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MINIMIZING THE ENERGY SPREAD WITHIN A SINGLE BUNCH BY SHAPING ITS CHARGE DISTRIBUTION*

GREGORY A. LOEW AND J. W. WANG
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Introduction

When electrons or positrons in a bunch pass through the periodic structure of a linear accelerator, they leave behind them energy in the form of longitudinal wake fields. The wake-fields thus induced by early particles in a bunch offset the energy of later particles. For a linear collider, the energy spread introduced within the bunches by this beam loading effect must be minimized because it limits the degree to which the particles can be focused to a small spot due to chromatic effects in the final focus system.† For example, for the SLC, the maximum allowable energy spread is ±0.5%.

It has been known for some time that partial compensation of the longitudinal wake field effects can be obtained for any bunch by placing it ahead of the accelerating crest (in space), thereby letting the positive rising sinusoidal field offset the negative beam loading field.[1] The work presented in this paper shows that it is possible to obtain complete compensation, i.e., to reduce the energy spread essentially to zero by properly shaping the longitudinal charge distribution of the bunch and by placing it at the correct position on the wave.

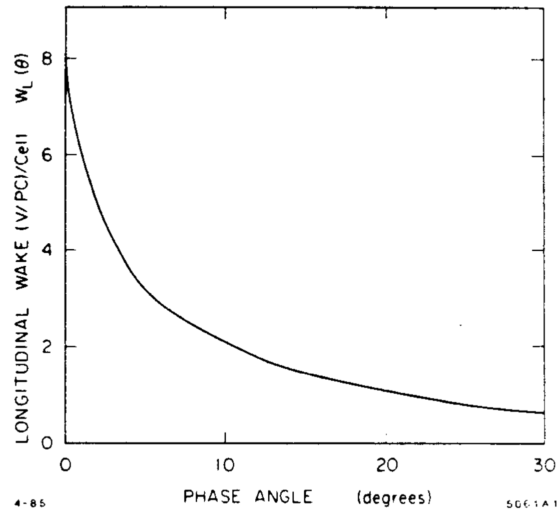

Fig. 1. Longitudinal wake function $w_L(\theta)$ for single average cavity of SLAC constant-gradient structure.

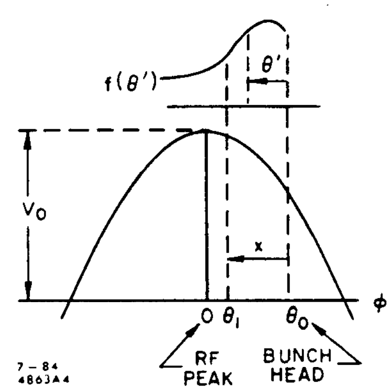

Fig. 2. Definitions of phase angles showing position of bunch with respect to accelerating wave. The charge distribution is $f(\theta')$ and the maximum energy gain is V_0.

Optimizing the Bunch Shape

The energy gained by a single particle riding at an angle θ_1 with respect to the crest of a traveling wave of accelerating gradient E_0 over a length L is

$$V = E_0 L \cos\theta_1 \quad . \tag{1}$$

In the case of a bunch consisting of many particles, this energy is modified by the presence of the wake fields left by particles ahead of θ_1. For the examples worked out in this paper, we will use the SLAC constant-gradient structure although the technique should be applicable to any structure for which the longitudinal wake function is known. This wake function, $w_L(\theta)$, is defined as the voltage excited by a unit charge traversing the structure. It is shown in Fig. 1 as calculated for a single average cavity of length d ($d = 3.5$ cm) of the SLAC constant-gradient structure.[2] At the cost of an error estimated to be on the order of 5%, we can neglect the fact that the actual cavities differ in iris diameter from this average cavity by about ±15%. We can then obtain the function W_L for the entire accelerator by simply multiplying $w_L(\theta)$ by N, the number of cavities (L/d). With these definitions and a bunch charge distribution $f(\theta')$ as illustrated in Fig. 2, Eq. (1) now becomes:

$$V(\theta_1) = V_0 \cos\theta_1 - \int_0^{(\theta_0-\theta_1)} f(\theta')\, W_L(\theta_0 - \theta_1 - \theta')\, d\theta' \tag{2}$$

where $V_0 = E_0 L$, θ_0 is the position of the head of the bunch with respect to the wave and θ', the coordinate within the bunch, is made to vary from 0 (the head of the bunch) to $\theta_0 - \theta_1$ (the position where we want to know the net energy).

In order to reduce the energy spread within the bunch to zero, we must make $V(\theta_1)$ independent of θ_1. This requires that

$$\frac{\partial V(\theta_1)}{\partial \theta_1} = 0 \quad . \tag{3}$$

By taking the partial derivative of Eq. (2) with respect to θ_1 and setting it to zero, we get:

$$-V_0 \sin\theta_1 - \int_0^{(\theta_0-\theta_1)} f(\theta') \frac{\partial W_L}{\partial \theta_1}(\theta_0 - \theta_1 - \theta')\, d\theta' + f(\theta_0 - \theta_1)\, W_L(0) = 0 \quad ,$$

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† There may also be measurements of particle resonances which would benefit from extremely narrow energy spreads.

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or

$$f(\theta_0-\theta_1)=\frac{V_0}{W_L(0)}\sin\theta_1+\int_0^{(\theta_0-\theta_1)}\frac{f(\theta')\,\frac{\partial W_L}{\partial\theta_1}(\theta_0-\theta_1-\theta')}{W_L(0)}\,d\theta' \;. \tag{4}$$

Letting $\theta_0-\theta_1=x$ where $x\geq\theta'$, Eq. (4) becomes:

$$f(x)=\frac{V_0}{W_L(0)}\sin(\theta_0-x)-\int_0^{x}\frac{\frac{\partial W_L}{\partial x}(x-\theta')f(\theta')}{W_L(0)}\,d\theta' \tag{5}$$

which is a Volterra integral equation of the second kind. This equation can be solved digitally through a multi-step method using Day's starting procedure in conjunction with Simpson's rule and the three-eighths rule.[3] The wake function can be fitted with a polynomial so as to be represented by an analytical expression.

Figures 3 and 4 give results for several examples. These examples were all worked out for a no-load energy V_0 of 54.75 GeV, an accelerator length L of 960 sections, each with 86 cavities (i.e., L = 2890 m, N = 82,560 cavities) and a bunch of integrated charge $5\times10^{10}e$. The value of V_0 was chosen so as to yield a final beam energy just over 50 GeV. Figure 3 shows six different bunch shapes with the corresponding θ_0's (positions of the head with respect to the wave) required to give essentially zero energy spread. The head of the bunch is on the left (zero-abscissa) and the tail defined as the point where an integrated

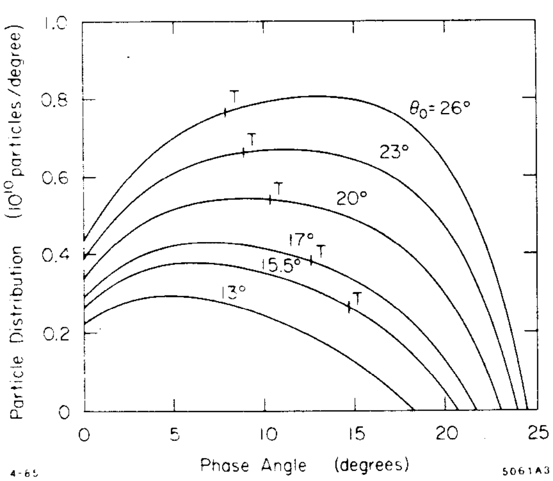

Fig. 3. Bunch shape, i.e. particle distribution as a function of phase angle which leads to negligible spectrum width, for various values of θ_0. The point marked "T" indicates where the integrated charge in the bunch reaches 5×10^{10} e.

charge of $5\times10^{10}e$ is reached, is at the abscissa corresponding to the letter "T" on each curve. An interesting aspect of these curves is that if the bunches are extended beyond the "T" points as shown, the energy spread continues to be zero even though the charge in the extended bunch is greater than 5×10^{10} e. The end points on the individual curves give the limits of how far one can go. The curve for $\theta_0=13°$ has no T because the integral under it does not quite reach 5×10^{10} particles: its charge is 3.96×10^{10} e. Figure 4 gives the respective energies of the bunches of Fig. 3 (except for the $\theta_0=13°$ case) as a function of angular position.

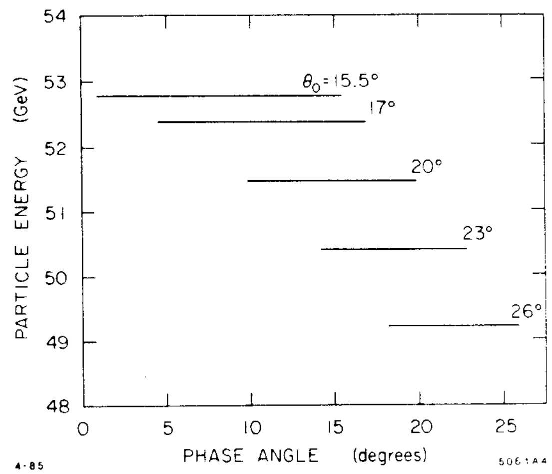

Fig. 4. Particle energy along bunches of Fig. 3 as a function of phase angle.

Table 1 gives a summary of the average energies ($\bar{E}$) and spectral qualities $[(E_{max}-E_{min})/\bar{E}$ and the fractional standard deviation $\sigma_E/\bar{E}]$ for the cases shown in Figs. 3 and 4.

Table 1
(All cases for 5×10^{10} particles.)

θ_0 (degrees)	$\bar{E}$ (GeV)	$(E_{max}-E_{min})/\bar{E}$ (%)	$\sigma_E/\bar{E}$ (%)
26	49.210	0.008	0.003
23	50.399	0.009	0.003
20	51.450	0.010	0.003
17	52.360	0.014	0.004
15.5	52.755	0.010	0.004
Truncated Gaussian 15.5	52.809	0.14	0.038
Gaussian ($6\sigma_z$) 20	52.795	3.5	0.352

The sixth example, shown also for $\theta_0=15.5°$, is that of a truncated Gaussian fitted to the shape of the "ideal" $\theta_0=15.5°$ case. It has a σ_z of 9° but is truncated at ±7.5°. These results all compare extremely favoarably with the energy spread σ_E/E of 0.35% which one obtains for a Gaussian bunch of $5\times10^{10}e$ with a length $6\sigma_z$, a σ_z of 4° and a θ_0 of 20° (the seventh example in Table 1), or for that matter, for the same bunch with charge reduced to $5\times10^{8}e$. The energy spectra for all the above cases are illustrated in Fig. 5. Note that the examples of Table 1 are so narrow in energy that they can only be represented by a line.

Discussion

If we rewrite Eq. (5) in terms of the gradient E_0 instead of the total energy V_0, it becomes:

$$f(x)=\frac{E_0 d}{w_L(0)}\sin(\theta_0-x)-\int_0^{x}\frac{\frac{\partial w_L}{\partial x}(x-\theta')}{w_L(0)}f(\theta')\,d\theta' \tag{6}$$

We see that for a structure with a given $w_L(\theta)$, once the gradient E_0 and the angular position θ_0 of the head are chosen, the shape is fixed by Eq. (6) and is independent of the total energy V_0 and length L. For a given gradient E_0, $f(\theta)$ starts at a higher value as θ_0 is made larger since

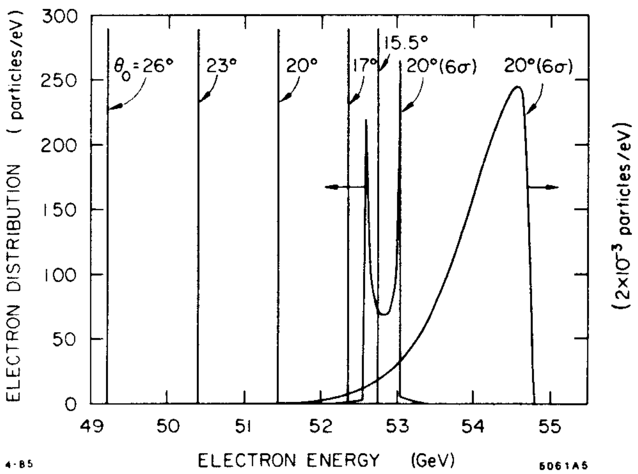

Fig. 5. Energy spectra for the examples of Table 1 with $5 \times 10^{10}\,e$ (left-hand scale) and for the Gaussian bunch with $5 \times 10^{8}\,e$ (right-hand scale).

$$f(0) = \frac{E_0 d}{w_L(0)} \sin\theta_0 \tag{7}$$

as shown in Fig. 3. Clearly, the more charge one wants, the higher gradient one needs, or the farther ahead of crest one must place the head.

Tolerances

To get a feeling for allowable tolerances, it is interesting to calculate the effect of changes in injection angle (θ_0) or bunch charge on $\bar{E}$ and $\sigma_E/\bar{E}$, assuming constant bunch shape. Table 2 shows the effect of varying θ_0 while keeping the total charge of the bunch equal to $5 \times 10^{10}\,e$ in the case of the truncated Gaussian bunch discussed earlier.

Table 2. Truncated Gaussian bunch.
($\theta_{total} = 15°$, $\sigma = 9°$, $5 \times 10^{10}\,e$)

θ_0 (degrees)	$\bar{E}$ (GeV)	$(E_{max} - E_{min})/\bar{E}$ (%)	$\sigma_E/\bar{E}$ (%)
11.5	53.210	1.78	0.51
12.5	53.135	1.31	0.38
13.5	53.043	0.84	0.26
14.5	52.934	0.38	0.13
15.5	52.809	0.14	0.038
16.5	52.668	0.55	0.14
17.5	52.510	1.02	0.26
18.5	52.335	1.50	0.39
19.5	52.145	1.90	0.52

Table 3 shows the effect of changing the bunch charge while keeping its shape and θ_0 constant.

Table 3. Truncated Gaussian bunch.
($\theta_{total} = 15°$, $\sigma = 9°$, $\theta_0 = 15.5°$)

Bunch Charge ($\times 10^{10}\,e$)	$\bar{E}$ (GeV)	$(E_{max} - E_{min})/\bar{E}$ (%)	$\sigma_E/\bar{E}$ (%)
3	53.314	1.52	0.43
3.5	53.187	1.12	0.33
4	53.061	0.81	0.23
4.5	52.935	0.45	0.12
5	52.809	0.14	0.038
5.5	52.683	0.27	0.10
6	52.557	0.64	0.21
6.5	52.431	1.00	0.31
7	52.305	1.36	0.42

The variations of $\sigma_E/\bar{E}$ in both tables are close to hyperbolic.

Conclusions

We have shown in this paper that it is theoretically possible to find bunch shapes for the SLC which yield 5×10^{10} or more particles within negligible energy spread at the end of the linear accelerator. As it turns out, these shapes depend only on the linac energy gradient and the angle at which the head of the bunch is placed with respect to the accelerating wave, and are independent of the total energy or length of the accelerator. Excursions away from this angle in parts of the linac, designed to cause Landau damping of the transverse wake field effect, are of course permissible as long as overall "phase closure" to preserve the desired average θ_0 is accomplished. Some of these theoretical bunch shapes are not too different from shapes that ought to be realizable from injectors or damping rings. How to realize them exactly is not the subject of this paper.

References

1. See for example, SLAC Linear Collider, Conceptual Design Report, SLAC-229, pp. 17 and 117.
2. *Ibid*, pp. 112-116.
3. Handbook of Applicable Mathematics, Vol. III. Numerical Methods. John Wiley & Sons (1982).