SLAC-PUB-3599
March 1985
(A)

# EFFECTS OF SLOTS/HOLES IN DISKS ON FREQUENCIES OF $TM_{01}$ AND $EH_{11}$ WAVES IN THE DISK-LOADED WAVEGUIDE*

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## Introduction

The increase in the BBU threshold current is very important for the high and medium energy electron linacs because of a larger operating current attainable or a smaller emittance available at certain operating current. For this purpose, many means can be used, among which the improvement in the accelerating structure itself is always fundamental.[1–4] SLAC's three-meter long section is unique with truly constant gradient performance. The theoretical analyses and operating experiences at SLAC have indicated that detuning the Nos. 3, 4 and 5 cavities of some sections by 2 or 4 MHz for $TM_{01}$ wave (about 2.5 or 5 MHz for $EH_{11}$ wave as $[(\partial f/\partial b)_{11,\pi}/(\partial f/\partial b)_{01,2\pi/3}] = 1.25$ according to our measurements) had considerably raised the BBU threshold current.[5–8]

The way to do this for SLAC was to squeeze the cavity wall inward. It had three disadvantages. First of all, it is troublesome to do this after the linac has been built. Secondly, after doing this the matching characteristic of the whole section would turn worse, and so it was necessary to readjust the match parameters of the couplers. Lastly, the detuning not only created a separation of $EH_{11}$ wave resonations in different sections which is desirable and useful, but also yielded a deviation in phase shift over the cavity from 120° for $TM_{01}$ wave which is hopeless and mischievous. A method of opening four holes symmetrically distributed on disks described below in detail will get the benefits in improvement of BBU threshold current and overcome the three shortcomings mentioned above.

## Effects of Slots in Disks on Frequency of $TM_{01}$ Wave

Two slots in each disk shown in Fig. 1 bring about a disturbance on cavity fields. The Slater's formula can be used for calculation of the effects if the slot half-width $h \le t$. The $t$ is the thickness of the disk.

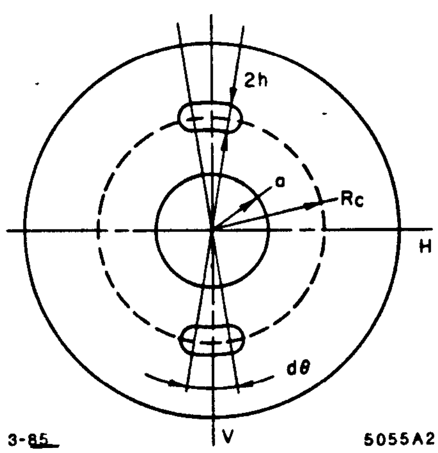


Fig. 1. The denotation of dimensions of slots in disk. The slots turn to holes if $d\theta = 0°$.

*Work supported in part by the Department of Energy, contract DE-AC03-76SF00515.

The field strength exponentially decays within the slots, that means the equivalent depth $L$ of the slot should be $L = 0.4343h$, rather than $t/2$. That means the frequency disturbance by holes is in proportion with $h^3$ instead of $h^2$. The experiments have verified that this conclusion is correct approximately. The Slater's formula is as follows:

$$\frac{df}{f} = \frac{\frac{1}{2}\int_\tau [E^2 - (Z_0 H)^2] d\tau}{\frac{1}{2}\int_v [E^2 + (Z_0 H)^2] dv} = \frac{D}{S} \tag{1}$$

where $\tau$ is the volume disturbed and $v$ the whole cavity volume.

The electric and magnetic stored energy are equal to each other when resonation occurs. For $TM_{01}$ wave the denominator $S$ of Eq. (1) becomes

$$S = 2\pi D \int_0^a r \sum_{m=-\infty}^{\infty} A_m^2 \left( \frac{\beta_m^2}{X_m^2} \frac{J_1^2(X_m r)}{J_0^2(X_m a)} + \frac{J_0^2(X_m r)}{J_0^2(X_m a)} \right) dr$$
$$+ \pi d \int_a^b r \sum_{s=0}^{\infty} a_s^2 (1 + \delta_{s0}) \left( \frac{F_0^2(X_s r)}{F_0^2(X_s a)} + \frac{\eta_s^2}{X_s^2} \frac{F_1^2(X_s r)}{F_0^2(X_s a)} \right) dr$$

Because of $E_r = 0$ on the disk surfaces the numerator $D$ of Eq. (1) can be computed by the formula below in the case of two slots in each disk:

$$D = 0.8686\, h \int_{R_c - h}^{R_c + h} \Bigg\{ F_e \Bigg[ \Bigg( \sum_{s=0,2,4,\ldots} a_s \frac{F_0(X_s r)}{F_0(X_s a)} (-1)^{s/2} \Bigg)^2 + \Bigg( \sum_{s=1,3,5,\ldots} a_s \frac{F_0(X_s r)}{F_0(X_s a)} (-1)^{(s-1)/2} \Bigg)^2 \Bigg]$$
$$- \Bigg[ \Bigg( \sum_{s=0,2,4,\ldots} a_s \frac{k}{X_s} \frac{F_1(X_s r)}{F_0(X_s a)} (-1)^{s/2} \Bigg)^2 + \Bigg( \sum_{s=1,3,5,\ldots} a_s \frac{k}{X_s} \frac{F_1(X_s r)}{F_0(X_s a)} (-1)^{(s-1)/2} \Bigg)^2 \Bigg] \Bigg\}$$
$$\left( d\theta + \frac{2\sqrt{h^2 - (R_c - r)^2}}{r} \right) r dr \ .$$

Where $F_e = 0.635$ is a factor which comes from electric distribution in slots because of boundary condition $E_t = 0$ on the slot wall.

The effect of four lots on fequency is a factor of two larger than that caused by two slots because of the circular symmetry of field distribution for $TM_{01}$ wave.

## Effects of Slots in Disks on Frequency of $EH_{11}$ Wave

The $EH_{11}$ wave is a polarization one. The fields are related to angular coordinate $\theta$. When there are two slots in each disk the denominator $S$ of Eq. (1) for the $EH_{11}$ wave becomes[10,11]

Poster paper presented at the 1985 Particle Accelerator Conference,
Vancouver, B.C., Canada, May 13-16, 1985.

$$
\begin{aligned}
S = \pi D \int_a^b \Bigg\{ & \sum_m A_m^2 \frac{J_1^2(X_m r)}{J_1^2(X_m a)} \\
& + \sum_m \left( A_m \frac{\beta_m}{X_m^2 r} \frac{J_1(X_m r)}{J_1(X_m a)} + B_m \frac{k}{X_m} \frac{J_1'(X_m r)}{J_1'(X_m a)} \right) \\
& + \sum_m \left( A_m \frac{\beta_m}{X_m} \frac{J_1'(X)mr)}{J_1(X_m a)} + B_m \frac{k}{X_m^2 r} \frac{J_1(X_m r)}{J_1'(X_m a)} \right)^2 \Bigg\} r dr \\
+ \frac{\pi d}{2} \int_a^b \Bigg\{ & \sum_s a_s^2 \left( \frac{F_1(X_s r)}{F_1(X_s a)} \right) (1 + \delta_{s0}) \\
& + \sum_s \left( a_s \frac{\eta_s}{X_s^2 r} \frac{F_1(X_s r)}{F_1(X_s a)} + b_s \frac{k}{X_s} \frac{f_1'(X_s r)}{f_1'(X_s a)} \right)^2 \\
& + \sum_s \left( a_s \frac{\eta_s}{X_s} \frac{F_1'(X_s r)}{F_1(X_s a)} + b_s \frac{k}{X_s^2 r} \frac{f_1(X_s r)}{f_1'(X_s a)} \right)^2 \Bigg\} r dr
\end{aligned}
$$

The numerator $D$ of the equation (1) has the following form due to $E_r = E_\theta = H_z = 0$ on the disk surfaces:

$$
\begin{aligned}
D = {} & 0.8686\, h\, F_e \int_{R_c-h}^{R_c+h} \int_{\theta_0-\theta(r)/2}^{\theta_0+\theta(r)/2} \cos^2\theta\, d\theta \Bigg\{ \Bigg[ \sum_{s=0,2,4,\ldots} a_s \frac{F_1(X_s r)}{F_1(X_s a)} (-1)^{s/2} \Bigg]^2 + \Bigg[ \sum_{s=1,3,5,\ldots} a_s \frac{F_1(X_s r)}{F_1(X_s a)} (-1)^{(s-1)/2} \Bigg]^2 \Bigg\} r dr \\
& - 0.8686\, h \Bigg\{ \int_{R_c-h}^{R_c+h} \int_{\theta_0-\theta(r)/2}^{\theta_0+\theta(r)/2} \cos^2\theta\, d\theta \\
& \Bigg\{ \Bigg[ \sum_{s=0,2,4,\ldots} \left( a_s \frac{k}{X_s} \frac{F_1'(X_s r)}{F_1(X_s a)} + b_s \frac{\eta_s}{X_s^2 r} \frac{f_1(X_s r)}{f_1'(X_s a)} \right) (-1)^{s/2} \Bigg]^2 \\
& \Bigg[ \sum_{s=1,3,5,\ldots} \left( a_s \frac{k}{X_s} \frac{F_1'(X_s r)}{F_1(X_s a)} + b_s \frac{\eta_s}{X_s^2 r} \frac{f_1(X_s r)}{f_1'(X_s a)} \right) (-1)^{(s-1)/2} \Bigg]^2 \Bigg\} r dr \\
& + \int_{R_c-h}^{R_c+h} \int_{\theta_0-\theta(r)/2}^{\theta_0+\theta(r)/2} \cos^2\theta\, d\theta \\
& \Bigg\{ \Bigg[ \sum_{s=0,2,4,\ldots} \left( a_s \frac{k}{X_s^2 r} \frac{F_1(X_s r)}{F_1(X_s a)} + b_s \frac{\eta_s}{X_s} \frac{f_1'(X_s r)}{f_1'(X_s a)} \right) (-1)^{s/2} \Bigg]^2 \\
& \Bigg[ \sum_{s=1,3,5,\ldots} \left( a_s \frac{k}{X_s^2 r} \frac{F_1(X_s r)}{F_1(X_s a)} + b_s \frac{\eta_s}{X_s} \frac{f_1'(X_s r)}{f_1'(X_s a)} \right) (-1)^{(s-1)/2} \Bigg]^2 \Bigg\} r dr \Bigg\}
\end{aligned}
$$

where

$$
\begin{aligned}
F_1(X_s r) &= J_1(X_s r)\, Y_1(X_s b) - Y_1(X_s r)\, J_1(X_s b) \quad , \\
F_1'(X_s r) &= J_1'(X_s r)\, Y_1(X_s b) - Y_1'(X_s r)\, J_1(X_s b) \quad , \\
f_1(X_s r) &= J_1(X_s r)\, Y_1'(X_s b) - Y_1(X_s r)\, J_1'(X_s b) \quad , \\
f_1'(X_s r) &= J_1'(X_s r)\, Y_1'(X_s b) - Y_1'(X_s r)\, J_1'(X_s b) \quad .
\end{aligned}
$$

$$
\frac{\theta(r)}{2} = \frac{d\theta}{2} + \frac{\sqrt{h^2 - (R_c - r)^2}}{r} \quad .
$$

$d\theta$ is the angular width of the slots and $\theta_0$ the angular coordinate of the slot center.

Two cases have to be considered as the $EH_{11}$ wave is a polarized one:

(a) $\theta_0 = 0°$: which means the polarization direction is parallel to the line through the centers of two slots.

(b) $\theta_0 = 90°$: which means the polarization direction is perpendicular to the line through the centers of two slots.

The four slots in each disk yield a frequency shift for $EH_{11}$ wave which can be computed by following this simple formula:

$$
df \ (4 \text{ slots}) \ = df(\theta_0 = 0°, 2 \text{ slots}) \ + df(\theta_0 = 90°, 2 \text{ slots})
$$

In this case there are four polarized waves with polarization directions $\theta_0 = 0°$, $\pm 45°$ and $90°$. Their frequency shifts caused by four slots are all the same and are computed by the above formula.

### The Results Computed and Measured

The term numbers of the series in the calculations are: $m = 25$ and $s = 13$ for $TM_{01}$ wave; $m = 50$ and $s = 25$ for $EH_{11}$ wave. Obviously, the $2\pi/3$ mode for $TM_{01}$ wave and $\pi$ mode for $EH_{11}$ wave are most interesting to us. All results given below belong to the case of circular holes ($d\theta = 0°$) for simplicity of machining.

Figure 2 shows the frequency shifts of $2\pi/3$ mode of $TM_{01}$ wave caused by two or four holes in each disk at different radial position $R_c$. It can be seen that the frequency will move down if holes are placed where the magnetic field is stronger than the electric one. The parameter in the figure is the hole diameter $2h$.

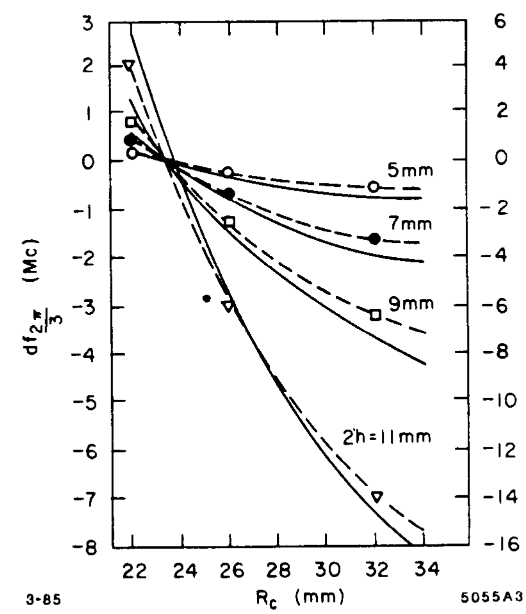


Fig. 2. The frequency shifts of $2\pi/3$ mode of $TM_{01}$ wave caused by two or four holes in each disk versus radial position $R_c$. $2h$ is hole diameter; — computed; - - - measured; cavity dimension: $a = 13.90$ mm; $t = 4.00$ mm.

The frequency shifts of $\pi$ mode of $EH_{11}$ wave with $\theta_0 = 0°$ and $90°$ caused by two holes in each disk at different $R_c$ are shown in Figs. 3 and 4, respectively. Figure 5 shows the frequency shifts of $\pi$ mode of $EH_{11}$ wave caused by four holes in each disk at different $R_c$ with respect to the case without holes in disks on the condition of keeping the frequency of $2\pi/3$ mode of $TM_{01}$ wave invariable that means the corresponding corrections in cavity diameters $2b$ have been made.

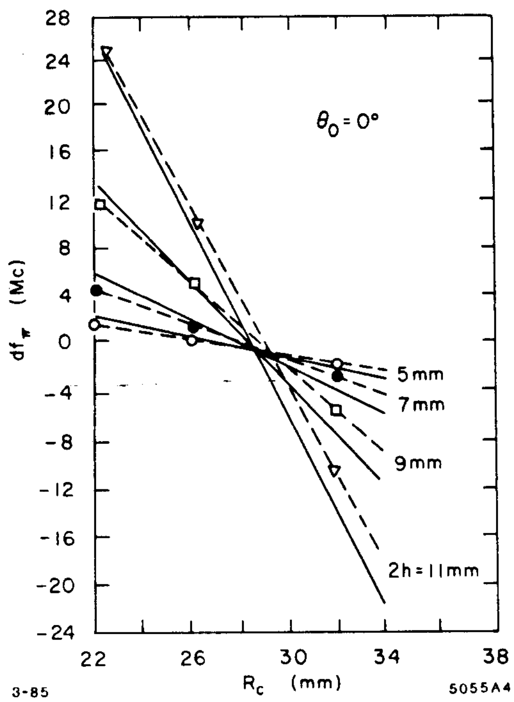


Fig. 3. The frequency shifts of $\pi$ mode of $EH_{11}$ wave with $\theta_0 = 0°$ caused by two holes in each disk versus radial position $R_c$. $2h$ is hole diameter; — computed; - - - measured; cavity dimension: $a = 13.90$ mm; $t = 4.00$ mm.

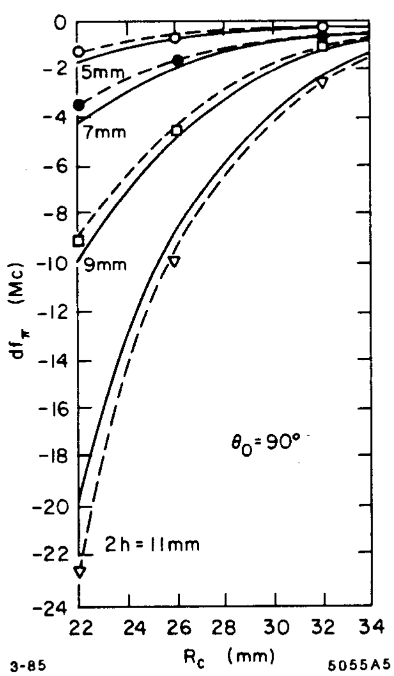


Fig. 4. The frequency shifts of $\pi$ mode of $EH_{11}$ wave with $\theta_0 = 90°$ caused by two holes in each disk versus radial position $R_c$. $2h$ is hole diameter; — computed; - - - measured; cavity dimension: $a = 13.90$ mm; $t = 4.00$ mm.

For example, opening four holes in each disk with $2h = 9$ or 11 mm at $R_c = 28$ mm and reducing the dimension $2B$ by $-0.09$ or $-0.26$ mm, the $f(2\pi/3)$ of $TM_{01}$ wave still maintains at 2856 MHz but the frequency bands of $EH_{01}$ wave will move up by about 4 or 9 MHz respectively. All computations and measurements given above are made with $2\pi/3$ mode cavities of light velocity section of our 20 MeV feed-back electron linac.[10] The dimensions are: $a = 13.90$ mm and $t = 4.00$ mm ($f_0 = 2856$ MHz) and denoted on the figures. We do the same experiments and calculations with the SLAC No. 4 cavity ($2a = 26.04$ mm and $t = 5.84$ mm) and existence of this effect is again verified.

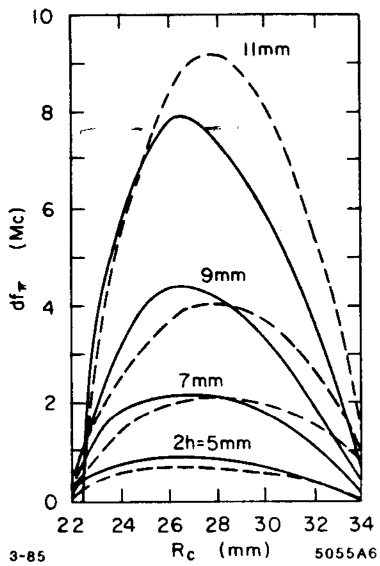


Fig. 5. The frequency shifts of $\pi$ mode of $EH_{11}$ wave caused by four holes in each disk versus $R_c$ on condition of $f(2\pi/3) = 2856$ MHz for $TM_{01}$ wave. $2h$ is hole diameter; — computed; - - - measured; cavity dimension: $a = 13.90$ mm; $t = 4.00$ mm.

The electric coupling through the central holes in disks is dominant in the case of disk-loaded waveguide. Opening holes in disks at $R_c = 28$ mm where the magnetic field is stronger than the electric one will degrade the electric coupling. The measurements show that the decrease in frequency band width is only 2–5%, so that its effect is negligible.

## Conclusion

This skill is a simple way to build up several kinds of different structures in light of BBU occurrence. The effect of holes in disks on both property of the tube and matching parameters of couplers is negligible if the hole diameter $2h$ is not too big. The skill can be used for SLAC structure to improve linac performance; perhaps it can also be used for other tube design with approximately constant gradient property instead of using several kinds of tube with different dimension variation range to some extent.

In a practical case, of course, the holes should be made with out corners and edges to prevent arcing, although all calculations and measurements mentioned above are done with holes of rectangular edge for simplicity.

## Acknowledgement

I wish to express my thanks to Professor J. L. Xie of IHEP in China and Dr. G. A. Loew of SLAC in the United States for their kind regards to this work and helpful discussions.

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