K_n and I_n are modified Bessel functions. This radial profile is parabolic near the origin but falls off exponentially for $r > a$. It was chosen to yield a simple parabolic dependence in Eq. (4) below.

To use the above results we need the divergence of the force due to the Hamiltonian in Eq. (2). This is given by

$$\nabla \cdot \vec{F} = 4\pi e^2 n_1 + \frac{e^2 E_0^2 k_p^2}{4m\omega^2} (1 - r^2/a^2) \cos(k_p z - \omega_p t) \quad r < a, \quad (4)$$

where Poisson's equation has been used to substitute for $\nabla^2 \phi_1$. Substituting into Eq. (1) yields

$$\frac{\partial^2 n_1}{\partial t^2} + \omega_p^2 n_1 = \begin{cases} - \left(\frac{\omega_p}{\omega}\right)^2 \frac{E_0^2 k_p^2}{16\pi m} (1 - r^2/a^2) \cos(k_p z - \omega_p t) & r < a \\ 0 & r > a \end{cases}, \quad (5)$$

which has a solution of the form

$$n_1(r, z, t) = f(r, z, t) \sin(k_p z - \omega_p t), \quad (6)$$

where

$$f(r, z, t) = \begin{cases} -\frac{E_0^2 k_p^2}{32\pi m \omega^2} (1 - r^2/a^2) (k_p z - \omega_p t) & r < a \\ 0 & r > a \end{cases} \quad (7)$$

With $n_1(r, z, t)$ in hand, we now must find the electric field $\vec{\mathcal{E}}_1$ due to the plasma oscillation. Since the magnetic field due to a linear plasma wave vanishes, we can simply use Poisson's equation,

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \phi_1 \right) + \frac{\partial^2 \phi_1}{\partial z^2} = -4\pi e n_1. \quad (8)$$

If we have a laser pulse of length τ , at the end of the pulse the amplitude of the plasma density wave will reach its peak value. From Eq. (7) this is given by

$$f_{max}(r) = \frac{\omega_p \tau E_0^2 k_p^2}{32\pi m \omega^2} (1 - r^2/a^2) \quad r < a, \quad (9)$$

and the potential can be shown to be

$$\phi_1 = R(r) \sin(k_p z - \omega_p t) \quad (10)$$

with

$$R(r) = \frac{\omega_p \tau e E_0^2}{4\omega^2 m} \begin{cases} K_2(k_p a) I_0(k_p r) + \frac{1}{2} \left(1 - \frac{r^2}{a^2} \right) - \frac{2}{(k_p a)^2} & , \quad r < a \\ I_2(k_p a) K_0(k_p r) & , \quad r > a \end{cases} \quad (11)$$

The longitudinal and transverse electric fields for $r < a$ for the PBWA are thus given by

$$\begin{aligned} \mathcal{E}_z &= -\frac{\omega_p \tau k_p e E_0^2}{4\omega^2 m} \left\{ K_2(k_p a) I_0(k_p r) + \frac{1}{2} \left(1 - \frac{r^2}{a^2} \right) - \frac{2}{(k_p a)^2} \right\} \cos(k_p z - \omega_p t), \\ \mathcal{E}_r &= -\frac{\omega_p \tau k_p e E_0^2}{4\omega^2 m} \left\{ K_2(k_p a) I_1(k_p r) - \frac{r}{k_p a^2} \right\} \sin(k_p z - \omega_p t). \end{aligned} \quad (12)$$

For the case of the PWFA the situation is very similar. We only need to change the laser source term in Eq. (5). For the case of a driving beam of density n_b , the divergence of the force is given by

$$\nabla \cdot \vec{F} = 4\pi e^2 (n_1 + n_b). \quad (13)$$

Following Ref. 4, consider a driving beam with density profile

$$n_b = \sigma(r)\delta(z - v_b t) . \quad (14)$$

Then the solution for the perturbed density is given by

$$n_1(r) = \begin{cases} k_p \sigma(r) \sin(k_p z - \omega_p t) & k_p z - \omega_p t < 0 \\ 0 & k_p z - \omega_p t > 0 . \end{cases} \quad (15)$$

To compare with the PBWA we use a parabolic distribution given by

$$\sigma(r) = \begin{cases} \frac{2N}{\pi a^2} (1 - r^2/a^2) & r < a \\ 0 & r > a \end{cases} , \quad (16)$$

where N is the total number of particles in the driving bunch. Once again it is possible to calculate the longitudinal and transverse electric fields due to the plasma wave.⁴ These are given by

$$\begin{aligned} \mathcal{E}_z &= \frac{-16eN}{a^2} \left\{ K_2(k_p a) I_0(k_p r) + \frac{1}{2} - \frac{2}{(k_p a)^2} - \frac{r^2}{2a^2} \right\} \cos(k_p z - \omega_p t) , \quad r < a \\ \mathcal{E}_r &= \frac{-16eN}{a^2} \left\{ K_2(k_p a) I_1(k_p r) - \frac{r}{k_p a^2} \right\} \sin(k_p z - \omega_p t) , \quad r < a . \end{aligned} \quad (17)$$

Thus the electric fields for the two schemes turn out to be remarkably similar.

For reasons which we will discuss later the transverse size of the driven beam must be somewhat smaller than the transverse size of the laser beams or the driving electron beam. In addition if $k_p a \gg 1$, then the electric fields for both schemes are of the following form:

$$\begin{aligned} \mathcal{E}_z &\simeq -A \left(1 - \frac{r^2}{a^2}\right) \cos(k_p z - \omega_p t) \\ \mathcal{E}_r &\simeq 2A \frac{r}{k_p a^2} \sin(k_p z - \omega_p t) \end{aligned} \quad (18)$$

where

$$A = \begin{cases} \frac{\omega_p r k_p e E_0^2}{8\omega^2 m} & PBWA \\ \frac{8eN}{a^2} & PWFA \end{cases} . \quad (19)$$

Other than different coefficients, the forces that the driven electrons experience share the same physical characteristics in both schemes. To be specific there is

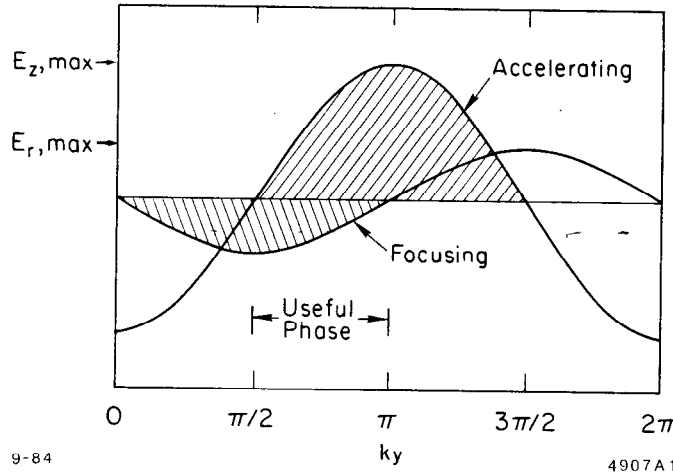


Fig. 1.

a longitudinal force $e\mathcal{E}_z$ that either accelerates or decelerates the driven bunch of electrons, and there is a transverse force $e\mathcal{E}_r$ shifted in phase which either will focus or defocus the driven bunch (see Fig. 1). From Fig. 1 it is clear that we have both acceleration and focusing over $1/4$ of the plasma wavelength.

ACCELERATOR PHYSICS ISSUES

In this section we discuss some accelerator physics issues which are relevant to both schemes of plasma accelerators. To begin we concentrate on the quality and intensity of a driven electron bunch with finite transverse extent. In particular we treat the transverse oscillations and the energy spread due to the transverse variation of the accelerating field. We then discuss other issues such as phase slippage, spot size and driving beam energy for the PBWA and PWFA. The details in the discussion of these issues are different for the two schemes since we choose to fix different parameters in the two cases. Finally, in order to address the question of intensity, we discuss the efficiencies of both schemes.

The Beta Function

In this paper the beta function is defined to be the wavelength/ 2π of the transverse oscillation at some instantaneous phase ϕ along the plasma wave. In the last section we saw that, except for a difference in coefficients, the PBWA and PWFA have the same electric fields. We also pointed out that there is a useful phase between $\pi/2$ and π along the plasma wave. In general there will be some phase slippage between the plasma wave and the driven beam. If this phase slippage is slow, then we can calculate the transverse focusing effects as if the beam were at a fixed phase on the wave. The differential equation governing

the transverse oscillations of a highly relativistic particle is

$$\frac{d^2 x}{dz^2} = e \frac{\mathcal{E}_x}{\gamma m c^2}, \quad (20)$$

where $\gamma m c^2$ is the particle's instantaneous relativistic mass.- Thus, for small radius from Eq. (18) we have

$$\frac{d^2 x}{dz^2} = e \left[\frac{e A \sin \phi}{k_p a^2 \gamma m c^2} \right] x. \quad (21)$$

Identifying the coefficient of x above with β^{-2} yields the beta function:

$$\beta = \left[\frac{k_p a^2 \gamma m c^2}{e A \sin \phi} \right]^{1/2}. \quad (22)$$

Energy Spread

From Eq. (18) it is evident that for a driving beam with finite transverse size, the longitudinal field varies transversely. Consider a driven bunch with transverse radius b which moves along the axis of the plasma wave. Since the field varies parabolically in the transverse direction, the average energy gain is reduced slightly and an energy spread is induced. If we assume that the beam is already very relativistic, then the average change in energy for one stage is

$$\Delta E_{ave} = \Delta E \left(1 - \frac{2}{3} \left(\frac{b}{a} \right)^2 \right), \quad (23)$$

where ΔE is the energy gain for a particle on the axis of the plasma wave. The corresponding energy spread induced in one stage for the model we have chosen is

$$\left[\frac{\delta(\Delta E)}{\Delta E} \right]_{rms} = \frac{\sqrt{2}}{3} \left(\frac{b}{a} \right)^2. \quad (24)$$

The Trapping Parameter

The trapping parameter is defined to be the ratio of the plasma density perturbation n_1 and the unperturbed density n_0 . Physically, this parameter indicates the linearity of the plasma oscillation. Since we work in the linear approximation for the plasma wave in both schemes, α should be kept reasonably small. For the case of the PBWA, we assume that the plasma oscillation

saturates at the end of the laser, which corresponds to⁵

$$\alpha \equiv \frac{n_1}{n_0} \simeq \frac{1}{4} \quad PBWA. \quad (25)$$

For the case of the PWFA we take L and \mathcal{E}_z as chosen parameters. In addition, to scale the transverse effects we fix the ratio between the transverse size of the driving bunch and the plasma wavelength: a/λ_p . This in turn determines the plasma wavelength and the plasma density. In order to check that the plasma wave so generated is indeed a linear wave, we must calculate α , which in this case is given by

$$\alpha = \frac{e\mathcal{E}_z}{mc\omega_p} \quad PWFA. \quad (26)$$

Phase Slippage

For both accelerator schemes the phase velocity of the plasma wave is not equal to the velocity of the driven bunch. This means that the driven bunch will slip in phase along the plasma wave as it is accelerated. For the PBWA we maximize \mathcal{E}_z for a given L by optimizing the phase shift δ . If we choose a laser frequency ω , an acceleration length L , and a phase slippage δ for speed of light particles; then the plasma frequency is given by²

$$\omega_p = \left(\frac{2\delta c\omega^2}{L} \right)^{1/3}. \quad (27)$$

On the other hand, the acceleration gradient that the driven bunch sees varies along L due to the phase slippage. If the total phase slippage over the entire acceleration length is δ , then the average acceleration gradient is related to the ideal gradient by a phase slip form factor $\sin \delta / \delta$, that is

$$e\mathcal{E}_z^{ave} = \alpha mc\omega_p \frac{\sin \delta}{\delta}. \quad (28)$$

Here the phase has been allowed to slip from the top of the cosine down one side so that the bunch is always in a focusing region. The average acceleration gradient can be maximized for a given L if

$$\delta \simeq \frac{5\pi}{16} \quad \text{and} \quad \frac{\sin \delta}{\delta} \simeq 0.85 \quad PBWA. \quad (29)$$

For the PWFA we consider only relativistic driving and driven bunches. In addition we require that the final energy of the driving bunch after the distance

L is still relativistic. In this case we can calculate the phase slippage along the plasma wave since the plasma wave phase velocity is equal to the velocity of the driving bunch. Following Ref. 4 we integrate the relative velocity along the length L to obtain

$$\delta \simeq \frac{\pi L}{\lambda_p} [(\gamma_{1i}\gamma_{1f})^{-1} - (\gamma_{2i}\gamma_{2f})^{-1}] \quad PWFA. \quad (30)$$

Since in an actual high energy accelerator the second term would be quite small, we will neglect it when using Eq. (30).

The Transverse Size

We need the transverse size to calculate the transverse dynamics of the driven bunch. For the PBWA to make the optimum use of the laser beam it is necessary to match the Rayleigh length R to the acceleration section. We choose the section to be twice the Rayleigh length. This in turn determines the diffraction limited spot size,

$$a^2 = \frac{R\lambda}{\pi} = \frac{L\lambda}{2\pi} = \frac{2\delta c^2 \omega}{\omega_p^3} \quad PBWA, \quad (31)$$

where Eq. (27) has been used to eliminate L . For the PWFA since we would like to fix the number of particles in the driving bunch, the transverse size is determined by the desired accelerating field,

$$a = \left[\frac{8r_e N_1 m c^2}{e \mathcal{E}_z} \right]^{1/2} \quad PWFA, \quad (32)$$

where r_e is the classical electron radius.

The Energy Requirement

In the PBWA the laser beam power for the beam profile given in Eq. (3) is

$$W = \frac{\pi a^2 E_0^2 c}{2 \cdot 8\pi}. \quad (33)$$

If we assume that we have a laser pulse length τ , the energy necessary to drive the plasma wave density to αn_0 is²

$$W\tau = \frac{\alpha \delta m^2 c^5}{e^2 \omega_p} \left(\frac{\omega}{\omega_p} \right)^3 \quad PBWA. \quad (34)$$

where Eq. (31) has been used to eliminate a^2 . On the other hand, the energy in the driving bunch for the PWFA is simply given by

$$W\tau = N_1 E_1 \quad PWFA. \quad (35)$$

The Efficiency

The overall efficiency of the accelerators here can be divided into three parts. The first part is the efficiency of conversion of 'wall plug' energy to either laser energy or electron beam energy. These two efficiencies may be quite different, however, we will not discuss them here. The second efficiency is the conversion of either laser or electron beam energy to plasma energy. The third efficiency is that for conversion of the plasma energy to the driven electron beam. The efficiency of the transfer of energy from the laser to the plasma has been calculated for the PBWA model we have chosen.² For a general phase shift δ the ratio of the plasma energy to the laser energy is given by

$$\eta_1 = \frac{P.E.}{W_T} = \frac{\alpha\delta}{4}. \quad (36)$$

If laser depletion is included in the analysis, this number will be reduced slightly.

The efficiency of the transfer of energy from an electron beam to the plasma is quite different. In this case one must consider the beam loading effects. If we could treat the bunch as a macro-particle, then for a very relativistic driving bunch we could extract nearly all of its energy before it's velocity changed enough to yield a phase slip. However, due to beam loading this is not possible since the leading edge of the driving bunch loses essentially no energy to the plasma while the trailing edge loses twice as much as that calculated for a point like particle. Thus, for very short bunches, we can only extract about 1/2 of the energy

$$\eta_1 = \frac{1}{2} PWF A. \quad (37)$$

For longer bunches of electrons, one can improve this factor and also improve the 'transformer ratio'⁶ at the expense of the peak field. Since this technique might be quite difficult to realize in the PWFA, we will not consider it here.

The final efficiency to calculate is that from the plasma to the driving bunch. This efficiency is the same for both cases provided that the characteristics of the plasma wave are the same. The total acceleration gradient experienced by a bunch with N_2 particles in a plasma wave is

$$G \equiv \frac{dE_2}{dz} = e\mathcal{E}_z f - 4e^2 \frac{N_2}{b^2}. \quad (38)$$

The second 'beam loading' term is due to the plasma wake induced by the trailing bunch. $e\mathcal{E}_z$ is the peak longitudinal electric field, and f is a factor less than unity which takes into account phase slippage or shifts in phase from the peak accelerating field. The efficiency is given by the total energy gained by the

bunch divided by the plasma energy,

$$\eta_2 = N_2 G L \left(\frac{\mathcal{E}_z^2 \pi a^2}{8\pi} L \right)^{-1} . \quad (39)$$

This efficiency has a maximum when

$$N_2 = \frac{f \mathcal{E}_z b^2}{8e} , \quad (40)$$

and the value is given by

$$\eta_2^{max} = f^2 \frac{b^2}{a^2} . \quad (41)$$

For the PWFA f can be taken to be essentially unity while for the PBWA f is given by Eq. (29). This yields

$$\begin{aligned} \eta_2^{max} &\simeq .72 \frac{b^2}{a^2} && PBWA \\ \eta_2^{max} &\simeq \frac{b^2}{a^2} && PWFA \end{aligned} \quad (42)$$

COMPARISONS AND DISCUSSION

Now we come to a detailed comparison between the PBWA and the PWFA. As mentioned earlier, our guide will be the self consistency among all relevant accelerator parameters within each scheme. Our approach is to choose a set of parameters in each scheme that we fix from the beginning. The remaining parameters in each scheme can then be calculated in terms of those chosen parameters. The scaling to different sets of chosen parameters is straight forward using the results of the previous section. To make a fair comparison we will study two sets of sample accelerators with the same acceleration gradient and the same length L . In addition to make the comparison meaningful to real experiments, we employ only those laser and electron beams that are presently available. Under these considerations, the parameters that should be fixed in the two schemes are quite different. In particular for the PBWA we need to fix the laser frequency ω by choosing a particular laser source. If we then fix the length L of the acceleration section, the phase slippage determines the plasma frequency ω_p . This means that the longitudinal electric field \mathcal{E}_z is a derivable quantity. On the other hand, the energy gradient in the PWFA is chosen so that the intensity and dimensions are not far from realizable values. As we shall see, in spite of this difference it is possible to match the acceleration gradients.

Numerical comparisons

To keep the dimensions to a laboratory scale, we select the acceleration lengths to be 10 cm and 100 cm. These two lengths are then combined with two different laser frequencies, the Nd : Glass laser and the CO_2 laser, to form four sets of sample calculations. For the PBWA the parameter α is chosen to be 0.25, which is approximately the saturation value⁵ and the phase slippage is taken to be the optimum value given in the previous section. Finally, we assume that the laser pulse length and the growth time for the plasma wave τ is about 159 cycles ($\omega_p \tau = 1000$).

Since the PWFA is not so restrictive in its design, we can now set the parameters to match some of those for the PBWA. In particular we use the same acceleration gradient and the same a/λ_p . The number of particles in the driving bunch is taken from the present number in the SLC and the bunch length is assumed to be somewhat less than the plasma wavelength. The initial and final energies of the driving bunch are selected so that the final energy of the bunch tail is 90% of its initial energy. As we can see from Tables 1 and 2, the phase slippage for the PWFA is much smaller than that for the PBWA. All parameters except the efficiency and the energy in the driving beam turn out to be quite comparable. In particular note that the focusing for both schemes is quite strong. The energy required for the driving bunch is consistently higher for the PBWA; however, because it is less efficient in these examples, the number of particles which can be driven is comparable to the PWFA.

Discussion

The examples above seem to favor the Plasma Wake Field Accelerator especially for the longer accelerator sections. This is due to the divergence of the laser. For longer Rayleigh lengths it is necessary to have a larger spot and thus more peak power to obtain the same intensity at the spot. On the other hand the particle beam is assumed not to diverge. This is true because the emittance of the beam is typically much smaller than the corresponding wavelength/ π for the laser. In addition it is possible to use magnetic focusing elements to define the size of a charged particle beam. The problem of the divergence of the laser beam might be solved by using lasers sufficiently intense to self focus in the plasma; however, this possibility was not considered since it lies outside the scope of the simple models given here. In addition, for the PBWA parameters chosen here, the laser power is somewhat below the critical value for relativistic self focusing.⁷

Table 1. Plasma Beat Wave Accelerator

Chosen Parameters	Values			
	Nd: Glass	1.78×10^{15}	CO ₂	1.78×10^{14}
ω [sec ⁻¹]				
L [cm]	10	100	10	100
α	0.25	0.25	0.25	0.25
δ [rad]	$5\pi/16$	$5\pi/16$	$5\pi/16$	$5\pi/16$
$\sin \delta/\delta$	0.85	0.85	0.85	0.85
$\omega_p \tau$	1000	1000	1000	1000
Derived Parameters				
ω_p [10^{13} sec ⁻¹]	2.65	1.23	.571	.265
n_0 [10^{16} cm ⁻³]	21.7	4.67	1.00	0.22
$e\mathcal{E}_z$ [GeV/m]	9.38	4.36	2.00	0.94
a [mm]	0.13	0.41	0.41	1.30
a/λ_p	1.82	2.70	1.25	1.82
β [$\sqrt{\gamma/\sin \phi}$ mm]	0.18	0.57	0.57	1.80
N [10^{10}]	$1.95\eta_2$	$9.04\eta_2$	$4.19\eta_2$	$1.95\eta_2$
$W\tau$ [J]	23.9	515.4	11.1	239.2

Unfortunately, for both schemes the efficiency η_2 and the energy spread induced are directly related. Thus, if a small energy spread is necessary, then η_2 will necessarily be small for both schemes. The efficiency η_1 of the PWFA was better in all cases because the energy transfer from the laser to the plasma is limited by Eq. (36) to quite a small value. There is a possible solution to this problem. Since the laser is not depleted very much, it might be possible to reuse the beam after a suitable amplification. This would yield a very high repetition rate and looks quite attractive; however, this possibility needs much more study.

There is one final problem for the PBWA. We have assumed that the plasma wave would grow over $1000/2\pi$ cycles. If there are density fluctuations greater than about .2%, then the wave would saturate much sooner. This case would require a much larger laser energy in order to drive the plasma to the desired field in a shorter time.

Table 2. Plasma Wake Field Accelerator

Chosen Parameters	Values			
L [cm]	10	100	10	100
$e\mathcal{E}_z$ [GeV/m]	9.38	4.36	2.00	0.94
N_1	5×10^{10}	5×10^{10}	5×10^{10}	5×10^{10}
E_1 [GeV]	1.04	4.84	0.22	1.04
a/λ_p	1.82	2.70	1.25	1.82
Derived Parameters				
a [mm]	0.25	0.36	0.54	0.78
δ [10^{-3} rad]	5.5	2.5	42	18
ω_p [10^{13} sec $^{-1}$]	1.37	1.41	.439	.438
n_0 [10^{16} cm $^{-3}$]	5.90	6.18	.606	.604
α	0.38	0.17	0.25	0.11
β [$\sqrt{\gamma/\sin\phi}$ mm]	0.28	0.59	0.73	1.52
N_2 [10^{10}]	$2.25\eta_2$	$2.25\eta_2$	$2.25\eta_2$	$2.25\eta_2$
$W\tau = N_1 E_1$ [J]	8.33	38.8	1.76	8.33

REFERENCES

1. T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).
2. R. D. Ruth and A. W. Chao, in *Laser Acceleration of Particles*, Ed. P. J. Channell, AIP Conference Proceedings No. 91, American Institute of Physics, New York, 1982.
3. P. Chen, R. W. Huff and J. M. Dawson, UCLA Report No. PPG-802, 1984, and Bull. Am. Phys. **29**, 1355 (1984); P. Chen, J. M. Dawson, R. W. Huff and T. Katsouleas, Phys. Rev. Lett. **54**, 693 (1985).
4. R. D. Ruth, A. W. Chao, P. L. Morton and P. B. Wilson, SLAC-PUB-3374, 1984 (to be published in Particle Accelerators).
5. M. N. Rosenbluth and C. S. Liu, Phys. Rev. Lett. **29**, 701 (1972); D. J. Sullivan and B. B. Godfrey, *ibid.*, Ref. 2.
6. K. L. F. Bane, P. Chen and P. B. Wilson, *Co-Linear Wake Field Acceleration for Linear Colliders*, to be presented at the 1985 Particle Accelerator Conference, Vancouver, B.C., Canada, May 13-16, 1985; P. Chen and J. M. Dawson, this Proceedings.
7. H. Hora, *Physics of Laser Driven Plasmas*, John Wiley and Sons, New York (1981) and P. Sprangle and C. M. Tang, this conference.